Table of Contents

```
%%Yisheng Leng && Yizhan Ao
```

Matlab 6

Problem 3 Part A

```
A = [3 -1;1 -2]
[xi, R] = eig(sym(A))

% x(t) = c1*[5/2 - 21^(1/2)/2;1]*exp((1/2 - 21^(1/2)/2)t) +
 c2[21^(1/2)/2 +
% 5/2;1]*exp((21^(1/2)/2 + 1/2)t)
% The solution will be approaching 0 when y(t) increases faster than
 x(t) for
% x(t)/y(t) and approach positive infinity if y(t) increases at a
 slower
% rate than x(t). These two solutions could approach either zero or
% positive infinity depending on whether the exponents attached to
 their
% eigenvectors is positive or negative.


A =

     3    -1
     1    -2


xi =

[5/2 - 21^(1/2)/2, 21^(1/2)/2 + 5/2]
[               1,               1]
```

```
R =

[1/2 - 21^(1/2)/2,                    0]
[             0, 21^(1/2)/2 + 1/2]
```

Problem 3 Part B

```
A = [3 3;-3 -2]
[xi, R] = eig(sym(A))

% x(t) = c1[5/6 - (11^(1/2)*i)/6;1]*exp((1/2 - (11^(1/2)*i)/2)t) +
 c2[ 5/6
% + (11^(1/2)*i)/6;1]*exp(( 1/2 + (11^(1/2)*i)/2)t)


A =

     3     3
    -3    -2


xi =

[(11^(1/2)*1i)/6 - 5/6, - (11^(1/2)*1i)/6 - 5/6]
[                    1,                        1]


R =

[1/2 - (11^(1/2)*1i)/2,                        0]
[                    0, (11^(1/2)*1i)/2 + 1/2]
```

Problem 3 Part C

```
A = [-2 -1 2;0 4 5;0 -1 0]
[xi, R] = eig(sym(A))
b = [7;5;5]
c = xi\b

% x(t) = (5/2 - (15*i)/2)*[1;i-2;1]*exp(2-i) + ((15*i)/2 +
% 5/2)*[1;-i-2;1]*exp(i+2)+ 2*[1;0;0]*exp(-2)


A =

    -2    -1     2
     0     4     5
     0    -1     0
```

```
xi =

[1,          1,          1]
[0, - 2 + 1i, - 2 - 1i]
[0,          1,          1]


R =

[-2,        0,        0]
[ 0, 2 - 1i,        0]
[ 0,        0, 2 + 1i]


b =

     7
     5
     5


c =

          2
5/2 - 15i/2
5/2 + 15i/2
```

Problem 3 Part D

```
syms x y z t
[x,y,z] = dsolve('Dx = -2*x-y+2*z','Dy = 4*y+5*z','Dz = -
y','x(0)=7','y(0)=5','z(0)=5','t')
```

Warning: Support of character vectors and strings will be removed in a future
release. Use sym objects to define differential equations instead.

```
x =

2*exp(-2*t) + 5*exp(2*t)*cos(t) - 15*exp(2*t)*sin(t)


y =

5*exp(2*t)*cos(t) + 35*exp(2*t)*sin(t)


z =

5*exp(2*t)*cos(t) - 15*exp(2*t)*sin(t)
```

