Date

Discrete Structures Assignment

Name: Nashwt Budhwani

Roll-No: 20K-0274

Section: BCS-3B

Ans1) $(\neg p \wedge q) \vee \neg(p \vee q) \equiv \neg p$

$(\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p$

$(\neg p \vee (\neg p \wedge \neg q)) \wedge (q \vee (\neg p \wedge \neg q)) \equiv p$

$(\neg p) \wedge ((q \vee \neg p) \wedge (q \vee \neg q))$

$(\neg p) \wedge ((q \vee \neg q) \wedge 1)$

$(\neg p) \wedge (q \vee \neg q)$

$(\neg p \wedge q) \vee (\neg p \wedge \neg p)$

$(\neg p \wedge q) \vee (\neg p)$

$\neg p (q \vee 1)$

$\neg p \equiv \neg p$

Ans2) $\neg(\neg p \rightarrow \neg q) \vee (p \wedge q) \equiv q$

$\neg(p \vee \neg q) \vee (p \wedge q)$

$(\neg p \wedge q) \vee (p \wedge q)$

$q (\neg p \vee p) \equiv q.$

$q \equiv q.$

Date

Ans3)

p	q	r	¬q	p→¬q	(p→¬q)→r
T	T	T	f	f	f
T	T	F	f	f	T
T	F	T	T	T	T
T	F	F	T	T	f
f	T	T	f	T	T
f	T	F	f	T	f
f	F	T	T	T	T
f	f	f	T	T	f

Ans4) $q \rightarrow (p \rightarrow q)$

p	q	p→q	q→(p→q)
T	T	T	T
T	F	F	T
F	T	T	T
f	F	T	T

tautalgy

Signature

No.

Date

Ans5)

				x	y	
p	q	¬p	¬q	p ↔ ¬q	¬p ∨ ¬q	x → y
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

tautology.

Quantifiers:

Ans 8) A ∪ C = B ∪ C then A = B is False.

Let $A = \{2\}$, $B = \{2, 4\}$, $C = \{2, 4, 3\}$

$A \cup C = \{2, 3, 4\}$

$B \cup C = \{2, 3, 4\}$

Since $A \cup C = B \cup C$ But $A \neq B$ so

hence Shown

Date

Ans 9) $p \rightarrow$ wearing a pink tie
$q \rightarrow$ wearing a red shirt
$r \rightarrow$ It is saturday

$\neg p \vee q$
$\neg r \rightarrow p$
$\neg p$
$\therefore r$

$\neg q \vee \neg r$
$\neg q$
$\neg r$

$\neg(\neg q \vee \neg r)$

$\neg q \rightarrow r$
$\neg q$
r modus ponen.

No.

Signature

Date

Set Theory:

Ans11) $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

$(\bar{A} \cup \bar{B}) = (\bar{A} \cup \bar{B})$

$(\bar{A} \cup \bar{B}) = \bar{A} \cup \bar{B}$ ∴ Demorgan's law.

Ans12) A = Triumph = 30

B = No of honda = 32

~~C~~ Owns neither = 15

Total Motorcycles 50

Since $(A \cup B)_n = |A|_n \rightarrow |B|_n - (A \cap B)$

$35 = 30 + 32 - (A \cap B)$

$(A \cap B) = 27$ Owns each motorcycle.

Signature

Date

Ans 14) $d = \gcd(3142, 900)$

$3142 = 900 \cdot 3 + 442$

$900 = 442 \cdot 2 + 16$

$442 = 16 \cdot 27 + 10$

$16 = 10 \cdot 1 + 6,$

$10 = 6 \cdot 1 + 4$

$6 = 4 \cdot 1 + 2$

$4 = 2 \cdot 2 + 0$

$\gcd(3142, 900) = 2.$

$2 = 6 - 4$

$2 = 6 - 4(10 - 6)$

$2 = 6 \cdot 2 - 10$

$2 = (16 - 10) \cdot 2 - 10$

$2 = 16 \cdot 2 - 10 \cdot 3$

$2 = 16 \cdot 2 - 3(442 - 16 \cdot 27)$

$2 = 16 \cdot 83 - 442 \cdot 3$

$2 = (900 - 442 \cdot 2) \cdot 83 - 442 \cdot 3$

$2 = 900 \cdot 83 - 4 - 169$

$2 = 900 \cdot 83 - 169(3142 - 900 \cdot 3)$

$2 = 900 \cdot 590 - 3142(169)$

Signature

No.

