MŰEGYETEM 1782

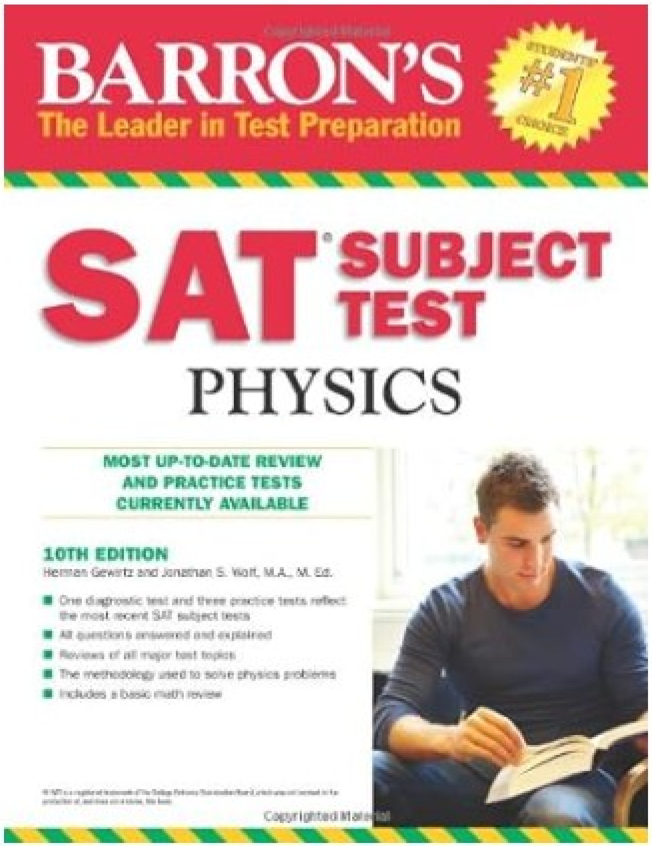
Physicist-Engineer BSc

Problem solving in Physics

1D and 2D motion (kinematics)

Budapest, 2024 September 19

Main resource: Barron's SAT physics textbook, by Herman Gewirtz and Jonathan S. Wolf M.A.

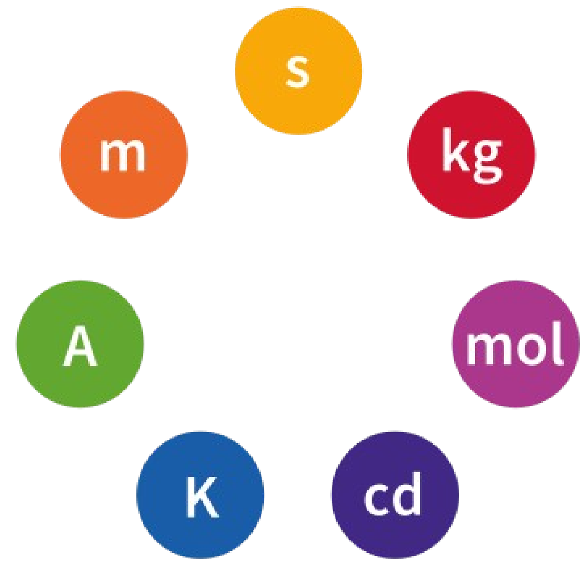


This week: chapters 1-4

Units, fundamental and derived

Fundamental SI Units Used in Physics

Quantity	Unit Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
plane angle	radian	rad
luminous intensity	candela	cd



wikipedia: interesting history
https://en.wikipedia.org/wiki/SI_base_unit

Derived units, a few examples:

Quantity	Equation	Unit
Velocity:	$v = s/t$	m/s
Acceleration:	$a = dv/dt$	m/s^2
Force:	$F = ma$	$N = kg\ m / s^2$
Charge:	$I = dQ/dt$	$C = A\ s$

For small or big numbers: Use prefixes and/or scientific notation

exa	E	10^{18}	1 000 000 000 000 000 000
peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto	h	10^{2}	100
deca	da	10^{1}	10

deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001
atto	a	10^{-18}	0.000 000 000 000 000 001

450,000,000 m = 4.5×10^{8} m = 4.5e8 m = 450 Mm

Coefficient × Base$^{\text{Exponent}}$

0.00000023 m = 2.3×10^{-7} m = 2.3e-7 m = 230 nm

Dimensional analysis: often, just looking at the unit helps you with the equation

$$F_{\mathrm{cp}} = m\omega^2 r$$

or

$$F_{\mathrm{cp}} = m\frac{\omega^2}{r}$$

Based on the units 2nd formula cannot work, 1st formula can be correct

Which of the following could be correct?
How does the velocity c of ocean waves depend on their wavelength, λ?

