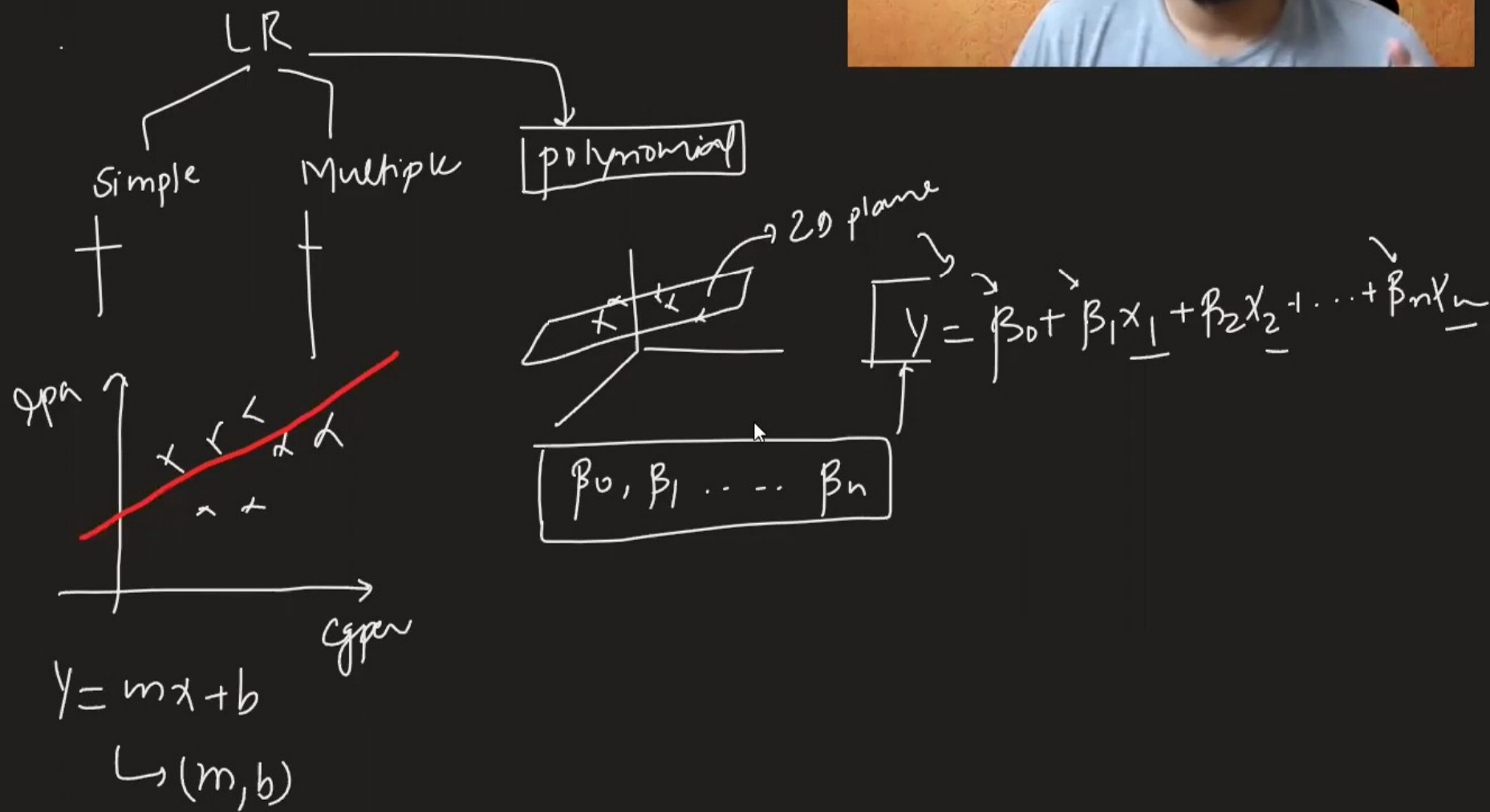




## Mathematical Formulation

Saturday, May 15, 2021 4:02 PM



### 100 Days of ML

Day 32 - Discrtizati...