Problem6 Part A

Finding Critical Points

```matlab
syms x y
syst1 = x*(1.5-x-0.5*y);
syst2 = y*(2-1.5*x-0.5*y);
[xc, yc] = solve(syst1, syst2, x, y);
disp('Critical points:');
disp([xc yc])

% Finding Stability
A = jacobian([syst1 syst2], [x y])
evals = eig(A);
disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,1):');
disp(double(subs(evals, {x, y}, {1, 1})))
disp('Eigenvalues at (3/2,0);');
disp(double(subs(evals, {x, y}, {3/2, 0})))
disp('Eigenvalues at (0,4):');
disp(double(subs(evals, {x, y}, {0, 4})))

% According to the eigenvalues we calculated, our stability points
 are:
% (0,0) - unstable critical point.
% (1,1) - asymptotically stable.
% (3/2,0) - asymptotically stable.
% (0,4) - asymptotically stable.

Critical points:
[  0, 0]
[  1, 1]
[3/2, 0]
[  0, 4]


A =

[3/2 - y/2 - 2*x,              -x/2]
[      -(3*y)/2, 2 - y - (3*x)/2]

Eigenvalues at (0,0);
    1.5000
    2.0000

Eigenvalues at (1,1):
   -1.6514
    0.1514

Eigenvalues at (3/2,0);
   -1.5000
   -0.2500
```
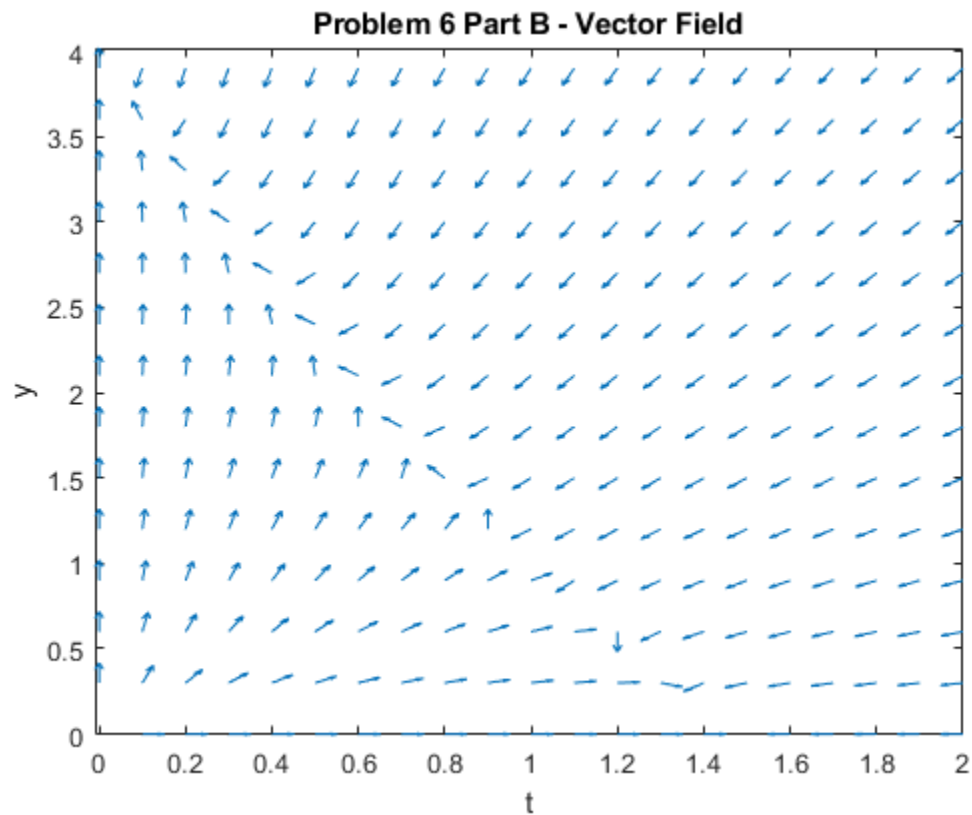
```
Eigenvalues at (0,4):
    -2.0000
    -0.5000
```

Problem 6 Part B

```matlab
[X, Y] = meshgrid(0:0.1:2, 0:0.3:4);
U = X.*(1.5-X-0.5*Y);
V = Y.*(2-1.5*X-0.5*Y);
L = sqrt((U/2).^2 + (V/4.5).^2);
vectfield = quiver(X, Y, U./L, V./L, 0.4);
axis tight
xlabel t, ylabel y
title 'Problem 6 Part B - Vector Field'
```


Problem 6 Part C

```matlab
f = @(t, x) [x(1)*(1.5-x(1)-.5*x(2)); x(2)*(0.75-0.125*x(1)-x(2))];
figure; axes; hold on
for a = .25:0.25:1.75
    for b = 0.5:0.5:4
        [t, xa] = ode45(f, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
```

```
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis([0 3/2 0 4])
xlabel t, ylabel y
title 'Problem 6 Part C Trajectories'
```

Warning: Failure at t=-1.787421e+00. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (3.552714e-15)
at time t.
Warning: Failure at t=-9.086971e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-6.193112e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-4.712359e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-3.807170e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-3.195186e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-2.753348e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-1.719958e+00. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (3.552714e-15)
at time t.
Warning: Failure at t=-8.915683e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.

