Computational Methods Summer 2021

HOMEWORK 6

Due Date: Thursday, June 10

1. Obtain the precise operation count (number of operations $+, -, *, /$) for computing a matrix-matrix product AB. Suppose each matrix is $n \times n$.

2. Suppose we have an $n \times n$ matrix. In class we discussed how the elimination step of Gaussian elimination is $O(n^3)$, while back-substitution (also forward-substitution) is only $O(n^2)$. The back-substitution steps for finding the components x_i of a solution can be concisely written as

$$x_n = b_n/u_{nn}, \qquad x_i = \frac{1}{u_{ii}}\left(b_i - \sum_{j=i+1}^{n} u_{ij}x_j\right), \quad \text{for } i = n-1, ..., 1.$$

   Show that the total operation count for constructing $\mathbf{x}$ is exactly n^2.

3. (Optional, not graded) Write down the forward substitution algorithm and pseudocode for solving an $n \times n$ lower triangular matrix system. [A matrix is lower triangular if $a_{ij} = 0$ for $i < j$.]