# TOA Assignment 3.

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Ans 1) Union:

| | | 0 | | 1 | |
|---|---|---|---|---|---|
| II | $z_1$ | $z_2$ | II | $z_3$ | II |
| | $z_2$ | $z_1$ | II | $z_4$ | I |
| | $z_3$ | $z_5$ | I | $z_1$ | II |
| I | $z_4$ | $z_6$ | II | $z_2$ | II |
| | $z_5$ | $z_3$ | II | $z_5$ | I |
| | $z_6$ | $z_4$ | I | $z_6$ | II |

| | | 0 | | 1 | |
|---|---|---|---|---|---|
| $z_1$ | III | $z_2$ | IV | $z_3$ | II |
| $z_2$ | IV | $z_1$ | III | $z_4$ | V |
| $z_3$, $z_6$ | II | $z_5$ | I | $z_1$ | III |
| | | $z_4$ | V | $z_6$ | II |
| $z_4$ | V | $z_6$ | II | $z_2$ | IV |
| $z_5$ | I | $z_3$ | II | $z_5$ | I |

| | | | | | |
|---|---|---|---|---|---|
| $z_1$ | III | $z_2$ | IV | $z_3$ | VI |
| $z_2$ | IV | $z_1$ | III | $z_4$ | V |
| $z_3$ | II | $z_5$ | I | $z_1$ | III |
| $z_6$ | VI | $z_4$ | V | $z_6$ | VI |
| $z_4$ | V | $z_6$ | VI | $z_2$ | IV |
| $z_5$ | I | $z_3$ | II | $z_5$ | I |

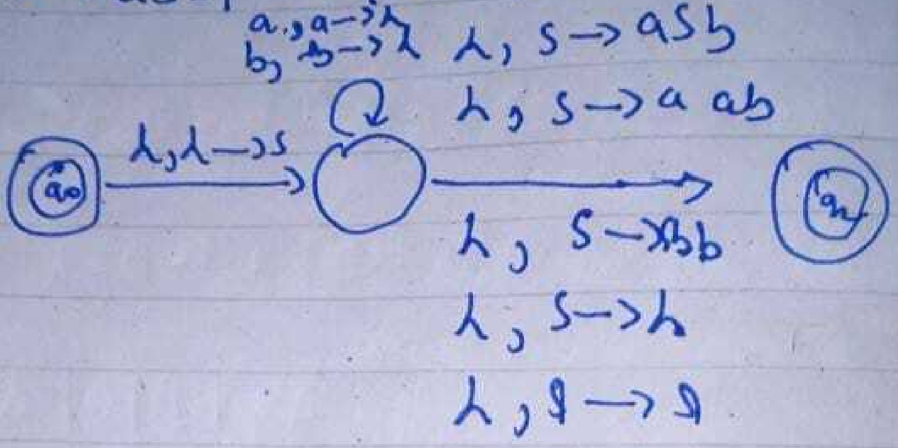
No state(s) were merge into one DFA nonminimized further.

(i) Clasure

| | | |
|---|---|---|
| $q_1$ ŞI | $s_2, s_1$ II | $s_1$ Ll |
| $s_1$ ŞI | $s_2, s_1$ II | $s_1$ ll |
| $s_2 s_1$ | $s_1$ I1 | $s_2, s_1$ ll |

Ans 2) (a) $\{a^n b^m \ (n+m) \text{ is even}\}$.

$S \to aSb \mid aaS \mid Sbb \mid \lambda$

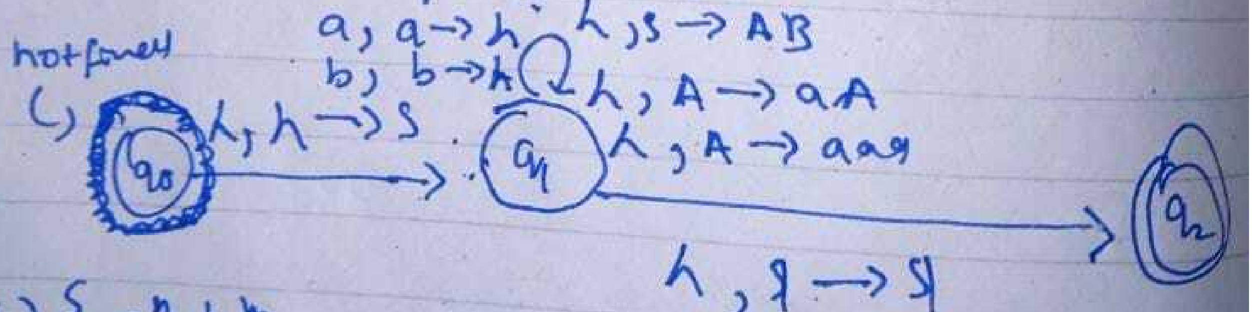



(b) $\{a^n b^m ; n \geq 4, m \leq 3\}$.

