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Problem 1

a) $A = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$P(A) = \det(A - \lambda I) = \det\left(\begin{bmatrix} -1-\lambda & 1 \\ 5 & 3-\lambda \end{bmatrix}\right)$

$P(\lambda) \Rightarrow \left((-1-\lambda)(3-\lambda) - 5\right) = -3 - 3\lambda + \lambda^2 + \lambda - 5 = \lambda^2 - 2\lambda - 8$

b) $\lambda^2 - 2\lambda - 8 = 0$ ($P(\lambda) = 0$)

Eigen Values: $(-2, 4)$

$(\lambda+2)(\lambda-4) = 0$

$\boxed{\lambda_1 = -2,}$ $\boxed{\lambda_2 = 4}$

c) when $\lambda = -2$

$\begin{bmatrix} -3 & 1 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -3 & 1 & 0 \\ 5 & 5 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$-x_1 = x_2$

Eigen Vector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

when $\lambda = 4$

$\begin{bmatrix} -5 & 1 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -5 & 1 & 0 \\ 5 & -1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 5 & -1 & 0 \end{bmatrix}$

$5x_1 = x_2$

$\therefore x_1 = \frac{1}{5} x_2$

Eigen Vector $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$

2) a) T b) T c) T d) F e) T