K200274

Date

Assignment #04

Ex 5.2 !

Ans 8) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\det(\lambda I - A) = \begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{bmatrix}$

$= ((\lambda-1)(\lambda-1)^2 - 1) = \lambda(\lambda-1)(\lambda-2)$

$\lambda = 0 \quad \lambda = 1 \quad \lambda = 2$

$-A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$

$I - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$2I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

Signature

No.

K200274

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x = 0$

$-y - z = 0$

$z = t$

$\boxed{y = -t}$

$x = 0$

$$S_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

λ_1

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = t$

$x_2 = 0$

$x_3 = 0$

$$S_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

K200274

Date ____

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad \begin{bmatrix}x\\ y-z\\ -y+z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x=0 \quad y=z=t$

$$s_3 = \begin{bmatrix}0\\1\\1\end{bmatrix}$$

$$P = \begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix} \quad P^{-1} = \begin{bmatrix}0 & 1/2 & -1/2\\ 1 & 0 & 0\\ 0 & 1/2 & 1/2\end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix}0 & 1/2 & -1/2\\ 1 & 0 & 0\\ 0 & 1/2 & 1/2\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}$$

Signature ____ No. ____

K200274

Date

Ans 10) $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$
(a)

$A - \lambda I = \begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{bmatrix}$

~~$\text{Det}(A - \lambda I) = (3-$~~

$\det(\lambda I - A) = \begin{bmatrix} \lambda-3 & 0 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & -1 & \lambda-2 \end{bmatrix} = (\lambda-3)(\lambda-2)^2$

has 2 eigen values with Algebric multiplicity.
2 and 3)

(b) $2I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ rank$(2I - A) = 2$.

$3I - A \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $(3I - A) = 2$.

Signature

No.

K200274

Date

(c) geometric multipicity of $\lambda=3$ is $3-2=1$ equal to algebric multplicity. However, geometric multipicity of eigen value $\lambda=2$ is $3-2=1$, which is less then the corresponding algebric multipicity (2), therefore by theorm A is not diagonalizable.

Ans 15(a) $(\lambda-1)(\lambda+3)(\lambda-5)=0$

Eigen values;

$\lambda=1,\ \lambda=-3,\ \lambda=5$

and all of them have algebric multipicity 1.

Since A is 3×3 with 3 distinct eigen values it follows from Theorm that it diagonizable.

Geometric multipicity = Algebric Multipicity

(b) $\lambda^2(\lambda-1)(\lambda-2)^3=0$

$\lambda=0,1,2.$

$\lambda=0$ Algebric multplicity = 2,

$\lambda=1$ Algebric multipicity = 1

$\lambda=2$ Algebric multipicity = 3

Signature

No.

K200274

Date

$\dim A_0 = 2$

$\dim A_1 = 1$

$\dim A_2 = 3$

Roots of this equation are eigen.

Ans 20) $\lambda_1 = \lambda_2 = -1 \quad \lambda_3 = -1$

$\cancel{A = PDP^{-1}}$

$D = P^{-1}AP$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = -1 \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Signature

No.

K200274

Date

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}.$$

$$A = PDP^{-1}$$

$$PDP^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^{1000} = PD^{1000}P^{-1}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

No.

Signature

K200274

$$\boxed{I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\boxed{A^{1000} = I}$$

(b) $\lambda_1 = \lambda_2 = -1 \quad \lambda_3 = -1$

$$A = PDP^{-1} = P \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = -4 \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

K200274

Date

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$A = PDP^{-1}$

$$PDP^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

No.

Signature

K200274

Date

$$A^{1000} = (PDP^{-1})^{1000}$$
$$= PD^{1000}P^{-1}$$

$$A^{-1000} = A^{-1 \cdot 1000} = (A^{1000})^{-1}$$

$$A^{1000} = PD^{1000}P^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boxed{A^{-1000} = I}$$

Signature

K200274

Date

(c) $\lambda_1 = \lambda_2 = -1 \quad \lambda_3 = -1$

$$A = PDP^{-1} = P\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = -1\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

Signature

No

K200274

Date

$$PDP^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{2301} = PD^{2301}P^{-1}$$

$$A^{2301} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{2301} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$$

$$\boxed{A^{2301} = A}$$

Signature

No.

K200274

Date

(d) $A = PDP^{-1} = P\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1}$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = -1\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

Signature

No.

K200274

Date

$$PDP^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$$

$$A^{2301} = PD^{2301}P^{-1}$$

$$A^{2301} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{2301} & 0 & 0 \\ 0 & (-1)^{2301} & 0 \\ 0 & 0 & (1)^{2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A^{1} = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$$

$$\boxed{A^{-2301} = A^{-1} = A} \quad A^{-1} = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Signature

No.

