



Problem solving in Physics

Forces (mechanics) 2.

attendance:

<https://forms.office.com/r/gHgLHJiqak>

Budapest, 2023 October 10

Significant Digits

Rules for Counting Significant Digits:

- All non-zero digits and any zeros contained between non-zero digits count.

300042 = 6 significant digits

- Leading zeros don't count.

0.000034 = 2 significant digits

- Trailing zeros count if there is a decimal point.

0.0002500 = 4 significant digits

- Trailing zeros may or may not count if there is no decimal point, so we go with the most conservative answer.

190000 = 2 significant digits (could be up to 6)

Precision of operations

Rules for Calculating With Significant Digits:

- When **adding** or **subtracting**, round the answer to the **least number of decimal places**.

$$\begin{array}{r} 1.457 \\ + 83.2 \\ \hline 84.657 \end{array}$$

rounds to 84.7

$$\begin{array}{r} 0.0367 \\ - 0.004322 \\ \hline 0.032378 \end{array}$$

rounds to 0.0324

- When **multiplying** or **dividing**, round the answer to the **least number of significant digits**.

$$\begin{array}{r} 4.36 \\ \times 0.00013 \\ \hline 0.0005668 \end{array}$$

rounds to 0.00057

$$\frac{12.300}{0.0230} = 534.78261$$

rounds to 535

Write numbers in scientific notation, with a precision of 3 significant digits

$$0.00012309991 = ?$$

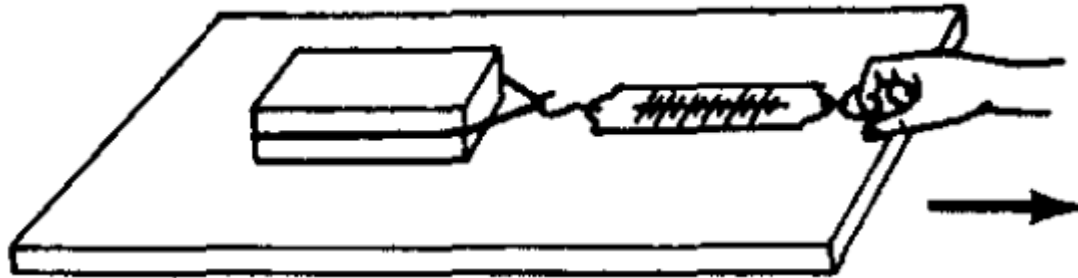
A) $12.3\text{e-}5 = 12.3 \cdot 10^{-5}$

B) $1.23\text{e-}4 = 1.23 \cdot 10^{-4}$

C) $1.231\text{e-}4 = 1.231 \cdot 10^{-4}$

D) $0.124\text{e-}3 = 0.123 \cdot 10^{-3}$

Force: push or pull exerted by an object on an another object. Can be measured by spring. Can lead to deformation, or change in motion. A vector quantity.



What causes them:

Elastic $\mathbf{F}_{el} = -k\mathbf{x}$

Frictional $\mathbf{F}_f = -\mu\mathbf{F}_N$

Air resistance (drag)

Gravitational $\mathbf{F}_G = G \frac{mM}{r^2} \frac{\mathbf{r}}{r}$

Electric (Coulomb) $\mathbf{F}_C = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \frac{\mathbf{r}}{r}$

Magnetic (Lorentz)? $\mathbf{F}_L = q\mathbf{v} \times \mathbf{B}$

$$\mathbf{F}_{\text{drag}} = -\rho C_d \frac{A}{2} v^2 \frac{\mathbf{v}}{v}$$

$$\mathbf{F}_{\text{Stokes}} = -6\pi\mu R v \frac{\mathbf{v}}{v}$$

What role they play:

Centripetal $\mathbf{F}_{cp} = m\omega^2 \mathbf{r}$

Normal \mathbf{F}_N

Newton's laws of motion (dynamics)

1. A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by a force.

2. When a body is acted upon by a net force, the body's acceleration multiplied by its mass is equal to the net force.

$$\sum_j \mathbf{F}_j = \mathbf{F}_{\text{total}} = m\mathbf{a}$$

3. If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.^[2]

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

[2]: “For every action, there is an equal and opposite reaction”