$S \to AB$

$A \to aA \mid aaa$

$B \to b \mid bb \mid bbb \mid \lambda$



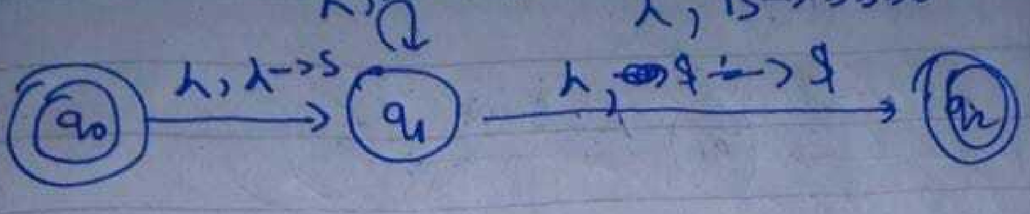

(c) $\{a^n b^m \ n < 4, m \leq 4\}$.

$S \to AB$

$A \to a \mid aa \mid aaa \mid \lambda$

$B \to b \mid bb \mid bbb \mid bbbb \mid \lambda$

$\lambda, A \to \lambda$
$\lambda, A \to aaa$
$\lambda, A \to aa$ $\lambda, B \to b$ $\lambda, B \to \lambda$.
$\lambda, B \to bb$
$\lambda, S \to AB$ $\lambda, B \to bbb$
$\lambda, A \to a$ $\lambda, B \to bbbb$



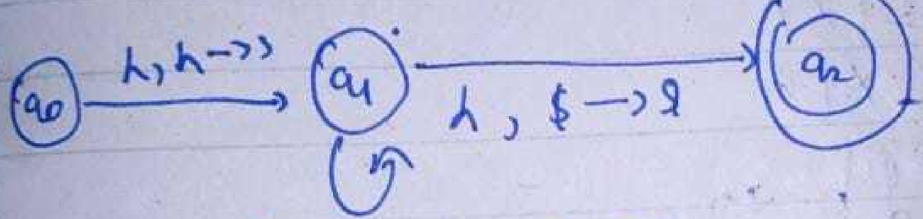

(d) having exactly one pair at consecutive zeros.

$(1+01)^* \; 00 \, (1+10)^*$

$S \to A00B$

$A \to 1A \mid 01A \mid \lambda$

$B \to 1B \mid 10B \mid \lambda$



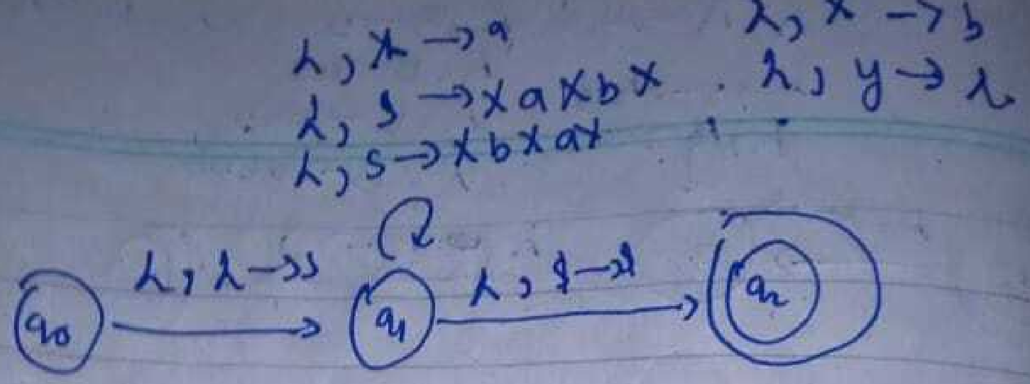

$\lambda, S \to A00B$ $\lambda, B \to 1$
$\lambda, A \to 1A$ $\lambda, B \to 10$
$\lambda, A \to 01A$ $\lambda, B \to \lambda$
$\lambda, A \to \lambda$

(c) all strings that contain atleast one occurrence of each symbol.

