Contents

```
clear all;
%Name Yizhan Ao MATH240 Project4
```

Problem 1

```
format rat
syms x
%(a)
e = [1 0 0 0 ; 0 1 0 0; 0 0 1 0; 0 0 0 1]
B = [1 1 2 4 ; 0 2 -1 -1; 0 0 3 0; 0 0 0 1]
C = [1 2 0 0 ; 3 1 3 3; 0 0 -1 0; 1 0 4 0]
%Since e is 4*4 I, B = eP(e<-B) = IP(E<-B)
%and C = EP(E<-C) =IP(E<-C)= P(E<-C)
P = B
Q = C

%(b)
A = inv(Q)
R= A*P

%(C)
X = [ 0 0 0 1]'
R*X

%(d)
pt_B = [0 3 2 1]'
pt_c = R*pt_B

%(e)
P * pt_B
%
```

```
e =

     1          0          0          0
     0          1          0          0
     0          0          1          0
     0          0          0          1


B =

     1          1          2          4
     0          2         -1         -1
     0          0          3          0
     0          0          0          1


C =

     1          2          0          0
     3          1          3          3
```

```
         0             0            -1             0
         1             0             4             0


P =

         1             1             2             4
         0             2            -1            -1
         0             0             3             0
         0             0             0             1


Q =

         1             2             0             0
         3             1             3             3
         0             0            -1             0
         1             0             4             0


A =

         0             0             4             1
       1/2             0            -2          -1/2
         0             0            -1             0
      -1/6           1/3          -7/3          -5/6


R =

         0             0            12             1
       1/2           1/2            -5           3/2
         0             0            -3             0
      -1/6           1/2         -23/3         -11/6


X =

         0
         0
         0
         1


ans =

         1
       3/2
         0
     -11/6


pt_B =

         0
         3
         2
         1


pt_c =

        25
        -7
        -6
     -47/3
```

```
ans =

     11
      3
      6
      1
```

Problem 2

```
format short
A = [163 34 -8; -522 -108 26; 990 210 -47]
%(a)
[P,D] = eig(A)

%(b)
P*D*inv(P)
A
%YEs they are identical

%(c)
E1 = D(1,1)
V1 = P(:,1)
E2 = D(2,2)
V2 = P(:,2)
E3 = D(3,3)
V3 = P(:,3)
%E1,E2,E3 are eigenvalues an v1,2,3 are the eigen vectors
```

```
A =

   163     34     -8
  -522   -108     26
   990    210    -47


P =

   -0.1229     0.1617    -0.1961
    0.3686    -0.5659     0.7845
   -0.9214     0.8085    -0.5883


D =

    1.0000         0         0
         0    4.0000         0
         0         0    3.0000


ans =

   163.0000    34.0000    -8.0000
  -522.0000  -108.0000    26.0000
   990.0000   210.0000   -47.0000


A =

   163     34     -8
  -522   -108     26
   990    210    -47


E1 =
```

```
    1.0000


V1 =

   -0.1229
    0.3686
   -0.9214


E2 =

    4.0000


V2 =

    0.1617
   -0.5659
    0.8085


E3 =

    3.0000


V3 =

   -0.1961
    0.7845
   -0.5883
```