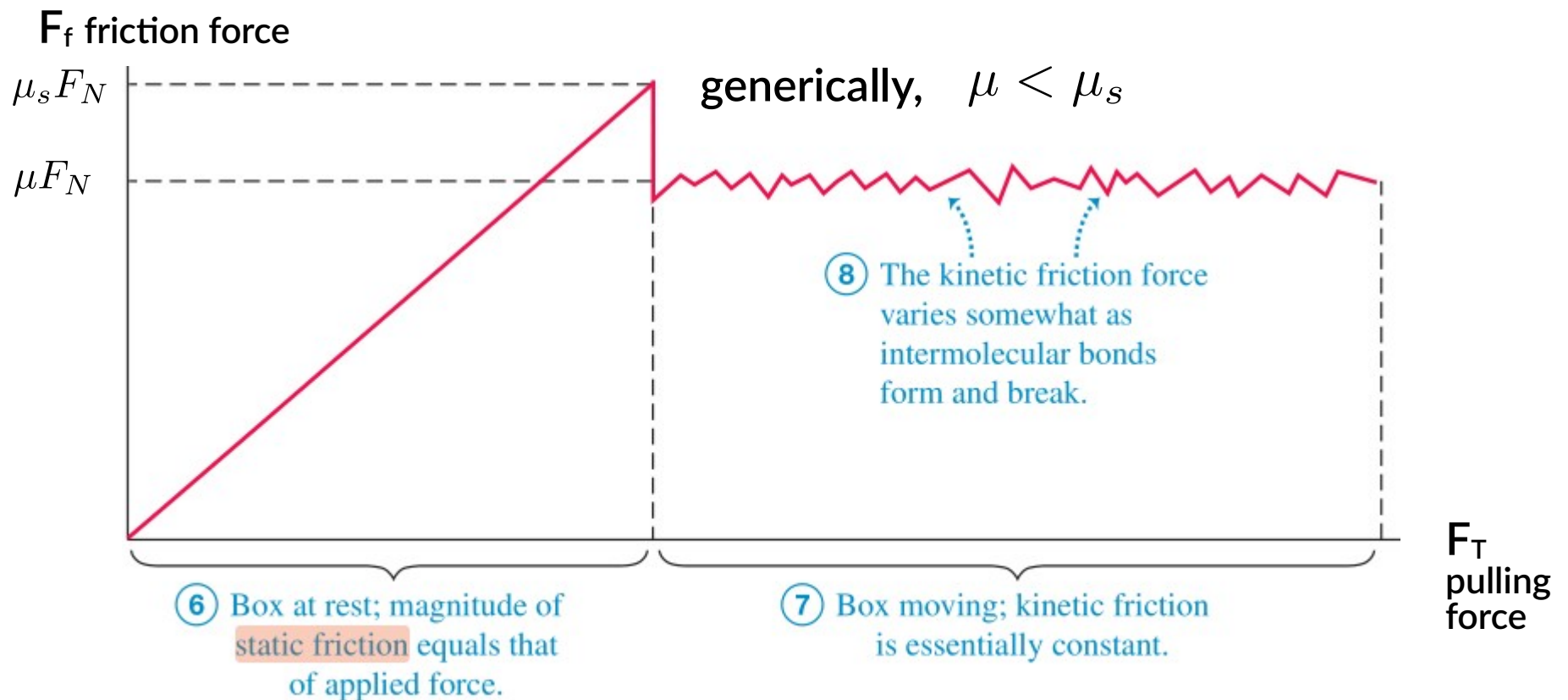
Static friction: can take any value up to the limit $\mu_s F_N$

