

10.5.4-5

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Question:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step=

$$V = \frac{1}{4} \cdot \frac{1}{2} \cdot 50 \quad (1)$$

solution: here

parameter	description	value
$x(0)$	first term	6.25
d	common difference	6.25
n	no of terms -1	14
$x(n)$	volume of $(n+1)$ th step	$(6.25 + 6.25n)u(n)$

TABLE 0: formula parameters

$$x(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad |z| > |1| \quad (2)$$

$$= \left(\frac{6.25}{(1 - z^{-1})^2} \right) \quad |z| > |1| \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$y(z) = x(z)u(z) \quad (5)$$

$$u(z) = \frac{1}{1 - z^{-1}} \quad |z| > |1| \quad (6)$$

$$y(z) = \left(\frac{6.25}{1 - z^{-1}} + \frac{6.25z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |z| > |1| \quad (7)$$

$$y(z) = \frac{6.25z^3}{(z - 1)^3} \quad |z| > |1| \quad (8)$$

contour integration to find inverse z transform

$$y(14) = \frac{1}{2\pi j} \oint_c y(z) z^{13} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_c \frac{6.25z^{16}}{(z - 1)^3} \quad (10)$$

pole at 1 repeated 3 times

$$m = 3 \quad (11)$$

$$R = \frac{1}{(m - 1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (12)$$

$$= \frac{1}{(2!)} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{6.25z^{16}}{(z - 1)^3} \right) \quad (13)$$

$$= 3.125 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{16}) \quad (14)$$

$$y(14) = 750 \quad (15)$$

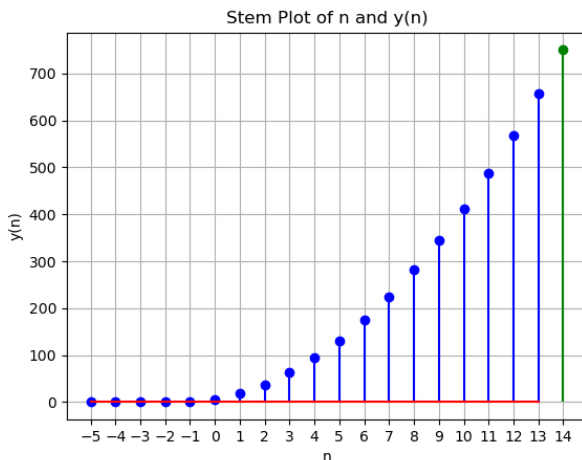


Fig. 0: plot $y(n)$ vs n