Problem 3

```matlab
A = [-23 -32 -10; 11 15 5; 18 26 7]
%(a)
A^2
A^3
A^4
A^5
A^6
A^7
A^8
%Pattern A^(4k) = I,  k>0
%(b)
[P,D] = eig(A)
%(c)
D
D^2
D^3
D^4
%we cam further observe the fact that D^4 will be the identity matrix and
%the pattern is D^4*k where the k is 1,2,3,4 ...
%(d)
% 10000001 = 4n +1, n = 250000
```

```
A =

   -23   -32   -10
    11    15     5
    18    26     7
```

```
ans =

    -3    -4     0
     2     3     0
    -2    -4    -1


ans =

    25    36    10
   -13   -19    -5
   -16   -22    -7


ans =

     1     0     0
     0     1     0
     0     0     1


ans =

   -23   -32   -10
    11    15     5
    18    26     7


ans =

    -3    -4     0
     2     3     0
    -2    -4    -1


ans =

    25    36    10
   -13   -19    -5
   -16   -22    -7


ans =

     1     0     0
     0     1     0
     0     0     1


P =

  -0.7559 + 0.0000i  -0.7559 + 0.0000i   0.5774 + 0.0000i
   0.3780 + 0.0000i   0.3780 - 0.0000i  -0.5774 + 0.0000i
   0.5292 + 0.0756i   0.5292 - 0.0756i   0.5774 + 0.0000i


D =

   0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -1.0000 + 0.0000i


D =

   0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 1.0000i   0.0000 + 0.0000i
```

```
   0.0000 + 0.0000i    0.0000 + 0.0000i   -1.0000 + 0.0000i


ans =

  -1.0000 + 0.0000i    0.0000 + 0.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i   -1.0000 - 0.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i    0.0000 + 0.0000i    1.0000 + 0.0000i


ans =

  -0.0000 - 1.0000i    0.0000 + 0.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i   -0.0000 + 1.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i    0.0000 + 0.0000i   -1.0000 + 0.0000i


ans =

   1.0000 - 0.0000i    0.0000 + 0.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i    1.0000 + 0.0000i    0.0000 + 0.0000i
   0.0000 + 0.0000i    0.0000 + 0.0000i    1.0000 + 0.0000i
```

Problem 4

```
%(a)
A = [3 1; 0 3]
[P,D] = eig(A)
%(b)
%from the part A we can tell P is not invertible
P*D*inv(P)
%A is not equal to pdp^-1
%(c)
%Basis is for the eigenspace = [1;0]
%(d)
%There is no such basis that for R^2 therefore part b is wrong
```

```
A =

     3     1
     0     3


P =

    1.0000   -1.0000
         0    0.0000


D =

     3     0
     0     3


ans =

     3     0
     0     3
```

Problem 5

```matlab
%(a)
V1 = [ 9 14 -11 3 0 ]'
V2 = [ -14 -4 -10 9 -5]'
V3 = [ 1 -10 4 -7 5]'
V4 = [ 6 8 -1 -12 -8]'
%(b)
A = [V1 V2 V3 V4]
rref(A)
rank(A)
%there is a pivot in each column therefore the v1 v2 v3 v4 are linearly
%independent So v1- v4 are also linearly independent and form a basis for W

%(c)
W1 = V1
W2 = V2 -(dot(V2,W1)/ dot(W1,W1))*W1
%(d)
W3 = V3 -(dot(V3,W1)/dot(W1,W1))*W1- (dot(V3,W2)/ dot(W2,W2))*W2
W4 = V4 -(dot(V4,W1)/dot(W1,W1))*W1- (dot(V4,W2)/ dot(W2,W2))*W2- (dot(V4,W3)/dot(W3,W3))*W3

%(e)
K1 = W1/norm(W1)
K2 = W2/norm(W2)
K3 = W3/norm(W3)
K4 = W4/norm(W4)
%(f)
Q = [K1 K2 K3 K4]
Q'*Q
%Q columns are orthogonal  since its transpose is equal to its identity
%matrix
%(g)
R = Q'*A
Q*R
A
%A = Q*R
%(h)
[Q1 R1] = qr(A,0)
Q
R
```

V1 =

```
     9
    14
   -11
     3
     0
```


V2 =

```
   -14
    -4
   -10
     9
    -5
```


V3 =

```
     1
   -10
     4
    -7
     5
```

```
V4 =

     6
     8
    -1
   -12
    -8


A =

     9   -14     1     6
    14    -4   -10     8
   -11   -10     4    -1
     3     9    -7   -12
     0    -5     5    -8


ans =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
     0     0     0     0


ans =

     4


W1 =

     9
    14
   -11
     3
     0


W2 =

  -13.0049
   -2.4521
  -11.2162
    9.3317
   -5.0000


W3 =

    1.4401
   -3.9922
   -4.6557
   -2.7611
    3.5029


W4 =

   -1.7788
    1.3028
   -2.5483
  -10.0873
   -9.1222
```

