EXAM 351 MIDTERM

1) 30 40 10 20

a) 6 Bubblesort comparison: 1+1+1+1+1+1

b) 7 Insertion sort with a sentinel: 1+1+1+1+1 1+1+1 1+1

c) 4 Mergesort: 30 40 10 20 1+1 +1+1

10 20 30 40

d) 5 Heap 40 30

e) 3

40 10 → 20 40 10 → 20 30 40 10

30

1+1+1+1+1+1+1+1

40

30 20

10

Quicksort

30 20 1+ 10 20

3.

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1 7) $T(n) = 2T(n/4) + 3n$ $\boxed{T(1)=5}$ $3n$

2 $T(n/4)$ $T(n/4)$ $2\cdot\left(\frac{n}{4}\right)\cdot 3$

4 $T(n/4^2)$ $T(n/4^2)$ $T(n/4^2)$ $T(n/4^2)$ $4\cdot\left(\frac{n}{4^2}\right)\cdot 3$

$8\cdot\left(\frac{n}{4^3}\right)\cdot 3$

8 $T(n/4^3)$ $T(n/4^3)$ $T(n/4^3)$ $T(n/4^3)$

$2^{(n-1)}$ $T\left(\frac{n}{4^{n-1}}\right)$ $T\left(\frac{n}{4^{n-1}}\right)$ $2^{n-1}\cdot\left(\frac{n}{4^{n-1}}\right)\cdot 3$

$5\cdot 2^{\log_4 n}$

2^n 5 5 5 5

$$T(n) = \sum_{i=1}^{\log_4 n - 1} 2^i \cdot 3\left(\frac{n}{4^i}\right) = 3n \sum_{0}^{\log_4 i} \left(\frac{1}{2}\right)^i = 3n\cdot\left(\frac{\frac{1}{2}^{\log_4 n} - 1}{\left(\frac{2}{4} - 1\right)}\right)$$

$$= 3n\cdot\left(\frac{n^{\log_4\left(\frac{1}{2}\right)} - 1}{-\frac{1}{2}}\right) = 3n\left(\frac{\frac{1}{\sqrt{n}} - 1}{-\frac{1}{2}}\right)$$

$$= -\frac{\frac{3n}{\sqrt{n}}}{2} + \frac{3n}{\frac{1}{2}} = \boxed{6n - \frac{6n}{\sqrt{n}}}$$

$$T_n = T(\theta) = \boxed{6n - \frac{6n}{\sqrt{n}}}$$

3) R[0], R[1], R[2] ... R[m-1]

m = s + 16

read(g, h, d), write(g, h, d)

a) Selection sort sedo code (i) (i, j, k, n).

s always divides (i)

```
for i=n downto 2 do
    if i≤s then read (1,1,i)        ← k ← 1
    for j=2 to i do
     if A[j] > A[k] then A[j] ↔ A[k]
     end if
    end for  ← A[k] ↔ A[i]
    write (1,1,i)
    else
        R[0] ← R[min(s,i)]
        r ← min(s,i)
        while r+s < i do
            read (1,r,s)
            for j=0 to s-1 do
                if R[j] > R[j+1]
            end if
            end for
            write (r,0,s)
            j[0] ← R[s]
            r ← r+s
        end while
    read (r,1,i-r)
     for j=0, to i-r do
      if R[j] > R[j+1]
```

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then $R[j] \leftrightarrow R[j+1]$
end if
end for
write $(r, 0, i-r)$
end if
end for

b) case of memory $f(n)=3$ ~~comp exchang~~

$n >> s$

α, β, s, n

$$\sum_{i=2}^{n} \sum_{k=1}^{i/s} \beta s = \beta \sum_{i=2}^{n} s \sum_{k=1}^{i/s} 1 = \beta \sum_{i=2}^{n} s i / s = \beta \sum_{i=2}^{n} i = \frac{\beta n^2}{2}$$

when $f(n)=3$

$$\sum_{i=2}^{n} \sum_{k=3}^{i/s} \alpha = \alpha \sum_{i=2}^{n} \sum_{k=3}^{i/s} 1 = \alpha \sum_{i=2}^{n} i/3s = \alpha/3s \sum_{2}^{n} i$$

$$= \frac{\alpha n^2}{6s}$$

When $f(n)=n$

$$\sum_{i=2}^{n} \sum_{k=1}^{i/s} \alpha k s = \alpha \sum_{i=2}^{n} s \sum_{k=1}^{i/s} k = \alpha \sum_{i=2}^{n} s \frac{i^2}{2s^2} = \frac{\alpha}{2s} \sum_{i=2}^{n} i^2$$

$$= \frac{\alpha n^3}{6s}$$

Problem 4.

