AMSC 460 - HW 3

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Problem 1

Find the multiplicity of the root r = 0 of f(x) = 1 _ cos(x). Find the forward and backward errors for the approximate root xˆ = 10^_5.

```
f = 1-cos(x);     % f(0) = 0, so r = 0 is a root.
diff(f);          % ans = sin(x), sin(0)=0, so r = 0 is a root of
 multiplicity at least 2
diff(sin(x));     % ans = cos(x), cos(x)=1, so r = 0 is a root of
 multiplicity 2.
```

```
          So we see that r = 0 has multiplicity of 2.
```

```
r = 0; x_bar = 10^-5;
f = @(x) 1-cos(x);
fwd_err = abs(r - x_bar)
bwd_err = abs(f(x_bar))

%   backward error
```

```
fwd_err =

   1.0000e-05


bwd_err =

   5.0000e-11
```

Problem 2

```
% In class it was mentioned how backward error and forward error are
 not
% always of similar sizes. Verify this by finding the relationship
 between
% the forward and backward error for the problem of finding the root
 of $f(x)=ax+b$.

%Since r0 = -n/a +r, f(r0) = ar. The backward error and the front
 error
```

```
%will have different sizes of ar and r. Therefore the accuracy of the
%changes will not be accurate. The 10^-10 to be accurate the absolute
%value of a will be smaller than 10^-2 since 10^(-10+8) (Assuming the
 tolerance is 10^-8)
```

Problem 3

Newton's method for solving f(x) = 0 requires computing the derivative of f each iteration. Suppose instead that the initial slope d = f'(x0#is kept throughout the iterations, i.e xn+1 = xn _ f(xn)/d. Suppose that the root r is simple (so that f'(r)# 0). Find a condition on the initial slope d that ensures the scheme will be locally convergent. What is the order of convergence?

```
%Accordint to our class note, Newton is an FPI, with g(x) = x - f(x)/
f'(x)
%In this case, we have g(x) = x - f(x)/d
%                 then g(r) = r - f(r)/d = r - f(r)/f'(x0)
%g'(r) = 1 - [ f'(x0)f'(r)- f(r)f''(x0)/[f'(x0)]^2] = 0 < 1
%since f(r)=0  g'(r) = 1 - [ f'(x0)f'(r)/[f'(x0)]^2] = 0 < 1
%                  1 =  f'(x0)f'(r)/[f'(x0)]^2
%        f'(x0)f'(r) = [f'(x0)]^2
%              f'(r) = f'(x0)
%Given that f'(r)# 0 so d = f'(x0) # 0, assume f(x)is a continuous and
%second derivative is also continuous and f(x) is defined on an
 interval
%I = [r-#,r+#], with #> 0. Given that f(r) = 0 and f''(r) # 0
%If the x0 is closing to the root r, then the sequence {xn} converges
%quadratically to the root r. Then, convergence order is 2.
%
%Proof:
%Assumeing that x_n# I. Since f(r) = 0,
%A Taylor expansion of f(x) at x = x_n, evaluated at x = r is:
%0 = f(r) = f(x_n)+(r - xn)f'(x_n) + [(r-x_n)^2 / 2] f''(#_n)
%Then we have r - x_n = [ -2f(x_n) - f''(#_n)(r - x_n)^2 ] / 2f'(x_n)
% Using Newton iterations we have
%r - x_n+1 = r - x_n + [f(x_n)/f'(x_n)] = - [f''(#_n)(r -
 x_n)^2]/2f'(x_n)
%| r - x_n+1 | # [(r - x_n)^2 / 2]*A # |r-x_n|/2  # ……# 2^(1-n) *|r-
x_0|
%
%x_n -> r as n -># 
%which implies the quadratic convergence of {x_n} to r.
```

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