Probability and Statistics Assignment 2.

Name : Nashit Budhwani
Roll No: 20K-0274 Sec : BCS-4B

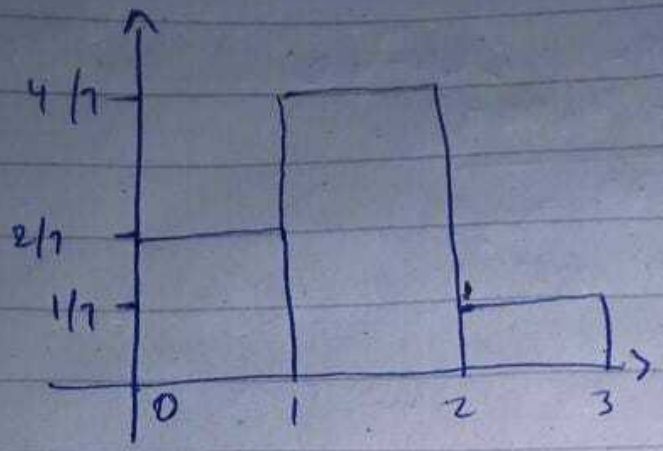
Q) 3.1
X = Discrete
Y = Continous
M = Continous
N = Discrete
P = Discrete
Q = Continous.

Q) 3.11
Let x be the number of defective sets
$x = 0, 1, 2.$

$$f(x) = \frac{\binom{2}{x}\binom{5}{3-x}}{\binom{7}{3}}$$

x	0	1	2
f(x)	2/7	4/7	1/7

And the histogram is made on next page.



Question 3.20 :

$$f(x) = \frac{2}{27}\int_2^x (1+t)\,dt = \frac{2}{27}\left[1 + \frac{t^2}{2}\right]_2^x$$

$$\frac{2}{27}\left[1 + x^2 - 1 + \frac{4}{2}\right] = \frac{2}{27}\left[\frac{x^2+4}{2}\right]$$

$$= \frac{(x+4)(x-2)}{27}$$

$$P(3 \le x < 4) = f(4) - f(3)$$

$$= \frac{8(2)}{27} - \frac{7(1)}{27} = \frac{9}{27} = \frac{1}{3} \text{ Ans.}$$

Question :

The cdf of x is

$$\begin{cases} 0 & x < 0 \\ 0.41 & 0 \le x < 1 \\ 0.78 & 1 \le x < 2. \\ 0.94 & 2 \le x < 3 \\ 0.99 & 3 \le x < 4 \\ 1 & x \ge 2. \end{cases}$$

Question 3.39:

we can select x oranges from 3, 4 form 2 apples and $4-x-y$ bannas from 3 in $\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}$. Random of 4 from 8 fruits can be done by this formula.

$$f(x) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}} \qquad \begin{array}{l} x = 0,1,2,3 \\ y = 0,1,2 \\ 1 \le x+y \le 4 \end{array}$$

b) $P\{X, Y \in A\} = P(X+Y \le 2) = f(1,0) + f(2,0) + f(0,1) + f(1,1) + f(0,2)$

$$f(1,0) = \frac{\binom{3}{1}\binom{2}{0}\binom{3}{3}}{\binom{8}{4}} = \frac{3}{70}$$

$$f(2,0) = \frac{\binom{3}{2}\binom{2}{0}\binom{3}{2}}{\binom{8}{4}} = \frac{9}{70}$$

$$f(0,1) = \frac{\binom{3}{0}\binom{2}{1}\binom{3}{3}}{\binom{8}{4}} = \frac{2}{70}$$

$$f(1,1) = \frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}} = \frac{18}{70}$$

$$f(0,2) = \frac{\binom{3}{0}\binom{2}{2}\binom{3}{2}}{\binom{8}{4}} = \frac{3}{70}$$

$$\frac{3}{70} + \frac{18}{70} + \frac{2}{70} + \frac{3}{70} + \frac{9}{70} = \frac{1}{2},$$