Date

$x = -169$ & $y = 590$.

Ans 15) $\gcd(2017, 122) = 1$

$2017 = 122 \cdot 16 + 65$

$122 = 65 \cdot 1 + 57$

$65 = 57 \cdot 1 + 57$

$57 = 8 \times 7 + 1$

$8 = 1 \times 8 + 0$

$\gcd(2017, 122) = 1$ Shown

reverse

$1 = 57 - 8 \cdot 7$

$1 = 57 - 7(65 - 57 \cdot 1)$

$1 = 57 \cdot 8 - 65 \cdot 7$

$1 = (122 - 65 \cdot 1) \cdot 8 - 65 \cdot 7$

$1 = 122 \cdot 8 - 65 \cdot 15$

$1 = 122 \cdot 8 - 15(2017 - 122 \cdot 16)$

$1 = 122 \cdot 248 - 2017 \cdot 15$

$x = -15$

$y = 248$

Signature

Date

Ans 16) $d = \gcd(578, 442)$

$578 = 442 \cdot 1 + 116$

$442 = 116 \cdot 3 + 94$

$116 = 94 \cdot 1 + 22$

$94 = 22 \cdot 4 + 6$

$22 = 6 \cdot 3 + 4$

$6 = 4 \cdot 1 + 2$,

$4 = 2 \cdot 2 + 0$

$d = \gcd(578, 442) = 2$

backward substitution,

$2 = 6 - 4$

$2 = 6 - (22 - 6 \cdot 3)$

$2 = 6 \cdot 4 - 22$

$2 = (94 - 22 \cdot 4) \cdot 4 - 22$

$2 = 94 \cdot 4 - 22 \cdot 17$

$2 = 94 \cdot 4 - 17(116 - 94 \cdot 1)$

$2 = 94 \cdot 21 - 116 \cdot 17$

$2 = (442 - 116 \cdot 3) \cdot 21 - 116 \cdot 17$

$2 = (442 \cdot 21 - 116 \cdot 63) - 116 \cdot 17$

$2 = 442 \cdot 21 - 116 \cdot 80$

Date

$2 = 442.101 - 558.80$

$2 = 442.101 - 558.80$

