Problem 8

a) $\cancel{\sum_{i=1}^{n}}$ $\boxed{IB:}$ when $n=1$

$LHS = \sum_{i=1}^{1} i(i+1) = 1 \times 2 = 2$

$RHS = \frac{n(n+1)(n+2)}{3} = \frac{1 \times 2 \times 3}{3} = 2$

$\because LHS = RHS$, Therefore I.B. Holds

$\boxed{I.H.}$ Assume $n=k\geq 1$ holds for $\sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$

$\boxed{I.S.}$ Prove equation holds $(n=k+1)$

$$\sum_{i=1}^{k+1} i(i+1) = \sum_{i=1}^{k} i(i+1) + (k+1)(k+2)$$

From $\boxed{IH}$ we have

$$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

$$= \frac{(k+3)(k+1)(k+2)}{3}$$

$$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$$

From $\boxed{IH}$ and above in $\boxed{IS}$, we have

$\sum_{i=1}^{k+1} i(i+1) = \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$ holds for $\forall$ $n=k+1$

DONE

b.) $\boxed{I.B}$ when $n=0$

$$LHS = \sum_{i=0}^{0} 2^i = 1$$

$$RHS = 2^{0+1} - 1 = 1$$

LHS = RHS, Therefore base case hold

$\boxed{IH}$ Assume $n = (k-1) \geq 1$ which holds

$$\sum_{i=0}^{k-1} 2^i = 2^k - 1$$

$\boxed{IS}$

$$\sum_{i=0}^{k} 2^i = \sum_{i=0}^{k-1} 2^i + 2^k = 2^k - 1 + 2^k$$

$$= 2 \cdot 2^k - 1$$

$$= 2^{k+1} - 1$$

Therefore, $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$ for $\forall n \in k$, Done

Problem 9

$\boxed{IB}$ When $n=0$,

LHS: $\sum_{i=0}^{0} 2^i = 1$

RHS: $a2^0 + b = a + b$

LHS = RHS, we have $a+b=1$

$\boxed{IH}$ Assume $n=(k-1)\geq 0$ holds in

$$\sum_{i=0}^{k-1} 2^i = a2^{k-1} + b$$

$\boxed{IS}$ $\sum_{i=0}^{k} 2^i = \sum_{i=0}^{k-1} 2^i + 2^k = a2^{k-1} + b + 2^k$

$$= \left(\frac{a}{2} + 1\right) 2^k + b$$

because $a = \frac{a}{2} + 1$ we now have $a = 2$

since $a+b=1$, $b=-1$