Computational Methods Summer 2021

HOMEWORK 7

Due Date: Friday, June 11

1. Suppose you have used a finite element method to convert a system of PDE to a $10^6 \times 10^6$ linear system of equations $A\mathbf{x} = \mathbf{b}$. To check runtime, you decide to first solve a 100×100 linear system $C\mathbf{x} = \mathbf{d}$ and find that it takes a total of 0.001 seconds using an LU decomposition method (elimination + backsolve).

   Give an estimate of the time it will take to solve the system $Ax = b$. Would you consider using LU to solve $A\mathbf{x} = \mathbf{b}$? Or is a faster method needed? [Assume A and C have similar structure, so the difference in runtime is solely due to the different matrix sizes.]

2. Find the $PA = LU$ decomposition (using partial pivoting) for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}.$$

   All calculations should be recorded and done by hand. Check your answer using MATLAB's `lu` command.

3. (Optional, not graded) Find the LU decomposition of

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

4. (Optional, not graded) Suppose L is a nonsingular lower triangular matrix, P is a permutation matrix, and $\mathbf{b}$ is a given vector. How would you efficiently solve the following two linear systems? Without using inverses of course... Comment on the operation counts involved. [Consider that permutation matrices P are orthogonal, so we know $P^{-1} = P^T$.]

   (a) $LP\mathbf{x} = \mathbf{b}$

   (b) $PL\mathbf{x} = \mathbf{b}$