$x = -80$ & $y = 101$

Ans 17) $m = 900$

$n = 189 = 3^3 \cdot 7$

a) $2^2 \times 3^2 \times 5^2$

b) $\gcd(m, n) = 2^0 3^2 5^2 7^0$ and

$LCM = 2^2 3^3 5^2 7$

(c) Smallest multiple of 189 such that $\gcd(m, k) = 45$ is 5×189

Ans 19) $a_1 = 4$

$a_n = a_{n-1} + 4$

$a_1 = a_0 + 4$

$4 = 7a_0 + 4$

$a_0 = 0$

$a_2 = 7a_1 + 4 = 7(4) + 4 = 32$

$a_3 = 7a_2 + 4 = 7(32) + 4 = 228$

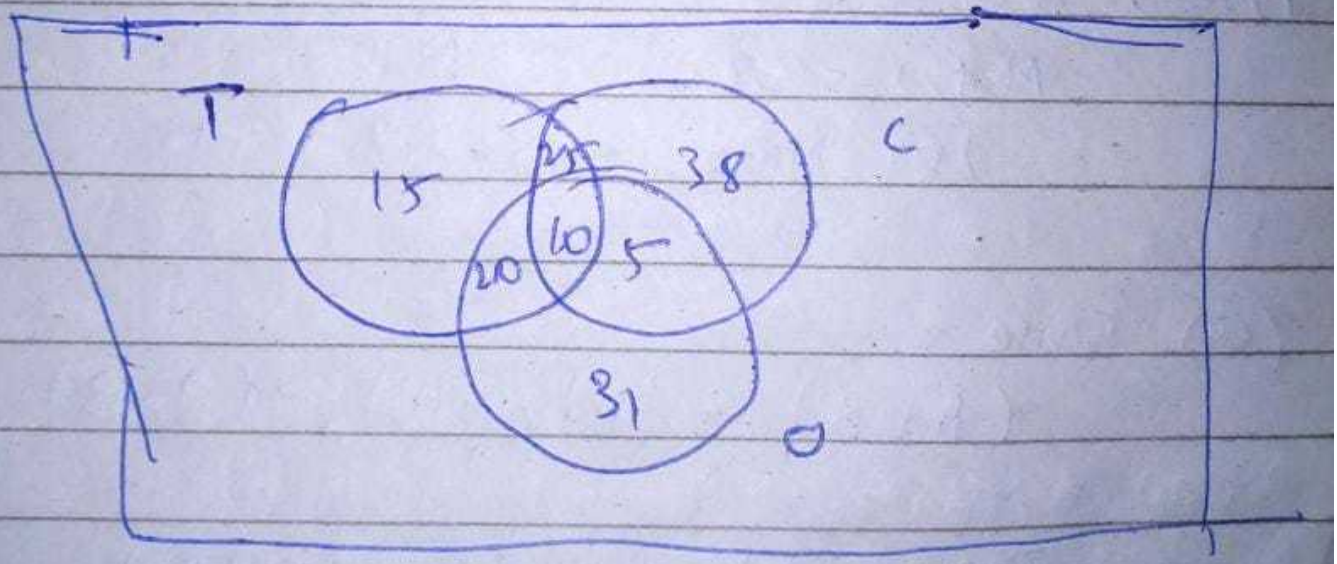
$a_4 = a_3 + 4 = 7(228) + 4 = 1600$

Signature

Date

Ans 20) Total = 200

Coffee = 78	(C)
tea = 70	(T)
Orange = 66	(O)
(C ∪ T) = 35	(C ∩ T)
(T ∪ O) = 30	(T ∩ O)
(C ∪ O) = 15	(C ∩ O)
(C ∩ T ∩ O) = 10	(C ∩ T ∩ O)



People who like orange juice only = 31

People who like no drink = 56

Date

Ans 21) Total customer = 50

$|M \cup R| = |M| + |R| + |M \cup R|' - |M \cap R|$

$50 = 30 + 35 - 15 - x$

$x = 30$

So 30 people have a mountain bike and a road bike.

Ans 22) Total = 348

Maths = 321 (M_1)

101 Math = 286 (M_2)

(a) Since

$(A \cup B)_n = |A|_n + |B|_n \Rightarrow (A \cap B)_n$

$332 = 321 + 286 - (A \cap B)_n$

$332 - 607 = -(A \cap B)_n$

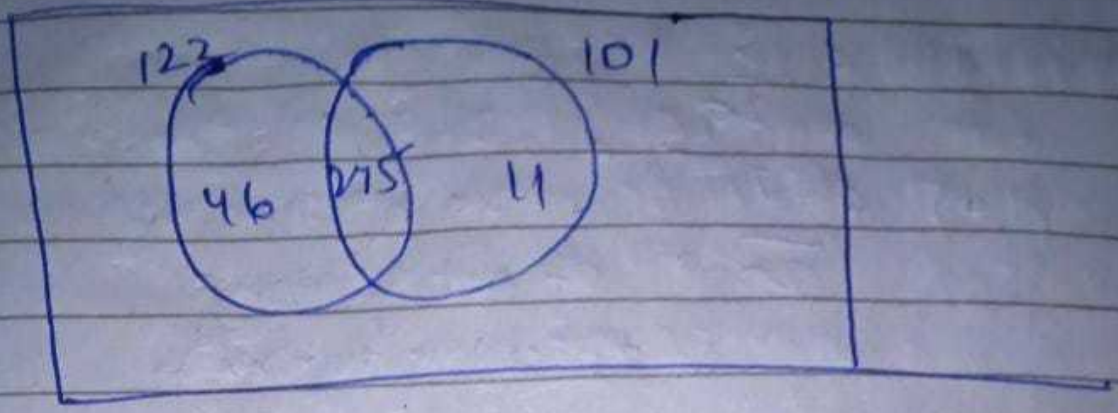
$275 = (A \cap B)_n$

Signature

No.

