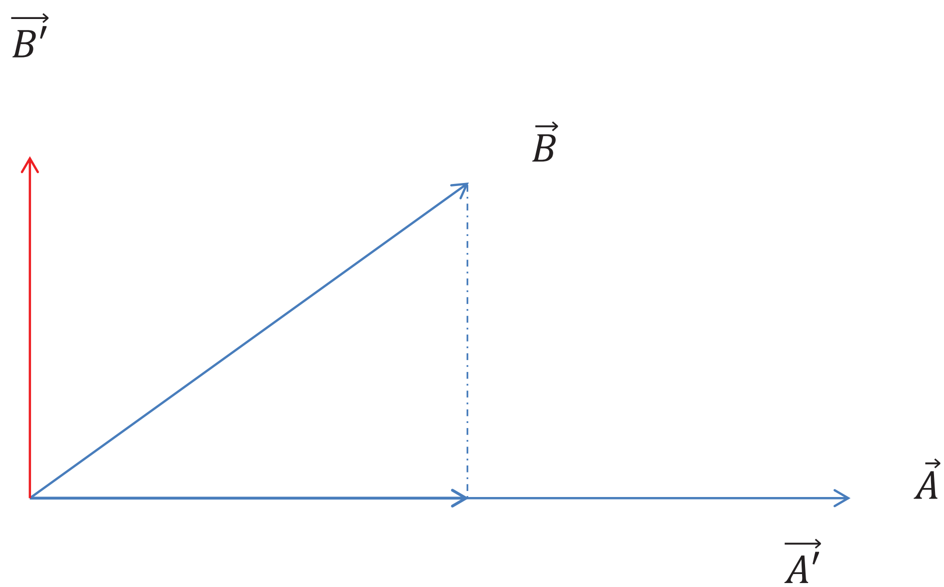
Gram Schmidt Orthogonalization

It is an algorithm of obtaining a set of orthogonal vectors from a set of linearly independent vectors



$$\overrightarrow{\boldsymbol{A}'} = \vec{\boldsymbol{A}} \ , \quad \overrightarrow{\boldsymbol{B}'} = \vec{\boldsymbol{B}} - \left(\vec{\boldsymbol{B}} \cdot \widehat{\boldsymbol{A}}'\right) \ \widehat{\boldsymbol{A}}'$$

$$\overrightarrow{\boldsymbol{A}'} = \overrightarrow{\boldsymbol{A}} \; , \qquad \overrightarrow{\boldsymbol{B}'} = \overrightarrow{\boldsymbol{B}} - \left(\overrightarrow{\boldsymbol{B}} \cdot \widehat{\boldsymbol{A}'}\right) \widehat{\boldsymbol{A}'}$$

$$\overrightarrow{\boldsymbol{B}'} = \overrightarrow{\boldsymbol{B}} - \frac{\left(\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}'}\right)}{\left|\overrightarrow{\boldsymbol{A}'}\right|} \frac{\overrightarrow{\boldsymbol{A}'}}{\left|\overrightarrow{\boldsymbol{A}'}\right|}$$

$$\overrightarrow{\boldsymbol{B}'} = \overrightarrow{\boldsymbol{B}} - \left(\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}'}\right) \frac{\overrightarrow{\boldsymbol{A}'}}{\overrightarrow{\boldsymbol{A}'} \cdot \overrightarrow{\boldsymbol{A}'}}$$

Vector space : Inner product

$$\mid \boldsymbol{A}' > = \mid \boldsymbol{A} > \quad \mid \boldsymbol{B}' > = \mid \boldsymbol{B} > - < \boldsymbol{A}' \mid \boldsymbol{B} > \frac{\mid \boldsymbol{A}' >}{< \boldsymbol{A}' \mid \boldsymbol{A}' >}$$

Inner product of two vectors belong to R^n are :

$$<\boldsymbol{A} \mid \boldsymbol{B}> = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{n}} \boldsymbol{a}_{\boldsymbol{i}} \boldsymbol{b}_{\boldsymbol{i}} = \boldsymbol{A}.\,\boldsymbol{dot}(\boldsymbol{B})$$

$$\mid \boldsymbol{A}'> = \mid \boldsymbol{A}>$$

$$\mid \boldsymbol{B}'> = \mid \boldsymbol{B}> - <\boldsymbol{A}' \mid \boldsymbol{B}> \frac{\mid \boldsymbol{A}'>}{<\boldsymbol{A}' \mid \boldsymbol{A}'>}$$

$$\mid C'> = \mid C> - <A' \mid C> \frac{\mid A'>}{<A' \mid A'>} - <B' \mid C> \frac{\mid B'>}{<B' \mid B'>}$$

Inner product of matrices :

$$<\boldsymbol{A} \mid \boldsymbol{B}> = \boldsymbol{trace}(\boldsymbol{A}^{\boldsymbol{T}} \boldsymbol{B})$$