static friction

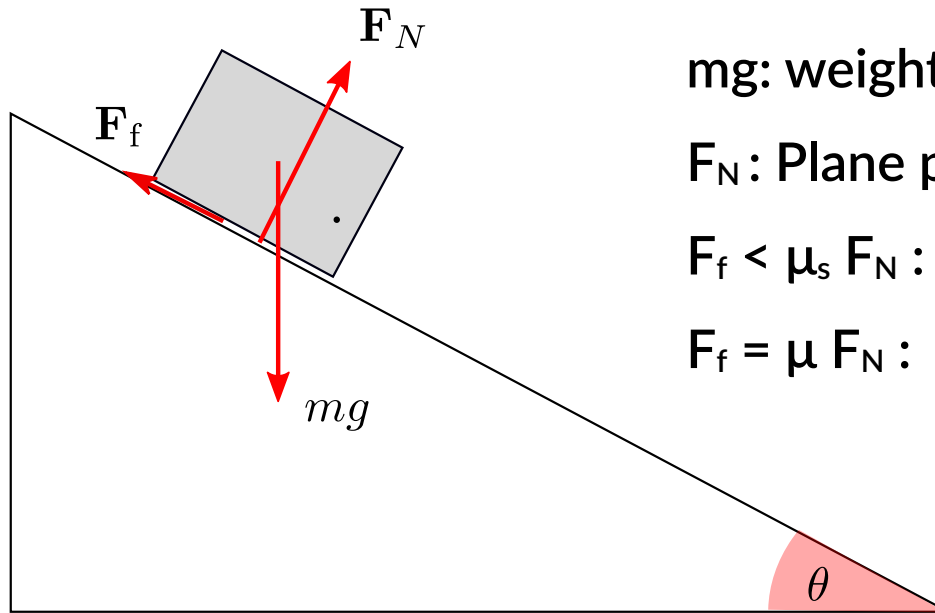
$$F_f < \mu_s F_N$$

kinetic friction

$$F_f = \mu F_N$$



Example: given μ_s , at what angle does the box start to slip?



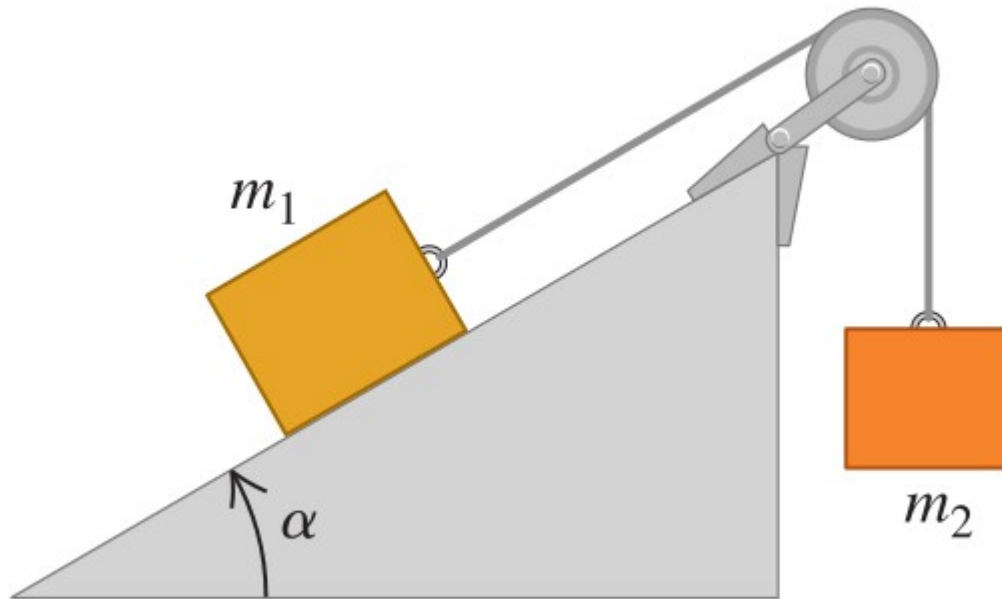
mg : weight, gravitational pull of Earth