Date

(b)



46 students did not like math 101

Ans 26) ISBN-10 : 12 5 9 7 3 1 23 6

$1 \cdot 10 + 209 + 315 + 409 + 507 + 603 + 701 + 802 + 903$

$= 10 + 18 + 15 + 36 + 35 + 18 + 7 + 16 + 27$

$= 182 \bmod 11$

$= 6$

$6 + x = 11$

$x = 5$ is the check digit.

No.

Signature

Date

Ans 27) $P(d) = \frac{3}{20}$

$P(d)' = \frac{17}{20}$ is the probability of not choosing defective footballs.

Functions and Relations:

Ans 28) Suppose that $g(x) = g(y)$

$g\{f(x)\} = g\{f(y)\}$

$g \circ f(x) = g \circ f(y)$

$x = y$

So f is one to one.

Ans 30) Reflexive:

$\frac{n}{n+2} \geq \frac{n}{n+2}$ $n \sim n$

It is reflexive

Date

ant: Symmetric suppose $a \sim b \wedge b \sim a$
want $a = b$, since $a \sim b$, $\frac{a}{b+2} \geq \frac{b}{a+2}$

since $b \sim a$, $\frac{b}{a+2} \geq \frac{a}{b+2}$

$\therefore \frac{a}{b+2} = \frac{b}{a+2}$ so $a(a+2) = b(b+2)$

$a^2 + 2a = b^2 + 2b$

so $a^2 + 2a + 1 = b^2 + 2b + 1$

$(a+1)^2 = (b+1)^2$

since $a+1 = b+1$ $\therefore a = b$ It is antisymetric

Ans 31) Symmetric: suppose $x \sim y$

- The smallest element of x equals the smallest element of y.
- The smallest element of y equals the smallest element of x,
- $y \sim x$ so $\sim$ is symmetric.

No.

Date

Reflexive:

for any $x \subseteq \{1, 2, 3, 4\}$ the smallest element of x equals the smallest element of x (except $x = \phi$)

∴ is reflexive

Transitive:

Suppose $x \sim y$ and $y \sim z$

The smallest element of x equals the smallest element of y and the smallest element of y equals the smallest element of z.

Hence the smallest element of x equals the smallest element of x.

so $x \sim z$ is an equivalence Relation.

Ans 3 2) Let $B = \{(1,2)\}$ and $S = \{(2,1)\}$

Then R and S are anti-symmetric

Let $R \cup S = \{(1,2)(2,1)\}$ is not antisymmetric

because $1 \neq 2$.

Signature

Date

Binomial Theorm:

Ans36) $1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n+1)! - 1$

let $n = k$

$1 \cdot 1! + 2 \cdot 2! + \cdots k \cdot k! = (k+1)! - 1$

let $n = k+1$

$1 \cdot 1! + 2 \cdot 2! + \cdots (k+1)(k+1)! = (k+2)! - 1$

$1 \cdot 1! + 2 \cdot 2! + (k+1)(k+1)! = (k+1)! - 1$

$(k+1)(k+1)!$

$(k+1)(k+1)! = (k+1)!(1+k+1)$

$(k+1)(k+1)! + 1 = (k+1)!(k+2)$

through property of factorial

$= (k+2)! = (k+2)(k+1)!$

$(k+1)(k+1)! = (k+2)! - 1$

now substituting $k = n-1$

$(n)(n)! = (n-1)! - 1$

Signature

No.

Date

Ans 38) $\left[\frac{2}{3}x+1\right]^3 = (1)\left(\frac{2}{3}x\right)^3(1)^0 + (1)(3)\left(\frac{2}{3}x\right)$