K1 =

     0.4461
     0.6940
    -0.5452
     0.1487
          0

K2 =

    -0.6399
    -0.1207
    -0.5519
     0.4592
    -0.2460

K3 =

     0.1866
    -0.5172
    -0.6032
    -0.3577
     0.4538

K4 =

    -0.1269
     0.0930
    -0.1819
    -0.7199
    -0.6510

Q =

     0.4461   -0.6399   0.1866   -0.1269
     0.6940   -0.1207   -0.5172   0.0930
    -0.5452   -0.5519   -0.6032   -0.1819
     0.1487   0.4592   -0.3577   -0.7199
          0   -0.2460   0.4538   -0.6510

ans =

     1.0000   0.0000   0.0000   0.0000
     0.0000   1.0000   0.0000   -0.0000
     0.0000   0.0000   1.0000   0.0000
     0.0000   -0.0000   0.0000   1.0000

R =

    20.1742   -2.2306   -9.7154   6.9891
    -0.0000   20.3230   -6.0853   -7.7946
    -0.0000   0.0000   7.7189   -1.7529
     0.0000   0.0000   0.0000   14.0115

ans =

     9.0000 -14.0000   1.0000   6.0000
    14.0000   -4.0000 -10.0000   8.0000
   -11.0000 -10.0000   4.0000   -1.0000
     3.0000   9.0000   -7.0000 -12.0000
    -0.0000   -5.0000   5.0000   -8.0000

```
A =

    9   -14     1     6
   14    -4   -10     8
  -11   -10     4    -1
    3     9    -7   -12
    0    -5     5    -8


Q1 =

  -0.4461    0.6399    0.1866   -0.1269
  -0.6940    0.1207   -0.5172    0.0930
   0.5452    0.5519   -0.6032   -0.1819
  -0.1487   -0.4592   -0.3577   -0.7199
        0    0.2460    0.4538   -0.6510


R1 =

  -20.1742    2.2306    9.7154   -6.9891
        0  -20.3230    6.0853    7.7946
        0         0    7.7189   -1.7529
        0         0         0   14.0115


Q =

   0.4461   -0.6399    0.1866   -0.1269
   0.6940   -0.1207   -0.5172    0.0930
  -0.5452   -0.5519   -0.6032   -0.1819
   0.1487    0.4592   -0.3577   -0.7199
        0   -0.2460    0.4538   -0.6510


R =

   20.1742   -2.2306   -9.7154    6.9891
   -0.0000   20.3230   -6.0853   -7.7946
   -0.0000    0.0000    7.7189   -1.7529
    0.0000    0.0000    0.0000   14.0115
```

Problem 6

```
W= [5 9 -2 2 -1; -5 -12 -4 3 -2; 0 2 4 -2 2; -4 -3 10 -4 5]
v1= [5 -5 0 -4 -3 4]'
v2= [9 -12 2 -3 -7 2]'
v3= [-2 -4 4 10 -2 -12]'
v4= [2 3 -2 -4 -2 7]'
v5 =[-1 -2 2 5 -1 -6]'
A = [v1 v2 v3 v4 v5]
%(a)
rank(W)
rref(W)
%(b)
%only v1 v2 v4 has basis for W
B = [ V1 V2 V4]
%(c)
[Q, R] = qr(B)
%(d)
e = Q*Q'
v = [1 1 1 1 1 ]'
e * v
%(e)
```

```
F = B'
rref(F)
%(f)
w1 = v1
w2 = v2 -(dot(v2,w1)/dot(w1,w1))*w1
w3 = v3 -(dot(v3,w1)/dot(w1,w1))*w1 -(dot(v3,w2)/dot(w2,w2))*w2
w4 = v4 -(dot(v4,w1)/dot(w1,w1))*w1 -(dot(v4,w2)/dot(w2,w2))*w2 - (dot(v4,w3)/dot(w3,w3))*w3
w5 = v5 -(dot(v5,w1)/dot(w1,w1))*w1 -(dot(v5,w2)/dot(w2,w2))*w2 - (dot(v5,w3)/dot(w3,w3))*w3 - (dot(v5,w4)/dot(w4,w4))*w4
%(g)
M = [w1 w2 w3 w4 w5]
A.*v1
```

