Computational Methods Summer 2021

HOMEWORK 21

Due Date: Tuesday, July 6

Homework should be handed in *individually*, though you may work with others and collaboration is encouraged. For MATLAB problems please follow the guidelines specified in ELMS.

1. Consider the integral $\int_0^1 e^{3x}\,\mathrm{d}x$.

   (a) If using the composite trapezoid rule, how many subintervals are needed to approximate the integral to within an absolute error of 10^{-8}?

   (b) Implement the composite trapezoid rule in MATLAB. For $n = 10^p$, with $p = 1, 2, 3, 4$, keep track of the error $E_T(n) = |I - T(f, n)|$, where $T(f, n)$ denotes the trapezoid rule with n subintervals. Plot E_T for the given values of n on the same plot (use logarithmic axes for better scaling). Does your plot agree with the analysis performed in part (a)?

   Pro tip: Vectorize your code (for the nodes, function values, summations) if possible, instead of using 'for loops', otherwise the run-time can be very slow - especially for the $p = 4$ case.

2. (Optional, not graded) Use the method of undetermined coefficients to derive a quadrature of the form

$$\int_0^1 f(x)\,\mathrm{d}x \approx Af(1/3) + Bf(3/4).$$

   Transform the quadrature to the new interval $[a, b]$. Apply this result to approximate $\int_0^\pi \sin x$. Compare the result of the approximation with the exact value of the integral.

3. (Optional, not graded) Derive a formula for approximating $\int_1^2 f(x)\,\mathrm{d}x$ in terms of $f(0)$, $f(1)$, and $f(3)$. Your quadrature should be exact for all polynomials of degree ≤ 2.

4. (Optional, not graded) Repeat Problem 1 but instead use composite Simpson's Rule.