HW 8

Problem 1

a) $T(n) = (n-1) + \frac{1-p}{n} \sum_{q=1}^{n} T(\max(q-1, n-q))$

$= n-1 + \frac{2}{n} \sum_{q=\frac{n}{2}}^{n-1} (1-p) T(q)$

If $T(n) \leq an$

$T(n) = (n-1) + \frac{2a}{n}(1-p) \sum_{q=\frac{n}{2}}^{n-1} q$

$= \frac{3}{4}[a(1-p)+1]n - \frac{a}{2}(1-p) - 1$

b) $\frac{3}{4}[a \cdot \frac{2}{3} + 1]n - \frac{a}{2} \cdot \frac{2}{3} - 1$

$= \frac{a}{2}n + \frac{3}{4}n - \frac{a}{3} - 1 \leq a_n$

$\frac{3}{4}n \leq \frac{a}{2}n \qquad a \geq \frac{3}{2} \qquad T(n) \geq \frac{3}{2}n$

c) $p = \frac{2}{3}$

$\frac{3}{4}[a \cdot \frac{1}{3} + 1]n - \frac{a}{2} \cdot \frac{1}{3} - 1$

$= (\frac{a}{4} + \frac{3}{4})n - \frac{a}{6} - 1 \leq a_n$

$\frac{a}{4}n + \frac{3}{4}n - \frac{a}{6} - 1 \leq a_n$

$\frac{3}{4}n \leq \frac{a}{6} \qquad T(n) \geq \frac{3}{2}n$

$n \leq \frac{2}{9}a$

$T(n) \geq \frac{1}{2}n$

Problem 2

a) median of a list

$$\frac{n(n-1)}{2} = \frac{15 \times 14}{2} = 105 \text{ Comparison}$$

b) (i) $n[\lg n] - 2^{\lg n} + 1 = 45 \text{ comparison}$

(ii) $= 15 \times 4 - 15 = 46$

(iii) Merge sort

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j = l
k = mid + l
while (p ≤ mid && k ≤ q)
    if ( a[p] < a[k]) {
        b[i] = a[p]
        p++;
        j++;
    } else {
        b[i] = a[k]
        k++
        j++
    }
```

Problem 2

a) $T(n) = (n-1) + (n-2) + \sim + \frac{n}{2}$

$n = 15 \quad T(n) = 14 + 13 + \cdots + 7 = 84$

b) $\dfrac{n\lg n - n + 1 - n + 1}{2} + n - 1$

$= \dfrac{n}{2}(\lg n - n + 1 + n - 1) = \dfrac{n}{2}\lg n$

$n = 15 \quad \dfrac{15}{2}\lg 15 \approx 4 * 7 = \boxed{28}$

Problem 4

a) $n - \frac{4n}{15} - \frac{4n}{15} = \frac{7}{15}n$

b) $\frac{8n}{15} \cdot \frac{1}{2} = \frac{8n}{30} = \frac{4}{15}n$

c) $T(n) = \frac{28n}{15} + T(\frac{n}{15}) + \frac{7}{15}n + T(\frac{11n}{15})$

d) If $T(n) \leq an$

$\leq \frac{an}{15} + \frac{a_{11}n}{15} + \frac{28n}{15} + \frac{7}{15}n$

$= \frac{12}{15}a_n + \frac{7}{3}n$

$= (\frac{4}{5}a + \frac{7}{3})n$

$(\frac{4}{5}a + \frac{7}{3}) \leq a$

$\frac{7}{3} \leq \frac{1}{5}a$

$\frac{35}{3} \leq a \qquad \Rightarrow T(n) \leq 11.67n$

Since 15 is faster $\quad 11.67n < 16n$