Section 0201 VON PETERSDORFF Spring 04

AMSC/CMSC 460 Final Exam

Show all your work. Use Gaussian elimination (not your calculator) to solve linear systems. Draw a box around your answers.

1. (20 pts) We are given the following data points (t_j, y_j): $(-2, 2)$, $(-1, 1)$, $(0, 2)$, $(1, 2)$. Find the best least squares fit $g(t)$ of the form $g(t) = c_1 |t| + c_2 t^2$.

2. (20 pts) We want to find x_1, x_2, x_3 which satisfy the nonlinear system

$$\begin{aligned} 4x_1 + x_2^2 - 1 &= 0 \\ 4x_2 + x_3^2 - 1 &= 0 \\ 4x_3 - x_1^2 - 1 &= 0 \end{aligned}$$

   Perform one step of the Newton method starting with the initial guess $\mathbf{x}^{(0)} = (1, 1, 1)^\mathsf{T}$.

3. (40 pts) Consider the initial value problem

$$y'' + yy' = t, \qquad y(1) = 1, \qquad y'(1) = 2$$

   We want to find $y(T)$ for $T = 2$.

   (a) (10 pts) Perform two steps of the Euler method with $h = \frac{1}{2}$. Give the resulting approximation for $y(T)$.

   (b) (10 pts) Perform one step of the RK2 method with $h = 1$. Give the resulting approximation for $y(T)$.

   (c) (10 pts) Consider now the initial values $y(1) = 1$, $y'(1) = v$. Write an m-file `solT.m` so that `z = solT(v)` returns an approximation for $y(T)$.

   (d) (10 pts) We try `solT(-2)` and get a negative value, then we try `solT(-1)` and get a positive value. Write an m-file which finds a value v so that $y(T) = 0$.

4. (20 pts) We want to approximate $I := \int_1^2 \frac{1}{x} dx$ using the composite trapezoid rule with N subintervals of equal size.

   (a) (10 pts) Find an approximation Q using $N = 2$ subintervals.

   (b) (10 pts) Use the error formula to determine a number N such that $|Q - I| \leq 10^{-15}$.