Computational Methods   Summer 2021

HOMEWORK 11

Due Date: Monday, June 21

1. Write down the polynomial that interpolates $f(x) = e^x$ at the points $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$ in Lagrange form.

2. The Vandermonde matrix can be badly conditioned and is not ideal for solving many interpolation problems. On the other hand, some of this ill-conditioning can be mitigated by scaling the data. Suppose we are given data points $(x_0, y_0), .., (x_n, y_n)$ with $x_0 < x_1 < ... < x_n$. Consider scaling the x values by letting

$$z_i = \frac{x_i - \alpha}{\beta},$$

where α and β are given numbers with $\beta > 0$. The data points (x_i, y_i) change to (z_i, y_i), and the interpolation polynomial changes to

$$p_n(z) = a_0 + a_1 z + \cdots + a_n z^n.$$

(a) The original data interval is $x_0 \le x \le x_n$. What is the data interval when using $z = (x - \alpha)/\beta$? What matrix equation must be solved to find the $a_i'{s}$ in the above formula for $p_n(z)$?

(b) Taking a hint from the previous step, the data will be scaled so that the new data interval is instead $-1 \le z \le 1$. What must α and β be here?

(c) Consider the following population data for the USA over the 100 year period between 1900 and 2000.

x	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
y	76.21	92.23	106	123.2	151.3	179.3	203.3	226.5	248.8	281.4	308.7

The y values represent the population of the USA in *millions*. Using the direct approach (Vandermonde), plot the interpolation function using the original x_i data. You should use MATLAB's `vander` command to construct the Vandermonde matrix V. Using MATLABs `cond` commmand, what is the condition number $\mathrm{cond}(V)$ of the associated Vandermonde matrix V?

(d) Using the same population data from part (c), scale the data to $[-1, 1]$ and find the coefficients for $p_n(z)$. What is the condition number in this case? Once the $a_i'{s}$ are computed the resulting (unscaled) polynomial is

$$p_n(x) = a_0 + a_1\left(\frac{x - \alpha}{\beta}\right) + \cdots a_n\left(\frac{x - \alpha}{\beta}\right)^n.$$

Plot this function and compare it with the function you found in part (c). Comment on the difference between the two.