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## AMSC 460 - HW11

```
clear all; format compact; close all; syms f(x) x y z
```

### Problem 1 (finished)(AFFRImative)

Write down the polynomial that interpolates  $f(x) = e^x$  at the points  $x_0 = 0$  and  $x_1 = 1$  in Lagrange form and Newton form (using divided differences). Check that these polynomials are the same.

$f(x) = e^x$ , so the three points are  $(0,1)$   $(1,e)$   $P(x) = 1((x-1)/(0-1)) + e((x-0)/(1-0))$  The final answer is  $P(x) = ex - x + 1$

### Problem 2 (WTh

The Vandermonde matrix can be badly conditioned and is not ideal for solving many interpolation problems. On the other hand, some of this ill-conditioning can be mitigated by scaling the data. Suppose we are given data points  $(x_0, y_0), \dots, (x_n, y_n)$  with  $x_0 < x_1 < \dots < x_n$ . Consider scaling the  $x$  values by letting  $z_i = (x_i - x_0) / (x_n - x_0)$  where  $x_0$  and  $x_n$  are given numbers with  $x_n > x_0$ . The data points  $(x_i, y_i)$  change to  $(z_i, y_i)$ , and the interpolation polynomial changes to  $p_n(z) = a_0 + a_1 z + \dots + a_n z^n$ .

(a) The original data interval is  $x_0 \leq x \leq x_n$ . What is the data interval when using  $z = (x - x_0) / (x_n - x_0)$ ? What matrix equation must be solved to find the  $a$ 's in the above formula for  $p_n(z)$ ?

(b) Taking a hint from the previous step, the data will be scaled so that the new data interval is instead  $0 \leq z \leq 1$ . What must  $x_0$  and  $x_n$  be here?

$$\begin{aligned} \text{Then } z_0 &= (x_0 - x_0) / (x_n - x_0) = 0 \Rightarrow x_0 = 0 \\ \text{and } z_n &= (x_n - x_0) / (x_n - x_0) = 1 \Rightarrow x_n = x_n \\ \text{Then } x_0 + x_n &= 0 + x_n = x_n \\ \Rightarrow x_n &= (x_0 + x_n) / 2 \\ \text{and } x_n - x_0 &= x_n - 0 = x_n \\ \Rightarrow x_n &= (x_n - x_0) / 2 \end{aligned}$$

(c) Consider the following population data for the USA over the 100 year period between 1900 and 2000. The  $y$  values represent the population of the USA in millions. Using the direct approach (Vandermonde), plot the interpolation

function using the original xi data. You should use MATLAB's vander command to construct the Vandermonde matrix V . Using MATLABs cond command, what is the condition number cond(V) of the associated Vandermonde matrix V ?

```
x = [1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000];
y = [76.21; 92.23; 106; 123.2; 151.3; 179.3; 203.3; 226.5; 248.8; 281.4; 308.7]*1000000;
V = vander(x)
```

```
V =
    1.0e+33 *
Columns 1 through 7
    0.6131    0.0003    0.0000    0.0000    0.0000    0.0000    0.0000
    0.6462    0.0003    0.0000    0.0000    0.0000    0.0000    0.0000
    0.6808    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.7171    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.7551    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.7950    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.8367    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.8804    0.0004    0.0000    0.0000    0.0000    0.0000    0.0000
    0.9261    0.0005    0.0000    0.0000    0.0000    0.0000    0.0000
    0.9739    0.0005    0.0000    0.0000    0.0000    0.0000    0.0000
    1.0240    0.0005    0.0000    0.0000    0.0000    0.0000    0.0000
Columns 8 through 11
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
```

