

Problem 1. Given the following list of numbers, [20, 0, 10, 6, 4, 9, 14, 15, 7, 13], answer the following questions:

1. How would the list look like after three complete passes of bubble sort?

**Solution:** [0, 4, 6, 9, 10, 7, 13, 14, 15, 20]

2. How many comparisons have been carried out after 5 passes of bubble sort? How did you find it? Describe in two or three sentences.

**Solution:** 35

There will be  $n-1(=9)$  comparisons in the first pass,  $n-2(=8)$  comparisons in the second pass and so on, for a total of,  $9 + 8 + 7 + 6 + 5 = 35$  comparisons after 5 passes.

Problem 2. An alien is lost on earth and wants to return home. The only way for the alien to return is to retrieve a key to the space shuttle. The key is hidden in one of the cells of an,  $n \times n$ , rectangular grid. Each cell in this rectangular grid contains a unique integer. One of these integers will help him retrieve the key. A kind soul feels bad for him and comes to his rescue by giving him three important pieces of information:

- the integer that would retrieve the key.
- that the numbers are stored in the increasing order from the top down in each column and left to right in each row.
- that he should start from the bottom left cell and try to reach the top right cell.

Even with this information the alien still has to find the cell that contains the integer and hence the key to the space shuttle. In order to find the cell containing his key the alien would have to compare the integer given to him with the number stored in a given cell. You are supposed to answer the following:

1. Describe the most optimal strategy (or pseudocode) in a sentence or two for the alien to find the cell that contains the key. What is the runtime of your approach?

**Solution:** Compare the integer,  $x$ , to the bottom-left element of the grid. If  $x$  is less than this element, move up one element. If  $x$  is greater than this element, move to the right. Continue until  $x$  is found.  $O(n)$

2. Find the number of comparisons in the worst case as a function of  $n$  if a single comparison returns whether the integer he knows, is smaller, larger, or the same as the number stored in the cell.

**Solution:**  $2n - 2$

3. Find the number of comparisons in the worst case as a function of  $n$  if two comparisons are needed to return whether the integer he knows, is smaller, larger, or the same as the number stored in the cell.

**Solution:**  $4n - 3$

4. If the alien's life decreases by 1 every time a comparison is made, how big a grid (2D) would the alien survive starting with a life of 50, for the case with one comparison (part 2 above) and the case with two comparisons (part 3 above). You may assume a square grid.

**Solution:** single comparison: grid size =  $26 \times 26$   
Two comparisons: grid size =  $13 \times 13$