Computational Methods Summer 2021

HOMEWORK 15

Due Date: Friday, June 25

1. Download the text file `OrbitData2.txt` from the course webpage, which contains (x, y, z) coordinates of a collection of satellites around a certain planet. Save the file in your MATLAB directory. Your task is to find the sphere best approximating the data in the least-squares sense.

   Create a new script. Read the coordinate data using the following command

   ```
   C = dlmread('OrbitData2.txt')
   ```

   This stores the coordinates in a matrix C. Next create a vector `X` consisting of the x-coordinates (first column of C), a vector Y consisting of the y-coordinates (second column of C), and finally a vector Z with the z-coordinates. Plot the coordinate pairs using the command `plot3(X,Y,Z,'.','MarkerSize',10)` to make sure the data is properly loaded. It should look somewhat spherical.

   (a) Center the X, Y, Z coordinates around zero. For example, to center the x-values you would compute $X - \overline{X}$ (where $\overline{X}$ is the mean of the x-values).

   (b) The general equation of a sphere centered at the origin can be written

   $$ax^2 + by^2 + c = z^2,$$

   where a, b, c are constant coefficients to be determined. By forcing the coordinates stored in C to fit the model, a linear system $A\mathbf{u} = \mathbf{w}$ is created, where $\mathbf{u} = [a, b, c]^T$. Create the matrix A and vector $\mathbf{w}$ in MATLAB. (Do not print these out.)

   (c) Solve the normal equations for the system $A\mathbf{u} = \mathbf{w}$ to get the least-squares solution $\mathbf{u}$. (It is wise to compute $\mathrm{cond}(A^TA)$ to check the quality of the solution!)

   (d) Plot the coordinate data points and the equation of the ellipse in the same figure. (Use `fimplicit3` to plot the sphere. Also if the viewing angle is bad when publishing, you can change it by using the command `view`.)

   (e) Compute the error for the fit using the Euclidean norm $\|\cdot\|_2$ of the residual $\mathbf{r} = \mathbf{w} - A\mathbf{u}$.