Question No. 3.57

$$1 = k^2 \int_0^1 \int_0^1 \left[\frac{x^2}{2} y^2 z \right]_0^1 dy\, dz$$

$$\frac{1}{1} = \frac{k}{3} \qquad \boxed{k = 3}$$

(b) $$P\left(x < \frac{1}{4}, y > \frac{1}{2}, 1 < z < 2\right)$$

$$3 \int_1^2 \int_{1/2}^1 \int_0^{1/4} xy^2 z \, dx\, dy\, dz = \frac{9}{2} \int_0^{1/4} \int_{1/2}^1 y^2 z$$

$$= \frac{21}{16} \int_1^2 z \, dz = \frac{21}{512} \text{ Ans.}$$

Question 3.62:

(a)

x	1	3	5	1
$f(x)$	0.4	0.2	0.2	0.2

(b) $P(4 < X \leq 7) = P(X \leq 7) - P(X \leq 4)$

$= f(7) - f(4) = 1 - 0.6 = 0.4$ Ans.

Question 4.1:

$M = E(x) = 0(0.41) + 1(0.37) + 2(0.16) + 3(0.05) + 4(0.01)$

$M = 0.88$

Question 4.7:

Expected gain $= E(x) = 4000(0.3) + (-1000)(0$

$E(x) = \$500$ Ans.

Question 4.12.

$$E(x) = \int_0^1 2x(1-x)\,dx = \frac{1}{3}$$

$$\left[x^2 - \frac{2x^3}{3} \right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

$\frac{1}{3}(5000) = \$1667.67$ Ans.

Question 4.20:

$$E(g(x)) = E(e^{2x/3}) = \int_0^{\infty} e^{2x/3} e^{-x}$$

$$E(x) = \int_0^{\infty} e^{-x/3} dx = \frac{e^{-x/3}}{1/3} = 3\left[e^{-x/3}\right]^{\infty}_0$$

$$E(g(x)) = 3 \text{ Ans.}$$

Question 4.34.

$$M = (-2)(0.3) + 3(0.2) + 5(0.5) = -0.6 + 0.6 + 2.5$$

$$M = 2.5$$

$$E(x^2) = (-2)^2(0.3) + (3)^2(0.2) + (5)^2(0.5)$$

$$E(x^2) = 15.05$$

$$\sigma^2 = E(x^2) - (M)^2$$

$$\sigma^2 = 15.5 - (2.5)^2$$

$$\sigma^2 = 9.25$$

$$\sqrt{\sigma^2} = \sqrt{9.25}$$

$$\boxed{\sigma = 3.041}$$

Question 4.45:

$\mu_X = \sum_x x\, g(x) = 2.45$, $\mu_Y = \sum_y y\, h(y) = 3.20$

$$E(XY) = \sum_x \sum_y xy\, f(x,y) = 1(0.05) + 2(0.05)$$

$$+ (0)(0.10) + 2(0.05) + 4(0.10) + 6(0.35) + 3(0) + 6(0.20) + 9(0.10) = 7.85$$

So, $\sigma_{XY} = 7.85 - (2.45)(3.20) = 0.01$

Question 4.58

Since $E(X) = \int_0^1 x^2\, dx + \int_1^2 x(2-x)\, dx$

$$E(X) = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \left[4 - \frac{8}{3} - 1 + \frac{1}{3}\right]$$

$$= \frac{1}{3} + \left[3 - \frac{7}{3}\right] = \frac{1}{3} + \frac{2}{3} = 1$$

$$E(X^2) = \int_0^1 x^4\, dx + \int_1^2 x^2(2-x)\, dx$$

$$= \frac{7}{6}.$$

$$E(Y) = 60\,E(x^2) + 39\,E(x)$$
$$= 60\left[\frac{7}{6}\right] + 39(1)$$
$$= 109 \text{ kilowatt hours.}$$

Question 5.12:

$n = 9$ and $p = 0.25$, $(x \le 4) = 0.8343$

Question 5.26:

Given $n = 8$ and $p = 0.60$

(a) $P(x = 6) = \binom{8}{6}(0.6)^6(0.4)^2 = 0.2090$

(b) $P(x = 6) = P(x \le 6) - P(x \le 5) =$
$0.8936 - 0.6816 = 0.2090$

Ans.