K200274

Date

Ex 6.1:

$$(u,v) = \frac{1}{2} \cdot 1 \cdot 3 + 5 \cdot 1 \cdot 2$$

$$(u,v) = \frac{3}{2} + 10$$

$$(u,v) = \frac{23}{2}$$

(b) $$(kv, w) = \frac{1}{2}(kv_1)w^1 + 5(kv_2)w^2.$$

$$(kv, w) = \frac{1}{2}(3 \cdot 3) \cdot 0 + 5(3 \cdot 2)(-1)$$

$$(kv, w) = 0 + (-30)$$

$$(kv, w) = -30$$

(c) $$(u+v, w) = \frac{1}{2}(u_1+v_1)w^1 + 5(u_2+v_2)w_2.$$

$$(u+v, w) = \frac{1}{2}(1+3) \cdot 0 + 5(1+2)(-1)$$

$$(u+v, w) = -15$$

Signature

No.

K200274

Date

(d) $||v|| = \sqrt{\frac{1}{2}(v_1 v_1) + 5(v_2 v_2)}$

$||v|| = \sqrt{\frac{1}{2}(3 \cdot 3) + 5(2 \cdot 2)}$

$||v|| = \sqrt{\frac{9}{2} + 20}$

$||v|| = \sqrt{\frac{49}{2}} = \frac{7}{\sqrt{2}}$

(e) $d(u, v) = ||u - v||$

$d(u, v) = ||(-2, -1)||$

$d(u, v) = \sqrt{\frac{1}{2}(-2(-2)) + 5(-1)(-1)}$

$d(u, v) = \sqrt{2+5}$

$d(u, v) = \sqrt{7}$

(f) $u - kv = (1, 1) - 3(3, 2)$

$u - kv = (1, 1) - (9, 6)$

$u - kv = (-8, -5)$

Signature

K200274

$$d(u,v) = ||u - v||$$

$$d(u,v) = ||(-8,-5)||$$

$$d(u,v) = \sqrt{\frac{1}{2}(-8\cdot(-8)) + 5(-5\cdot(-5))}$$

$$d(u,v) = \sqrt{157}$$

Ans 13) $U = (u_1, u_2) \in R^2$

$V = (v_1, v_2) \in R^2$

$$\langle u,v \rangle = c_1 u_1 v_1 + c_2 u_2 v_2$$

$$\begin{bmatrix} \sqrt{c_1} & 0 \\ 0 & \sqrt{c_2} \end{bmatrix}$$

Matrix of weighted inner product:

$$\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{bmatrix}$$

K200274

Date

Ans 15) $p(x_0) = p(-2) = -2 + (-2)^3 = -2-8 = -10$

$p(x_1) = p(-1) = -1 + (-1)^3 = -1-1 = -2$

$p(x_2) = p(0) = 0 + 0^3 = 0 + 0 = 0$

$p(x_3) = p(1) = 1 + 1^3 = 1+1 = 2.$

$q(x_0) = q(-2) = 1 + (-2)^2 = 1+4 = 5$

$q(x_1) = q(-1) = 1 + (-1)^2 = 2.$

$q(x_2) = q(0) = 1 + 0^2 = 1$

$q(x_3) = q(1) = 1 + 1^2 = 2.$

inner product on p3 :-

$\langle p, q \rangle = p(x_0)\,q(x_0) + p(x_1)\,q(x_1) + p(x_2)\,q(x_2) + p(x_3)\,q(x_3)$

$= -10(5) + (-2)(2) + (0)(1) + (2)(2)$

$= -50 - 4 + 0 + 4 = -50$

Signature

No.

K200274

Date

Ans22) $\langle U, V \rangle = tr(U^T V) = U_1V_1 + U_2V_2$

$$U = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 & V_2 \\ V_3 & V_4 \end{bmatrix}$$

$\langle U, U \rangle = 1^2 + 2^2 + (-3)^2 + 5^2 = 1$

$||U|| = \sqrt{\langle U, U \rangle} = \sqrt{39}$

$$U - V = \begin{bmatrix} 1-4 & 2-6 \\ -3-0 & 5-8 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}$$

$\langle U - V, U - V \rangle = (-3)^2 + (-4)^2 + (-3)^2 + (-3)^2$

$= 9 + 16 + 9 + 9 = 43$

$d(U, V) = ||U - V|| = \sqrt{\langle U-V, U-V \rangle}$

$= \sqrt{43}$

$||U|| = \sqrt{39}$ and $d(U, V) \sqrt{43}$

Signature

No.

