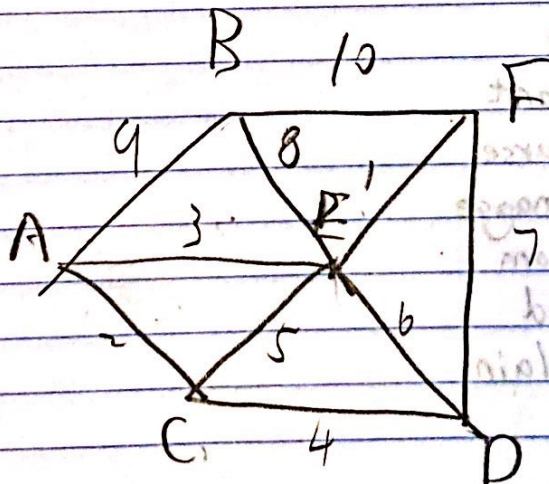


1.a)

Top	Top	Top	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> OPS OPT POP POT SOP SOT TOO TOP </div>
POT	POP	POT	
Top	SOP	SOP	
OPT	OPS	POT	
OPS	POT	SOT	
SOT	OPT	OPS	
POP	SOT	OPT	
SOP			

✓

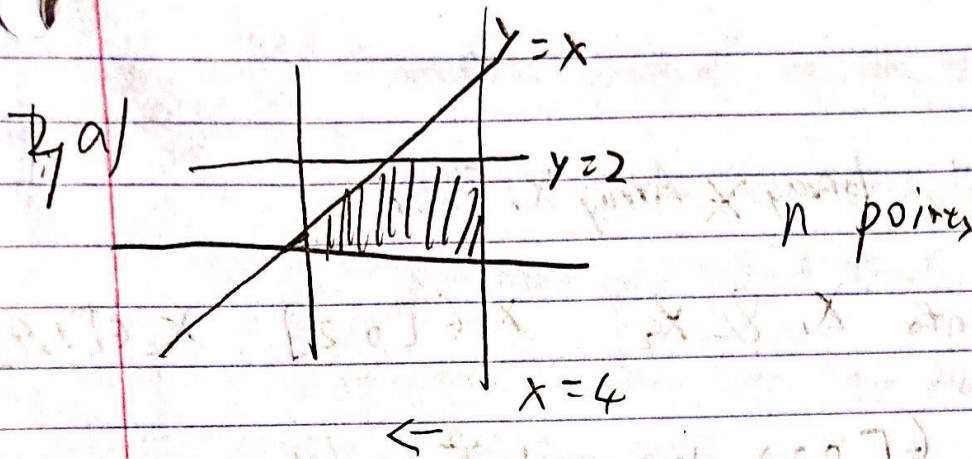
2 a) 12348



b) 23148

c) 23149

CMSC351 Final Yizhan Ao



Show some points by their distance to y axis in any linear time

Assume bucket sort works in linear time. $n = 6k$

We can divide the points into 6 intervals based on

the n value, there will be points (x, y) that

$$\frac{2}{3}(i-1) \leq x_i \leq \frac{2}{3}i$$

There will be points (x, y) that $\frac{2}{3}(i-1) \leq x_i \leq \frac{2}{3}i$

the using an empty array and then to use the bucket sort to sort the Array from 0 to $n-1$ that we append each point to be one of the elements.

b. Pseudo code

T. Input Array X Array Y , Int n

Split x into X_1 & X_2 $X_1 \in [0, 2]$, $X_2 \in [2, 4]$

For $X_1 \in [0, 2]$ we need $\frac{n}{3}$ buckets

for $i = 1$ to $\frac{n}{3}$ }

put $X_1[i]$ into Bucket $B[\frac{n}{3} \cdot X_1[i]]$

end for

sort each Bucket

Making each Bucket

For $X_2 \in [2, 4]$ use $\frac{2n}{3}$ Buckets

for $i = 1$ to $\frac{2n}{3}$ }

put $X_2[i]$ to $B[\frac{1}{3}n \cdot (X_2[i] - 2)]$

sort each Bucket

Make each Bucket

end for

sort X_1, X_2

end for

5a) Since each pair of neighbors can only be involved with 1 communication, we can have every other column of processors send their number to the right neighbor. Then we can have every other row of processors to be able to move up. Then move to the right neighbor. Then we have every other row of processors to move up. Then move to the right of each added other column to its neighbor column then decrease by a factor of 2. Then move to the up added every other row to its neighbor decrease the factor by 2 again. Repeat the process until we add up all numbers.