RE. $(a+b)^* a \, (a+b)^* b (a+b)^* + (a+b)^* b$
$(a+b) + (a+b)^*$

$S \to XaXbX \mid XbXaX$

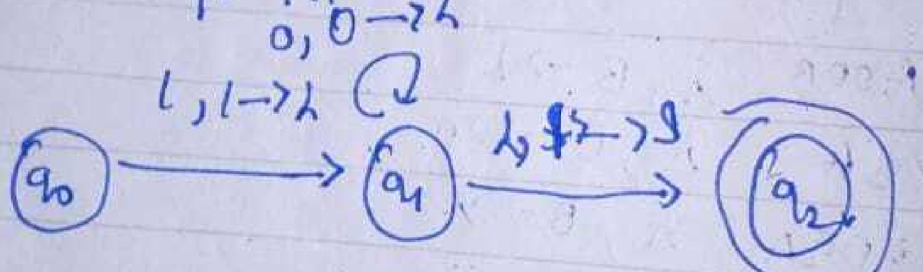
$X \to aX \mid bX \mid \lambda$




(f) All strings not ending in 01

R.E $\lambda + 0 + 1 + (0+1)^* (00 + 10 + 11)$

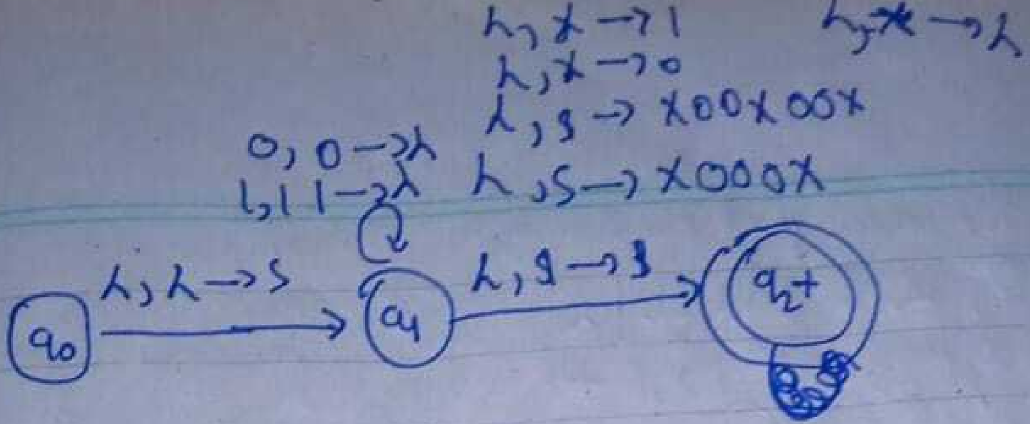
$S \to \lambda \mid 0 \mid 1 \mid XA$
$X \to 0X \mid 1X \mid \lambda$
$A \to 00 \mid 10 \mid 11$




(g) all strings having atleast two occurrences of substring 00

R.E: $(1+0)^* 00 (1+0)^* 00 (1+0)^* + (1+0)^* 000 (1+0)^*$

$S \to X00X00X \mid X000X$
$X \to 1X \mid 0X \mid \lambda$



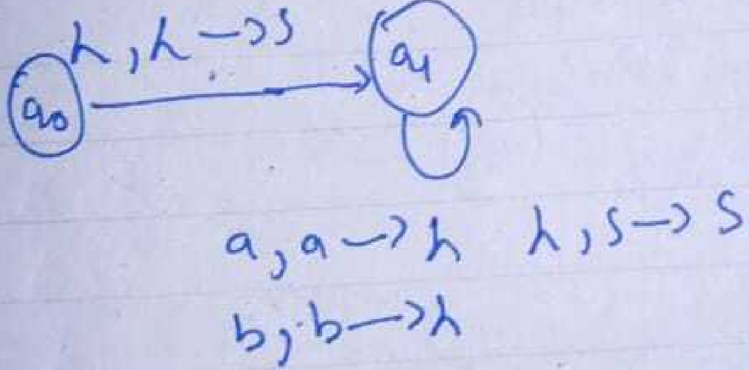

(h) $\{ w : n_a(w) \bmod 3 = 0 \}$.

