Acharya Prafulla Chandra College

Physics Honours   Semester IV

Paper: PHSACOR01P

Assignment for Mathematical Physics III Lab

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1. **ODE initial value problems by RK2 & RK4**
   (a) Find numerical solution of the equation $\dot{x} = \cos(\pi t)$within [0, 2] with initial condition $x(t = 0) = 0$ and $\Delta t = 0.1$ using i) Euler method, ii) RK2 method and iii) RK4 method. Plot all the solutions on same on the same graph using matplotlib.pyplot for comparison. Use different line-types for multiple plots on same grapgh.
   (b) Reduce the step size to one fifth of the previous one and repeat the work.
   (c) Solve the Hamilton's equations of one dimensional motion of a particle with unit mass attached with a spring (of spring constant π^2) fixed with a rigid support at the other end, taking $\Delta t = 0.02$and draw its phase space plot applying Euler method and RK4 method on same graph paper along with the plot from analytical solution. Plot to be done for at least one complete cycle

2. **Use of numpy ndarray objects for various matrix operations**
   (a) Create the following matrices using 'eye' , 'one' and 'zeros' of numpy array:
   $A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
   and perform following matrix operation: i) AB, ii) Create a (3×3) matrix D from A and C^T, iii) Create matrix E,multiplying B and D element wise, iv) Replace 2^{nd}column of E by C.
   (b) Create a 6×2 integer array from a range between 100 to 220 such that the difference between each element is 10. Split the array into three equal-sized sub-arrays.
   (c) Create a (3×3) numpy array with the following complex entities: [3, 2i, 6, 7, 4, 5i, 2, 6, 3]. Generate Hermintian and unitary matrices from it and check.

3. **Solution of Linear system of equations by Gauss elimination method and by Gauss Jordan method.**
   (a) Write a python program to implement the Gauss elimination method (partial pivoting is preferable) to determine the determinant of the co-efficient matrix of a system of linear equation. If the determinant becomes non-zero solve the equations by back-substitution. Apply this program to the following system of linear equations: $2x + 2y + z = 6$; $4x + 2y + 3z = 4$; $x + y + z = 0$.
   (b) Modify the above program to implement it for Gauss-Jordan method to determine the determinant of the co-efficient matrix of a system of linear equation and if the determinant is non-zero solve the system of equations and also find the inverse of the co-efficient matrix. Apply it for the following system of equations: $2x + 4y - 6z = -8$; $x + 3y + z = 10$; $2x - 4y - 2z = -12$.

4. **Solution of linear system of equations by Gauss-Seidal iterative method.**
   (a) Write a python program to implement the Gauss Gauss-Seidal iterative method to solve a system of linear equations. Apply this program to the following system of linear equations: $2x + y + z = 5$; $3x + 5y + 2z = 15$; $2x + y + 4z = 8$.
   (b) Choosing suitable set of constants in inhomogeneous part of linear equations, find the inverse of the co-efficient matrix.
   (c) Use the code for determinant calculation through Gauss elimination method to show that the determinant of inverse matrix is the reciprocal of the same of the original matrix.

5. **Gram-Schmidt orthogonalisation method**
   (a) Write a python program to implement the Gram-Schmidt orthogonalisation method to get a set of orthogonal vectors from any number of arbitrarily chosen linearly independent vectors (R^n) and verify the result explicitly with inner products.[***Take input from user***]Apply the method to the following sequence of vectors in R^3 : (1, 2, 0), (8, 1, -6), (0, 0, 1)
   (b) Write a python program to implement the Gram-Schmidt orthogonalisation method to get a set of orthogonal vectors from any number of arbitrarily chosen linearly independent vectors (R^n) and verify the result explicitly with inner products. [***Take input from file***] Apply the method to the following sequence of vectors in R^4 : (1, -1, 1, 1), (1, 0, 1, 0), (0, 1, 0, 1).

6. **Explicit calculation of largest eigenvalue calculation by power iterative method for real symmetricmatrix and corresponding eigenvector.**
   (a) Write a python program to check whether an input matrix is symmetric or nor and if the matrix is symmetric ind the largest eigen-value by magnitude and corresponding eigenvector. Apply it to the matrix A=([9, 10, 8], [10, 5, -1], [8, -1, 3])
   (b) Modify the above program to accept the matrix from a file. Apply it to the matrix B=([1, 2, 0], [2, 1, 0], [0, 0, 1])