# 10.5.4-5

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Question:
A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$V = \frac{1}{4} \cdot \frac{1}{2} \cdot 50 \tag{1}$$

solution: here

| parameter | description | value |
|---|---|---|
| $x(0)$ | first term | 6.25 |
| $d$ | common difference | 6.25 |
| $n$ | no of terms $-1$ | 14 |
| $x(n)$ | volume of $(n+1)$th step | $(6.25 + 6.25n)\,u(n)$ |

TABLE 0: formula parameters

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \qquad |z| > |1| \tag{2}$$

$$= \left(\frac{6.25}{(1 - z^{-1})^2}\right) \qquad |z| > |1| \tag{3}$$

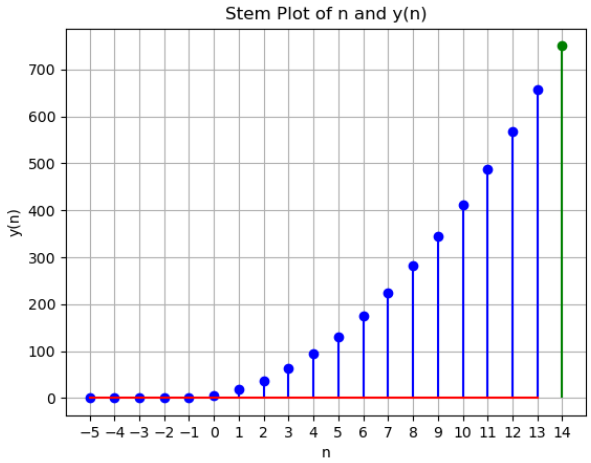


Fig. 0: plot y($n$) vs n

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)\,U(z) \tag{5}$$

$$U(z) = \frac{1}{1 - z^{-1}} \qquad |z| > |1| \tag{6}$$

$$Y(z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \qquad |z| > |1| \tag{7}$$

$$Y(z) = \frac{6.25z^3}{(z - 1)^3} \qquad |z| > |1| \tag{8}$$

contour integration to find inverse z transform

$$y(14) = \frac{1}{2\pi j}\oint_c Y(z)\,z^{13}dz \tag{9}$$

$$= \frac{1}{2\pi j}\oint_c \frac{6.25z^{16}}{(z - 1)^3} \tag{10}$$

pole at 1 repeated 3 times

$$m = 3 \tag{11}$$

$$R = \frac{1}{(m-1)!}\lim_{z \to a}\frac{d^{m-1}}{dz^{m-1}}\left((z - a)^m f(z)\right) \tag{12}$$

$$= \frac{1}{(2!)}\lim_{z \to 1}\frac{d^2}{dz^2}\left((z - 1)^3\,\frac{6.25z^{16}}{(z - 1)^3}\right) \tag{13}$$

$$= 3.125\lim_{z \to 1}\frac{d^2}{dz^2}\left(z^{16}\right) \tag{14}$$

$$y(14) = 750 \tag{15}$$