

Q.  $\Rightarrow$

$$V_n = \text{span} \{1, x, x^2\}$$

$$W = e^{-x}$$

$$x_1 = 1, x_2 = x, x_3 = x^2$$

$$v_1 = x_1 = 1$$

$$\bullet V_2 = x_2 - \frac{\langle x_2, v_1 \rangle}{\|v_1\|^2} \cdot v_1$$

$$= x - \int_0^{\infty} x \cdot e^{-x} dx = x - 1$$

$$\bullet V_3 = x_3 - \frac{\langle x_3, v_1 \rangle}{\|v_1\|^2} \cdot v_1 - \frac{\langle x_3, v_2 \rangle}{\|v_2\|^2} \cdot v_2$$

$$= x^2 - \frac{\int_0^{\infty} e^{-x} \cdot x^2 dx}{1} - \frac{\int_0^{\infty} (x-1) e^{-x} \cdot x^2 dx}{1} - (x-1)$$

$$= x^2 - 2 - 4x - 4$$

$$= x^2 - 4x - 6$$

$$\{1, x-1, x^2-4x-6\}$$

$\{1, x-1, x^2-4x-6\}$  is orthogonal on  $[0, \infty]$

$\star$



$$f(x) = e^x \quad [0, 2]$$

$$P_1(x) = a_0 + a_1 x$$

$$P_1(f) = \int_0^2 [f(x) - p(x)]^2 dx$$

$$\Rightarrow \int_0^2 e^x \cdot x dx = (x-1)e^x \Big|_0^2 = e^2 + 1$$

$$\Rightarrow \int_0^2 e^x dx = e^x \Big|_0^2 = e^2 - 1$$

$$a_0 = (e^2 + 1) \frac{3}{2} = \frac{3e^2 + 3}{2}$$

$$a_1 = (e^2 - 1) \frac{1}{2} = \frac{e^2 - 1}{2}$$

$$P(x) = \frac{3}{2}(e^2 + 3) + \frac{e^2 - 1}{2} x$$