R.E = $b(b^* a b^* a b^* a b^*)^*$

$S \to SX$

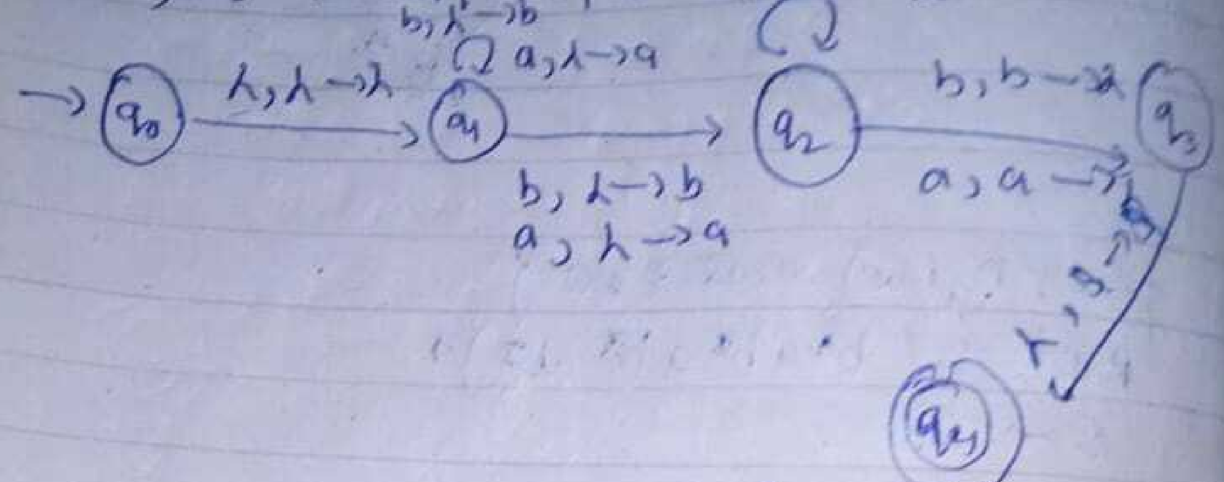
$X \to BaBaBaB$

$B \to bB \mid \lambda$



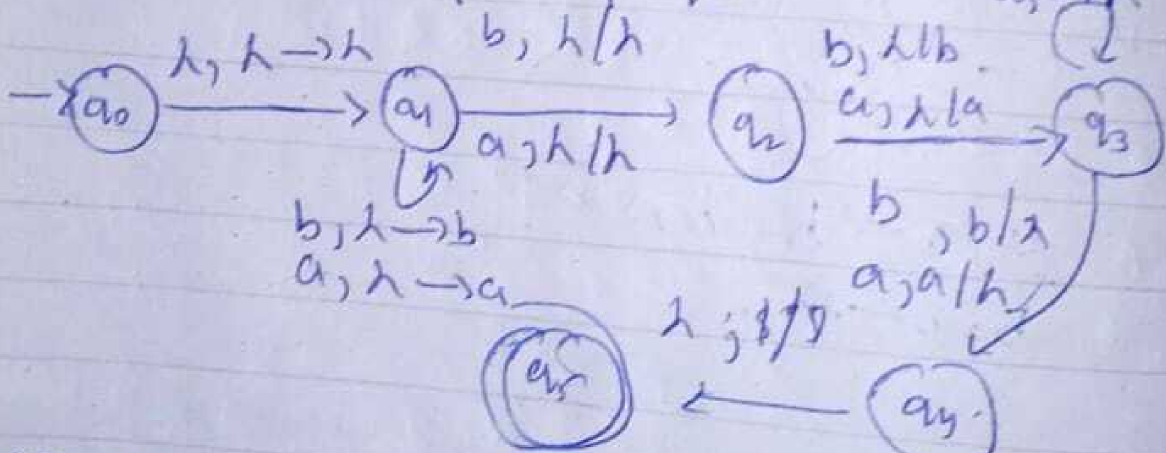
The language of even length palindromes

Ans 3a) $S \to aSa \mid bSb \mid \lambda$



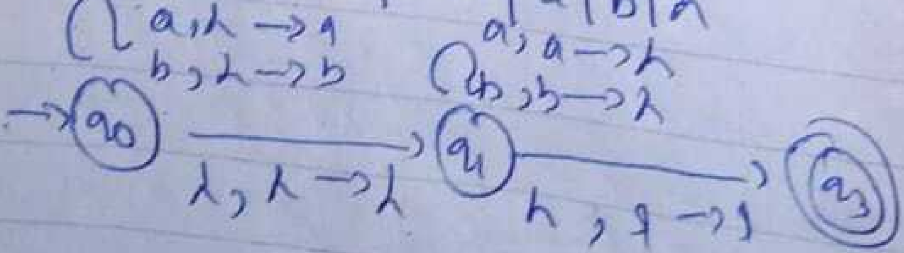

(b) odd palindrome:

$S \to aSa \mid bSb \mid a \mid b$




(c) all palindrome

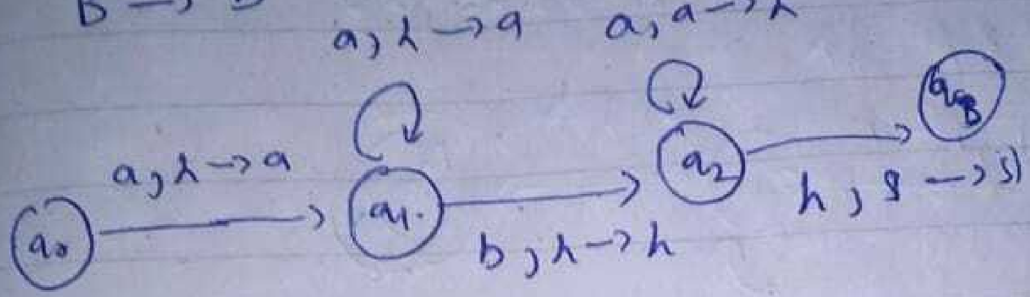
$S \to aSa \mid bSb \mid a \mid b \mid \lambda$



(d) $a^n b a^n$

$S \to aSa \mid B$

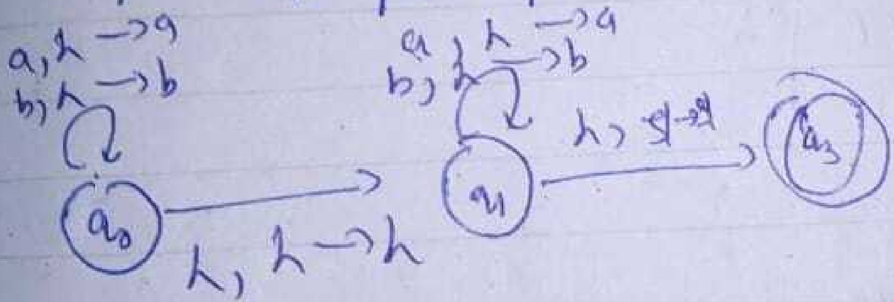
$B \to b$




(e) $ww$: $w \in \{a, b\}^*$

$S \to aAb \mid \lambda$

$A \to aA \mid bA \mid \lambda$

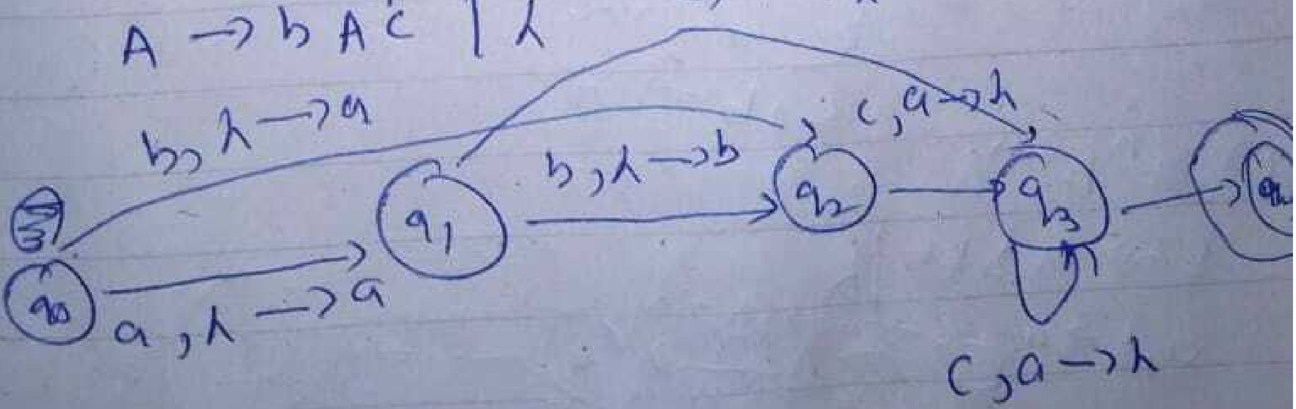



(f) $a^n b^m c^{n+m}$

$a^n b^m c^m c^n$

$S \to aSa \mid A$

$A \to bAc \mid \lambda$



(g) $a^{n+m} b^n c^m$

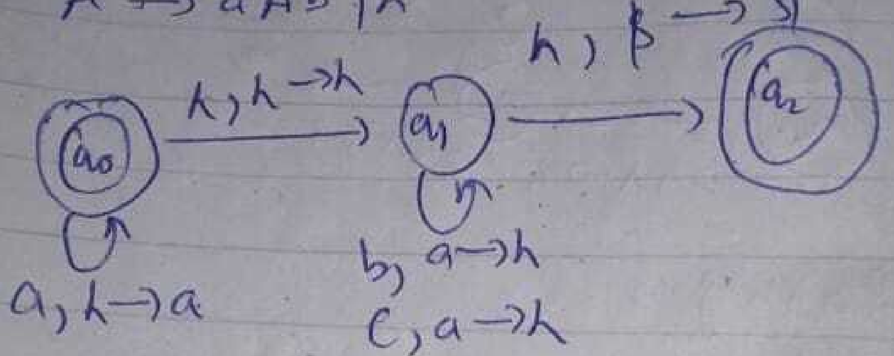
$a^m a^n b^n c^m$

$S \to aSc \mid A$

$A \to aAb \mid \lambda$

$S \to aAC \mid \lambda$

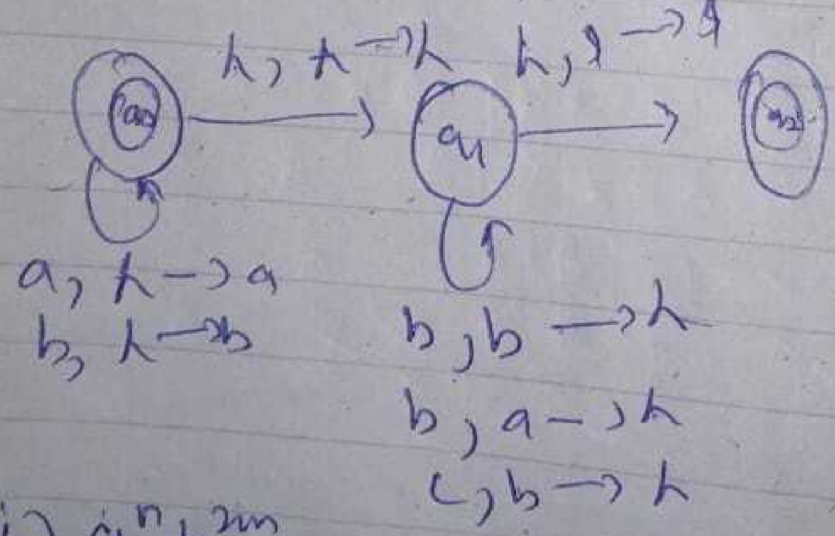