$$5.6) O(n^3)$$

$$5.7) O(n^3)$$

Problem 6 Room form hold 3 students



$H(i, j, k)$

$n \times n \times n$ Matrix
 n is multiple of 3

partition $\{1, \dots, n\}$
into T to maximize H

- 1) Optimization $H(i, j, k)$ n is $3k$ $k \in \mathbb{N}^{29}$



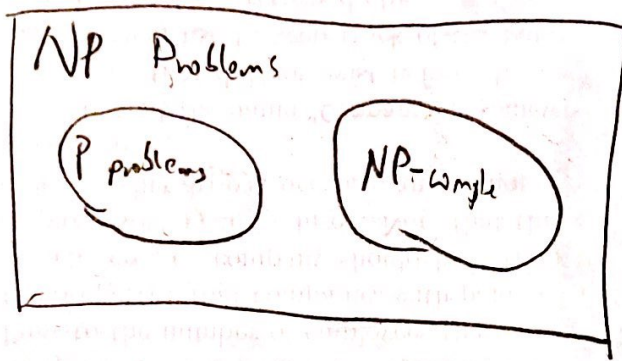
Assume: i, j, k rankings are going from top to low ①

2) Decision version of roommate assignment $n \times n \times n$

$H(i, j, k) \geq G$ "goal" $T \Rightarrow \{1, \dots, n\}$

Q: is there T into sum $H(i, j, k)$ At least G

a) SHOW Decision of Roommate assignment Problem is in NP



We are proving this is in P
Therefore since P is inside of NP problem, we can tell this problem is a NP problem

Prove the verification is in a polynomial time

Given a matrix of $H(i, j, k)$

Assume there are 5 students representing A, B, C, D, E, F

H Matrix	i	j	k
	A:	(B, C, D)	
	B:	(C, E, D)	
	C:	(B, E, D)	
	D:	(A, E, C)	
	E:	(A, B, C)	

Assume (i, j, k) are different people &

Assume they can't choose themselves to be (i, j, k)

Don't consider where i, j are same. ★

We don't know what G is

To make i, j, k meaningful we make $i=1, j=0, k=-1$, not in ijk is -2

Algorithm: Firstly, we let each person ask their favorite people (i) to live together, then move on to the 2nd favourite (j)
Each response will be "no" or "yes", If response Yes, then live together. No move on

* Room(1, 2)

Secondly. If the (i) says no, then each person ask their
Second choice if it is "Yes" together. If Yes, live together.
If no, move on.

Thirdly. ~~Ask~~ If (j) says no, then move on to (k). ~~which~~

The rest will be move to a same room if the answer
is "No" again. The rest of (k) people will form a room

The complexity so far is (n^k) Therefore, it is a polynomial
algorithm. Therefore it is a NP problem. - completeness

If G is solvable

b) Show solve optimization in Polynomial time.

```

function optimisation(matrix H(i,j,k), int n) {
    if threepeopleInHouse == true then
        return true
    end
    else
        return false
    end
    Graph X = (V, E);
    if (V = (n mod 3)) // n is divisible by 3
        if (each(T) contains 3E) // then is a V into n
            sets of H (T)
            for (each T set {x1, x2, x3})
                Make Ti = {x1, x2, x3}
                {x1, x2}, {x2, x3}, {x1, x3} ∈ E.
                threepeopleInHouse == true
            end if
        end if
    }
}

```

Runtime $O(n^3)$ If optimization is solved by this therefore decision is also in NP-completeness

Q) If solve decision in Poly then solve opt poly

function decision (matrix $H(i,j,k)$; int n , int G) {
 if three people in the house == true; \wedge three people in the house == false

return true;

end if

else

return false;

$M = H$;

if M matches $I(G)$ // $I(G)$ is the solution of perfect matching
 three people in the house = true; so that every one can get a house guaranteed
 end if

else $I = [a, b, c, d]$ // a, b, c, d are in the m
 a, b, c, d are people representing here

$I = \text{outgoe } [a, b, c, d]$ // d is the match

AA

match M for I with $\{a, b, c\}$ & $\{e, x, y\}$

three people in the house = true

// based on their level in $\{i, j, k\}$

for (each element in I)

add (i, j, k) ; // add value to represent likeness
 end for

end else

}
 runtime $O(\sum_{i=1}^m r_i) = O(n^2)$