K200274

Date

Ans 26) $\langle u, u \rangle = (Au) \cdot (Au)$

$$= \left(\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right) \cdot \left(\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 \\ 7 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = 3^2 + 7^2 = 9 + 49 = 58$$

$u - v = (-1, 2) - (2, 5) = (-1-2, 2-5) = (-3, -3)$

$\langle u-v, u-v \rangle = (A(u-v)) \cdot (A(u-v))$

$$= \left(\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix} \right) \left(\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -9 \\ -6 \end{bmatrix} \begin{bmatrix} -9 \\ -6 \end{bmatrix} = (-9)^2 + (-6)^2$$

$$= 81 + 36 = 117.$$

$d(u, v) = \| u - v \| =$

$\sqrt{\langle u-v, u-v \rangle} = 117$

Signature

No.

K200274

Date

Ans 28) (a) $\langle u - v - 2w, 4u + v\rangle$

$= 4\langle u, u\rangle + \langle u, v\rangle - 4\langle v, u\rangle - \langle v, v\rangle$
$- 8\langle w, u\rangle - 2\langle w, v\rangle$

$= 4\|u\|^2 + 2 - 4\langle u, v\rangle - \|v\|^2 - 8\langle u, w\rangle$
$- 2\langle v, w\rangle$

$= 4 + 2 - 4(2) - 2^2 - 8(-3) - 2(-6)$

$= 4 + 2 - 8 - 4 + 24 + 12.$

$= 30.$

$\langle u - v - 2w, 4u + v\rangle = 30$

(b) $\|2w - v\| = \sqrt{\langle 2w - v, 2w - v\rangle}$

$= \sqrt{4\langle w, w\rangle - 2\langle w, v\rangle - 2\langle v, w\rangle + \langle v, u\rangle}$

$= \sqrt{4\|w\|^2 - 2\langle v, w\rangle - 2\langle v, w\rangle + \|v\|^2}$

$= \sqrt{4(7)^2 - 2(-6) - 2(-6) + 2^2}$

$= \sqrt{196 + 12 + 12 + 4}$

$= \sqrt{224}$

$\|2w - 4\| = \sqrt{224}$

No.

Signature

K200274

Date

Ex 6.2:

Ans 2) (a) $u = (-1, 0)$ $v = (3, 8)$.

$$\cos\theta = \frac{u \cdot v}{\|u\| \, \|v\|} = \frac{(-1) \cdot 3 + 0}{\sqrt{(-1)^2 + 0^2}\sqrt{73}}$$

$$= -\frac{3\sqrt{73}}{73}$$

$$\cos\theta - \frac{3\sqrt{73}}{73}$$

Ans 4) $p = x - x^2$. $q = 7 + 3x^2$.

$$\cos\theta = \frac{0 \cdot 7 + 1 \cdot 3 + (-1) \cdot 3}{\sqrt{0^2 + 1^2 + (-1)^2}\sqrt{7^2 + 3^2 + 3^2}}$$

$$= \frac{0}{\sqrt{2}\sqrt{67}} = 0$$

Signature

K200274

Date

Ans 6) $A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$

$$\cos\theta = \frac{\langle A, B \rangle}{\|A\| \|B\|} = \frac{2\cdot(-3) + 4\cdot 1 + (-1)\cdot 4 + 3\cdot 2}{\sqrt{2^2+4^2+(-1)^2+3^2}\sqrt{(-3)^2+1^2+4^2+2^2}}$$

$= 0$

$\cos\theta = 0$

Ans 8)(a) $U = (u_1, u_2, u_3)$ $V = (0,0,0)$.

$\langle u, v \rangle = u_1 \cdot 0 + u_2 \cdot 0 + u_3 \cdot 0 = 0$

Orthogonall

(b) $(-4, 6, -10, 1)$ $v = (2, 1, -2, 9)$.

$\langle u, v \rangle = (-4)\cdot 2 + 6 \cdot 1 + (-10)\cdot(-2) + 1\cdot 9 = 27$

not orthogonal

(c) $U = (a, b, c)$ $V = (-c, 0, a)$

$\langle u, v \rangle = a\cdot(-c) + b\cdot 0 + c\cdot a = -ac + 0 + ac = 0$.

Orthogonall.

Signature

No.

K200274

Date

Ans 12) $u = \begin{bmatrix} 5 & -1 \\ 2 & -2 \end{bmatrix}$ $v = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$

$\langle u, v \rangle = 5\cdot 1 + (-1)\cdot 3 + 2(-1) + (-2)0$

$= -5 - 3 - 2 = 0.$

orthogonal.

Ans 18) $\langle u, v \rangle = (Au)\cdot(Av)$

$= \left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right) \left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -8 \end{bmatrix} \right)$

$= \begin{bmatrix} 9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 9(2) + 6(-3) = 0.$

orthogonal

$\boxed{(Au)(Av) = 0}$

Signature