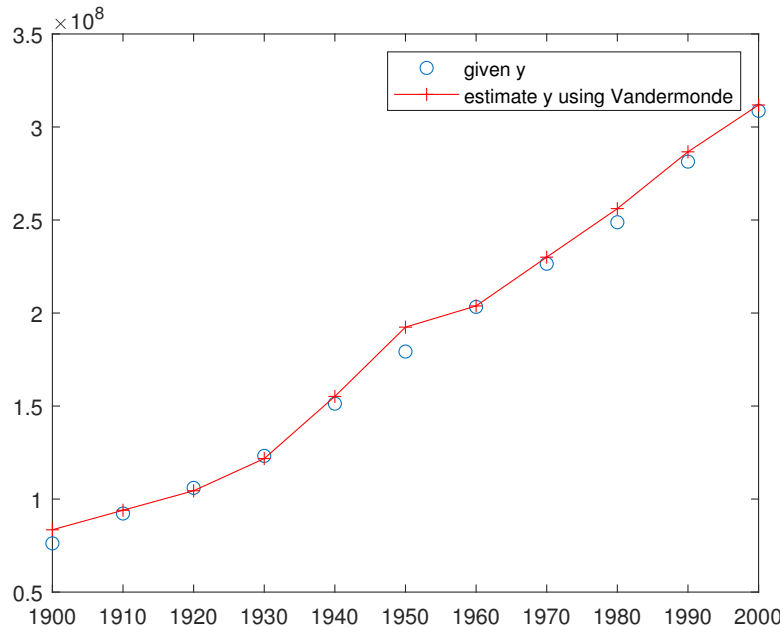
```
cond(V)
```

```
ans =
    6.5717e+48
```

```
p = V\y;
t = linspace(1900,2000,11);
```

```
f = polyval(p,t);
plot(x,y,'o',t,f,'r-+')
legend({'given y', 'estimate y using Vandermonde'})
```

Warning: Matrix is close to singular or badly scaled. Results may be inaccurate.  
RCOND = 2.119618e-49.

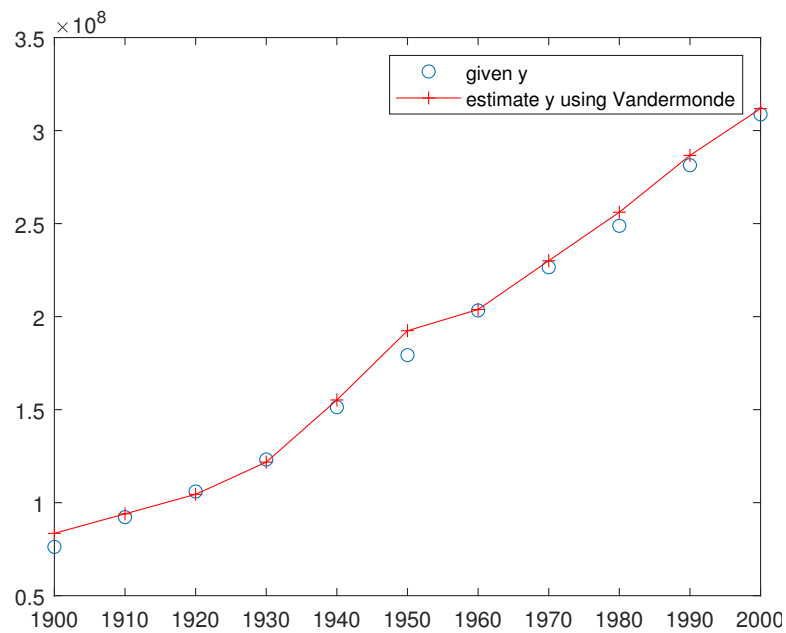


(d) Using the same population data from part (c), scale the data to  $[1, 1]$  and find the coefficients for  $p_n(z)$ . What is the condition number in this case? Once the  $a$ 's are computed the resulting (unscaled) polynomial is  $p_n(x) = a_0 + a_1(x/a) + \dots + a_n(x/a)^n$ .

