Math 401 - Homework #2
2D Computer Graphics
due on Gradescope Wednesday, 9/15

Instructions: Work through the problems below. Write down your solutions on paper. You can use MATLAB for computations unless the problem says "Must do all computations by hand." For problems in which you use MATLAB to do computations, I do not want you to turn in any MATLAB code. You need to clearly present all steps of your solution so that I can follow your thought process without guessing what you are thinking (try writing in complete sentences.)

1. **(Must do all computations by hand.)** Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ which first reflects through the y-axis, and then rotates counterclockwise about the origin by $\pi/4$ radians (45 degrees). Find the standard 2×2 matrix for this linear transformation in two ways:
   (a) Find the standard matrix of f by directly determining the images of the standard basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ under f.
   (b) Find the standard matrix of f by first finding the standard matrices of the reflection and of the rotation, and then use matrix multiplication.
2. Find the point obtained when $(7, 9)$ is rotated counterclockwise about the origin by 50 degrees ($5\pi/18$ radians). (Do the computation in MATLAB, but make it clear what computation you are doing.)
3. **(Must do all computations by hand.)**
   (a) Do Exercise 2.5 from the text. (The matrix should be 3×3, for homogeneous coordinates.)
   (b) Do Exercise 2.6 from the text. (The matrix should be 3×3, for homogeneous coordinates.)
4. Do Exercise 2.9 from the text.
5. Do Exercise 2.11 from the text.
6. Find the 3×3 matrix (for homogeneous coordinates) for the transformation which first rotates counterclockwise about the point $(4, 8)$ by $\pi/5$ radians, and then rotates counterclockwise about the point $(7, 1)$ by $4\pi/11$ radians.
7. **(Must do all computations by hand.)**
   (a) Let L denote a line in the plane that does **not** go through the origin. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ denote the transformation which reflects through the line L. Give a reason why f is not a linear transformation.
   (b) Reflection through a line which does not go through the origin can be represented as a 3×3 matrix (which operates on homogeneous coordinates). Give the matrix for reflection through the vertical line $x = 5$.
   (c) Give a formula for the image of the generic point (x, y) under reflection through the line $x = 5$.
8. **(Must do all computations by hand.)** Let L_θ denote the line in the plane through the origin that makes an angle of θ radians with the positive x-axis. In class, we saw that reflection through the

line L_θ has standard matrix $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$. Now let θ and φ be two angles. Show that the composition of reflection through L_φ followed by reflection through L_θ is equal to a rotation about the origin, and find the angle of rotation. (Multiply the matrices and use trig identities to recognize the result as a rotation matrix.)

9. **(Optional - don't have to turn in)**

   (a) Let L_m denote the line through the origin with slope m. Find the 2×2 matrix for the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects through the line L_m. Simplify your answer so that there are no trig or inverse trig functions. Then give the 3×3 matrix for this transformation (that operates on homogeneous coordinates).

   (b) Let $L_{(a,b),m}$ be the line through the point (a, b) with slope m. Find the 3×3 matrix (for homogeneous coordinates) for the transformation of reflection through the line $L_{(a,b),m}$.