$A \to aAb \mid ab$




(h) $a^n b^{n+m} c^m$

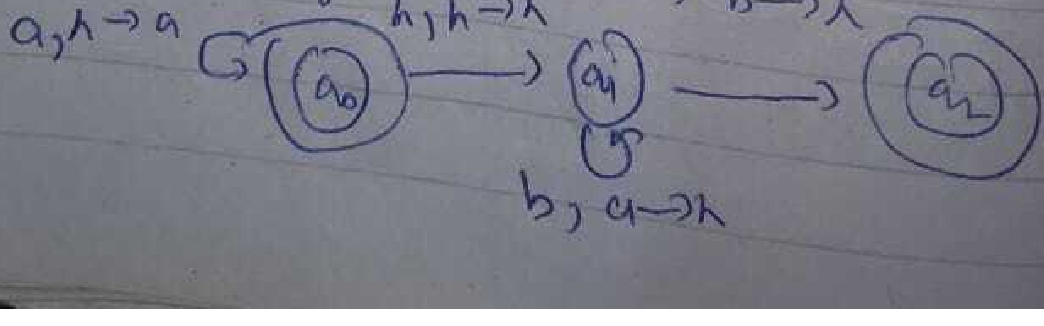
$S \to aAC \mid \lambda$

$A \to bAb \mid bb$

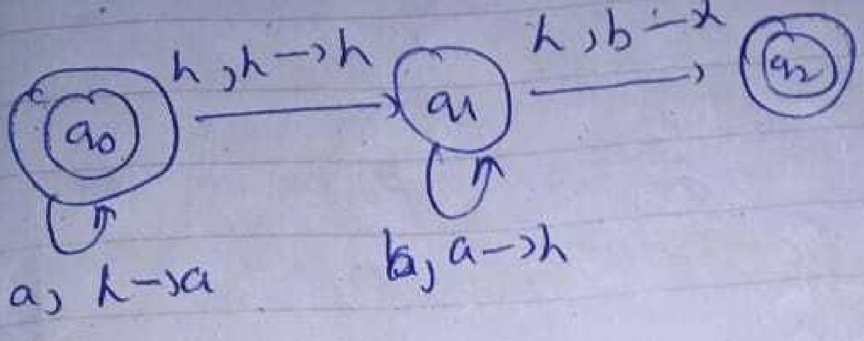



(i) $a^n b^{2m}$

$S \to aSbb \mid \lambda$



(j) $S \to aaSbbb \mid \varepsilon$.




Q4) (i). $S \to abS \mid abA \mid abB$

$A \to Cd$

$B \to aB$

$C \to dC$

Removing C not reachable

$S \to abS \mid abA \mid abB$

$A \to Cd$

$B \to aB$ Ans.

(ii) $S \to ABC \mid a$

$A \to b$

$B \to c$

$C \to d$

$E \to e$

$F \to f$

$G \to g$

$S \to ABC \mid a$

$A \to b$

$B \to c$

$C \to d$

(iii) $S \to AB \mid bX$

$A \to Bad \mid bSX \mid a$