Warning: Failure at t=-6.113275e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-4.666047e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.776913e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.173867e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-2.737515e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-1.642053e+00. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (3.552714e-15)
at time t.
Warning: Failure at t=-8.721853e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-6.025396e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-4.616012e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.744643e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.151338e-01. Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)

```
at time t.
Warning: Failure at t=-2.720899e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-1.543278e+00.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (3.552714e-15)
at time t.
Warning: Failure at t=-8.493366e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-5.926630e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-4.561281e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.709941e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-3.127393e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-2.703389e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (8.881784e-16)
at time t.
Warning: Failure at t=-1.293918e+00.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (3.552714e-15)
at time t.
Warning: Failure at t=-8.193270e-01.  Unable to meet integration
  tolerances
without reducing the step size below the smallest value allowed
  (1.776357e-15)
at time t.
Warning: Failure at t=-5.811584e-01.  Unable to meet integration
  tolerances
```

without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-4.500312e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-3.672211e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-3.101754e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-2.684836e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-1.441192e+00. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (3.552714e-15)
at time t.
Warning: Failure at t=-9.995400e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-7.450494e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-5.664643e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (1.776357e-15)
at time t.
Warning: Failure at t=-4.430286e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.
Warning: Failure at t=-3.630526e-01. Unable to meet integration tolerances
without reducing the step size below the smallest value allowed (8.881784e-16)
at time t.

Warning: Failure at t=-3.074028e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.
Warning: Failure at t=-2.665047e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.
Warning: Failure at t=-1.000853e+00. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (3.552714e-15)
at time t.
Warning: Failure at t=-8.021999e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (1.776357e-15)
at time t.
Warning: Failure at t=-6.529736e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 *(1.776357e-15)*
at time t.
Warning: Failure at t=-5.336554e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (1.776357e-15)
at time t.
Warning: Failure at t=-4.343823e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.
Warning: Failure at t=-3.583255e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.
Warning: Failure at t=-3.043621e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.
Warning: Failure at t=-2.643750e-01. Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (8.881784e-16)
at time t.

Problem 6 Part C Trajectories
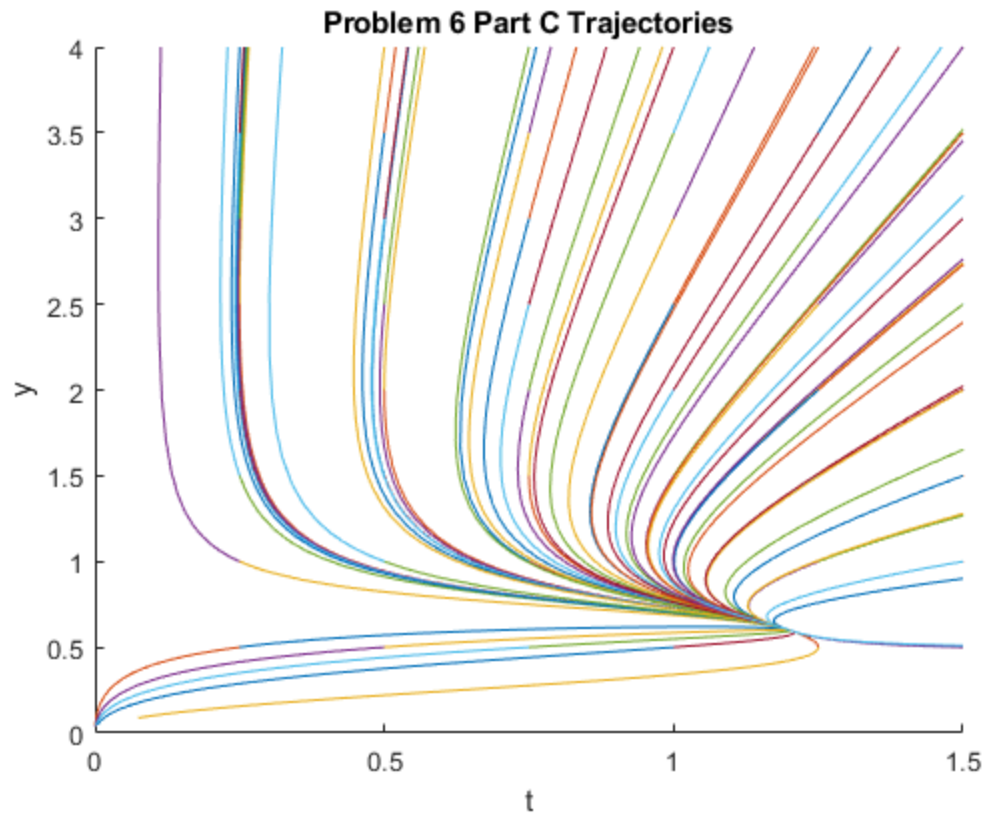
Problem 6 Part D

