AMSC460/CMSC460 – Gil Ariel. Final, May 16th 2016

Answer four out of the five questions and show work.

Good luck.

1. Find a function of the form $y = a\cos x + b\cos(2x)$ that goes through the three points $(0,1)$, $(\pi/2,1)$ and $(\pi,-1)$. Solve the equations by hand in a least squares sense.

2. A function $f(x)$ is known to have two distinct roots in the interval $[-1,1]$. Write a Matlab function that gets as input a tolerance *tol*>0 and returns a vector with the two roots with accuracy *tol*.

   Any method or built in Matlab function can be used.

3. Ultra-spherical polynomials are the orthogonal polynomials on the interval $[-1,1]$ with weight function $w(x) = (1-x^2)^{\alpha}$, where $\alpha > 0$ is a parameter. Write a Matlab function that receives as input a number $\alpha > 0$ and an integer $n \ge 0$ and returns a vector holding the coefficients of the *n*'th degree ultra-spherical polynomial of degree *n*.

   All integrations and equation-solving should be done numerically.
   Any method or built-in Matlab function can be used.

4. Simpson's 3/8 rule approximates integrals of the form $\int_0^1 f(x)dx$ as

$$\frac{1}{8}\left[f(0)+3f(1/3)+3f(2/3)+f(1)\right].$$

   Find the degree of the rule.

5. A theorem on the rate of new records:
   Suppose X_1, X_2, ... is a sequence of (independent) continuous random numbers. We say that round *k* sets a new record if X_k is larger than all previous numbers $X_1,\ldots,X_{k-1}$. Let N_k denote the number of new records up to round *k*. Then, a theorem in probability states that N_k grows logarithmically, i.e. for large values of *k* it can be approximated as $N_k = a\ln(k)$, where $a > 0$ is a constant.

   a. Write a Matlab program that draws a sequence of random numbers X_1, X_2, ..., X_N from a given continuous distribution of your choice and calculates N_k for several large values of *k*.
   b. Write a Matlab program that numerically evaluates *a*.