A) long waves travel faster, $c = \sqrt{\frac{g\lambda}{2\pi}}$

B) long waves travel slower, $c = \sqrt{\frac{g}{2\lambda}}$

C) velocity independent of λ $c = \sqrt{\frac{g}{2\pi}}$

D) None of the above is possible, velocity must depend on density as well

Precision, accuracy

Coordinates, vectors

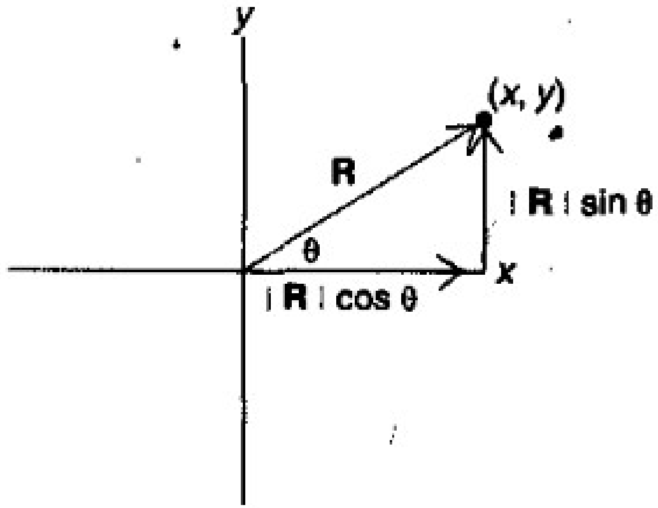


Figure 2.2

$$x = r\cos\theta; \qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \qquad |\mathrm{R}| = r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\frac{y}{x} = \arctan(y/x) = \arctan_2(x, y)$$

Addition, subtraction, multiplication:

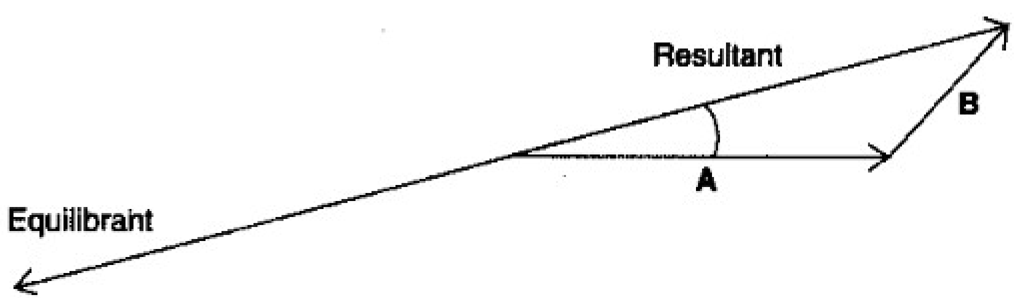


Figure 2.6

$$a\mathbf{R}_1 + b\mathbf{R}_2 =$$

$$= a\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + b\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \end{pmatrix}$$

A girl walks 2 meters north, 4 meters west, 2 meters south. Her final displacement is...

A) 4 m east

B) 4 m west

C) 2 m north

D) 5 m northwest

Which pair of forces can produce a resultant of 15 newtons?

A) 20 N, 20 N

B) 25 N, 5 N

C) 5 N, 5 N

D) more than one combination above

Coordinates, vectors: summary

- Make sure you know what frame of reference a problem is referring to.
- Make sure you know the difference between vector and scalar quantities. Vectors are quantities that have both magnitude and direction. Scalar quantities have only magnitude. You should be able to cite a few examples of each.
- Make sure you know how to combine vectors graphically so you can recognize the correct procedure on a multiple-choice question. Remember that vectors can be added by constructing them head to tail.
- Concurrent vectors are vectors acting at the same place and at the same time.
- You should be able to use the Pythagorean theorem to find the resultant of two vectors.
- You should be able to apply the rules of trigonometry for right triangles (SOHCAHTOA) to find angles and components of vectors.

