AMSC/CMSC 460 Dr. Wolfe FINAL EXAM May 17, 2004

1.(10 points) Let $f(x) = x^2 - 6.1x + 1.5$. Then $f(4.71) = -5.0469$. Do the following in three digit arithmetic with rounding:

(a) Compute $(4.71)^2 - (6.1)(4.71) + 1.5$. Compute the relative error in the result.

(b) Evaluate $f(4.71)$ using Horner's method (synthetic division). Compute the relative error. What do you conclude?

2.(10 points)

(a) Let $x = 3 \cdot 10^{20}$, $y = 4 \cdot 10^{20}$. Compute $z = \sqrt{x^2 + y^2}$.

(b) In IEEE single precision floating point arithmetic the largest machine number is about $2^{127} \approx 1.70 \cdot 10^{38}$. What would happen if we tried to compute z according to the formula above in this arithmetic ?

(c) Can you rewrite the formula for z in such a way that we can compute it in the above arithmetic?

3. (15 points) Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

(a) Show that we <u>cannot</u> write $A = LU$ where U is upper triangular and L is lower triangular with ones on the diagonal.

(b) Find a permutation matrix P, upper triangular matrix U and a lower triangular matrix L such that $PA = LU$.

(c) Explain how the result of part (b) is used to solve $A\mathbf{x} = \mathbf{b}$.

4. (15 points) Given the data points $(0, 2)$, $(\frac{1}{2}, 5)$, $(1, 4)$.

(a) Find $p_2(x)$, the polynomial of degee ≤ 2 interpolating this data.

(b) Find the function $P(x)$ of the form

$$P(x) = A + B\cos \pi x + C \sin \pi x$$

interpolating the data.

5. (12 points) Find constants A and B such that the integration rule

$$\int_0^h f(x)\sqrt{x}\, dx \approx h(Af(0) + Bf(h))$$

is exact for all first degree polynomials.

6. (13 points)

(a) What are the solutions α, if any, of the equation $x = \sqrt{1+x}$?

(b) Does the iteration $x_{n+1} = \sqrt{1 + x_n}$ converge to any of these solutions (assuming x_0 is chosen sufficiently close to α)?

7. (15 points)

(a) Transform the second order system of differential equations

$$\begin{gathered} u'' + 3v' - 2u = \sin t \\ v'' + u' - u + v = \cos t \\ u(0) = 1, \quad u'(0) = 2, \quad v(0) = 3, \quad v'(0) = 4 \end{gathered} \tag{1}$$

into a first order system. (Here $' = \frac{d}{dt}$).

(b) Compute an approximation to the solution of (1) at $t = 0.1$ using Euler's method with $h = .1$.

8. (10 points)Let

$$A = \begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 14 \\ 12 \end{bmatrix}.$$

As you can easily check, the solution of $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = (3, 2)^T$.

(a) Find an approximate solution of $A\mathbf{x} = \mathbf{b}$ by doing three Jacobi iterations starting at $\mathbf{x}^{(0)} = (0, 0)^T$.

(b) Find an approximate solution of $A\mathbf{x} = \mathbf{b}$ by doing three Gauss-Seidel iterations starting at $\mathbf{x}^{(0)} = (0, 0)^T$.