Day 33 - Working-...

Day 34 - Working...

Day 35 - Complete...

Day 36 - Handling...

Day 37 - Handling...

Day 38 - Missing Indi...

Day 39 - KNN Impu...

Day 40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day 44 - Outlier De...

Day 45 - Feature C...

Day 46 - Curse of...

Day 47 - PCA

Day 48 - Simple Li...

Day 49 - Regressio...

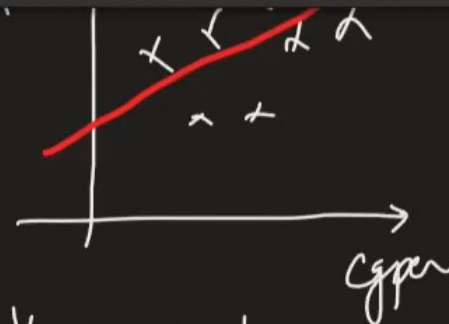
Day 50 - Multiple L...

+ Add section

+ Add page

100 Days of ML

Day 32 - Discrtizati...	Multiple Linear Regression
Day 33 - Working-...	Code Example
Day 34 - Working...	Mathematical Formulation
Day 35 - Complete...	
Day 36 - Handling...	
Day 37 - Handling...	
Day38-Missing Indi...	
Day39 - KNN Impu...	
Day40 - Iterative I...	
Day 41 - Outliers in...	
Day 42 - Outlier De...	
Day 43 - Outlier de...	
Day44 - Outlier De...	
Day 45 - Feature C...	
Day 46 - Curse of...	
Day 47 - PCA	
Day 48 - Simple Li...	
Day 49 - Regressio...	
Day 50 - Multiple L...	



$$\boxed{\beta_0, \beta_1 \dots \beta_n}$$

$$y = mx + b$$

$\hookrightarrow (m, b)$

$$y = mx + b$$
$$y = \beta_0 + \beta_1 x$$

cgpa | iq | gender | lpa

$x_1$   $x_2$   $x_3$   $\otimes$   $\rightarrow$  actual

4D

premium  $\rightarrow$   $\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

100 students  
(100, 4)




 $(m, b)$ 

$$y = mx + b$$

$$y = \beta_0 + \beta_1 x$$

 $\beta_0, \beta_1, \beta_2, \beta_3$ 

$$\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_{100} \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 x_{11} & \beta_2 x_{12} & \beta_3 x_{13} \\ \beta_0 & \beta_1 x_{21} & \beta_2 x_{22} & \beta_3 x_{23} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_0 & \beta_1 x_{1001} & \beta_2 x_{1002} & \beta_3 x_{1003} \end{bmatrix}$$

1 row  
1 column

# 100 Days of ML

Day 32 - Discrtizati...

Day 33 - Working-...

Day 34 - Working...

Day 35 - Complete...

Day 36 - Handling...

Day 37 - Handling...

Day38-Missing Indi...

Day39 - KNN Impu...

Day40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day44 - Outlier De...

Day 45 - Feature C...

Day 46 - Curse of...

Day 47 - PCA

Day 48 - Simple Li...

Day 49 - Regressio...

Day 50 - Multiple L...

+ Add section

+ Add page





$$\begin{bmatrix} \vdots \\ \hat{y}_{100} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \vdots \\ \beta_0 & \beta_1 x_{1001} & \beta_2 x_{1002} \\ \vdots \end{bmatrix}$$

100 rows  $\rightarrow$  n rows     4 cols  $\rightarrow$  (3) cols  $\rightarrow$  m cols

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 x_{11} & \beta_2 x_{12} & \beta_3 x_{13} & \dots & \beta_m x_{1m} \\ \beta_0 & \beta_1 x_{21} & \beta_2 x_{22} & \beta_3 x_{23} & \dots & \beta_m x_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_0 & \beta_1 x_{n1} & \beta_2 x_{n2} & \beta_3 x_{n3} & \dots & \beta_m x_{nm} \end{bmatrix}$$

## 100 Days of ML

Day 32 - Discrtizati...