\mathbf{F}_N : Plane pushing on block, normal force

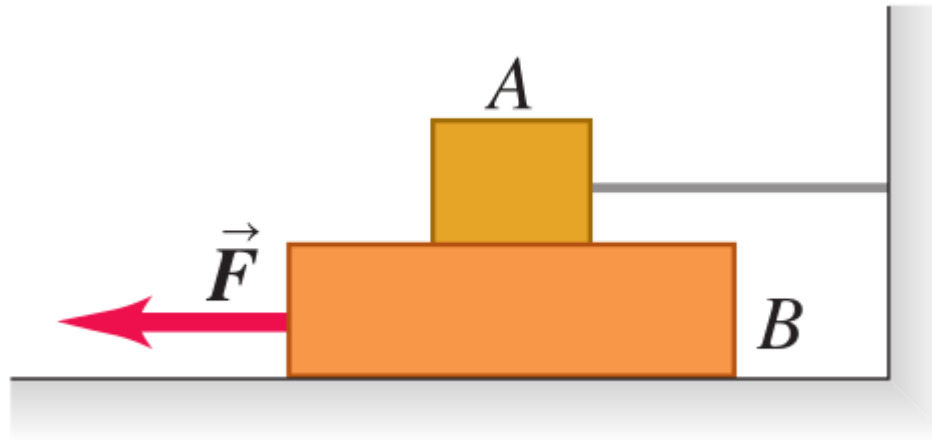
$F_f < \mu_s F_N$: static friction force

$F_f = \mu F_N$: (kinetic) friction force

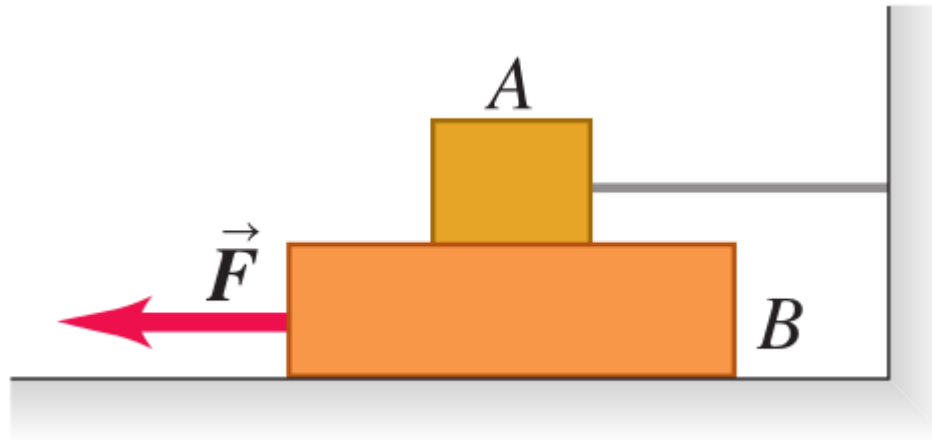
Example: given μ_s , at what angle does the box start to slip, if $m_1 = m_2$?



What is the force required to get box B moving, if box A is tied to the wall? The static coefficient of friction is $\mu_s=0.33$ between all surfaces, $m_B=2m_A=2\text{ kg}$?



How does the force required to get box B moving, change, if masses of both boxes are increased by a factor of 2?



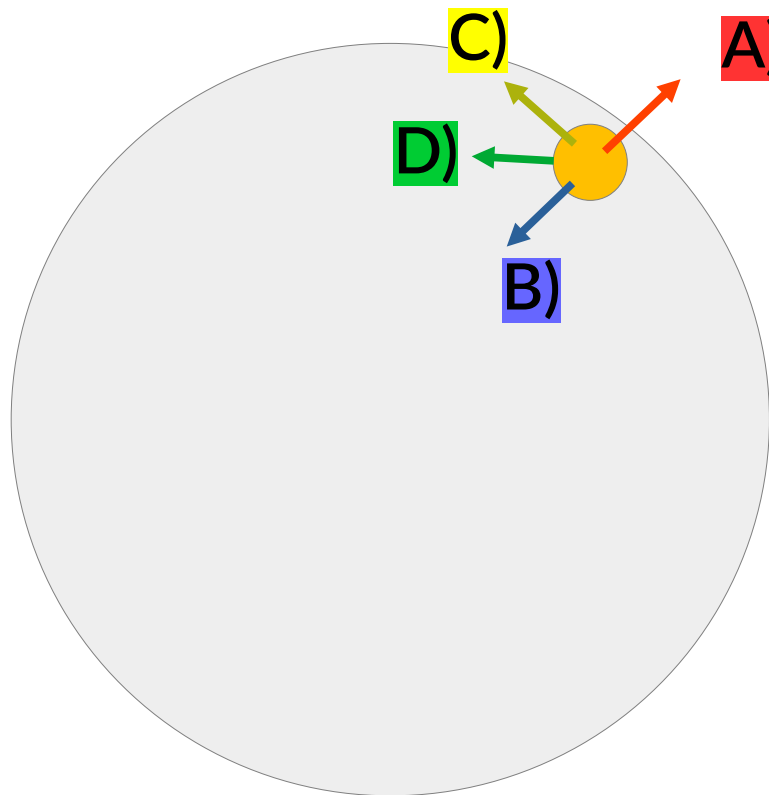
- A) Increase 2-fold
- B) Increase 4-fold
- C) Decrease to $\frac{1}{2}$ of original
- D) Does not change

