AMSC 460 - HW 5

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Problem 1

Compute the relative error d = `x # fl(x)`/|x| exactly as a base-10 number, and show that d satisfies the upper bound d ##_mach/2.

```
d = abs(0.4 * 2^(-49))/abs(12.8)
eps/2 - d
```

```
d =

   5.5511e-17


ans =

   5.5511e-17
```

```
%ans = 5.551115123125783e-17 > 0 so the d satisfies the upperbound
%d ##_mach/2.
```

Problem 2

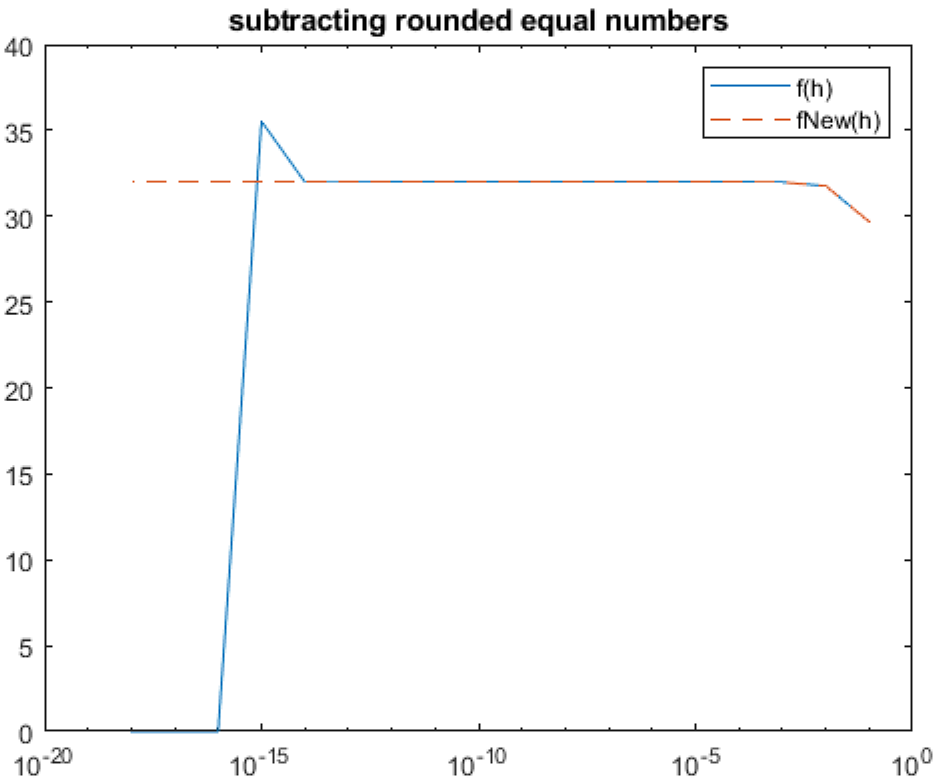
Let x = 2. To avoid subtraction of nearly equal numbers, find an alternative form f~(h) ≡ f(h) to evaluate f(h) = x^4 _ (x _ h)^4 /h for small h. Compute f(h) using MATLAB based on the formula (1) and the alternative form f~(h) you propose, and report your results for h = 10^_1, 10^-2, ..., 10^-18 on a semilogx plot (both functions should be on the same graph). What is lim_h#0 f(h)? Does your modified function compute more accurately for small h?

```
x = 2;
f = @(h) (x^4 - (x-h).^4)./(h);

          fNew(h) = [x^4 # (x # h)^4 /h] * [(x^4 + (x # h)^4 )/(x^4 + (x # h)^4)]
                  = (x^8 - (x # h)^8 )/h(x^4 + (x # h)^4)
          factor (x^8 - (x # h)^8 ) = h(x^4+(x-h)^4)(x^2+(x-h)^2)(2x-h)
          Cancel the common factor : h(x^4 + (x # h)^4)
          fNew(h) = (2x^2-2xh+h^2)(2x-h) = 4x^3 - 6x^2h + 4xh^2 - h^3

fNew = @(h) ( 4*(x^3) - 6*(x^2)*h + 4*x*(h.^2) - (h.^3));
x = 10*ones(1,18); y = 1:1:18; h = x.^ (-y);
semilogx(h,f(h),h,fNew(h),'--')
```

```
title 'subtracting rounded equal numbers'; legend({'f(h)','fNew(h)'});
```




```
%Since h#0, the lim_h#0 f(h) should be 32. However, for very small h
 the error starts
%increasing due to the subtraction of nearly equal numbers.
%Modified function computes more accurately for smaller h,the dash
 line
%represent the modified function and lim_h#0 fNew(h) = 32.
```

Problem 3 (Optional)

Consider a right triangle whose legs are of length 3344556600 and 1.2222222 (seven 2's). Using MATLAB to compute, how much longer is the hypotenuse than the longer leg? Explain how you arrived at your answer.

```
a = 3344556600; b = 1.2222222;
c = sqrt(vpa(a)^2 + vpa(b)^2)
c-a
```

```
c =

3344556600.0000000002233221447311


ans =
```

```
0.000000000223322144731059430844800074784292
```

```
%Thus the length of the hyphotenuse os greater than the length of the
 longer side by
%2.233221447310594e-10
%vpa(x) uses variable-precision floating-point arithmetic (VPA) to
 evaluate each element
%of the symbolic input x to at least d significant digits, where d is
 the value of the digits
%function. The default value of digits is 32. If we calculate c =
 sqrt(a^2+b^2) directly
%Matlab will "ignore" b and return c=sqrt(a^2)# since a will be much
 bigger than b
%
%
```

If we can not use vpa(x) to solve this problem, we can do:

```
          c - a = #(a^2 +b^2) - a = [#(a^2 +b^2) - a]× [#(a^2 +b^2) + a]/[#(a^2 +b
                                  = [(a^2 + b^2) - a^2] / [#(a^2 +b^2) + a]
                                  = b^2 / [#(a^2 +b^2) + a]
```

```
b^2 / (sqrt(a^2+b^2)+a)
```

```
ans =

   2.2332e-10
```

```
          c - a = 2.233221447310594e-10
```

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