Final Exam : AMSC/CMSC 460

Section: 0101, Spring 2017

May 19, 2017 (8:00-10:00am)

Instructor: D. Dahiya Total points: 80

Do any eight problems. 10 points for each problem.

1. Let x and x_A denote the exact and approximate values of a number: $x = 998/999$, $x_A = 0.9990000$. Let y and y_A be the exact and approximate values of a number computed using the relation $y = (1-x)/0.001$, $y_A = (1-x_A)/0.001$. Find the number of significant digits of (i) x in x_A and (ii) y in y_A.

2. The function
$$g(x) = \left(\frac{10}{4+x}\right)^{1/2}$$
has a fixed point in the interval $[1,2]$. Without computing the successive approximations, show that the fixed point iterations given by $x_{n+1} = g(x_n)$ converge to the fixed point.

3. Consider the system of equations $Ax = b$ with
$$A = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}.$$
Find an upper bound in the relative error in the solution when vector b changes from $(1\ 1)^T$ to $(0.9\ 1)^T$. Use l_∞ norm for vectors in $\mathbb{R}^2$ and use matrix norm subordinate to the l_∞ vector norm.

4. Using Cholesky factorization solve the linear system $AX = b$, with
$$A = \begin{pmatrix} 4 & 2 \\ 2 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 5 \end{pmatrix}.$$

5. Find the interpolating polynomial $Q_2(x) \in \Pi_2$ for $f(x) = \sin(\pi x)$ in the interval $[-1,1]$, interpolating $f(x)$ at Chebyshev points. What is the upper bound for the absolute value of interpolation error ?

6. Find the Hermite interpolation polynomial $p(x)$ that satisfies $p(0) = f(0)$, $p(1) = f(1)$, $p'(1) = f'(1)$, $p''(1) = f''(1)$, where $f(x) = e^x$. Make the divided difference table for this interpolation with all the entries.

7. Find the linear least squares polynomial approximation to $f(x) = 1/x$ in interval $[1,3]$.

8. Find the constants x_0, x_1, and c_1 so that the quadrature formula

$$\int_0^1 f(x)dx = \frac{1}{2}f(x_0) + c_1 f(x_1)$$

has the highest possible degree of precision. What is the degree of precision ?

9. The initial value problem $y' = y$, $y(0) = 1$ has exact solution $y = e^x$. Compute the approximate solution $y(0.03)$ using Euler's method and Runge-Kutta method of order 2. Take $h = 0.01$. Which one of the two methods is more accurate (as compared to the exact solution) ?