10.5.4-5

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Question:
A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$V = \frac{1}{4} \cdot \frac{1}{2} \cdot 50 \tag{1}$$

solution:

parameter	description	value
l	length	$50m$
b	breadth	$0.25m$
h	height	$0.5m$
$y(n)$	sum of volume	$6.25m^3$

TABLE 0: input parameters

$$x(n+1) - x(n) = 6.25m^3 \tag{2}$$

$$y(n) = \frac{n+1}{2}\left[2x(0) + (n)\,d\right] \tag{3}$$

$$n = 0 \quad 1 \quad 2 \quad 3 \quad \ldots \tag{4}$$

here

parameter	description	value
$x(0)$	first term	6.25
d	common difference	6.25
n	no of terms -1	14
$x(n)$	volume of $(n+1)$th step	$x(0) + nd$

TABLE 0: formula parameters

$$y(14) = \frac{14+1}{2}\left[12.5 + (14)\,6.25\right] \tag{5}$$

$$= \frac{15}{2}\left[12.5\,(14)\,6.25\right] \tag{6}$$

$$= \frac{15}{2}\left[12.5 + 87.5\right] \tag{7}$$

$$= (7.5) \cdot 100 \tag{8}$$

$$= 750m^3 \tag{9}$$

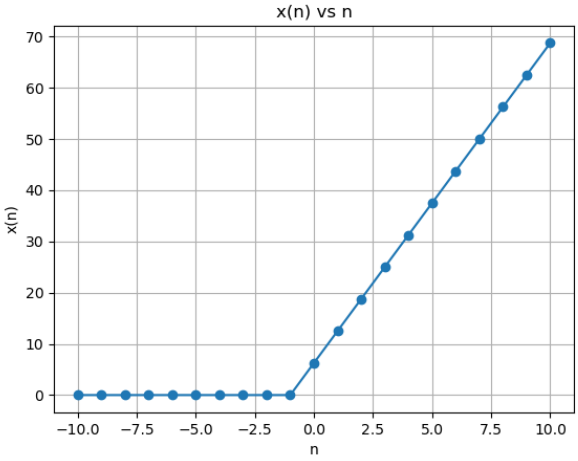

Fig. 0: plot x(n) vs n

$$x(n) \overset{Z}{\longleftrightarrow} X(Z) \tag{10}$$

$$x(n) = (x(0) + nd)\,u(n) \tag{11}$$

$$X(Z) = \sum_{-\infty}^{\infty} x(n)\,Z^{-n} \tag{12}$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{\left(1 - z^{-1}\right)^2} \qquad |z| > |1| \tag{13}$$

$$= \frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{\left(1 - z^{-1}\right)^2} \qquad |z| > |1| \tag{14}$$

convolution for y(n):

$$y(n) = x(n) * u(n) \tag{15}$$

$$x(n) \overset{Z}{\longleftrightarrow} X(Z) \tag{16}$$

$$u(n) \overset{Z}{\longleftrightarrow} U(Z) \tag{17}$$

$$y(n) \overset{Z}{\longleftrightarrow} Y(Z) \tag{18}$$

$$Y(z) = X(z)\,U(z) \tag{19}$$

$$X(z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2}\right) \quad |z| > |1| \tag{20}$$

$$U(z) = \frac{1}{1 - z^{-1}} \quad |z| > |1| \tag{21}$$

$$Y(z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{22}$$