GATE-2022, BM-37

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Question: Solution of the differential equation $\frac{dy}{dx} - y = cos\left(x\right)$ is

$$(A) \quad y = \frac{sin\left(x\right) - cos\left(x\right)}{2} + ce^{x} \tag{1}$$
$$(B) \quad y = \frac{sin\left(x\right) + cos\left(x\right)}{2} + ce^{x} \tag{2}$$
$$(C) \quad y = \frac{sin\left(x\right) + cos\left(x\right)}{2} + ce^{-x} \tag{3}$$
$$(D) \quad y = \frac{sin\left(x\right) - cos\left(x\right)}{2} + ce^{-x} \tag{4}$$

Solution:

$$\frac{dy}{dx} - y = cos\left(x\right) \tag{5}$$

compare with,

$$\frac{dy}{dx} + P\left(x\right)y = Q\left(x\right) \tag{6}$$
$$\implies -1 = P\left(x\right) \tag{7}$$
$$\implies cos\left(x\right) = Q\left(x\right) \tag{8}$$

integrating factor=

$$IF = e^{\int P(x)dx} \tag{9}$$
$$IF = e^{\int -1dx} \tag{10}$$
$$= e^{-x+c} \tag{11}$$

solve for y

$$y\left(IF\right) = \int Q\left(x\right)\left(IF\right)dx + c \tag{12}$$
$$\implies y\left(e^{-x+c}\right) = \int cos\left(x\right)e^{-x+c}dx + c \tag{13}$$
$$y\left(e^{-x}\right) = \int cos\left(x\right)e^{-x}dx + c \tag{14}$$

let,

$$J = \int cos\left(x\right)e^{-x}dx \tag{15}$$
$$= cos\left(x\right)\left(-e^{-x}\right) + \int sin\left(x\right)\left(-e^{-x}\right)dx + c \tag{16}$$
$$J = cos\left(x\right)\left(-e^{-x}\right) + sin\left(x\right)e^{-x} - \int cos\left(x\right)e^{-x}dx + c \tag{17}$$
$$2J = sin\left(x\right)e^{-x} - cos\left(x\right)e^{-x} + c \tag{18}$$
$$J = \frac{sin\left(x\right)e^{-x} - cos\left(x\right)e^{-x}}{2} + c \tag{19}$$

So,

$$y\left(e^{-x}\right) = \frac{sin\left(x\right)e^{-x} - cos\left(x\right)e^{-x}}{2} + c \tag{20}$$
$$y = \frac{sin\left(x\right) - cos\left(x\right)}{2} + ce^{x} \tag{21}$$
$$\tag{22}$$

option A is correct