Day 33 - Working-...

Day 34 - Working...

Day 35 - Complete...

Day 36 - Handling...

Day 37 - Handling...

Day38-Missing Indi...

Day39 - KNN Impu...

Day40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day44 - Outlier De...

Day 45 - Feature C...

Day 46 - Curse of...

Day 47 - PCA

Day 48 - Simple Li...

Day 49 - Regressio...

Day 50 - Multiple L...

Multiple Linear Regression

Code Example

Mathematical Formulation

+ Add section

+ Add page

100 Days of ML

- Day 32 - Discrtizati...
- Day 33 - Working-...
- Day 34 - Working...
- Day 35 - Complete...
- Day 36 - Handling...
- Day 37 - Handling...
- Day38-Missing Indi...
- Day39 - KNN Impu...
- Day40 - Iterative I...
- Day 41 - Outliers in...
- Day 42 - Outlier De...
- Day 43 - Outlier de...
- Day44 - Outlier De...

Multiple Linear Regression

Code Example

Mathematical Formulation

100 rows  $\rightarrow$   $n$  rows      4 cols  $\rightarrow$  (3) cols  $\rightarrow$   $m$  cols

$A B = A^T B$

$$\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 x_{11} & \beta_2 x_{12} & \beta_3 x_{13} & \dots & \beta_m x_{1m} \\ \beta_0 & \beta_1 x_{21} & \beta_2 x_{22} & \beta_3 x_{23} & \dots & \beta_m x_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_0 & \beta_1 x_{n1} & \beta_2 x_{n2} & \beta_3 x_{n3} & \dots & \beta_m x_{nm} \end{bmatrix}$$

$A$        $B$

$$= \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \dots & x_{1m} \\ 1 & x_{21} & x_{22} & x_{23} & \dots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n3} & \dots & x_{nm} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$







100 Days of ML

Day 32 - Discrtizati...	Multiple Linear Regression
Day 33 - Working-...	Code Example
Day 34 - Working...	Mathematical Formulation
Day 35 - Complete...	
Day 36 - Handling...	
Day 37 - Handling...	
Day38-Missing Indi...	
Day39 - KNN Impu...	
Day40 - Iterative I...	
Day 41 - Outliers in...	
Day 42 - Outlier De...	
Day 43 - Outlier de...	
Day44 - Outlier De...	

1

$x_{11}$	$x_{12}$	$x_{13}$
$x_{21}$	$x_{22}$	$x_{23}$
$x_{31}$	$x_{32}$	$x_{33}$

$[df] \rightarrow x, y$

$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

$e = y - \hat{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$

$e = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}$







100 Days of ML

- Day 32 - Discrtizati...
- Day 33 - Working-...
- Day 34 - Working...
- Day 35 - Complete...
- Day 36 - Handling...
- Day 37 - Handling...
- Day38-Missing Indi...
- Day39 - KNN Impu...
- Day40 - Iterative I...
- Day 41 - Outliers in...
- Day 42 - Outlier De...
- Day 43 - Outlier de...
- Day44 - Outlier De...

Multiple Linear Regression

Code Example

Mathematical Formulation

$$\begin{bmatrix} (y_1 - \hat{y}_1) & (y_2 - \hat{y}_2) & (y_3 - \hat{y}_3) & \dots & (y_n - \hat{y}_n) \end{bmatrix} \begin{bmatrix} (y_1 - \hat{y}_1) \\ (y_2 - \hat{y}_2) \\ (y_3 - \hat{y}_3) \\ \vdots \\ (y_n - \hat{y}_n) \end{bmatrix} \rightarrow \textcircled{1} \times 1$$

$(1 \times n)$

$(n \times 1)$

$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2 + \dots + (y_n - \hat{y}_n)^2$$

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2$$


100 Days of ML

Day 32 - Discretizati...