$B \to aSB \mid bBX$

$X \to SBD \mid aBX \mid ad$

Q3 (iii)

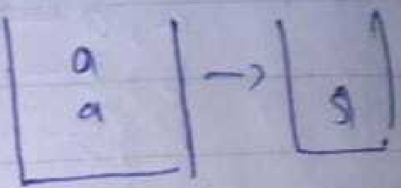
stack operations for

d) Let w = a a b a a

→ push all a's

→ when b is encountered read it but donot ~~put~~ push / pop anything

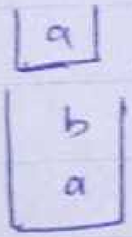
→ pop all a's



(f) Let w = a b c c

→ push all a's

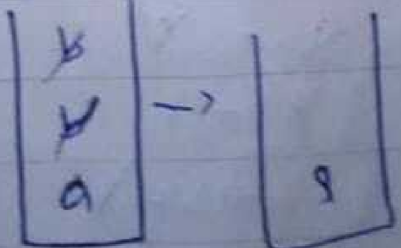
→ push all b's



→ Read all c's and for every c, pop an a or b out of the stack.

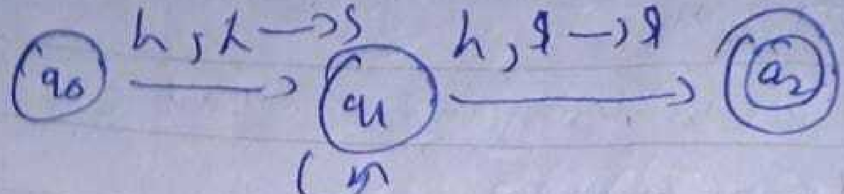
(a) Let w = a b b b b a

→ Read and push all letters until an assumed midpoint (p.d.a is non deterministic)



Q4(a) $S \to SS \mid \varepsilon$.

