Final Exam : AMSC/CMSC 460

Section: 0101, Fall 2017

Dec. 15, 2017 (8:00-10:00am)

Instructor: D. Dahiya Total points: 80

Do any eight problems. 10 points for each problem.

1. Solve the linear system $AX = b$ using LU factorization of matrix A

$$A = \begin{pmatrix} 1 & -1 \\ 3 & -2 \end{pmatrix}, b = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

where L is a unit lower triangular matrix (that is, L has ones in the main diagonal).

2. Construct the Lagrange interpolating polynomial for the function $f(x) = \sin x$ at $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$. Find a bound for the absolute value of interpolation error in the interval $[x_0, x_2]$.

3. Find the Hermite interpolation polynomial $p(x)$ in Newton form for the function $f(x) = \sin x$, such that $p(0) = f(0)$, $p'(0) = f'(0)$, $p''(0) = f''(0)$, $p'''(0) = f'''(0)$. Make the divided difference table.

4. The first two orthogonal polynomials in interval $[0, 2]$ are $P_0(x) = 1$ and $P_1(x) = x - 1$. Obtain the first two orthonormal polynomials in the given interval and use these to find the linear least squares approximation to the function $f(x) = x^3$ in interval $[0, 2]$.

5. Use the values of $f(x)$ at $x-h$ and $x+h$ to obtain the most accurate approximation of $f'(x)$. What is the truncation error in this approximation?

6. The quadrature formula

$$\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$$

is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

7. Find the approximation to the integral

$$\int_0^2 x^2 dx$$

using Simpson's rule and Trapezoidal rule. Compare the obtained approximations with the exact solution. Which one is more accurate?

8. Use Euler's method to solve the initial value problem $y' = -y + x + 1$, in interval $[0, 0.5]$ with step size $h = 0.1$ and initial condition $y(0) = 1$.

9. Use Taylor's method of order 2 to approximate the solution of the initial value problem

$$y' = xe^{3x} - 2y, \quad 0 \leq x \leq 0.5, \quad y(0) = 0, \quad h = 0.5.$$