Direction of static friction force: as required to keep the object still on the surface

Free-body diagram for an accelerating car

Direction of static friction force: as required to keep the object still on the surface

Coin on a uniformly rotating record player, held in place by static friction, top view. Which vector represents the static friction force on the coin?

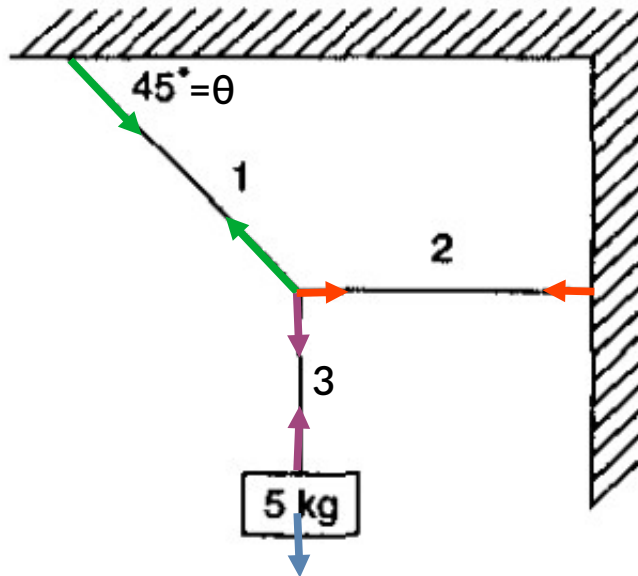


Action-reaction counterpart of static friction

A coin on a rotating record player is held in place by static friction. Which force is the action-reaction counterpart of the static friction force exerted by the record player on the coin?

- A)** static friction force exerted by coin on record player, pointing away from center
- B)** static friction force exerted by coin on record player, pointing towards center
- C)** centrifugal force pointing away from center
- D)** air resistance

Static equilibrium: sum of all forces on a body must be 0.
Can be used to deduce some forces



All ropes have negligible mass.

Body in equilibrium:
 $mg = F_3$

Knot in equilibrium:
 $F_1 \sin \theta = F_3$
 $F_1 \cos \theta = F_2$

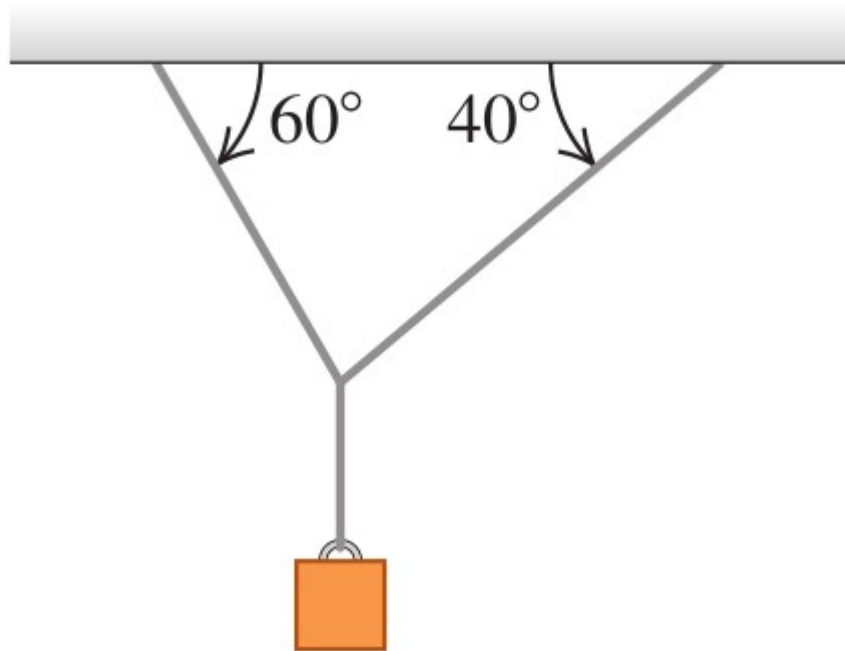
Ropes have 0 mass, Force/counterforce:
Colored forces are equal in magnitude