index (mod 3) 1 , 2 , 3

insertion

a) sedo code Not recursive

```
for i=4 to n do
    if i=1 (mod 3)
        temp ← A[i]
        j ← i-1  j ← i-3
        while j>0 and A[j] > temp do begin
            A[j+1] ← A[j]
            j ← j-3
        end while
        A[j+3] ← temp
    end if
end for

for i=5 to n do
    if i=2 (mod 3)
        temp ← A[i]
        j ← i-1  j ← i-3
    while j>0 and A[j] > temp do begin
        A[j+1] ← A[j]
        j ← j-3
        end while
        A[j+3] ← temp
        end if
end for

for i=6 to n do
    if i=3 (mod 3)
        temp ← A[i]
        j ← i-1  j ← i-3
    while j>0 and A[j] > temp do begin
        A[j+1] ← A[j]
        j ← j-3
        A[j+3] ← temp     ← end while
```

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end if
end for
for i = 7 to n do
    if i = 0 (mod 3)
        temp ← A[i]
        j ← i+1; j ← i-3
        while j > 0 and A[j] > temp do begin
            A[j+1] ← A[j]
            j ← j-3
        end while
    end if
end for

b) n = 12 Best case comparison

1 2 3 | 4 5 6 7 | 8 9 10 11 12

1 4 7 10 mod 3 $\boxed{1}$ — 3 —
2 5 8 11 mod 3 $\boxed{2}$ — 3 —
3 6 9 12 mod 3 0 — 3 —

$\boxed{9 + 11 = \boxed{20}}$

c) n = 12 worst case

$$\sum_{i=2}^{n} \sum_{j=1}^{i-1} 1 = \sum_{i=2}^{n} (i-1)$$

$$= \sum_{i=2}^{n} i - \sum_{i=2}^{n} 1$$

$$= 3\left(\frac{n(n-1)}{2}\right)$$

10 4 1

10 7 4 1 — $\boxed{6}$
11 8 5 2 — $\boxed{6}$
12 9 6 3 — $\boxed{6}$ + $\left(\frac{12-11}{2}\right) = \boxed{84}$ Final Answer

$3 \cdot \left(\frac{4 \cdot 3}{2}\right) = 3 \cdot (6) = \boxed{18} + \boxed{\cancel{\ }} = \boxed{\cancel{\ }}$

d) worst case $n = 3k \ (k \in \mathbb{N}^{\geq 0})$

$n \quad n-1 \quad n-2 \quad \text{———} \quad 2 \quad 1$

$n, \quad \frac{2n}{3} \quad \frac{n}{3}, \quad \text{———}$

$n-1 \quad \frac{2n}{3}-1 \quad \frac{n}{3}-1$

sort 1 index comparison $2n$

2 index $2n+1$

3 $n+1$

$$1+2+2+\sum_{i=4}^{n/3} 2i + \sum_{i=5}^{n/3} 2i+1 + \sum_{i=6}^{n/3} i+1$$

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Array

5) Assume (i, j) ← MinMax (A, p, r)

smallest largest

only: 1 comparesion

a) test(A, p, r){

B[ ].clearall; // Meaning B[1]~B[2]~B[n]=0

k=1

while A.length >1

(i, j) ← minmax (A, 1 , A.length)

B[k] = i

B[n-k] = j

k++;

A[i, j] Delete from Array

End while.

if a.length =1 then

B[k] = A[n]

End if

Return B;

}

b) no. comparison: B is even $n/2$

B is odd $n-1/2$

c) $\frac{n\left(\frac{3n}{2}-2\right)}{2} = \frac{3n^2}{4} - n$

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