Plot this function and compare it with the function you found in part (c). Comment on the difference between the two.

```
a = (2000+1900)/2; % = (x0 + xn) / 2
b = (2000-1900)/2; % = (xn - x0) / 2
disp('Scale the data to [1, 1]')
z = (x-a)/b
```

```
Scale the data to [1, 1]
z =
Columns 1 through 7
   -1.0000   -0.8000   -0.6000   -0.4000   -0.2000         0     0.2000
Columns 8 through 11
    0.4000    0.6000    0.8000    1.0000
```



```
disp('The new Vandermode matrix V')
Vz = vander(z)
```

The new Vandermode matrix V

Vz =

Columns 1 through 7

1.0000	-1.0000	1.0000	-1.0000	1.0000	-1.0000	1.0000
0.1074	-0.1342	0.1678	-0.2097	0.2621	-0.3277	0.4096
0.0060	-0.0101	0.0168	-0.0280	0.0467	-0.0778	0.1296
0.0001	-0.0003	0.0007	-0.0016	0.0041	-0.0102	0.0256
0.0000	-0.0000	0.0000	-0.0000	0.0001	-0.0003	0.0016
0	0	0	0	0	0	0
0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0016
0.0001	0.0003	0.0007	0.0016	0.0041	0.0102	0.0256
0.0060	0.0101	0.0168	0.0280	0.0467	0.0778	0.1296
0.1074	0.1342	0.1678	0.2097	0.2621	0.3277	0.4096
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Columns 8 through 11

-1.0000	1.0000	-1.0000	1.0000
-0.5120	0.6400	-0.8000	1.0000
-0.2160	0.3600	-0.6000	1.0000
-0.0640	0.1600	-0.4000	1.0000
-0.0080	0.0400	-0.2000	1.0000
0	0	0	1.0000
0.0080	0.0400	0.2000	1.0000

0.0640	0.1600	0.4000	1.0000
0.2160	0.3600	0.6000	1.0000
0.5120	0.6400	0.8000	1.0000
1.0000	1.0000	1.0000	1.0000

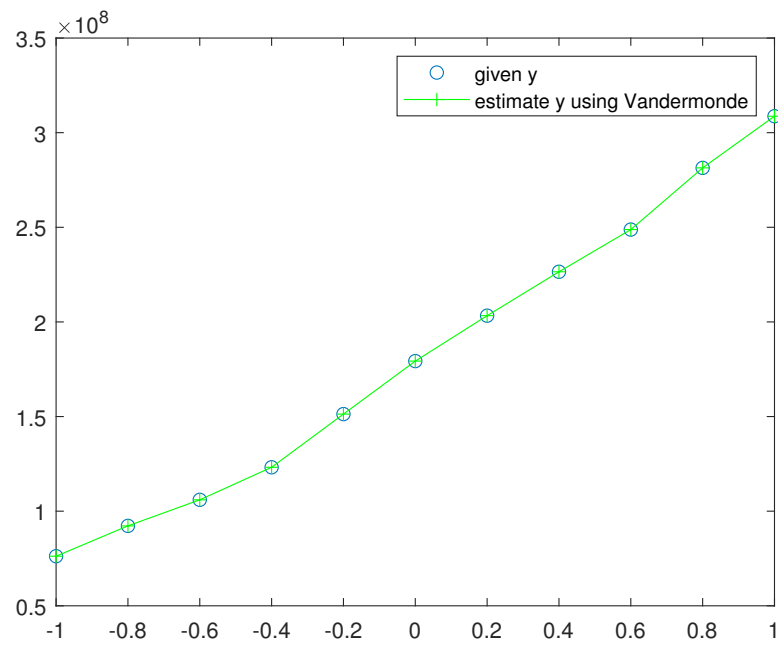
```
disp('The new condition number')
cond(Vz)
```

```
The new condition number
ans =
    1.3952e+04
```

```
disp('the coefficients for pn(z)')
p2 = Vz\y
```

```
the coefficients for pn(z)
p2 =
    1.0e+09 *
   -0.1803
   -0.5373
    0.4123
    1.0368
   -0.4142
   -0.5896
    0.2549
    0.0787
   -0.0596
    0.1277
    0.1793
```

```
t = linspace(-1,1,11);
f2 = polyval(p2,t);
plot(z,y,'o',t,f2,'g-+')
legend({'given y', 'estimate y using Vandermonde'})
```



The y using interpolation function in 4(d) is more accurate,  
 Also the condition number in 4(d) is way smaller than the condition number in 4(c)