351 HW1

Problem 1:

a) $\sum_{i=1}^{4} i(i+1) = 1(2) + 2(3) + 3(4) + 4(5) = 20$

b) $\sum_{i=0}^{4} 2^i = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 31$

Problem 2: $3\sum_{i=1}^{n}(5i^2-4) - 2\sum_{i=1}^{n}(3i^2-1)$

$= \sum_{i=1}^{n}(15i^2-12) - \sum_{i=1}^{n}(6i^2-2)$

$= \sum_{i=1}^{n} 9i^2 - 10$

Problem 3:

a) $b^x = a \Rightarrow \log_b b^x = \log_b a \Rightarrow x = \log_b a$

b) $c^x = ab$ ① $y = \log_c a \Rightarrow c^y = a$

② $z = \log_c b \Rightarrow c^z = b$

From ① & ② we have $a \cdot b = c^y \cdot c^z = c^{y+z}$

$\log_c a \cdot b = \log_c c^{y+z} = y+z$

Therefore $\log_c a \cdot b$, Done

c) $a^{\log_b n} = \left(b^{\log_b a}\right)^{\log_b n}$

$= b^{(\log_b a)\cdot(\log_b n)}$

$= \left(b^{\log_b n}\right)^{\log_b a}$

$= n^{\log_b a}$

Done

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Problem 4.

a) $\frac{df(\ln x^2+5)}{dx} = \frac{2x}{x^2+5}$

b) $\frac{df}{dx}(\lg(x^2+5)) = \frac{2x\log(e)}{x^2+5}$

c) $\frac{df}{dx}\left(\frac{1}{\ln(x^2+5)}\right) = \frac{df}{dx}\left((\ln(x^2+5))^{-1}\right) = -\frac{2}{x(\ln(x^2+5))^2}$

Problem 5: a) $\int \frac{1}{x}dx = \ln|x| + C$

b) $\int \frac{1}{7x+3}dx = \frac{\ln|7x+3|}{7} + C$

c) $\int \ln(x)dx = \int uv' = uv - \int u'v = x\ln x - x + C$

d) $\int x\ln x\,dx = \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$

e) $\int x\lg x\,dx = \log(e)\left(\frac{x^2\ln x}{2} - \frac{x^2}{4}\right) + C$

Problem 6: a) $\frac{1}{n}$

b) $\frac{2}{n}$

c) $\frac{1}{n}$

d) $\frac{2}{n}$

e) $\frac{1}{n} \cdot \frac{1}{n}$

Problem 7:

a) $3n^4$

b) $7n^3\log n$

c) $\Theta(n^4)$

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