20K-0274 NASHIT BUDHWANI

Assignment #02 . LA Date

Ex 4.1

Q2) $u+v = ((u_1+v_1+1), (u_2+v_2+1))$;
$ku = (ku_1, ku_2)$

(a) $u = (0,4)$ $k = 2$
$v = (1,-3)$

$u+v = (0,4)+(1,-3) = (0+1+1, 4-3+1)$
$= (2,2)$

$ku = 2(0,4) = (0,8)$

(b) $u+0 \Rightarrow (0,0)+(u_1, u_2)$
$\Rightarrow (0+u_1+1, 0+u_2+1)$
$\Rightarrow (u_1+1, u_2+1)$

(c) $u+0 = u$

$(u_1, u_2) + (-1,-1) = u$ (?)
$(u_1+1-1, u_2+1-1) = u$
$(u_1, u_2) = u$

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(d) $0 - u = -u$

$(-1, -1) - (u_1, u_2)$

$(-u_1, -1 - 1, -u_2 - 1 - 1)$

$(-u_1 - 2, -u_2 - 2) = -u$

$u + (-u) = 0$

$(u_1, u_2) = (-u_1 - 2, -u_2 - 2)$

$(u_1 - u_1 - 2 + 1, -u_2 + u_2 - 2 + 1)$

$(-1, -1) = 0$

(e) Axiom # 7 : $k(u + v) = ku + kv$

$k(u_1 + v_1 + 1, u_2 + v_2 + 1)$

$(ku_1 + kv_1 + k, ku_2 + kv_2 + k)$

but, $ku + kv$

$(ku_1, ku_2) + (kv_1, kv_2)$

$(ku_1 + kv_1 + 1, ku_2 + kv_2 + 1)$

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Axiom #8 $(K+m)U = KU + mU$

$$(K+m)U = ((K+m)U, (K+m)U_2)$$

$$= (KU_1 + mU_1, KU_1 + mU_2)$$

but ; $KU + mU$

$$(KU_1, KU_2) + (mU_1, mU_2)$$

$$(KU_1 + mU_1 + 1, KU_2 + mU_2 + 1)$$

Axiom 7 and 8 fail to hold.

Q9) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Axiom 1 :

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix}$$

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Axiom 2:

$$u+v = v+u$$

$$\begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} = \begin{bmatrix} c+a & 0 \\ 0 & d+b \end{bmatrix}$$

Same $\in V$.

Axiom #5:

$$(u+v)+w = u+(v+w)$$

$$\begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$$

$$= \begin{bmatrix} a+c+e & 0 \\ 0 & b+d+f \end{bmatrix}.$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c+e & 0 \\ 0 & d+f \end{bmatrix} = \begin{bmatrix} a+c+e & 0 \\ 0 & b+d+f \end{bmatrix}$$

yes $\in V$.

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Axiom #4

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

Axiom # 5 ;

$$U + (-U) = 0$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Axiom # 06.

$$K \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} Ka & 0 \\ 0 & Kb \end{bmatrix}$$

Axiom # 07

$$K(U+V) = KU + KV$$

$$K \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix} + \begin{bmatrix} kc & 0 \\ 0 & kd \end{bmatrix}$$

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$$\begin{bmatrix} ka+kc & 0 \\ 0 & kb+kd \end{bmatrix} = \begin{bmatrix} ka+kc & 0 \\ 0 & kb+kd \end{bmatrix}$$

Axiom #8.

$(k+m)U = kU + mU.$

$$\begin{bmatrix} ka+ma & 0 \\ 0 & kb+mb \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix} + \begin{bmatrix} ma & 0 \\ 0 & mb \end{bmatrix}$$

Axiom #9.

$k(mU) = km(U)$

$$k\begin{bmatrix} ma & 0 \\ 0 & mb \end{bmatrix} = \begin{bmatrix} kma & 0 \\ 0 & kmb \end{bmatrix}$$

$$\begin{bmatrix} kma & 0 \\ 0 & kmb \end{bmatrix}.$$

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Axiom # 10:

$1U = u$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = U$$

It is Vector

(It holds all Axioms)

Q 11) $U+V = (1, U+V)$; when U and V must be $(1, x)$

$KU = (1, KU)$

Axiom # 1:

$(1, U) + (1, V) = (1, U+V)$

Axiom # 2

$U+V = V+U$

$(1, U+V) = (1, V+U)$

↳ same.

