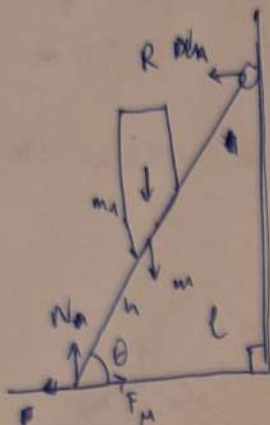


9.3.



$$\begin{aligned} m &= 15 \text{ kg} \\ l &= 5 \text{ m} \\ \mu &= 0.5 \\ m_1 &= 70 \text{ kg} \end{aligned}$$

$$W = mg = 15 \cdot 9.81 = 145.15 \text{ N}$$

$$W_1 = m_1 g = 70 \cdot 9.81 = 686.7 \text{ N}$$

Equilibrium:

$$\Rightarrow N = W + W_f$$

$$\Rightarrow R = F_f = \mu N$$

$$\tau_{\text{ladder}} = W \frac{l}{2} \cos \theta$$

$$\tau_{\text{person}} = W_1 h \cos \theta$$

$$\tau_{\text{wall}} = R l \sin \theta$$

$$\tau_{\text{ladder}} + \tau_{\text{person}} = \tau_{\text{wall}}$$

$$\Rightarrow W \frac{l}{2} \cos \theta + W_1 h \cos \theta = R l \sin \theta$$

$$\Rightarrow W \frac{l}{2} \cos \theta + W_1 h \cos \theta = \mu (W + W_1) l \sin \theta$$

$$\Rightarrow h = \frac{\mu (W + W_1) l \sin \theta - W \frac{l}{2} \cos \theta}{W_1 \cos \theta}$$

$$= \frac{208.4625 \sin \theta - 367.875 \cos \theta}{686.7 \cos \theta}$$

9.1.

$$m_{\text{mudake}} = 50 \text{ kg}$$

$$m_{\text{adder}} = 10 \text{ kg}$$

$$m_{\text{water}} = 60 \text{ kg}$$

$$\text{depth} = 70 \text{ cm} = 0.7 \text{ m}$$

$$H = \text{height} = 190 \text{ cm} = 1.9 \text{ m}$$

$$\theta = 30^\circ$$



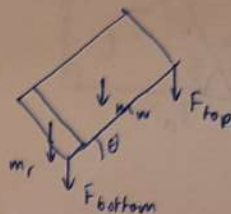
Upright orientation:

$$F_{\text{bottom}} + F_{\text{top}} = W \sin 30^\circ$$

$$F_{\text{top}} \cdot H = W \sin 30^\circ y_{\text{com}}$$

$$\Rightarrow F_{\text{top}} = \frac{60 \cdot 9.81 \cdot 0.792}{2 \cdot 1.9} \approx 123 \text{ N (3s.f.)}$$

$$\Rightarrow F_{\text{bottom}} = \frac{60 \cdot 9.81}{2} - 123 \approx 171 \text{ N (3s.f.)}$$



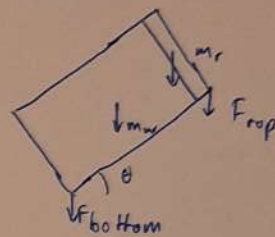
Inverse orientation:

$$F_{\text{bottom}} + F_{\text{top}} = W \sin 30^\circ$$

$$F_{\text{bottom}} F_{\text{top}} \cdot H = W \sin 30^\circ (H - y_{\text{com}})$$

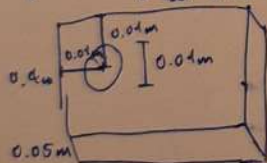
$$\Rightarrow F_{\text{top}} F_{\text{bottom}} = \frac{60 \cdot 9.81}{2 \cdot 1.9} (1.9 - 0.792) \approx 172 \text{ N (3s.f.)}$$

$$F_{\text{top}} = \frac{60 \cdot 9.81}{2} - 172 \approx 123 \text{ N (3s.f.)}$$



$$\text{width} = \text{height} = \text{length} = 40 \text{ cm}$$

9.2.



$$V_{\text{plank}} = 40 \cdot 40 \cdot 5 = 8000 \text{ cm}^3$$

$$V_{\text{disk}} = \pi r^2 \cdot 5 = \pi \cdot 5^2 \cdot 5 = 125\pi \approx 392.7 \text{ cm}^3$$

$$M_{\text{plank}} = \rho V_{\text{plank}} = \rho 8000$$

$$M_{\text{disk}} = \rho V_{\text{disk}} = \rho 392.7$$

$$M_{\text{remain}} = \rho 8000 - \rho 392.7 = \rho 7607.3$$

$$x_{\text{com}} = \frac{(\rho 8000 \cdot 20) - (\rho 392.7 \cdot 10)}{\rho 7607.3} \approx 20.5 \text{ cm}$$

$$y_{\text{com}} = \frac{(8000 \rho \cdot 20) - (392.7 \rho \cdot 10)}{7607.3 \rho} \approx 20.5 \text{ cm}$$

$$\Rightarrow (x_{\text{com}}, y_{\text{com}}) \approx (20.5 \text{ cm}, 20.5 \text{ cm})$$

```
import numpy as np
from scipy.integrate import solve_ivp
```

```
rho = 1.293 # air density
C_d = 0.295 # drag coefficient
diameter = 0.00782 # m (7.82 mm)
A = np.pi * (diameter / 2) ** 2 # cross sectional area of bullet
m_bullet = 0.00603 # mass of bullet
v_initial = 1330 * 1000 / 3600 # initial velocity
target_height = 500 # height of the target
```

```
g = 9.81
```

```
def drag_force(v):
    return 0.5 * rho * C_d * A * v**2
```

```
def equations_with_theta(theta):
    def equations(t, y):
        x, y_pos, v_x, v_y = y
        v = np.sqrt(v_x**2 + v_y**2)
        F_drag = drag_force(v)
        F_drag_x = F_drag * (v_x / v)
        F_drag_y = F_drag * (v_y / v)
        dxdt = v_x
        dydt = v_y
        dvxdt = -F_drag_x / m_bullet
        dvydt = -g - F_drag_y / m_bullet
        return [dxdt, dydt, dvxdt, dvydt]
    return equations
```

```
def reach_target_height(t, y):  
    return y[1] - target_height  
reach_target_height.terminal = True
```

```
def minimum_distance(theta_degrees):  
    theta = np.radians(theta_degrees)  
    initial_conditions = [0, 0, v_initial * np.cos(theta), v_initial * np.sin(theta)]
```

```
    solution = solve_ivp(  
        equations_with_theta(theta), [0, 100], initial_conditions,  
        dense_output=True, events=reach_target_height  
    )
```

```
    if solution.status == 1:  
        distance = solution.y[0, -1]  
        return distance  
    else:  
        return None
```

```
    angle_1, angle_2 = None, None  
    for angle in range(1, 90):  
        dist = minimum_distance(angle)  
        if dist:  
            if angle_1 is None:  
                angle_1 = angle  
            elif angle_2 is None:  
                angle_2 = angle  
            break
```

```
print(f"Two angles for hitting the target: {angle_1}° and {angle_2}°")  
print(f"Minimum distance to hit the window with drag included: {minimum_distance(angle_1):.3f} m")
```

Output:

Two angles for hitting the target: 38° and 39°

Minimum distance to hit the window with drag included: 834.230 m