Contents

AMSC 460 - HW11

```
clear all; format compact; close all; syms f(x) x y z
```

Problem 1

Find a quartic Hermite polynomial that interpolates

$$p(0) = 1, \quad p'(0) = -1, \quad p(1) = -2, \quad p'(1) = 2, \quad p(2) = 2.$$

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x - 1)(x - 2)}{(-1)(-2)} = \frac{1}{2}(x^2 - 3x + 2)$$

now differentiate the equation above

$$L_0'(x) = (1/2)(x^2 - 3x + 2) * \frac{d}{dx} \;\; = \frac{1}{2} * (2x - 3)$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_0)} = \frac{(x - 0)(x - 2)}{(1)(-1)} = -x^2 + 2x$$

now differentiate the equation above

$$L_1'(x) == -2x - 2$$

similarly, we can have

$$L_2(x) = \frac{1}{2}(x^2 - 2)$$

$$L_2'(x) = \frac{1}{2}(2x - 1)$$

$$L_2(x_2) = \frac{1}{2}(2 * 2 - 1) = \frac{3}{2}$$

Now

$$p4(x) = f[0] + f_[0, 0, 1](x^2) + f[0, 0, 1, 1](x^2)(x - 1) + f[0, 0, 1, 1, 2]x^2(x - 1)^2$$

$$= -10x^4 + 27x^3 - 19x^2 - x + 1$$

Problem 2 (

Consider the function

$$f(x) = \frac{e^{3x}\sin(200x^2)}{1 + 20x^2}$$

on the interval $0 \leq x \leq 1$. The goal of this problem is to observe the error reduction in cubic spline interpolation when increasing the number of nodes.

(a)Show that $|J(x)| \leq 1$, $|J'(x)| \leq 1$, $|J''(x)| \leq 1$, and in general that $|J^{(k)}(x)| \leq 1$ for any positive integer k.

```
syms x n
f(x) = (exp(3*x)*sin(200*x^2)/(1+20*x^2))
ezplot(f(x))
```

```
f(x) =
(exp(3*x)*sin(200*x^2))/(20*x^2 + 1)
```

$|J(x)| \leq 1$

(b)Suppose we would like to approximate J with a Chebyshev interpolant. Determine how many interpolation points are required on the interval $[0, 10]$ so that the error (in the max-norm) is no more than 10^{-6}. [You don't have to write down the interpolant.]