Mathematical Formulation

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n_dim_dataset 3-cols

$$y_{(lpa)} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$(cgpa) \quad (iq)$$

cgpa | iq | lpa

x_1	x_2	y
8.1	93	3.2
7.5	95	3.5

(2,3) .

{m, b}

{ $\beta_0, \beta_1, \beta_2$ }

$L(\beta_0, \beta_1, \beta_2)$ 4D

1) Random values

$\beta_0 = 0$, $\beta_1, \beta_2 = 1$

2) epoch = 100, lr = 0.1

$$\frac{\partial L}{\partial \beta_0} = \frac{\partial L}{\partial \beta_1} \quad \frac{\partial L}{\partial \beta_2}$$

$$\begin{cases} \beta_0 = \beta_0 - \eta \, \text{slope} \\ \beta_1 = \beta_1 - \eta \, \text{slope} \\ \beta_2 = \beta_2 - \eta \, \text{slope} \end{cases}$$

n_dim

$(n+1)\ \beta_0 - \beta_n$

$$L = \frac{1}{2n}\sum_{i=1}^{n}(y_i - \hat{y}_i)$$

$$= \frac{1}{2}\left[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2\right]$$

$$L = \frac{1}{2}\left[(y_1 - \beta_0 - \beta_1 x_{11} - \beta_2 x_{12})^2 + (y_2 - \beta_0 - \beta_1 x_{21} - \beta_2 x_{22})^2\right]$$

$$\frac{\partial L}{\partial \beta_0} = \frac{1}{2}\left[2(y_1 - \hat{y}_1)(-1) + 2(y_2 - \hat{y}_2)(-1)\right]$$

$$\frac{\partial L}{\partial \beta_0} = -\frac{2}{2}\left[(y_1 - \hat{y}_1) + (y_2 - \hat{y}_2)\right]$$

$$y_i = \beta_0 + \ldots$$

	x_1	x_2	y
1	8.1	93	3.2
2	7.5	95	3.5

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$\hat{y}_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12}$$

$$\hat{y}_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22}$$

$$L = \frac{1}{2}\left[(y_1 - \beta_0 - \beta_1 x_{11} - \beta_2 x_{12})^2 + (y_2 - \beta_0 - \beta_1 x_{21} - \beta_2 x_{22})^2\right]$$

$$\frac{\partial L}{\partial \beta_0} = \frac{1}{2}\left[2(y_1 - \hat{y}_1)(-1) + 2(y_2 - \hat{y}_2)(-1)\right]$$

$$\frac{\partial L}{\partial \beta_0} = -\frac{2}{2}\left[(y_1 - \hat{y}_1) + (y_2 - \hat{y}_2)\right]$$

$$= \frac{-2}{n}\left[(y_1 - \hat{y}_1) + (y_2 - \hat{y}_2) + (y_3 - \hat{y}_3) + \ldots + (y_n - \hat{y}_n)\right]$$

$$= \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i) = \frac{\partial L}{\partial \beta_0}$$

1.5	95	3.5

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$\hat{y}_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12}$$

$$\hat{y}_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22}$$

$$L = \frac{1}{2}\sum_{i=1}^{2}(y_i - \hat{y}_i)^2$$

$$L = \frac{1}{2}\left[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2\right]$$

$$L = \frac{1}{2}\left[(y_1 - \beta_0 - \beta_1 x_{11} - \beta_2 x_{12})^2 + (y_2 - \beta_0 - \beta_1 x_{21} - \beta_2 x_{22})^2\right]$$

$\frac{\partial}{\partial \beta}$ $-\beta_1 x_{11} = -x_{11}$

	β_1 (x_1)	β_2 (x_2)	1 (y)
1	8.1	93	3.2
2	7.5	95	3.5
3			
4			

$$\frac{\partial L}{\partial \beta_1} = \frac{1}{2}\left[2(y_1 - \hat{y}_1)(-x_{11}) + 2(y_2 - \hat{y}_2)(-x_{21})\right]$$

1!, 2!, 3! ... n!

$$\frac{\partial L}{\partial \beta_1} = \frac{-2}{n}\left[(y_1 - \hat{y}_1)(x_{11}) + (y_2 - \hat{y}_2)(x_{21}) + (y_3 - \hat{y}_3)(x_{31}) + \dots + (y_n - \hat{y}_n)(x_{n1})\right]$$

$$\boxed{\frac{\partial L}{\partial \beta_1} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i1}}$$

x_{i1} → 1 col data
β_1 → values of 1 col.

x_{11}, x_{21}, x_{31}, x_{n1}

$$\frac{\partial L}{\partial \beta_1} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)\,x_{i1}$$

$$\frac{\partial L}{\partial \beta_2} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)\,x_{i2}$$

$$\frac{\partial L}{\partial \beta_m} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)\,x_{im}$$

x_{i1} → 1 col data

β_1 → values of 1 col.

m cols

$\beta_0 - \beta_m$

code