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# AMSC 460 - EXAM 1

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clear all; format compact; close all; syms f(x) x y
```

# Problem 1 (finished)

Consider a base 2 normalized floating point system with mantissa of length 4, and exponent in the range $-5 \leq E \leq 5$.

(a) What is the distance between the number 8 and the next largest floating point number in this system?

The next largest # 1.1111*2^5 = 111110 111110_2 =62_10 62-8 =54

(b) What is the distance between the number 8 and the next smallest floating point number?

The next largest # 1.0001*2^3 = 1000.1 1000.1_2 =8.5_10 8.5-8 =0.5

(c) Is 1.05 exactly representable in this system? Explain why or why not

Not representable. Since 1.05_10 = 1.00001100110_2, and non terminating binary it will be rounded off

# Problem 2 (Yup)

(a) Pivoting is not necessary in Gaussian elimination for wellconditioned systems.

False because naive Gaussian elimination with no pivoting will sometimes fail (divide by zero)on a well-conditioned matrix.

(b) If xa is an approximate solution of a linear system, obtained using PA = LU, then xa can be considered accurate provided |Ax|_inf < 10^-16.

FALSE

(c) Suppose we have computed a factorization A = LU where A is an n*n matrix. How would you use this to solve the system AT x = b in O(n2) operations? Be precise about the steps you would take, and why your method ends up being O(n2).

IF A = LU then the Ly= b, y1 = b1/L11, y2 = (b2-L21*y1)/L22 The O(n) time will be needed

O(n^2) calculations we can get y = Ly = b Similarly, Ux = y. U(n,n)= yn/xn, x(n-1) = (y(n-1)-U(n-1,n)*xn)/U(n-1,n-1>>>>) Therefore the n^2 time can be resolved

# Problem 3 (Yup)

(a) Prove that g has a root in the interval (2, 3).

```
g(x) = 4^x-6*x^2;
double(g(2))
double(g(3))

ans =
    -8
ans =
    10
```

Since g(x) is continuous on [2, 3], and g(2) = -8 < 0 and f(3) = 10 > 0. By the Intermediate Theorem, g(x) has at least one root on the interval [2, 3].

(b) Suppose we decide to find the root using the bisection method. How many iterations are needed to guarantee the root is accurate to within 10^-6, starting with the bracket (2,3)?

```
g = @(x) (4^x)-6*x^2
g(2)*g(3)
a =2
tol = 10^(-6)
(log(3-a)-log(tol))/log(2)-1

g =
  function_handle with value:
    @(x)(4^x)-6*x^2
ans =
   -80
a =
     2
tol =
   1.0000e-06
ans =
   18.9316
```

19 iterations in total since 18.93 rounded in is 19, we have to take x0 as the first iteration

(c) Show that finding a root of g is equivalent to solving x = log_4(6*x^2)

4^x -6*x^2 = 0 -> 4^x = 6x^2 log_4(4^x)=log_4(6x^2) x = log_4(6*x^2) QED

(d) Explain whether or not fixed point iteration will converge for the problem in part (c), using any initial guess from the interval (2; 3). [Note that it is not enough to check a few values, since the question is about all starting guesses from this interval.]

Guess x is between 2,3 x = log_4(6x^2), x = log_4(6x^2)=f(x) Updating using x_n+= f(xn) According to fixed point thereom, f(x) = log_4(6x^2) is continuous on (2,3) f'(x) = 12x *1/(6x^2*ln4)= 2/(ln4*x) exist for all x in (2,3)

```
%plug in the number for x
% Then lim xn = x, so it converges to a fixed point x for any xi in
 (2,3)
```

# Problem 4(>.<::::()

Suppose we attempt to solve the equation x^2 +x^12 = 4 by rewriting as the following nonlinear system: x^2 +y^2 =4, y= x^8

(a) Set up and explain the Newton method to solve this system. Calculate the Jacobian and right hand side of the method.

```
f1(x,y) = x^2 +x^12 - 4;
f2(x,y) = x^2 +y^2 - 4;
f3(x,y) = -x^6 + y;
F = @(x,y) [f2(x,y); f3(x,y)];
DFx = jacobian([ x^2 +y^2,-x^6 + y], [x,y])
```

```
DFx =
[   2*x,   0]
[     0, 2*y]
[-6*x^5,   1]
```

Jacobian (x0) will be [2 2; -6 1] [2 2; -6 1]*x^(0) = [ -10; 2 ] This is the newton method first step my initial guess for x^(0) is [1, 2] Using Gauss elimination we can have [1 2; 0 12]* x^(0)= [-5 ; 7] We can put the solution in then The right hand side is from x_n+1 = x_n -[DF(xn)]^-1*F(xn) xn is the starting guess

(b) Explain what a good starting guess would be for the solution(s).

There is no best initial guess (that would be the root itself) instead, a suitable initial guess is needed if quickly possible, plot the function to compute a numerical approximation to a particular root, choose an initial guess close enough to that root close enough means a position x0, if evaluation of the function is expensive and/or computing its derivative is not directly possible, in such a case, use evaluated function values for finding the root and simultaneously plotting the function to get a better understanding of its behavior. wouse auxiliary or alternative methods, if necessary

(c) Write down the details for the system of equations needed to make the first Newton iteration. Do this without using matrix inverses. [You do NOT have to solve this system.]

To avoid computing inverses method is to get s = DF(xn^-1)*F(xn) Fix x0 starting guess Solve for b in the linear system Update x1=x0-s Repeat the iteration

# Problem 5 (Yup)

Write down the Lagrange interpolating polynomial that interpolates the function f(x) = x4 at the points x = -1, x = 0, and x = 2.

f(x) = x^4, so the three points are (-1,1) (0,0) (2,16) P(x)= (x)(x-2)/((-1)(-3))+0(x+1)(x-2)/((1)(-2))+16*(x +1)(x-0)/((2+1)(2)) The final answer is P(x)=3x^2+2x

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