Question No. 5.30:

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - h(0, 15, 3, 6)$$

So we can rewrite as

$$1 - \frac{\binom{6}{0}\binom{9}{3}}{\binom{15}{3}} = 1 - \frac{8}{65} = \frac{53}{65} \text{ Ans.}$$

Question No. 5.33:

We are applying hypergeometric dist.

(a) $$\frac{\binom{12}{2}\binom{40}{5}}{\binom{52}{7}} = 0.3246$$

Ans.

(b) $$1 - \frac{\binom{48}{7}}{\binom{52}{7}} = 0.4496$$

Question No 5.34:

using hypergeometric distribution we get the following values.

$$h(2;9,5,4) = \frac{\binom{4}{2}\binom{9}{3}}{\binom{9}{5}} = \frac{10}{21}$$

Question 5.50

(a) $b^*(7;3,1/2) = \binom{6}{2}\left(\frac{1}{2}\right)^7 = 0.1172$

(b) $g(4;1/2) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

Question No. 5.55:

Geometric distribution $- q(1-q)^x$.

(a) $P(X \leq 3) = g(3;0.7) = (0.7)(0.3)^2 = 0.0630$

(b) $P(X < 4) = \sum_{x=1}^{3} g(x;0.7) = \sum_{x=1}^{3} (0.7)(0.3)^{x-1}$

$= (0.7)(0.3)^0 + (0.7)(0.3)^1 + (0.7)(0.3)^2$

$= 0.9730$. Ans

Question No. 5.57:

(a) $P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.8571 = 0.1429.$

(b) $P(0; 2) = 2(-1)^0 = 0.1353$ Ans.

Question No. 6.8:

(a) Calculating $Z = (17-30)/6 = -2.17$
Area $= 1 - 0.0150 = 0.9850$ Ans.

(b) $Z = -0.76$; $X = (2.5)(-0.76) + 18 = 16.1$

(c) $Z = 0.91$; $X = (2.5)(0.91) + 18 = 20.0225$

$Z_1 = (17-18)\,2.5 = -0.4$
$Z_2 = (21-18)\,2.5 = 1.2$
$P(17 < x < 21) = 0.8849 - 0.3446 = 0.5403$ Ans.

Question No. 6.11 :-

(a) Calculating $Z = (224-200)/15 = 1.6$
fraction of Cup more than 224mm is
$P(Z < 1.6) = 1 - $ or $1 - P(Z < 1.6)$
$= 0.1151$ Ans.

(c) $Z = (230 - 200)/15 = 2$

$P(X > 230) = P(Z > 2) = 0.0228$ Ans.

So,

$(1000)(0.0228) = 22.8$ and we can say that approx 23 cup's

(d) $Z = -0.67$

$x = (15)(-0.67) + 200 = 18.95$ mm Ans

Question No 6.19!

(a) Given $M = \$15.90$ and $\sigma = \$1.50$

$1 - 49\% = 51\%$ so,

$P(13.75 < x < 16.22) = P\left[\frac{13.75 - 15.9}{1.5} < x < \frac{16.22 - 15.9}{1.5}\right] = P(-1.437 < x < 0.217)$

$= 0.5871 - 0.0749 = 0.5122$ Ans.

(b) Given $MS = \$18.36$, so $P(Z > 1.645) = 0.05$

$x = (1.645)(0.05) + 15.90 + 0.005$

$\boxed{x = 18.137.}$

Question No. 6.21 :-

(a) calculating $z = \frac{(10,175 - 10,000)}{1000}$

$z = 1.75$

$P(x > 10,175)$ or $P(z > 1.75) = 0.0401$

(b) Calculating z_1 and z_2.

$z_1 = \frac{(9,775 - 10,000)}{100} = -2.25$

$z_2 = \frac{(10,225 - 10,000)}{100} = 2.25$

Now calculating the component less than 9,775 and greater than 10,225 so we get $P(x \leq 9,775)$ + $P(x > 10,225)$ that is equal to:

$P(z < -2.25) + P(z > 2.25) = 2(P(z > 2.25))$

$= 2(0.0122) = 0.244$