W =

```
    5      9     -2     2     -1
   -5    -12     -4     3     -2
    0      2      4    -2      2
   -4     -3     10    -4      5
```

v1 =

```
    5
   -5
    0
   -4
   -3
    4
```

v2 =

```
    9
  -12
    2
   -3
   -7
    2
```

v3 =

```
   -2
   -4
    4
   10
   -2
  -12
```

v4 =

```
    2
    3
   -2
   -4
   -2
    7
```

v5 =

```
   -1
   -2
    2
    5
   -1
```

```
     -6


A =

     5      9     -2      2     -1
    -5    -12     -4      3     -2
     0      2      4     -2      2
    -4     -3     10     -4      5
    -3     -7     -2     -2     -1
     4      2    -12      7     -6


ans =

     3


ans =

     1      0     -4      0     -2
     0      1      2      0      1
     0      0      0      1      0
     0      0      0      0      0


B =

     9    -14      6
    14     -4      8
   -11    -10     -1
     3      9    -12
     0     -5     -8


Q =

  -0.4461     0.6399    -0.1491     0.5892    -0.1485
  -0.6940     0.1207     0.1565    -0.6652    -0.1920
   0.5452     0.5519    -0.1056    -0.3439    -0.5183
  -0.1487    -0.4592    -0.6700     0.0756    -0.5590
        0     0.2460    -0.7023    -0.2937     0.5999


R =

 -20.1742     2.2306    -6.9891
        0   -20.3230     7.7946
        0          0    14.1208
        0          0          0
        0          0          0


e =

   1.0000    -0.0000     0.0000     0.0000     0.0000
  -0.0000     1.0000    -0.0000    -0.0000    -0.0000
   0.0000    -0.0000     1.0000    -0.0000     0.0000
   0.0000    -0.0000    -0.0000     1.0000    -0.0000
   0.0000    -0.0000     0.0000    -0.0000     1.0000


v =

     1
     1
     1
     1
```

```
         1


ans =

    1.0000
    1.0000
    1.0000
    1.0000
    1.0000


F =

     9     14    -11      3      0
   -14     -4    -10      9     -5
     6      8     -1    -12     -8


ans =

    1.0000        0        0    2.6076    2.6773
         0   1.0000        0   -3.8328   -3.2515
         0        0   1.0000   -3.0174   -1.9477


w1 =

     5
    -5
     0
    -4
    -3
     4


w2 =

    0.9780
   -3.9780
    2.0000
    3.4176
   -2.1868
   -4.4176


w3 =

   1.0e-14 *

   -0.0222
   -0.0888
    0.0444
    0.0888
   -0.2665
   -0.1776


w4 =

    0.1920
    0.5964
    0.4486
    2.3190
   -5.8373
   -1.3514
```

```
w5 =

    0.2647
    1.1615
   -1.0276
   -2.9059
    8.0477
    3.8453


M =

    5.0000    0.9780   -0.0000    0.1920    0.2647
   -5.0000   -3.9780   -0.0000    0.5964    1.1615
         0    2.0000    0.0000    0.4486   -1.0276
   -4.0000    3.4176    0.0000    2.3190   -2.9059
   -3.0000   -2.1868   -0.0000   -5.8373    8.0477
    4.0000   -4.4176   -0.0000   -1.3514    3.8453


ans =

    25    45   -10    10    -5
    25    60    20   -15    10
     0     0     0     0     0
    16    12   -40    16   -20
     9    21     6     6     3
    16     8   -48    28   -24
```