Math 401 - Homework #2
2D Computer Graphics
due on Gradescope Wednesday, 9/15

1. **(Must do all computations by hand.)** Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ which first reflects through the y-axis, and then rotates counterclockwise about the origin by $\pi/4$ radians (45 degrees). Find the standard 2×2 matrix for this linear transformation in two ways:

   (a) Find the standard matrix of f by directly determining the images of the standard basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ under f.

   (b) Find the standard matrix of f by first finding the standard matrices of the reflection and of the rotation, and then use matrix multiplication.

2. Find the point obtained when $(7, 9)$ is rotated counterclockwise about the origin by 50 degrees ($5\pi/18$ radians). (Do the computation in MATLAB, but make it clear what computation you are doing.)

3. **(Must do all computations by hand.)**

   (a) Do Exercise 2.5 from the text. (The matrix should be 3×3, for homogeneous coordinates.)

   (b) Do Exercise 2.6 from the text. (The matrix should be 3×3, for homogeneous coordinates.)

4. Do Exercise 2.9 from the text.

5. Do Exercise 2.11 from the text.

6. Find the 3×3 matrix (for homogeneous coordinates) for the transformation which first rotates counterclockwise about the point $(4, 8)$ by $\pi/5$ radians, and then rotates counterclockwise about the point $(7, 1)$ by $4\pi/11$ radians.

7. **(Must do all computations by hand.)**

   (a) Let L denote a line in the plane that does **not** go through the origin. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ denote the transformation which reflects through the line L. Give a reason why f is not a linear transformation.

   (b) Reflection through a line which does not go through the origin can be represented as a 3×3 matrix (which operates on homogeneous coordinates). Give the matrix for reflection through the vertical line $x = 5$.

   (c) Give a formula for the image of the generic point (x, y) under reflection through the line $x = 5$.

8. **(Must do all computations by hand.)** Let L_θ denote the line in the plane through the origin that makes an angle of θ radians with the positive x-axis. In class, we saw that reflection through the line L_θ has standard matrix $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$. Now let θ and φ be two angles. Show that the composition of reflection through L_φ followed by reflection through L_θ is equal to a rotation about the origin, and find the angle of rotation. (Multiply the matrices and use trig identities to recognize the result as a rotation matrix.)

9. **(Optional - don't have to turn in)**

(a) Let L_m denote the line through the origin with slope m. Find the 2×2 matrix for the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects through the line L_m. Simplify your answer so that there are no trig or inverse trig functions. Then give the 3×3 matrix for this transformation (that operates on homogeneous coordinates).

(b) Let $L_{(a,b),m}$ be the line through the point (a, b) with slope m. Find the 3×3 matrix (for homogeneous coordinates) for the transformation of reflection through the line $L_{(a,b),m}$.