Computational Methods Summer 2021

HOMEWORK 22

Due Date: Friday, July 9

Homework should be handed in *individually*, though you may work with others and collaboration is encouraged. For MATLAB problems please follow the guidelines specified in ELMS.

1. Is the Improved Euler Method

$$w_{i+1} = w_i + \frac{h}{2}(K_1 + K_2), \quad K_1 = f(t_i, w_i), \quad K_2 = f(t_i + h, w_i + hK_1),$$

   stable for the equation $y' = -20y$ with $h = 0.05$? Explain why or why not.

2. Consider the integro-differential equation on the interval $0 \leq t \leq 20$:

$$y' = 1.3y - 0.25y^2 + 2\sin(t) - 0.005y \int_0^t y(s)\,\mathrm{d}s; \qquad y(0) = 8.$$

   Discretize the equation using the forward Euler method with $h = 0.2$. To approximate the integral term, use the trapezoidal quadrature rule. Implement the method in MATLAB to obtain the approximate solution $w_i \approx y(t_i)$, and create a plot of your solution.

$$y' = -20y, y = ae^{(} - 20x)$$
$$K_1 = f(x_0, y_0) = f(x_0, ae^{(} - 20x_0)) = -20y_0$$
$$K_2 = f(x_0 + h, y_0 - 20hy_0) = -20(y_0 - 20hy_0)$$