AMSC 460 - HW10

```
clear all; format compact; close all; syms f(x) x y z
```

Problem 1 (finished)(AFFRImative)

Consider the following system of equations. f1(x, y, z) = x^2 + y^2 + z^2 _ 1 = 0, f2(x, y, z) = 2x^2 + y^2 _ 2z = 0, f3(x, y, z) = 3x^2 _ 4y + z^2 = 0 This system can be concisely represented as F(x) = 0, where F(x) = (f1, f2, f3)^T, x = (x, y, z)^T and 0 = (0, 0, 0)^T.

(i) Find the Jacobian matrix DF(x).

```
f1(x, y, z) = x^2 + y^2 + z^2 - 1;
f2(x, y, z) = 2*x^2 + y^2 - 2*z;
f3(x, y, z) = 3*x^2 - 4*y + z^2;
F = @(x,y,z) [f1(x,y,z);f2(x,y,z); f3(x,y,z)];
DFx = jacobian([x^2 + y^2 + z^2,2*x^2 + y^2 - 2*z, 3*x^2 - 4*y + z^2],
 [x, y, z])
```

```
DFx =
[2*x, 2*y, 2*z]
[4*x, 2*y,  -2]
[6*x,  -4, 2*z]
```

(ii) (MATLAB) Starting with the initial condition x0 = (0.5, 0.5, 0.5)^T, implement 5 steps of the multi-variable Newton method to find the approximation x5.

```
x0 = [0.5; 0.5 ; 0.5]
i = 0;
while i < 5
    DFx0(1,1) = 2*x0(1); DFx0(1,2) = 2*x0(2); DFx0(1,3) = 2*x0(3);
    DFx0(2,1) = 4*x0(1); DFx0(2,2) = 2*x0(2); DFx0(2,3) = -2;
    DFx0(3,1) = 6*x0(1); DFx0(3,2) = -4; DFx0(3,3) = 2*x0(3);
    Fx0 =[ (x0(1)^2 + x0(2)^2 + x0(3)^2 -1) ; (2*x0(1)^2 + x0(2)^2 -
 2*x0(3)); (3*x0(1)^2 - 4*x0(2) + x0(3)^2)];
    S = DFx0\Fx0;
    x1 = x0 - S;
    x0 = x1
    i = i+1;
    fprintf('\tAfter %g steps\n', i)
end
```

```
x0 =
    0.5000
    0.5000
    0.5000
x0 =
    0.7308
    0.4423
    0.5769
 After 1 steps
x0 =
    0.6918
```

```
    0.4401
    0.5739
 After 2 steps
x0 =
    0.6907
    0.4401
    0.5739
 After 3 steps
x0 =
    0.6907
    0.4401
    0.5739
 After 4 steps
x0 =
    0.6907
    0.4401
    0.5739
 After 5 steps
```

(iii) Compute the absolute backward error of the approximate solution x5 in the 2-norm.

```
x5= [0.6907;0.4401;0.5739]
a = f1(0.6907,0.4401,0.5739)
b = f2(0.6907,0.4401,0.5739)
c = f3(0.6907,0.4401,0.5739)
Fdx = F(a,b,c)
M = norm(Fdx,2)
ans = double(norm(Fdx,2))
```

```
x5 =
    0.6907
    0.4401
    0.5739
a =
11571/100000000
b =
2099/100000000
c =
4017/25000000
Fdx =
-4999999801762767/5000000000000000
  -3213327818117/10000000000000000
   -838940155253/10000000000000000
M =
(50000000000000000000000000000000^(1/2)*50000001549903543986244883452427^(1/2))/50
ans =
    1.0000
```

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