Suppose the initial state of the population is given by x(0) = 0.1, y(0) = 0.1. Find the state of the population t = 1, 2, 3, 4, 5, ... , 20.

```
[t, xa] = ode45(f, [0:20], [2.5, 2]);
disp([t xa])

% The population jumpes down from t = 0 to 3. Aftet that, the
 population
% continues decreasing to 0.6 at t = 20.
```

```
          0     2.5000     2.0000
     1.0000     1.2224     0.9553
     2.0000     1.1312     0.7569
     3.0000     1.1417     0.6811
     4.0000     1.1619     0.6447
     5.0000     1.1770     0.6254
     6.0000     1.1866     0.6147
     7.0000     1.1922     0.6085
     8.0000     1.1955     0.6050
     9.0000     1.1973     0.6029
    10.0000     1.1984     0.6017
    11.0000     1.1991     0.6010
    12.0000     1.1995     0.6006
    13.0000     1.1997     0.6003
```

```
 14.0000    1.1998    0.6002
 15.0000    1.1999    0.6001
 16.0000    1.1999    0.6001
 17.0000    1.2000    0.6000
 18.0000    1.2000    0.6000
 19.0000    1.2000    0.6000
 20.0000    1.2000    0.6000
```

Problem 6 Part E

```
[t, xa] = ode45(f, [0, -10], [0.99 0.99]);
separatrix(1) = plot(xa(:,1), xa(:,2), ':');
[t, xa] = ode45(f, [0, -10], [1.01 1.01]);
separatrix(1) = plot(xa(:,1), xa(:,2), ':');
xlabel t, ylabel y
title 'Problem 6 Part E Trajectories and Separatrices'
hold on
axis([0 2 0 4.5])
xlabel t, ylabel y
hold off
```

```
Warning: Failure at t=-1.575462e+00.  Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (3.552714e-15)
at time t.
Warning: Failure at t=-1.512567e+00.  Unable to meet integration
 tolerances
without reducing the step size below the smallest value allowed
 (3.552714e-15)
at time t.
```
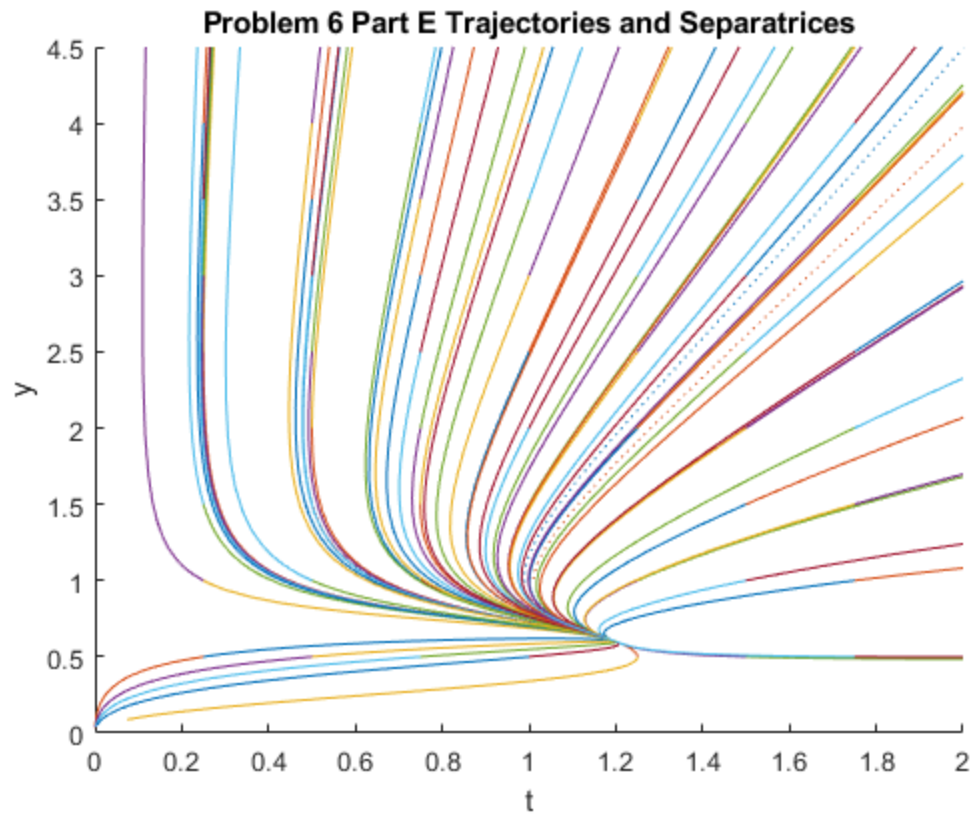
Problem 6 Part E Trajectories and Separatrices
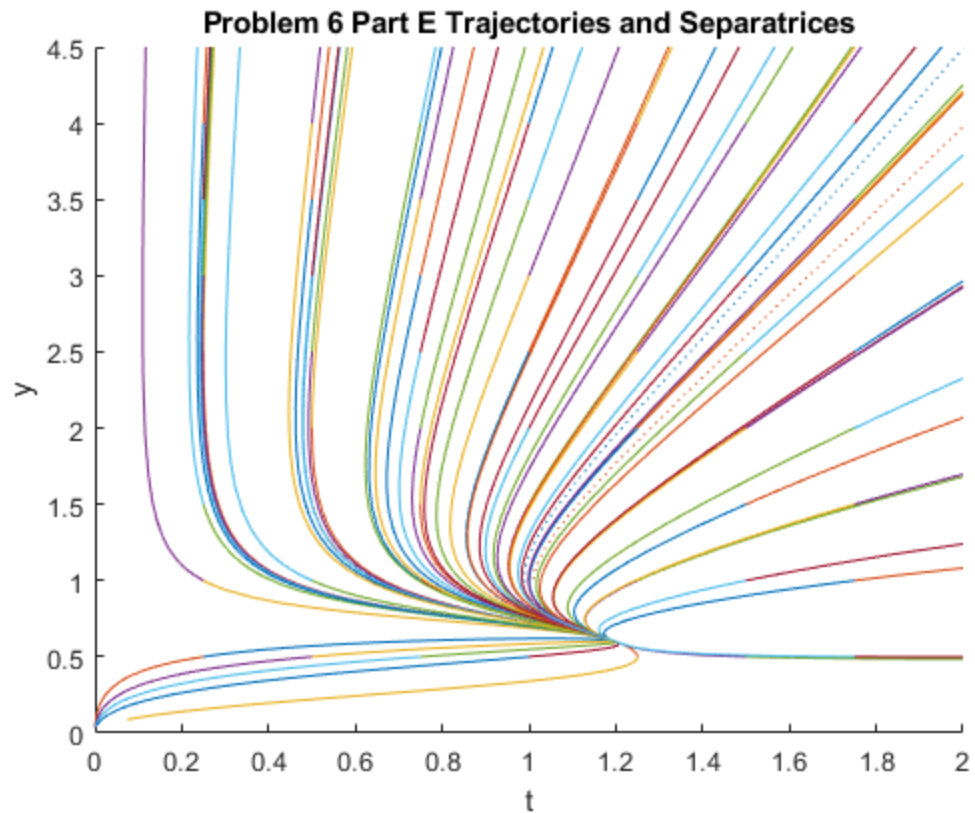
Problem 6 Part F

```
hitline = @(t, x) eval('deal(x(1) - 2.5, 1, 1)', 'x(1) - 2.5');
options = odeset('Events', hitline, 'relTol', 1e-4);
[t, xa] = ode45(f, [0, -10], [1.001 1.001], options);
xa(length(t),:)
```

ans =

2.5000   5.7116

Problem 6 Part E Trajectories and Separatrices

Published with MATLAB® R2020b