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Axiom #3: $(u+v)+w = u+(v+w)$

$(1, u+v) + (1, w) = (1, u+v+w)$

$(1, u) + (1, v+w) = (1, u+v+w)$

Axiom # 4

$u + 0 = 0$

$(1, u) + (1, 0)$

$(1, u+0)$

$(1, u) = u.$

~~Axiom #4:~~

Axiom # 5:

$-u = (1, -u)$

$(1, u) + (1, -u) = 0 \rightarrow (1, 0)$

$(1, u - u)$

$(1, 0) = 0$

Axiom # 6:

$k(1, u) = (1, ku)$

$\in V$

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Axiom #07:

$$k(1, u+v) = (1, ku) + (1, kv)$$

$$(1, ku+kv) = (1, ku+kv)$$

Axiom # 8:

$$(1, (k+m)u) = ku+mu$$

$$(1, ku+mu)$$

$$(1, ku) + (1, mu)$$

$$\downarrow$$

$$ku + mu$$

Axiom # 9;

$$k(1, mu) = km(u)$$

$$(1, kmu) = (1, kmu)$$

Axiom # 10:

$$1u = u$$

$$1(1, u) = u$$

$$(1, u) = u$$

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Q12)

$a = a_0 + a_1 x.$

$b = b_0 + b_1 x.$

Axiom # 1:

$a+b = (a_0 + b_0) + (a_1 + b_1) x$

Axiom # 6:

$ka = ka_0 + ka_1 x.$

Axiom #1:

$a+b = (a_0 + b_0) + (a_1 + b_1) x.$

Axiom # 6:

$ka = ka_0 + ka_1 x.$

Axiom # 2:

$U + V = V + U$

$(a_0 + b_0) + (a_1 + b_1) x = (b_0 + a_0) + (b_1 + a_1) x$

Axiom # 3:

$(U+V) + W = U + (V+W)$

$\left[(a_0 + b_0) + (a_1 + b_1) x\right] + c_0 + c_1 x$

$\Rightarrow (a_0 + b_0 + c_0) + (a_1 + b_1 + c_1) x$

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$$a_0 + a_1 x + [(b_0 + c_0) + (b_1 + c_1)x]$$

$$(a_0 + b_0 + c_0) + (a_1 x + b_1 x + c_1 x)$$

Axiom #4:

$$0 = 0 + 0x.$$

$$a + 0 = a$$

$$(a_0 + a_1 x) + (0 + 0x)$$

$$a_0 + (a_1)x = a.$$

Axiom # 05

$$u + (-u) = 0$$

$$(a_0 + a_1 x) + (-a_0 - a_1 x)$$

$$0 + 0x = 0$$

Axiom # 7:

$$k(a_0 + b_0) + k(a_1 + b_1)x.$$

$$(ka_0 + kb_0) + (ka_1 + kb_1)x.$$

$$k(a_0 + a_1 x) + k(b_0 + b_1 x).$$

$$ka_0 + ka_1 x + kb_0 + kb_1 x.$$

$$(ka_0 + kb_0) + (ka_1 x + kb_1 x)$$

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Axiom # 8:

$(k+m)v = kv + mv$

$(k+m)(a_0 + a_1 x)$

$\cancel{(ka_0 + kb_0) + (ka_1 x + kb_1 x)}$

$ka_0 + ka_1 x + ma_0 + ma_1 x.$

$k(a_0 + a_1 x) + m(a_0 + a_1 x)$

$ka_0 + ka_1 x + ma_0 + ma_1 x$

Axiom # 9

$k(ma_0 + ma_1 x) = km(a_0 + a_1 x)$

$kma_0 + kma_1 x = kma_0 + kma_1 x.$

Axiom # 10:

$1(a_0 + a_1 x) = a$?

$a_0 + a_1 x = a$

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Ex 4.2:

Q3(a) $\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$

$= \begin{bmatrix} a_1+b_1 & 0 & 0 \\ 0 & a_2+b_2 & 0 \\ 0 & 0 & a_3+b_3 \end{bmatrix}$

Axiom # 6

$k \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} ka_1 & 0 & 0 \\ 0 & ka_2 & 0 \\ 0 & 0 & ka_3 \end{bmatrix}$

$\in M_{nn}$.

(b) Axiom #1:

$\begin{bmatrix} b_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3+b_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$

$\det(A+B) \neq 0$

No

(c) Axiom # 1

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$$

$$= \begin{bmatrix} b_1+a_1 & 0 & 0 \\ 0 & b_2+a_2 & 0 \\ 0 & 0 & b_3+a_3 \end{bmatrix}$$

if $a_1 + a_2 + a_3 = 0$ & $b_1 + b_2 + b_3 = 0$

then $b_1 + a_1 + b_2 + a_2 + b_3 + a_3$

$\Rightarrow 0 + 0 = 0$

Axiom # 6 :

$$k \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \rightarrow \text{if } a_1 + a_2 + a_3 \neq 0$$