1D motion

velocity $$v = \frac{dx}{dt}$$ $$\frac{\text{infinitesimal increment of displacement}}{\text{infinitesimal increment of time}}$$

average velocity $$\overline{v} = \frac{\Delta x}{\Delta t}$$ $$\frac{\text{Total displacement}}{\text{Total time}}$$

acceleration $$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$ $$\frac{\text{infinitesimal increment of velocity}}{\text{infinitesimal increment of time}}$$

uniform acceleration with initial velocity v_0: $$s = \Delta x = v_0 t + \frac{a}{2}t^2$$

Graphical analysis: $x(t)$, $v(t)$, $a(t)$ curves

An example: car ride to Vienna

We want to go by car from Budapest to Vienna (240 km). For the first 160 km, until Mosonmagyaróvár, we go fast, average speed 100 km/h. We then make a short stop of 20 minutes. How fast do we have to drive for the rest of the way to have an overall average speed of 110 km/h?

1) list the data, choose units $x_{\mathrm{tot}} = 240\mathrm{km}$ $\quad \overline{v} = 110\mathrm{km/h}$ $\quad t_2 = 20\mathrm{min} = 0.333\mathrm{h}$

2) collect main equations $\overline{v} = \frac{x_{\mathrm{tot}}}{t_{\mathrm{tot}}}$ $\quad t_{\mathrm{tot}} = \frac{160\mathrm{km}}{100\mathrm{km/h}} + 20\mathrm{min} + \frac{80\mathrm{km}}{v_3}$

3) make a sketch

4) calculate – attention to units!

$$t_{\mathrm{tot}} = \frac{x_{\mathrm{tot}}}{\overline{v}} = \frac{240}{110} = 2.18\mathrm{h}$$

$$2.18 = 1.6 + 0.333 + \frac{80}{v_3} \Longrightarrow \underline{\underline{v_3 = 321\mathrm{km/h}}}$$

5) (write out result as a sentence)

6) check by inserting into equations as close to the text as possible.

7) interpret – does the result make sense?

The answer would have been negative if our break at M'óvár had been more than 42 minutes

A bomb is dropped from an airplane moving horizontally with a speed of 200 kilometers per hour. If the air resistance is negligible, the bomb will reach the ground in 5 seconds when the altitude of the plane is approximately

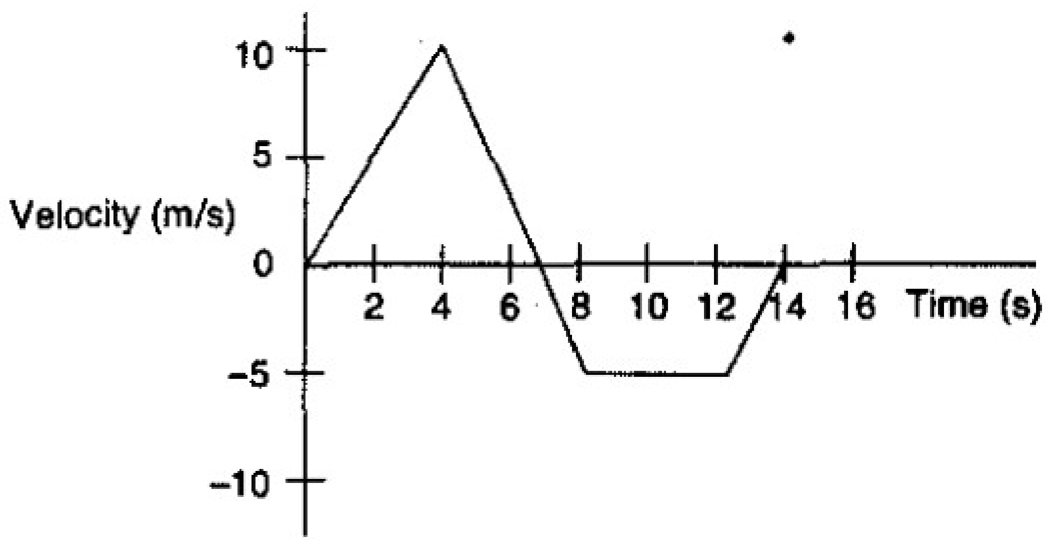
A) 50 m

B) 75 m

C) 125 m

D) 250 m

Given the velocity-time plot below, sketch the displacement-time and acceleration-time plots! Don't forget axis labels and, numbers, units!