$A \to aSb \mid Ab \mid ab$




$a, a \to \lambda$

$b, b \to \lambda$

$\lambda, S \to SS$

$\lambda, S \to \varepsilon$

$\lambda, A \to ab$

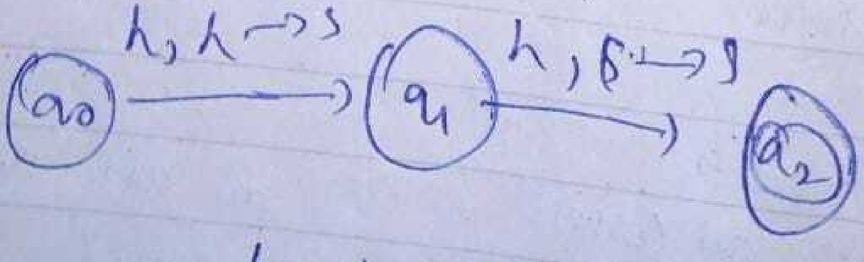
$\lambda, A \to aSb$

$\lambda, A \to Ab$

(b) $S \to S+X \mid X$

$X \to X*Y \mid Y$

$Y \to S$




$\lambda, X \to X*Y$

$\lambda, X \to Y$

$\lambda, Y \to S$

$\lambda, S \to S+X$

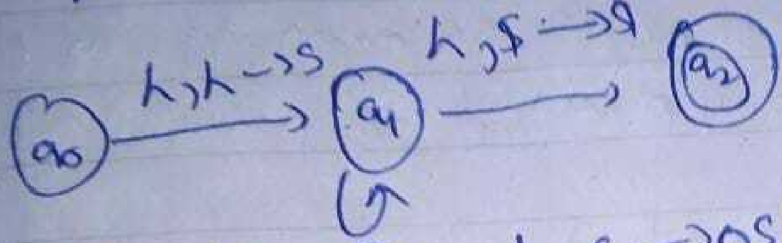
$\lambda, S \to X$

(c) S → 0S1 | 1S0 | λ

S → 0SX | 1SY | λ

X → 1

Y → 0




| | |
|---|---|
| λ, Y → 0 | λ, S → 0S1 |
| λ, X → 1 | 1,1 → λ |
| λ, S → 0SX | 0,0 → λ |
| λ; S → 1SY | λ, S → 1S0 |
| | λ, S → λ |

Q6(i) S → aS | ε.

| | |
|---|---|
| S → ABC | w = ABC |
| A → a | w = aBC |
| B → b | w = abC |
| C → c | w = abc |

no Ambiguity as there is only one word in language and only one way to derive it

(ii) $S \to aS \mid \varepsilon$

$w = aS$

$w = aaS$

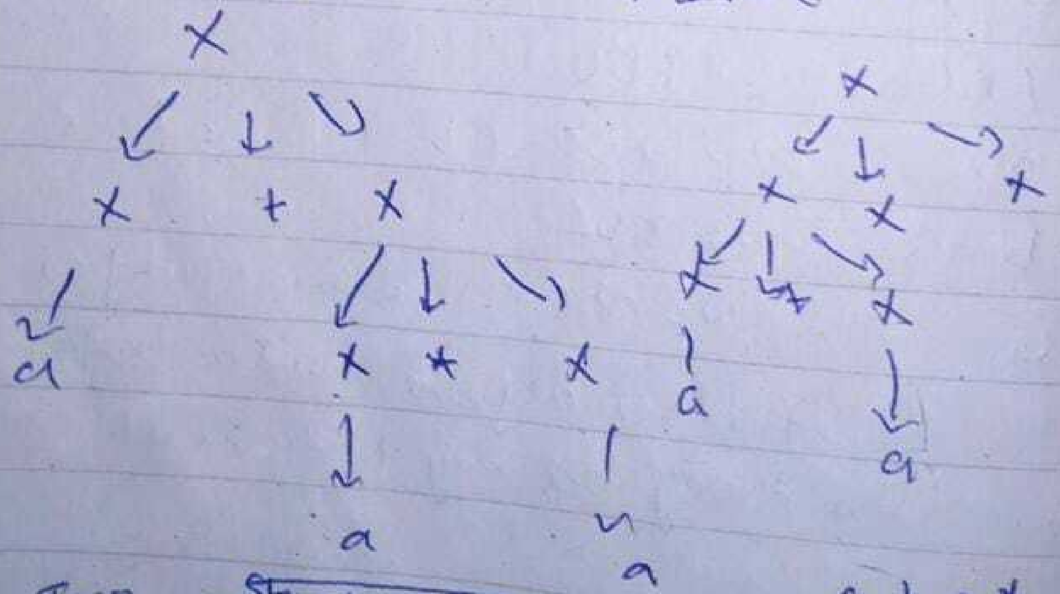
$w = aaaS$

$w = aaa$ no ambiguity as there is only one way to produce every word

(iii) $X \to X+X \mid X*X \mid X \mid a$.

ambigous as the example word '$a*a+a$' can be derived in multiple ways.

Structure 1 Structure 2



Two ~~structure fore string ambiguity~~ $a+a*a$

present.