① $f(x) = f(x) + f'(x) + \frac{1}{2!}f''(x) + \frac{1}{3!}f'''(x) + O(h^4)$

a) $f(x+h) = f(x) + hf'(x) + \frac{1}{2!}h^2 f''(x) + \frac{1}{3!}h^3 f'''(x) + O(h^4)$

$f(x+2h) = f(x) + 2hf'(x) + \frac{1}{2!}(2h)^2 f''(x) + \frac{1}{3!}(2h)^3 f'''(x) + O(h^4)$

$f(x+3h) = f(x) + 3hf'(x) + \frac{1}{2!}(3h)^2 f''(x) + \frac{1}{3!}(3h)^3 f'''(x) + O(h^4)$

$\frac{1}{h^3}\left(f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)\right)$

$= \frac{1}{h^3}\Big(0 f(x) + (-6)hf'(x) + \left(\frac{9}{2} - 6 + \frac{3}{2}\right)h^2 f''(x) + (27-24+3)\frac{1}{6}h^3 f'''(x)$

$+ (81-48+3)\frac{1}{24}h^4 f''''(x)\Big)$

$= f'''(x) + \frac{3}{2}hf''''(x) \leq f'''(x)$ Given $p=1$, $n=4$, $C=\frac{3}{2}$

$Ch^p \|f^{(n)}\|_\infty$

(4) b)

Give $f(x-h)$ and $f(x-2h)$ and $f(x+h)$

$$\frac{1}{2h^3}\left(f(x+2h) - 2f(x+h) + 2f(x-h) - f(x+2h)\right)$$

$$= \frac{1}{2h^3}\left((1-2+2-1)f(x) + (2-2-2+2)hf'(x) + 0 \cdot \frac{1}{2}h^2 f''(x)\right.$$

$$+ 0 \frac{1}{3!} f'''(x) \cdot h^3 + 0 f''''(x) \cdot h^4 = f'''(x)$$

$$\frac{1}{2h^3}\left(f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)\right)$$

$$= f'''(x) + \frac{1}{2h^3}(32-2+2+32)\frac{1}{120}h^5 f^5(x) = f'''(x) + \frac{1}{4}h^2 f^5(x)$$

$C = \frac{1}{4}$ $p = 2$ $n = 5$

$$\boxed{\frac{1}{4}h^2 f^{(5)}}$$