Cooper, J

AMSC 460

HOUR EXAM II April 30, 2003

1. Let a quadrature rule on the interval $[-1, 1]$ be given by

$$Q(f) = A_1 f(t_1) + A_2 f(t_2).$$

a) Find the weights A_1, A_2 and the nodes t_1, t_2 so that the method has maximum degree of precision. What is that degree? You may assume symmetry: $A_1 = A_2$ and $t_1 = -t_2$.

b) Transform this rule to the general interval $[a, b]$.

c) Use the rule to estimate $\int_0^1 e^x dx$. Compute the actual error.

2. a) State the compound trapezoid rule for the approximation of the integral $I = \int_a^b f dx$ with n panels. Use the parameter $h = (b-a)/n$ and call the rule T_n.

b) What is the degree of precision of this rule?

c) If you double the number of panels in the trapezoid rule how does the error change?

d) Show how you can use the results of T_n and T_{2n} to estimate the error $T_n - \int_a^b f dx$.

3. a) Use Newton's method to derive an iterative formula to calculate the cube root of a positive number A.

b) Use the formula to approximate the cube root of 9 . Start with $x_0 = 3$ and calculate x_1 and x_2.

4. Let $f(x) = x^3 - 4x^2 + 5x - 2$. f has roots at $x = 1$ and at $x = 2$.

a) Sketch the graph of f. Which root is the simple root and which root is the double root?

b) What is a safe interval on which to start Newton's method to guarantee convergence to the root at $x = 2$? Explain.

c) Which method, bisection or Newton's method, gives the faster convergence to the root at $x = 1$? Explain.

5. What combination of software routines would you use to approximate the value of c such that

$$\int_0^1 e^{-cx^2} dx = 1/2?$$

Explain how you would do the computation using pseudo code and names mfiles.