$+3\left(\frac{2}{3}x\right)(1)^2 + 3\left(\frac{2}{3}x\right)^0(1)$

$= \frac{8}{27}x^3 + \frac{4}{3}x^2 + 2x + 3$

Ans 39) Coefficient of $(1/y^{10})$ in $\left(y^3 + \frac{a^7}{y^5}\right)^{10}$

at term = 5

$T_5 = (y^3)^5 \times \left(\frac{a^7}{y^5}\right)^5 \times 10C_5$

$= \frac{252\, a^{35}}{y^{10}}$

Ans 40) $(x-y)^{15}$; $x^3y^{12} = ?$; $x^2y^{13} = ?$

$T_{12} = 15C_{12}(x)^{15-12}(-y)^{12}$

$= 455\, x^3 y^{12}$ coefficient 455

$T_{13} = 15C_{13}(x)^{15-13}(-y)^{13} = -105\, x^2 y^{13}$

Signature

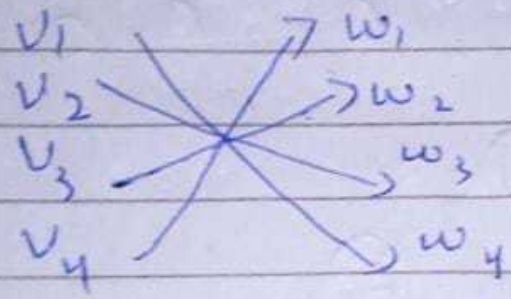
Date

Graph:

Ans 43) $G(\text{vertex}) \cong G'(\text{vertex})$

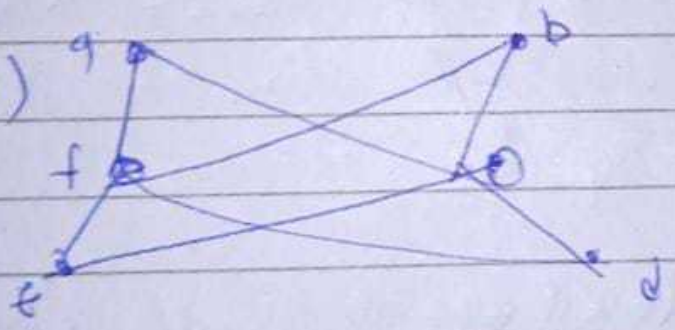
$G(\text{edge}) \cong G'(\text{edge})$

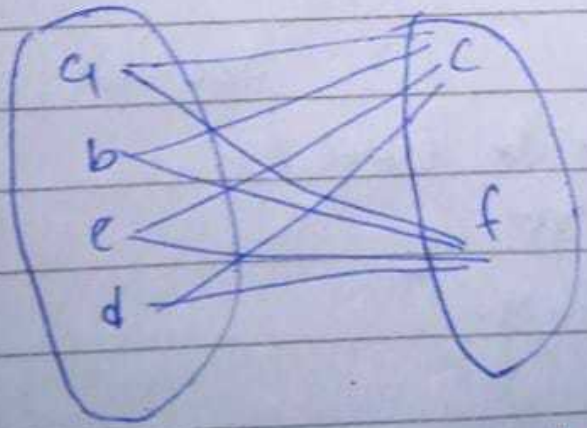
$G(\text{degree}) \cong G'(\text{degree})$



hence Isomorphic.

Ans 44)





v_1 Vertices are not connected with each other so bipartite graph, save with v_2

No.

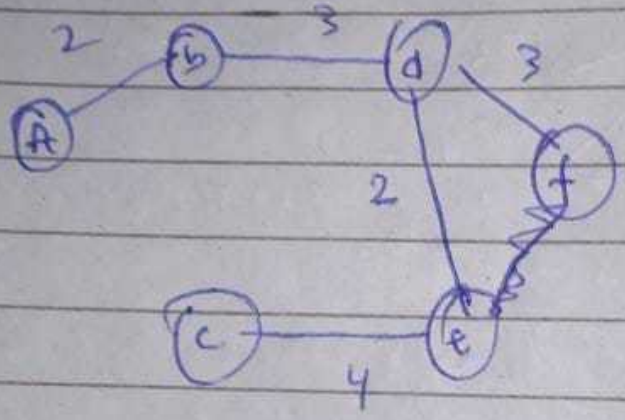
Signature

Date

Ans 45) incidence matrix.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	0	0	0	0	1	0
v_2	1	1	1	1	0	0	0
v_3	0	0	0	1	1	0	1
v_4	0	0	0	0	1	1	0
v_5	0	1	1	0	0	0	1

Ans 46)



Total lowest weight 2+3+3+4+2
=14.

Ans 48) Hamilton Circuit
(All Node to be Cover)
ABCDA = 140,
ABCDA = 125
ACBDA = 155
so
ABCDA = 125 is
minimum

Signature