Computational Methods Summer 2021

HOMEWORK 9

Due Date: Tuesday, June 15

1. Rearrange the following equations to form a strictly diagonally dominant system. Apply two steps of Jacobi and Gauss-Seidel methods starting with the zero vector. (Do the computations by hand.)

$$
\begin{aligned}
u + 3v &= -1, \\
5u + 4v &= 6.
\end{aligned}
$$

2. Componentwise the Jacobi method for solving $A\mathbf{x} = \mathbf{b}$ reads

$$
x_i^{(k+1)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right], \tag{1}
$$

where $x_i^{(k)}$ denotes the i^{th} component of the k^{th} Jacobi iterate, for $i = 1, ..., n$.

In many practial problems the matrix A is *sparse*, with many of its entries being zero. Iterative methods can work well for such problems since the zeros don't need to be stored, and operation counts are reduced. A usual example are banded matrices such as the following tridiagonal matrix with bandwidth 3,

$$
A = \begin{pmatrix}
d_1 & u_1 & 0 & 0 & 0 \\
l_2 & d_2 & u_2 & 0 & 0 \\
0 & l_3 & d_3 & u_3 & 0 \\
0 & 0 & l_4 & d_4 & u_4 \\
0 & 0 & 0 & l_5 & d_5
\end{pmatrix}. \tag{2}
$$

Only the d_i, l_i, u_i entries need to be stored. The notation is extended in an obvious way to matrices of size $n \times n$.

For this problem, rewrite the Jacobi method (1) for a tridiagonal matrix of order n, using the notation introduced in (2). In doing so, suppose that only the tridiagonal (nonzero) terms are stored and can be accessed. As an example, neither $a_{1,3}$ nor $a_{3,1}$ should be accessed, and your revision should reflect that.

3. (Optional, not graded) For Problem 1, compute the relative forward error, relative backward error, and error magnification factor (condition number), using 1-norms.