Final Exam : AMSC/CMSC 460

Section: 0101, Spring 2018

May 14, 2018 (8:00-10:00am)

Instructor: Daisy Dahiya — Total points: 80

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1. Assume computer word of length 4. What integers(decimal numbers) do the following 2's complement binary numbers represent.
   (a) 0111
   (b) 1100
   (c) 1111

2. Use the fixed point theorem to show that the fixed point iterations $x_{n+1} = g(x_n)$, where $g(x) = (1+x)^{(1/5)}$ converge in the interval $[0, 2]$.

3. Find an upper bound on the absolute value of the relative error in the solution of the linear system $AX = b$ when A^{-1} is perturbed to B. Use l_∞ norm.
$$A = \begin{pmatrix} 1 & 2 \\ -2 & -9 \end{pmatrix}, \quad B = \begin{pmatrix} 9/5 & 2/5 \\ 0 & -1/5 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 6 \end{pmatrix}.$$

4. Using Gram Schmidt process obtain the first three orthonormal polynomials in the interval $[-1, 1]$.

5. Use differentiation via interpolation algorithm with nodes 0, $\pi/2$, and π to approximate the derivative of the function $f(x) = x\cos x - x^2 \sin x$ at $\pi/2$. Write the formula for truncation error at $\pi/2$.

6. The backward difference formula can be expressed as
$$f'(x_0) = D(h) + c_1 h + c_2 h^2 + c_3 h^3 + \ldots$$
where $h > 0$, and $c_i's$ are constants. Use Richardson's extrapolation to derive an $\mathcal{O}(h^3)$ formula for $f'(x_0)$ using $D(h)$, $D(2h)$, and $D(4h)$ where
$$D(h) = \frac{f(x_0) - f(x_0 - h)}{h}$$

7. The quadrature formula $\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

8. Compute the approximation to the integral $\int_0^2 x^4 dx$ using
   (a) Trapezoidal rule
   (b) Composite Trapezoidal rule using four subintervals of equal length

   Compare the two approximations with the exact solution.