1D motion: summary

- You should remember that distance and speed are scalar quantities while displacement, velocity, and acceleration are vector quantities.
- The slope of a graph of displacement versus time is equal to the average velocity of the motion.
- The slope of a graph of velocity versus time is equal to the average acceleration.
- The slope of the line tangent to a curve is equal to the instantaneous rate of change of that quantity.
- The area under a graph of velocity versus time is equal to the change in displacement.
- The value of the acceleration due to gravity (**g**) is a constant near the surface of Earth.

2D motion

velocity (vector) $\mathbf{v} = \dfrac{d\mathbf{R}}{dt}$ $\quad \dfrac{\text{infinitesimal increment of displacement}}{\text{infinitesimal increment of time}}$

acceleration (vector) $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = \dfrac{d^2\mathbf{r}}{dt^2}$ $\quad \dfrac{\text{infinitesimal increment of velocity}}{\text{infinitesimal increment of time}}$

relative motion, changing reference frame: $\mathbf{v}_{\text{boat:river}} = \mathbf{v}_{\text{boat}} - \mathbf{v}_{\text{river}}$

launched projectile: $x = v_0 \cos(\theta)t; \qquad y = v_0 \sin(\theta)t - \dfrac{g}{2}t^2$

uniform circular motion: $\theta(t) = \omega t = \dfrac{2\pi}{T}t$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

constant speed, changing velocity: $|\mathbf{v}(t)| = v_0 = r\omega$

$$|\mathbf{a}(t)| = v_0\omega = r\omega^2 = \frac{v_0^2}{r}$$

Boat in river: relative velocity

The river flows east, with a speed of 4 m/s. A boat is driven northwards, towards the shore, at a speed of 10 m/s relative to the river. What is its speed in the reference frame of someone standing on the shore?

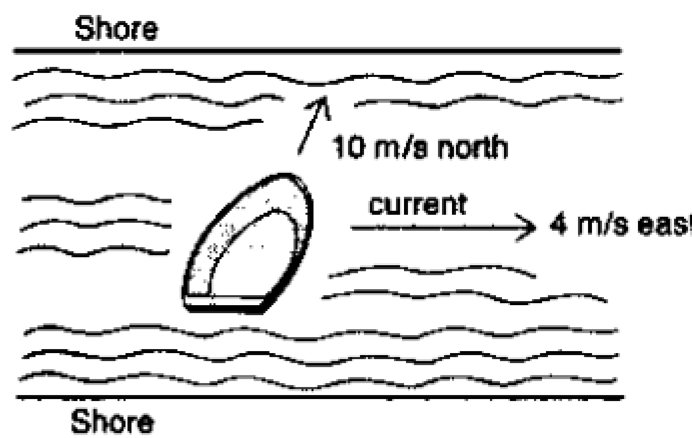


Figure 4.2

Launched projectile: treat x and y components of motion separately

I want to throw a stone as far as possible. At what angle should I throw it? Neglect air resistance, and my height.

