Computational Methods Summer 2021

HOMEWORK 14

Due Date: Thursday, June 24

1. Find a quartic Hermite polynomial that interpolates

$$p(0) = 1, \quad p'(0) = -1, \quad p(1) = -2, \quad p'(1) = 2, \quad p(2) = 2.$$

2. Consider the function

$$f(x) = \frac{e^{3x}\sin(200x^2)}{1 + 20x^2}$$

on the interval $0 \leq x \leq 1$. The goal of this problem is to observe the error reduction in cubic spline interpolation when increasing the number of nodes.

   (a) Plot the function in MATLAB.

   (b) Write a short script using the MATLAB `spline` command, that interpolates $f(x)$ at equidistant points $x_i = i/n$ ($i = 0, 1, ..., n$), where n is an arbitary fixed number of subintervals prescribed by the user.

   (c) For $n = 2^j$, $j = 4, 5, ..., 14$, run your script and record the maximum value of the error $e(x) := |f(x) - s(x)|$ at the points `x=0:0.001:1`, where $s(x)$ denotes the cubic spline interpolant. In other words, for each n, compute $\max_{x\in 0:0.001:1} e(x)$ and store this value. (For efficient coding, you shouldn't loop through all x. Vectorize f and use the `max` command to compute $\max_{x\in 0:0.001:1} |f(x) - s(x)|$ in one shot.)

   (d) Plot the errors against n on a `loglog` plot and make observations.

3. (Optional, not graded) A natural cubic spline is defined as a cubic spline for which the second derivative is zero at the first and last knots. Find a natural cubic spline function whose knots are $-3, 0, 1$ and that take the corresponding y-values $1, -2, 4$.

4. (Optional, not graded) A spline of degree k requires having continuous derivatives of order up to and including $k - 1$ at the knots. How many additional conditions are required to define the spline uniquely?