## Multiple Linear Regression

Day 33 - Working...

### Code Example

Day 34 - Working...

## Mathematical Formulation

Day 35 - Complete...

## Day 36 - Handling...

## Day 37 - Handling...

Day38-Missing Indi...

Day39 - KNN Impu...

Day40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day44 - Outlier De...

$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2 + \dots$$

$$+ (y_n - \hat{y}_n)^2$$

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$E = e^T e = (y - \hat{y})^T (y - \hat{y})$$

$$= (y^T - \hat{y}^T) (y - \hat{y})$$

(4x)

$$(A + B)^T = A^T + B^T$$

$$(A - B)^T = A^T - B^T$$



100 Days of ML ▾



Day 32 - Discrtizati...

Multiple Linear Regression

Day 33 - Working-...

Code Example

Day 34 - Working...

Mathematical Formulation

Day 35 - Complete...

Day 36 - Handling...

Day 37 - Handling...

Day38-Missing Indi...

Day39 - KNN Impu...

Day40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day44 - Outlier De...



$$\sum_{i=1}^n (y_i - \hat{y}_i)$$

$$E = e^T e = (y - \hat{y})^T (y - \hat{y})$$
$$= (y^T - \hat{y}^T) (y - \hat{y})$$

$$= \left[ y^T - (X\beta)^T \right] \underline{(y - X\beta)}$$

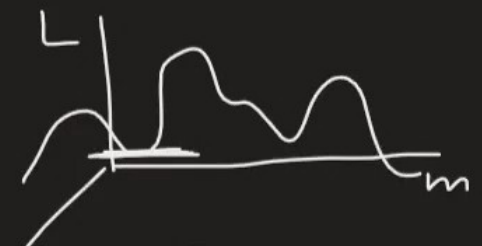
$$\bar{E} = y^T y - y^T X\beta - (X\beta)^T y + (X\beta)^T X\beta$$





100 Days of ML

Day 32 - Discrtizati...	Multiple Linear Regression
Day 33 - Working-...	Code Example
Day 34 - Working...	Mathematical Formulation
Day 35 - Complete...	Detour
Day 36 - Handling...	
Day 37 - Handling...	
Day38-Missing Indi...	
Day39 - KNN Impu...	
Day40 - Iterative I...	
Day 41 - Outliers in...	
Day 42 - Outlier De...	
Day 43 - Outlier de...	
Day44 - Outlier De...	

$$= \left[ y^T - (x\beta)^T \right] (y - x\beta)$$


$$\bar{E} = y^T y - \underline{y^T x \beta} - \underline{(x\beta)^T y} + (x\beta)^T x \beta$$

$$E = y^T y - 2y^T x \beta + \beta^T x^T x \beta$$

$$\frac{dE}{d\beta} = \frac{d}{d\beta} \left[ y^T y - 2y^T x \beta + \beta^T x^T x \beta \right] = 0$$



100 Days of ML

- Day 32 - Discrtizati...
- Day 33 - Working-...
- Day 34 - Working...
- Day 35 - Complete...
- Day 36 - Handling...
- Day 37 - Handling...
- Day38-Missing Indi...
- Day39 - KNN Impu...
- Day40 - Iterative I...
- Day 41 - Outliers in...
- Day 42 - Outlier De...
- Day 43 - Outlier de...
- Day44 - Outlier De...

