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AMSC 460 - HW11

clear all; format compact; close all; syms f(x) x y z

Problem 1

Suppose in designing a natural logarithm function for a calculator on the interval $[1, e]$, we are using a Chebyshev polynomial approximation. What is the smallest degree n of the polynomial that ensures an accuracy of 10^{-6} over the interval $[1, e]$?

$$P(x) = 1/3*(x+1)-1/12(x+1)(x-2) + 1/24(x+1)(x-2)(x-3)$$

Problem 2 (

Special functions appear in physics and applied mathematics, often as a solution to some ODE. The following function is in the `\emph{Bessel}` family `(\url{https://en.wikipedia.org/wiki/Bessel_function})[J(x)` = `\frac{1}{\pi}\int_0^\pi \cos(x\sin(s))\,ds.`

(a) Show that $|J(x)| \leq 1$, $|J'(x)| \leq 1$, $|J''(x)| \leq 1$, and in general that $|J^{(k)}(x)| \leq 1$ for any positive integer k .

(b) Suppose we would like to approximate J with a Chebyshev interpolant. Determine how many interpolation points are required on the interval $[0, 10]$ so that the error (in the max-norm) is no more than 10^{-6} . [You don't have to write down the interpolant.]