

10.5.4-5

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Question:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$V = \frac{1}{4} \cdot \frac{1}{2} \cdot 50 \quad (1)$$

solution:

parameter	description	value
l	length	50m
b	breadth	0.25m
h	height	0.5m
$y(n)$	sum of volume	$6.25m^3$

TABLE 0: input parameters

$$x(n+1) - x(n) = 6.25m^3 \quad (2)$$

$$y(n) = \frac{n+1}{2} [2x(0) + (n)d] \quad (3)$$

$$n = 0, 1, 2, 3, \dots \quad (4)$$

here

parameter	description	value
$x(0)$	first term	6.25
d	common difference	6.25
n	no of terms -1	14
$x(n)$	volume of $(n+1)$ th step	$x(0) + nd$

TABLE 0: formula parameters

$$y(14) = \frac{14+1}{2} [12.5 + (14)6.25] \quad (5)$$

$$= \frac{15}{2} [12.5 (14) 6.25] \quad (6)$$

$$= \frac{15}{2} [12.5 + 87.5] \quad (7)$$

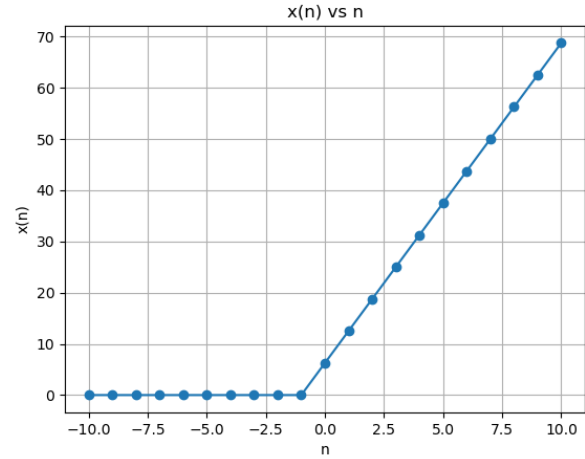


Fig. 0: plot $x(n)$ vs n

$$= (7.5) \cdot 100 \quad (8)$$

$$= 750m^3 \quad (9)$$

z transform of $x(n)$:

$$x(n) \xrightarrow{Z} X(Z) \quad (10)$$

$$x(n) = (x(0) + nd)u(n) \quad (11)$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n} \quad (12)$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (13)$$

$$= \frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (14)$$

convolution for $y(n)$:

$$(15)$$

$$(x * u)[n] = \sum_{-\infty}^{\infty} u(\tau) x[n - \tau] \quad (16)$$

$$y(n) = x(n) * u(n) \quad (17)$$

$$y[n] = \sum_{-\infty}^{\infty} u[\tau] (x(0) + (n - \tau) (6.25)) \quad (18)$$

$$x(n) \xleftrightarrow{Z} X(Z) \quad (19)$$

$$u(n) \xleftrightarrow{Z} U(Z) \quad (20)$$

$$y(n) \xleftrightarrow{Z} Y(Z) \quad (21)$$

$$Y(z) = X(z) U(z) \quad (22)$$

$$Y(z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2} \right) U(z) \quad (23)$$

$$|z| > 1 \quad (24)$$