4352 — y

× 3748 — x

$$xy = 10^4(37\times43) + 10^2(37+48)(43+52) + 48\times52$$

$37 \times 43$

$$10^2(4\times3) + 10(4+3)(3+7) + 3\times7$$

$07 \times 10$

$$10^2(0\times1) + 10(1\times7) + 7\times0$$

$85 \times 95$

$$10^2(9\times8) + 10(9+5)(8+5) + 5\times5$$

$14 \times 13$

$$10^2(1\times1) + 10(1+3)(1+4) + 4\times3$$

$48 \times 52$

$$10^2(5\times4) + 10(5+2)(4+8) + 2\times8$$

$07 \times 12$

$$10^2(1\times0) + 10(3\times7) + 2\times7$$

(1) (2) (3) (4) (5) (6) (7) 8 9 10 11 12

(Atomic Multiplications)

Total # of atomic multiplications:

12

(we don't count multiplications with a zero)

# HWK 5

2 (1)

$$y \quad \underbrace{y_{n-1}\ y_{n-2} \cdots y_{n/3}}_{d}\ \underbrace{y_{n/3+1} \cdots y_{2n/3}}_{e}\ \underbrace{y_{2n/3+1} \cdots y_2\ y_1\ y_0}_{f}$$

$$\times$$

$$x \quad \underbrace{x_{n-1}\ x_{n-2} \cdots x_{n/3}}_{a}\ \underbrace{x_{n/3+1} \cdots x_{2n/3}}_{b}\ \underbrace{x_{2n/3+1} \cdots x_2\ x_1\ x_0}_{c}$$

$$x = 10^{2n/3}a + 10^{n/3}b + c$$

$$y = 10^{2n/3}d + 10^{n/3}e + f$$

$$(x \cdot y) = \left(10^{2n/3}a + 10^{n/3}b + c\right)\left(10^{2n/3}d + 10^{n/3}e + f\right)$$

$$= 10^{4n/3}ad + 10^{n}(ae + bd) + 10^{2n/3}(be + af + cd) + 10^{n/3}(bf + ce) + cf$$

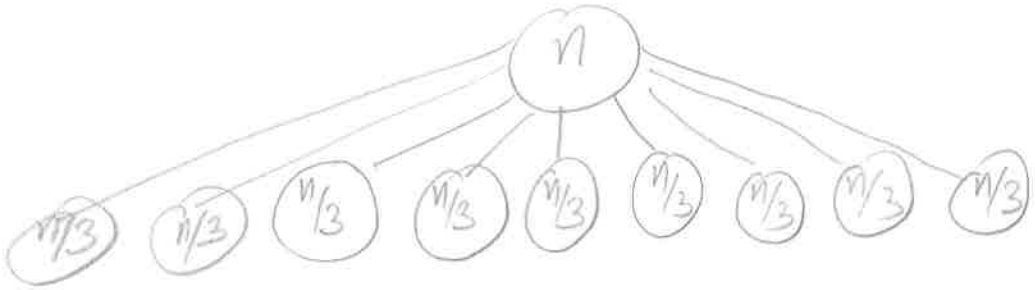
$2\alpha n/3$ — $4\alpha n/3$ — $2\alpha n/3$

Total Additions : $4\alpha n$

Total Multiplications : 9

$$M(n) = 9M(n/3) + 4\alpha n, \quad M(1) = \mu$$

2(2) $M(n) = 9M(n/3) + 4\alpha n$




$4\alpha n$

$9 \cdot 4\alpha \frac{n}{3}$

$9^{\log_3 n - 1} \cdot 4\alpha \frac{n}{3^{\log_3 n - 1}}$


$\mu 9^{\log_3 n}$

$$M(n) = \mu n^2 + \sum_{i=0}^{\log_3 n - 1} 9^i \cdot 4\alpha \frac{n}{3^i}$$

$$= \mu n^2 + 4\alpha n \sum_{i=0}^{\log_3 n - 1} 3^i$$

$$= \mu n^2 + 4\alpha n \left[1 + 3 + \ldots + 3^{\log_3 n - 1}\right]$$

$$= \mu n^2 + 4\alpha n \left[\frac{3^{\log_3 n} - 1}{3 - 1}\right]$$

$$= \mu n^2 + 2\alpha n [n - 1]$$

2(3) $O(n^2)$

2(4) When # of multiplications is 8

Runtime = $O(n^{\lg_3 8})$

$= O(n^{1.89})$

2(5)

| # of multiplications | Runtime |
|---|---|
| 7 | $O(n^{\lg_3 7}) = O(n^{1.77})$ |
| 6 | $O(n^{\lg_3 6}) = O(n^{1.63})$ |
| 5 | $O(n^{\lg_3 5}) = O(n^{1.46})$ |

We need 5 multiplications.

2(6)

| # of multiplications | Runtime |
|---|---|
| 9 | $O(n^{\lg_4 9}) = O(n^{1.58})$ |
| 8 | $O(n^{\lg_4 8}) = O(n^{1.5})$ |

9 multiplications is almost equivalent, 8 multiplications definitely gives a better runtime.