Problem 1

a) $T(n) = (n-1) + \frac{1-p}{n} \sum_{q=1}^{n} T(\max(q-1, n-q))$

$= n-1 + \frac{2}{n} \sum_{\frac{n}{2}}^{n-1} (1-p)\, T(q)$

If $T(n) \leq an$

$T(n) = (n-1) + \frac{2a}{n}(1-p) \sum_{\frac{n}{2}}^{n-1} q$

$= \frac{3}{4}\left[a(1-p)+1\right]n - \frac{a}{2}(1-p) - 1$

b) $\frac{3}{4}\left[a \cdot \frac{2}{3} + 1\right]n - \frac{a}{2} \cdot \frac{2}{3} - 1$

$= \frac{a}{2}n + \frac{3}{4}n - \frac{a}{3} - 1 \leq a_n$

$\frac{3}{4}n \leq \frac{a}{2}n \quad a \geq \frac{3}{2} \quad T(n) \geq \frac{3}{2}n$

c) $p = \frac{2}{3}$

$\frac{3}{4}\left[a \cdot \frac{1}{3} + 1\right]n - \frac{a}{2} \cdot \frac{1}{3} - 1$

$= \left(\frac{a}{4} + \frac{3}{4}\right)n - \frac{a}{6} - 1 \leq a_n$

$\frac{a}{4}n + \frac{3}{4}n - \frac{a}{6} + 1 \leq a_n$

$\frac{3}{4}n \leq \frac{a}{6}n$ $\quad T(n) \geq \frac{3}{2}n$

$n \leq a$

$T(n) \leq n$

Problem 2

a) median of a list

$$\frac{n(n+1)}{2} = \frac{15 \times 14}{2} = 105 \text{ comparison}$$

b) (i) $n\lceil \lg n \rceil - 2^{\lg n} + 1 = 45$ comparison

(ii) $= 15 \times 4 - 15 = 46$

(iii) Merge sort

```
j=L
k=mid+1
While (p≤mid && k≤q)
  if (a[p] < a[k]){
    b[i] = a[p]
    p++;
    j++;
  }else{
    b[i] = a[k]
    k++
    j++
  }
```

Problem 2

a) $T(n) = (n-1) + (n-2) + \cdots + \frac{n}{2}$

$n = 15 \quad T(n) = 14 + 13 + \cdots + 7 = 84$

b) $\frac{n \lg n - n + 1 - n + 1}{2} + n - 1$

$= \frac{n}{2}(\lg n - n + 1 + n - 1) = \frac{n}{2} \lg n$

$n = 15 \quad \frac{15}{2} \lg 15 \approx 7 = 4 * 7 = \boxed{28}$

Problem 4

a) $n - \frac{4n}{15} - \frac{4n}{15} = \frac{7}{15}n$

b) $\frac{8n}{15} \cdot \frac{1}{2} = \frac{8n}{30} = \frac{4}{15}n$

c) $T(n) = \frac{28n}{15} + T\left(\frac{n}{15}\right) + \frac{7}{15}n + T\left(\frac{11n}{15}\right)$

d) If $T(n) \leq an$

$$\leq \frac{an}{15} + \frac{a_{11}n}{15} + \frac{28n}{15} + \frac{7}{15}n$$

$$= \frac{12}{15}a_n + \frac{7}{3}n$$

$$= \left(\frac{4}{5}a + \frac{7}{3}\right)n$$

$$\left(\frac{4}{5}a + \frac{7}{3}\right) \leq a$$

$$\frac{7}{3} \leq \frac{1}{5}a$$

$$\frac{35}{3} \leq a \Rightarrow T(n) \leq 11.67n$$

Since 15 is faster $11.67n < 16n$