Multiple Linear Regression

Code Example

Mathematical Formulation

Detour



$E = Y^T Y - 2 Y^T X \beta + \beta^T X^T X \beta$  *max diff*

$\frac{dE}{d\beta} = \frac{d}{d\beta} \left[ \underbrace{Y^T Y}_{\Downarrow 0} - \underbrace{2 Y^T X \beta}_{\text{red underline}} + \beta^T \underbrace{X^T X \beta}_{\text{red underline}} \right] = 0$

$= 0 - 2 Y^T X + \frac{d}{d\beta} \left[ \beta^T \underbrace{X^T X}_{\text{red underline}} \beta \right] = 0$

$\rightarrow \frac{dy}{d\lambda} = 2 X^T$

$y = \frac{A^T X A}{\uparrow}$

$= -2 Y^T X + 2 X^T X \beta^T = 0$

$= X^T X \beta^T = Y^T X$

100 Days of ML

Day 32 - Discretizati...

## Multiple Linear Regression

Day 33 - Working-...

### Code Example

Day 34 - Working...

## Mathematical Formulation

Day 35 - Complete...

## Detour

## Day 36 - Handling...

## Day 37 - Handling...

Day38-Missing Indi...

Day39 - KNN Impu...

Day40 - Iterative I...

Day 41 - Outliers in...

Day 42 - Outlier De...

Day 43 - Outlier de...

Day44 - Outlier De...

$$y = \frac{A^T X A}{\uparrow}$$

$$= -2Y^T X + 2X^T X \beta^T = 0$$

$$= \cancel{X^T X} \beta^T = \cancel{Y^T X} \frac{1}{[X^T X]}$$

$$\beta^T = Y^T X (X^T X)^{-1}$$

$$(\beta^T)^T = \left[ \underline{y^T x} \quad \underline{(x^T x)^{-1}} \right]^T$$

$$\beta = [(X^T X)^{-1}]^T (Y^T X)^T$$





100 Days of ML

- Day 32 - Discrtizati...
- Day 33 - Working-...
- Day 34 - Working...
- Day 35 - Complete...
- Day 36 - Handling...
- Day 37 - Handling...
- Day38-Missing Indi...
- Day39 - KNN Impu...
- Day40 - Iterative I...
- Day 41 - Outliers in...
- Day 42 - Outlier De...
- Day 43 - Outlier de...
- Day44 - Outlier De...

Multiple Linear Regression

Code Example

Mathematical Formulation

Detour

$$\beta' = [Y' \quad \lambda' \quad (\lambda' \quad \lambda)']$$

$$\beta = [(X^T X)^{-1}]^T (Y^T X)^T$$

$$\beta = [(X^T X)^{-1}]^T X^T Y$$

$$\beta = (X^T X)^{-1} X^T Y$$

$$\boxed{\beta = (X^T X)^{-1} X^T Y}$$

$X = n \times (m+1)$

$(m+1) \times n$   $n \times (m+1)$

$(m+1) \times (m+1)$

$X \rightarrow \begin{cases} X_{\text{train}} \\ Y = Y_{\text{train}} \end{cases}$



100 Days of ML

- Day 32 - Discrtizati...
- Day 33 - Working-...
- Day 34 - Working-...
- Day 35 - Complete...
- Day 36 - Handling...
- Day 37 - Handling...
- Day38-Missing Indi...
- Day39 - KNN Impu...
- Day40 - Iterative I...
- Day 41 - Outliers in...
- Day 42 - Outlier De...
- Day 43 - Outlier de...
- Day44 - Outlier De...

Multiple Linear Regression

Code Example

Mathematical Formulation

Detour

Why Gradient Descent

$\beta = (X^T X)^{-1} X^T y$

$Y = y_{train}$

$(X^T X)^{-1}$

$\beta = (X^T X)^{-1} X^T y$

$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$

$(m+1) \times 1$

$\beta$

$[(m+1) \times (m+1)] [(m+1) \times n] [n \times 1]$

$[(m+1) \times n] [n \times 1]$

$1 \times (m+1) \times 1$

$10000000000$

$\eta = 1000$

$\eta^3$

$1000 \times 1000$

$\eta \times (m+1)$

$(m+1) \times n$

$\eta \times 1$

$X^T X$

$m \rightarrow \text{cols}$

$999$