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(d) $\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} = \begin{bmatrix} a_1+b_1 & 0 & 0 \\ 0 & a_2+b_2 & 0 \\ 0 & 0 & a_3+b_3 \end{bmatrix}$$

→ also symmetric

Axiom # 6:

$$K \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} = \begin{bmatrix} Ka_1 & 0 & 0 \\ 0 & Ka_2 & 0 \\ 0 & 0 & Ka_3 \end{bmatrix}$$

Ans (e) as Axiom # 1:

$$A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix} = \begin{bmatrix} 0 & a+b \\ -a+b & 0 \end{bmatrix}$$

→ yes its still

$(AB)^t = -(A+B)$

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Axiom #6:

$$kA = k\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & ka \\ -ka & 0 \end{bmatrix}$$

$\hookrightarrow$ also $(kA)^T = -kA$

(b) Axiom #1:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + B = \begin{bmatrix} -2 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

holds. | $A(x_1 + x_2) =$
$Ax_1 + Ax_2$
$0 + 0$
0 //

Axiom #6: $kv \Rightarrow kv_1 = k(0)$
$= 0$

is a subspace.

(c) Axiom #1

$$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$$
$$= \begin{bmatrix} a_1 + b_1 & 0 \\ 0 & a_2 + b_2 \end{bmatrix}$$

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Axiom # 6:

$$k \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \Rightarrow \begin{bmatrix} ka_1 & 0 \\ 0 & ka_2 \end{bmatrix}$$

It is a subspace.

(d) $$\begin{bmatrix} a_1 & a_3 \\ 0 & a_2 \end{bmatrix} + \begin{bmatrix} -a_1 & 0 \\ 0 & -a_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & a_3 \\ 0 & 0 \end{bmatrix}$$

?.

Axiom I failed

this is not invertible

Q4)(a) $A x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$(A+B)x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Ax + Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

not a subspace

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(b) $Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$(A+B)x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Ax + Bx = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Axiom #6;

$$(KA)x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$K\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Q16) (a) $A = 2 + 4x + 6x^2$
$B = 12 + 24x + 8x^2$.

$A + B = 14 + 28x + 14x^2$

even. Axiom # 1 holds.

Axiom # 06:

$KA = K2 + 4Kx + 6Kx^2$

even

holds.

(b) <u>Axiom # 1:</u>

$A = 2 + (-4x) + 2x^2$.

$B = -8 + 6x + 2x^2$.

$(A + B) = -6 + 2x + 4x^2$.

$= -6 + 2 + 4 = 0$

Axiom # 6

$KA = 2K + (-4K)x + (2K)x^2$

$= 2K + 2K - 4K = 0$

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Axiom # 1:

(c) $A = a_1 + a_2 x + a_3 x^2$

$B = b_1 + b_2 x + b_3 x^2$

$A + B = (a_1 + b_1) + (b_1 + b_2) x + (a_3 + b_3) x^2$

↳ still even

Axiom # 06:

$kA = ka_1 + ka_2 x + ka_3 x^2$

↳ still even

Ex 4.3:

Q15(a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$R_2 - R_1 \to R_2 \Rightarrow A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$(-1 \times R_2 \to R_2) \Rightarrow A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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$(R_1 - R_2 \to R_1) \Rightarrow A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x + z = 0$

$y + w = 0$

$0 = 0$

$x = s$

$z = -s$

$y = t$

$w = -t$

$(x, y, z, w) = (s, t, -s, -t)$

$= (s + 0, 0 + t, -s + 0, 0 - t)$

$= (s, 0, -s, 0) + (0, t, 0, -t)$

$\Rightarrow (x, y, z, w) = s(1, 0, -1, 0) + t(0, 1, 0, -1)$

Solution space is spanned by vectors
$u = (1, 0, -1, 0)$ and $v = (0, 1, 0, -1)$

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(b) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$(R_2 - R_1 \to R_2) \Rightarrow A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$(-1 \times R_2 \leftrightarrow R_2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$(R_3 - R_2 \to R_3)\, A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$(R_1 - R_2 \to R_1)\, A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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$x + z = 0$

$y + w = 0$

$0 = 0$

$x = s$

$z = -s$

$y = t$

$w = -t$

$(x, y, z, w) = (s, t, -s, -t)$

$= (s+0, 0+t, -s+0, 0-t)$

$= (s, 0, -s, 0) + (0, t, 0, -t)$

$(x, y, z, w) = s(1, 0, -1, 0) + t(0, 1, 0, -1)$

$u = (1, 0, -1, 0) \quad v = (1, 1, -1, -1)$

$(1, 0, -1, 0) + (0, 1, 0, -1) = a + b.$

the set $\{u, v\}$ spans W

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$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$$(R_2 - 2R_1 \rightarrow R_1) \Rightarrow A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$$(R_3 - 3R_1 \rightarrow R_1) \Rightarrow A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$y - z + w = 0$$
$$0 = 0$$
$$0 = 0$$

