Friday, December 11, 2020 11:24 AM

Problem 10. Random pick 2 numbers from $\overbrace{(n)}^{\text{numbers}}$ list

win if at least 1 is among i smallest

a) $P = \frac{i}{n} \cdot \frac{n-i}{n-1} = \frac{n \cdot (i)}{n(n-1)}$

First number from list i, the second number from list of i $\frac{n-i}{n} \frac{i}{n-1} = \frac{(n-i)i}{n(n-1)}$

Both number from list of i

$\frac{1}{n} \frac{i-1}{n-1} = \frac{(i-1)i}{n(n-1)}$

$$P = \frac{(n-i)i}{n(n-1)} + \frac{(n-i)i}{(n-1)n} + \frac{(i-1)i}{n(n-1)}$$

$$= \frac{2ni - i^2 - i}{n(n-1)}$$

b) Both number belong to the list of i

$$P = \frac{(i-1)i}{n(n-1)}$$

c) let $i = n/2$

$$P = \frac{((\frac{n}{2})-1)\,n/2}{n(n-1)} = \frac{n-2}{2} \cdot \frac{n}{2} \cdot \frac{1}{n(n-1)}$$

$$P = \frac{n-2}{4n-4}$$

Problem 11

a) $(1-P) + 2P(1-P) + 3P^2(1-P) + \ldots$

$= P(1-P) + 2P^2(1-P) + 3P^2(1-P) + \ldots$

b) $(1-P)E(x) = 1(1-P) + P(1-P) + P^2(1-P) + \ldots$

$= 1 + P + P^2 + P^3 + \ldots + P^k$

$= \frac{1}{1-P}$

c) $1(1-r) + 2r(1-r^2) + 3r^2(1-r^3) + \ldots$

$= (1-r) + 2r^2(1-r^2) + 3r^3(1-r^3) + \ldots$

Average i

$(1-r) + 2r^2(1-r^2) + 3r^3(1-r^3) + \ldots$