

Physics Practice Homework #5

"""

Calculations without air resistance:

Time to reach max height:

$$v_f = v_0 - g \cdot t$$

$$\Rightarrow t = v_0 / g = 37.66$$

Max height:

$$x = 0.5 (v_0 + v_f) t = 0.5 v_0 t = 0.5 v_0^2 / g = 6956.47$$

Velocity upon impact:

$$v_f^2 = v_0^2 - 2gx$$

$$\Rightarrow v_f = -\sqrt{2gx} = 369.44$$

(can also be trivially proven by conservation of energy, meaning the kinetic energy and hence the velocity at launch and at impact is the same)

"""

```
import numpy as np
import matplotlib.pyplot as plt
from mpmath import mp
import time
mp.dps = 10
```

```
# Constants
```

$$g = 9.81$$

$$\text{mass} = \text{mp.mpf}(0.00603)$$

$$C_d = \text{mp.mpf}(0.295)$$

$$\rho = \text{mp.mpf}(1.293)$$

$$\text{diameter} = \text{mp.mpf}(7.82\text{e-}3)$$

$$A = \text{mp.pi} * (\text{diameter} / 2) ** 2$$

$$v_0 = \text{mp.mpf}(369.44)$$

$$\text{dt} = \text{mp.mpf}(0.001)$$

```
def drag_force(v):
```

$$\text{return } 0.5 * C_d * \rho * A * v ** 2$$

```
def simulate_motion_without_drag(v0, dt):
```

$$y = 0$$

$$v = v_0$$

$$t = 0$$

```

positions = []
velocities = []
times = []

while y >= 0:
    F_gravity = mass * g
    F_net = F_gravity

    a = -F_net / mass

    v += a * dt
    y += v * dt
    t += dt

    positions.append(float(y))
    velocities.append(float(v))
    times.append(float(t))

return np.array(times), np.array(positions), np.array(velocities)

```

```

def simulate_motion_with_drag(v0, dt):
    y = 0
    v = v0
    t = 0
    positions = []
    velocities = []
    times = []

    while y >= 0:
        F_gravity = mass * g
        F_drag = drag_force(v)
        if v >= 0:
            F_net = F_gravity + F_drag
        elif v < 0:
            F_net = F_gravity - F_drag

        a = -F_net / mass

        v += a * dt
        y += v * dt
        t += dt

        positions.append(float(y))
        velocities.append(float(v))
        times.append(float(t))

    return np.array(times), np.array(positions), np.array(velocities)

```

```
drag_bool = input("With air resistance (True or False): ").strip().lower() == 'true'
```

```
if drag_bool:
```

```
    times, positions, velocities = simulate_motion_with_drag(v0, dt)
```

```
elif not drag_bool:
```

```
    times, positions, velocities = simulate_motion_without_drag(v0, dt)
```

```
max_height = np.max(positions)
```

```
time_to_ground = times[-1]
```

```
velocity_on_impact = velocities[-1]
```

```
print(f"Maximum height: {max_height:.3f} m")
```

```
print(f"Time to hit the ground: {time_to_ground:.3f} s")
```

```
print(f"Velocity upon impact: {abs(velocity_on_impact):.3f} m/s")
```

```
# Plot the height vs time graph
```

```
plt.plot(times, positions)
```

```
plt.title('Bullet Height vs Time with Air Resistance')
```

```
plt.xlabel('Time (s)')
```

```
plt.ylabel('Height (m)')
```

```
plt.grid(True)
```

```
plt.show()
```

Bullet Height vs Time with Air Resistance

