10.5.4-5

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Question:
A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$\text{volume} = 1/4 \cdot 1/2 \cdot 50 \tag{1}$$

solution

$$\text{dimensions of any step} = length \cdot breadth \cdot height \tag{2}$$

length of first step is l
breadth of first step is b
height of first step is h.

parameter	value
ł	$50m$
b	$0.5m$
h	$0.25m$

$$\text{dimensions of first step} = 50m \cdot 0.25m \cdot 0.5m \tag{3}$$

=volume of first step is $6.25m^3$
=All the dimensions except height are same for all 15 steps .
=The height difference between any 2 consecutive steps is 0.25 m.
=so, the height of the second step is 0.25m+0.25m=0.5m
=So, the volume of the second step is 50m*5m*0.5m= $12.5m^3$
=in the similar way the volume of the third step is $18.75m^3$
=so, we can clearly notice that the volume of the steps are in arthimetic progression.
let the first term of the AP be $x(n) = 6.25$
=the common difference is 6.25
=we have to find the sum of first 15 terms
=the formula of sum of first n terms in an AP is

$$S_n = \frac{n+1}{2}[2a + (n)d] \tag{4}$$

here n starts from 0 (5)

=n= number of terms
=a is first term of the AP
d is the common difference
here

parameter	value
a	6.25
d	6.25
n	14

$$S_n = \frac{14+1}{2}[12.5 + (14)6.25] \tag{6}$$

$$S_n = \frac{15}{2}[12.5(14)6.25] \tag{7}$$

$$S_n = \frac{15}{2}[12.5 + 87.5] \tag{8}$$

$$volume = (7.5) \cdot 100 \tag{9}$$

volume is 750
hence, the volume of the total concerate is 750 m^3

x(n) and u(n)
let x(n) be the n'th term of the above AP
x(n)=a+nd
here
a is the first term of the AP
d is the common difference
n starts from zero
n'th term in terms of u(n)

$$\text{x(n)} = (a + n \cdot d) \cdot u(n) \tag{10}$$

n	u(n)	x(n)	relation
$n < 0$	0	0	x(n)=u(n)=0
0	1	a	x(n)=(a)(u(n))
$n > 0$	1	(a+nd)	x(n)=(a+nd)u(n)