Contents

AMSC 460 - EXAM 1

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clear all; format rational; close all; syms f(x) x y
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Problem 1 (finished)

```
A = [1 0 0 1 2; -1 1 1 1 0; 1 -2 2 1 2/3; -1 3 3 1 0]
rref(A)
```

```
A =
       1              0              0              1              2
      -1              1              1              1              0
       1             -2              2              1              2/3
      -1              3              3              1              0
ans =
       1              0              0              0              1
       0              1              0              0              1/3
       0              0              1              0             -1/3
       0              0              0              1              1
```

Problem 2 (Yup)

Problem 3 (Yup)

(a) first we can write two equations base on $y(t) = xt$

$$3 = x$$

$$2 = 2x$$

so write the two equations in matrix form

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ x \end{pmatrix}$$

so we can get that the matrix $A = \begin{pmatrix} 1, & 0 \\ 0, & 2 \end{pmatrix}, b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$r = \begin{pmatrix} 1, & 0 \\ 0, & 2 \end{pmatrix} \begin{pmatrix} x \\ x \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} x-3 \\ 2x-2 \end{pmatrix}$$

Therefore:

$$||r||_2^2 = (x-3)^2 + (2x-2)^2 = 5x^2 - 14x + 3$$
$$f(x) = ||r||_2^2 = 5x^2 - 14x + 3$$

$$f^{'}(x) = 10x - 14$$

$$f'(x) = 0 = 10x - 14, x = \frac{7}{5}$$

$$||r||_2^2$$
$$x = \frac{7}{5}$$

((b)

$$r = \begin{pmatrix} x-3 \\ 2x-2 \end{pmatrix}$$

so

$$||r||_4^4 = (x-3)^4 + (2x-2)^4 = 17x^4 - 76x^3 + 150x^2 - 172x + 97$$

Problem 4

Compute the order of the local truncation error for this method.

$$f_{i+1} = f(t_i + h, y(t_i + h))$$

$$y_{i+1} = y(t_i + h) = y_i + hf_i + \frac{h^2}{2} y^{''} + \frac{h^3}{6} y^{'''} + O(h^4)$$

$$y_{i-1} = y(t_i - h) = y_i - hf_i + \frac{h^2}{2} y^{''} - \frac{h^3}{6} y^{'''} + O(h^4)$$

$$y^{''} = \frac{d^2y}{dt^2} = \frac{d}{dt}(\frac{dy}{dt}) = \frac{df}{dt} = f_t + ff_y$$

$$f(t_i+h, y(t_i+h)) = f_i+h(\frac{\partial}{\partial t}+\frac{\partial y}{\partial t}\frac{\partial}{\partial y})f+\frac{h^2}{2}(\frac{\partial}{\partial t}+\frac{\partial y}{\partial t}\frac{\partial}{\partial y})^2 f+\frac{h^3}{6}(\frac{\partial}{\partial t}+\frac{\partial y}{\partial t}\frac{\partial}{\partial y})^3 f+O(h^4)$$

so we can get:

$$y^{''} = \frac{df}{dt}, y^{'''} = \frac{d^2 f}{dt^2}$$

$$f(t_i + h, y(t_i + h)) = f_i + hy^{''} + \frac{h^2}{2} y^{'''} + \frac{h^3}{6} y^{(4)} + O(h^4)$$

so we can get that:

$$3y_{i+1} - 4y_i + y_{i-1}$$

$$= 3y_i + 3hf_i + \frac{3h^2}{2} y^{''} + \frac{h^3}{2} y^{'''} + O(h^4) - 4y_i$$

$$+ y_i - hf_i + \frac{h^2}{2} y^{''} - \frac{h^3}{6} y^{'''} + O(h^4)$$

$$= 2hf_i + 2h^2 y^{''} + \frac{h^3}{3} y^{'''} + O(h^4)$$

then:

$$2hf_{i+1} = 2hf_i + 2h^2 y^{''} + h^3 y^{'''} + \frac{h^4}{3} y^{(4)} + O(h^5)$$

so we can get

$$3y_{i+1} - 4y_i + y_{i-1} - 2hf_{i+1}$$

$$= 2hf_i + 2h^2 y^{''} + \frac{h^3}{3} y^{'''} + O(h^4) - (2hf_i + 2h^2 y^{''} + h^3 y^{'''} + \frac{h^4}{3} y^{(4)} + O(h^5))$$

$$= -\frac{2h^3}{3} y^{'''} + O(h^4)$$

The truncation error of the equation below:

$$w_{i+1} = \frac{1}{3}(4w_i - w_{i-1} + 2hf_{i+1})$$

is:

$$LTE = \frac{2}{9} h^3 y^{'''} + O(h^4)$$