

# GATE-2022, BM-37

EE23BTECH11033- JASWANTH KILLANA

Question: Solution of the differential equation let,  
 $\frac{dy}{dx} - y = \cos(x)$  is

$$(A) \quad y = \frac{\sin(x) - \cos(x)}{2} + ce^x \quad (1)$$

$$(B) \quad y = \frac{\sin(x) + \cos(x)}{2} + ce^x \quad (2)$$

$$(C) \quad y = \frac{\sin(x) + \cos(x)}{2} + ce^{-x} \quad (3)$$

$$(D) \quad y = \frac{\sin(x) - \cos(x)}{2} + ce^{-x} \quad (4)$$

$$J = \int \cos(x) e^{-x} dx \quad (15)$$

$$= \cos(x) (-e^{-x}) + \int \sin(x) (-e^{-x}) dx + c \quad (16)$$

$$J = \cos(x) (-e^{-x}) + \sin(x) e^{-x} - \int \cos(x) e^{-x} dx + c \quad (17)$$

$$2J = \sin(x) e^{-x} - \cos(x) e^{-x} + c \quad (18)$$

$$J = \frac{\sin(x) e^{-x} - \cos(x) e^{-x}}{2} + c \quad (19)$$

Solution:

$$\frac{dy}{dx} - y = \cos(x) \quad (5)$$

So,

$$y(e^{-x}) = \frac{\sin(x) e^{-x} - \cos(x) e^{-x}}{2} + c \quad (20)$$

$$y = \frac{\sin(x) - \cos(x)}{2} + ce^x \quad (21)$$

compare with,

option A is correct

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (6)$$

$$\implies -1 = P(x) \quad (7)$$

$$\implies \cos(x) = Q(x) \quad (8)$$

integrating factor=

$$IF = e^{\int P(x) dx} \quad (9)$$

$$IF = e^{\int -1 dx} \quad (10)$$

$$= e^{-x+c} \quad (11)$$

solve for y

$$y(IF) = \int Q(x) (IF) dx + c \quad (12)$$

$$\implies y(e^{-x+c}) = \int \cos(x) e^{-x+c} dx + c \quad (13)$$

$$y(e^{-x}) = \int \cos(x) e^{-x} dx + c \quad (14)$$