10.5.4-5

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Question:
A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$V = \frac{1}{4} \cdot \frac{1}{2} \cdot 50 \tag{1}$$

solution: here

parameter	description	value
$x(0)$	first term	6.25
d	common difference	6.25
n	no of terms -1	14
$x(n)$	volume of $(n+1)$th step	$(6.25 + 6.25n)\,u(n)$

TABLE 0: formula parameters

$$x(Z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \qquad |z| > |1| \tag{2}$$

$$x(Z) = \frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2} \qquad |z| > |1| \tag{3}$$

$$= \left(\frac{6.25}{(1 - z^{-1})^2} \right) \qquad |z| > |1| \tag{4}$$

$$\tag{5}$$

convolution for y(n):

$$Y(n) = x(n) * u(n) \tag{6}$$

$$Y(Z) = x(Z)\,U(Z) \tag{7}$$

$$U(Z) = \frac{1}{1 - z^{-1}} \qquad |z| > |1| \tag{8}$$

$$Y(Z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \qquad |z| > |1| \tag{9}$$

$$Y(z) = \frac{6.25}{(1 - z)^{-3}} \qquad |z| > |1| \tag{10}$$

$$\tag{11}$$

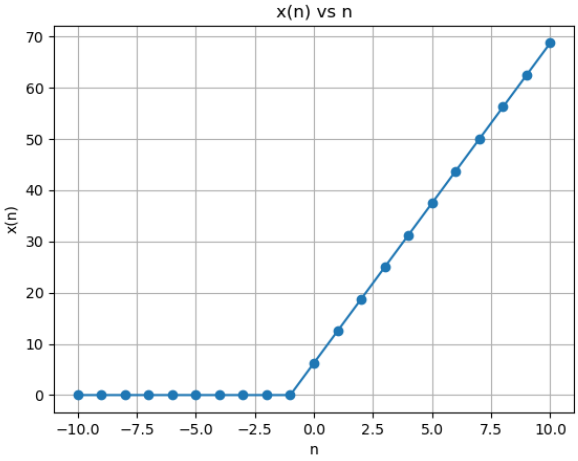

Fig. 0: plot x(n) vs n

contour integration to find inverse z transform

$$y(15) = \frac{1}{2\pi j} \oint_c Y(Z)\, z^{14} dz \tag{12}$$

$$= \frac{1}{2\pi j} \oint_c \frac{6.25z^{14}}{(1 - z^{-1})^3} \tag{13}$$

$$\tag{14}$$

pole at 1 repeated 3 times

$$m = 3 \tag{15}$$

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z) \right) \tag{16}$$

$$= \frac{1}{(2!)} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{6.25z^{16}}{(z - 1)^3} \right) \tag{17}$$

$$= 3.125 \lim_{z \to 1} \frac{d^2}{dz^2} \left(z^{16} \right) \tag{18}$$

$$y(15) = 750 \tag{19}$$