Bungee Jumping: A Physics Exploration

Problem Statement

Starting from rest, a person of mass m bungee jumps from a tethered hot-air balloon of height h above the ground. The bungee cord has negligible mass and unstretched length l_0. One end is tied to the basket of the balloon and the other end to a harness around the person's body. The cord is modeled as a spring that obeys Hooke's law with a spring constant of k, and the person's body is modeled as a particle. The hot-air balloon does not move.

Hint: You are encouraged to introduce appropriate notation, such as defining $y_0 = \frac{mg}{k}$ or any other relevant parameters, to make your parametric calculations more concise.

(a) Express the gravitational potential energy of the person–Earth system as a function of the person's variable height y above the ground. (1 point)

(b) Express the elastic potential energy of the cord as a function of y. (1 point)

(c) Express the total potential energy of the person–cord–Earth system as a function of y. (1 point)

(d) Plot a graph of the gravitational, elastic, and total potential energies as functions of y. (2 point)

(e) Assume air resistance is negligible. Determine the minimum height of the person above the ground during his plunge. (2 point)

(f) Does the potential energy graph show any equilibrium position or positions? If so, at what elevations? Are they stable or unstable? Explain your answer! (2 point)

(g) Determine the jumper's maximum speed. (1 point)

Submission Deadline: March 13, 2025.