1) sketch

2) solve

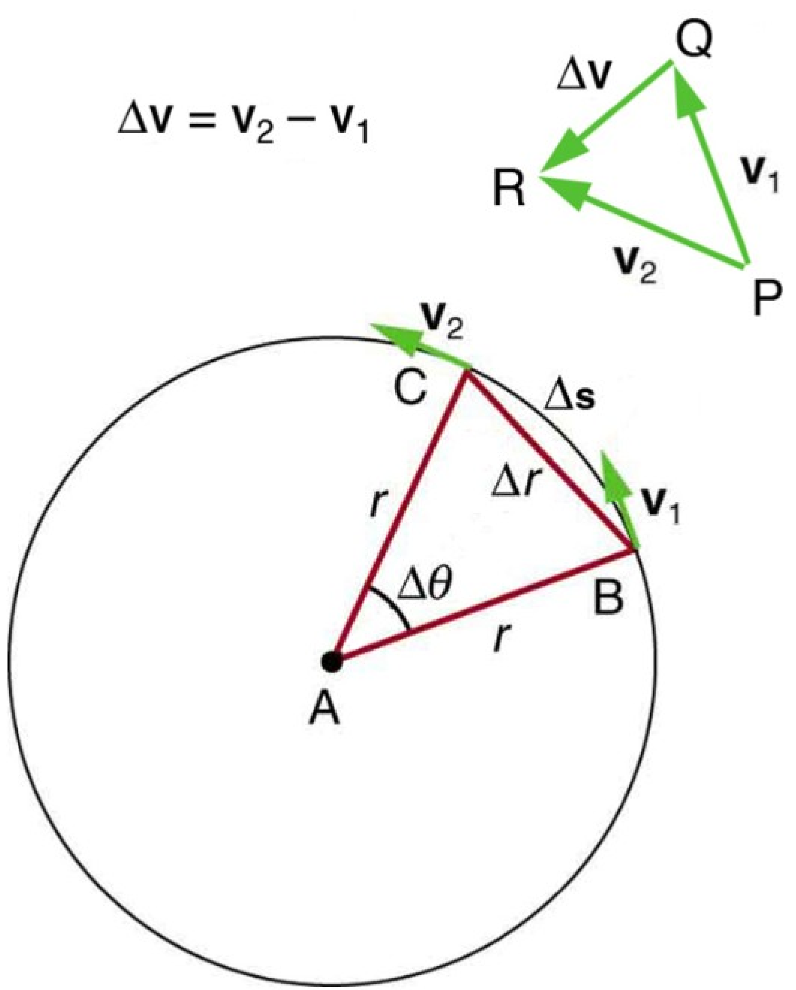
vertical motion: $y(t) = v_0 \sin(\theta) t - \frac{g}{2} t^2$

horizontal motion: $x(t) = v_0 \cos(\theta) t$

equation of trajectory: $y(x) = \tan(\theta) x - \frac{g}{2 v_0^2 \cos^2 \theta} x^2$

hitting the ground when y=0: $x = \frac{v_0^2}{g} \sin(2\theta)$ $\Rightarrow$ max distance when $\sin(2\theta) = 1 \Leftrightarrow \theta = 45^\circ = \frac{\pi}{4}$

Uniform circular motion: centripetal acceleration



https://phys.libretexts.org/
https://shorturl.at/xHNT5

$$\theta(t) = \omega t = \frac{2\pi}{T} t$$

angular frequency =
angular speed = ω
how many radians per sec?

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$|\mathbf{v}(t)| = v_0 = r\omega$$

$$|\mathbf{a}(t)| = v_0\omega = r\omega^2 = \frac{v_0^2}{r}$$

acceleration always points towards the center

2D motion: summary

- All motion is observed relative to a given frame of reference.
- The path of a projectile is a parabola.
- The horizontal and vertical components of projectile motion are independent of each other.
- Uniform circular motion involves acceleration toward the center of the circle. This is called the centripetal acceleration.
- An object undergoing uniform circular motion, with a constant speed, is still accelerating since its velocity vector is constantly changing direction.

While an arrow is being shot from a bow, it is accelerated over a distance of 2 meters. At the end of this acceleration it leaves the bow with a speed of 200 meters per second. The average acceleration imparted to the arrow is

A) 400 m/s^2

B) 500 m/s^2

C) 1000 m/s^2

D) 10000 m/s^2

A projectile is launched at an angle of 45° with a velocity of 250 meters per second. If air resistance is neglected, the magnitude of the horizontal velocity of the projectile at the time it reaches maximum altitude is equal to

A) 0 m/s

B) 175 m/s

C) 250 m/s

D) 300 m/s

A projectile is launched horizontally with a velocity of 25 meters per second from the top of a 75 meter cliff. How many seconds will the projectile take to reach the bottom?

A) 9.75

B) 6.31

C) 4.27

D) 3.91