$$F_2 = F_1 \cos \theta = \frac{F_3}{\sin \theta} \cos \theta = \frac{mg}{\tan \theta}$$

in this case, rope 2 pulls the wall with the same force as the weight of body:
 $F_2 = mg = 5 \times 9.81 \text{ N} = \underline{\underline{49.05 \text{ N}}}$

Discussion: What if angle θ is really small?

Which rope has greater tension (larger force in it?)



- A) The rope on the left
- B) The rope on the right
- C) Equal tensions
- D) Depends on how heavy the mass is

Uniform circular motion: sum of all forces has to point to the center (centripetal force)

A steel ball of mass 500 g is attached to a spring, with unstretched length $l_0=20$ cm, spring constant $k=200$ N/m. We are swinging it around on a roughly horizontal plane, with a frequency of 100 RPM.

What is the elongation of the spring?

Hooke's law for force exerted by spring elongated by x : $\mathbf{F}_{\text{el}} = -k\mathbf{x}$

Newton's 2nd law is very important: it allows us to predict the future

$$\sum_j \mathbf{F}_j = m\mathbf{a}$$

from time t to $t+dt$, the object

1. moves because it has a velocity (need to update its position)
2. accelerates because it has an acceleration (update velocity)
3. experiences forces (according to position, velocity)
4. $\sum_j \mathbf{F}_j = m\mathbf{a}$ (update acceleration)

Aim: plot trajectory, `plot(t, x)`, `plot(t, y)`, `plot(x, y)`. Algorithm:

- 1) Preparation: empty lists for t , x , y values.
- 2) Initial conditions: set t , x , y , v_x , v_y

3) *Loop*:

- * 0) $t_{\text{new}} = t + dt$
- * 1) $x_{\text{new}} = x + v_x * dt$; $y_{\text{new}} = y + v_y * dt$;
- * 3) $v_{x,\text{new}} = v_x + a_x * dt$ $v_{y,\text{new}} = v_y + a_y * dt$
- * 2) $a_x = \text{sum}F_x(x,y,v_x,v_y)/m$; $a_y = \text{sum}F_y(x,y,v_x,v_y)/m$
- * 4) append t , x , y values to the lists
- * 5) $x=x_{\text{new}}$, $y=y_{\text{new}}$, $v_x=v_{x,\text{new}}$, $v_y=v_{y,\text{new}}$

4) Plot the trajectories

Summary

- Forces are vector quantities that change the state of motion of objects.
- The inertia of an object is the tendency of the object to resist a force changing its state of motion.
- Inertia is proportional to the mass of the object.
- The sum of all the forces acting on a mass is called the net force.
- If the net force acting on a mass is equal to zero, the mass is in equilibrium.
- If an object is not moving in a given frame of reference, it is in static equilibrium.
- An object can be in dynamic equilibrium if it has a constant velocity in a given frame of reference.
- The centripetal force is a net force that acts toward the center of the circle.
- The centrifugal force is a fictitious force that is observed only in an accelerated frame of reference.
- Friction is a force that opposes motion as an object slides along a surface or as two masses slide against each other.
- The normal force is a force directed perpendicularly away from a surface.

Write numbers in scientific notation, with a precision of 3 significant digits

Please practice this

0.001200340056 =

1200340056 =

0.001295503056 =

129550340056 =