CMSC/MAPL 460 Dr. Wolfe FINAL EXAM July 27, 2001

1. (10 points) Suppose you have a computer with machine epsilon $\epsilon = 10^{-7}$. Let

$$f(x) = \ln(1 + x^2).$$

(a) If $x = 10^{-4}$ what result would the computer give for $f(x)$?
(b) Find a way to compute $f(10^{-4})$ to full machine accuracy and carry out the computation.

2. (15 points) Let

$$A = \begin{pmatrix} 4 & -2 \\ 2 & 5 \end{pmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}.$$

(a) Solve $A\mathbf{x} = \mathbf{b}$, using the LU factorization, forward elimination and back substitution.
(b) Find an approximate solution to $A\mathbf{x} = \mathbf{b}$ by doing two Jacobi iterations starting at $\mathbf{x}^{(0)} = (3, 2)^T$.
(c) Find an approximate solution to $A\mathbf{x} = \mathbf{b}$ by doing two Gauss-Seidel iterations starting at $\mathbf{x}^{(0)} = (3, 2)^T$.

3. (15 points) Determine by two methods the polynomial of degree ≤ 2 whose graph passes through the points $(1, 2), (2, 6)$ and $(3, 0)$. Verify that both methods give you the same answer.

4. (20 points) Let

$$I = \int_1^2 \frac{dx}{x}.$$

(a) Compute T_4, the 4-panel trapezoid rule approximation to I. Compare your answer with the exact value of I.
(b) Compute CT_4, the 4-panel corrected trapeziod rule approximation to I. Compare your answer with the exact value of I.
(c) Suppose you computed T_{16}, the 16-panel trapezoid rule approximation to I. What answer would you expect (approximately) ? Do not actually carry out the computation.
(d) How many panels would you need to compute I with an error of $< 10^{-6}$ using the trapezoid rule ?

5. (10 points) Let

$$g(x) = \frac{x^3 + 6x}{3x^2 + 2}.$$

(a) Show that $\alpha = \sqrt{2}$ is a fixed point of g.
(b) Let $x_0 = 1.5$. Compute x_1, x_2 and x_3 for the fixed point iterations $x_{n+1} = g(x_n)$.
(c) Prove that if x_0 if chosen sufficiently close to α, the fixed point iterates converge to α (at least) quadratically.

6. (15 points) Consider the nonlinear system

$$x^2 + y^2 = 4$$
$$x^2 - y^2 = 1$$

(a) Find all solutions to the system. (It might help to draw a picture.)
(b) We wish to find a solution to the system by Newton's method. If $(x_0, y_0) = (1, 1)$ what is (x_1, y_1) ? Do not reduce to a single equation.

7. (15 points) Consider the initial value problem

$$\frac{dy}{dt} = ty^2, \qquad y(1) = 2.$$

(a) Verify that the solution is $Y(t) = \frac{2}{2-t^2}$.
Find approximations to $Y(1.2)$ by using
(b) two steps of the Euler method with $h = .1$.
(c) one step of the Improved Euler method with $h = .2$.
In (b) and (c) compare your answers with the exact solution.