$w = x_1 \quad z = x_2.$

$$y - (x_2) + (x_1) = 0$$
$$y = x_2 - x_1$$

$\cancel{x = x_3}$

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$$(x,y,z,w) = (k_3, k_2-k_1, k_2, k_1)$$
$$= (k_3,0,0,0) + (0,-k_1,0,k_1)$$
$$+ (0,k_2,k_2,0)$$

$$(x,y,z,w) = k_3(1,0,0,0) + k_1(0,-1,0,1)$$
$$+ k_2(0,1,1,0)$$

$$(x,y,z,w) = (1,1,1,0) + k_1(0,-1,0,1)$$

$\{u, v\}$ spans w

Ans 16 b) $A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$

$(R_2 - 2R_1 \to R_2) \Rightarrow A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$

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$(R_3 - 3R_1 \to R_1) \Rightarrow A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$y - z + w = 0$

$0 = 0$

$0 = 0$

$w = k_1 \quad z = k_2$

$y - (k_2) + (k_1) = 0$

$y = k_2 - k_1$

$x = k_3$

$(x, y, z, w) = (k_3, k_2 - k_1, k_2, k_1)$

$= (k_3, 0, 0, 0) + (0, -k_1, 0, k_1) + (0, k_2, k_2, 0)$

$\Rightarrow (x, y, z, w) = k_3(1, 0, 0, 0) + k_1(0, -1, 0, 1) + k_2(0, 1, 1, 0)$

W is not spamed by V

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Ex #4.4 Q

Ans 11) $a\left(\lambda, -\frac{1}{2}, -\frac{1}{2}\right) + b\left(-\frac{1}{2}, \lambda, -\frac{1}{2}\right) + c\left(-\frac{1}{2}, -\frac{1}{2}, \lambda\right) = 0$

$$a\lambda - \frac{1}{2}b - \frac{1}{2}c = 0$$

$$-\frac{1}{2}a + \lambda b - \frac{1}{2}c = 0$$

$$-\frac{1}{2}a - \frac{1}{2}b + \lambda c = 0$$

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix} = \lambda \begin{bmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{bmatrix}$$

$$+ \frac{1}{2} \det \begin{bmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{bmatrix} - \frac{1}{2} \det \begin{bmatrix} -\frac{1}{2} & \lambda \\ -1/2 & -1/2 \end{bmatrix}$$

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$$= \lambda\left(\lambda^2 - \frac{1}{4}\right) + \frac{1}{2}\left(-\frac{2}{\lambda} - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{4} + \frac{\lambda}{2}\right)$$

$$= \lambda^3 - \frac{\lambda}{4} - \frac{1}{8} - \frac{1}{8} - \lambda$$

$$= \lambda^3 - \frac{3}{4}\lambda - \frac{1}{4}$$

$$\lambda^3 - \frac{3}{4}\lambda - \frac{1}{4} = 0$$

$$4\lambda^3 - 3\lambda - 1 = 0$$

$$-(\lambda - 1)(2\lambda + 1)^2 = 0$$

$$\lambda - 1 = 0 \qquad 2\lambda + = 0$$

$$\lambda_1 = 0$$

$$2\lambda + 1 = 0 \quad 2\lambda = -1$$

$$\lambda_2 = -\frac{1}{2}$$

$$\lambda = 1 \quad \lambda = -\frac{1}{2}$$

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$\lambda = 1 \quad \lambda = -\frac{1}{2}$ with multiplicity of 2.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$$

$V_1 = (1,0,0) \quad V_2 = (2,-1,1) \quad \& \quad V_3 = (0,1,1)$

$$T_A(V_1) = AV_1 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$T_A(V_2) = AV_2 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$T_A(V_3) = AV_3 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$T_A(V_1), \; T_A(V_3) = \{ (1,1,2), (3,-1,2), (3,-3,2$

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$a(1,1,2)+b(3,-1,2)+c(3,-3,2)=(0,0,0)$

$a(1,1,2)+b(3,-1,2)+c(3,-3,2)$

$=(a,a,2a)+(3b,-2b)+(3c,-3c,2c)$

$(a+3b+3c,\ a-b-3c,\ 2a+2b+2c)$

$a+3b+3c=0$

$a-b-3c=0$

$2a+2b+2c=0$

$a+3b+3c+a-b-3c-(2a+2b+2c)$

$=0+0-0$

$-2c=0$

$\boxed{c=0}$

$a-b=0$

$a+3b=0$

$2a+2b=0$

$\therefore a=b=c=0$

Linearly independent