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0101

AMSC 460

FINAL EXAM **May 17, 2003**

1. Let $f(x) = e^x - 1 - x$.

a) What computational problem arises when we attempt to evaluate $f(x)$ for $x = .01$? To illustrate, evaluate $f(.01)$ using 4 digit, chopped arithmetic at each stage. Compare the result computing this way with the result you get using the 10 digit arithmetic of your calculator.

b) Find an alternate expression for $f(x)$ and evaluate again using 4 digit, chopped arithmetic. Show that this alternate method gives 4 accurate digits.

2. Let the matrix A and its inverse A^{-1} be

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9.1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 8\frac{1}{3} & -19\frac{1}{3} & 10 \\ -18\frac{2}{3} & 39\frac{2}{3} & -20 \\ 10 & -20 & 10 \end{bmatrix}$$

a) What is the condition number of A in the max norm?

b) Let z be the computed solution by Gaussian elimination of the equation $Ax = b$. How can you estimate the residual $Az - b$ in terms of the unit round off u of your machine?

c) How can you estimate the relative error $||x - z||/||z||$ in terms of the condition number of A and the unit roundoff?

3. Suppose we must solve the linear system $Ax = b$ for 500 different right-hand sides b. How would you use the LU factorization to do this in the most efficient manner? Write a short code to show how this is done. You do not have to use precise MATLAB instructions.

4. We are given 20 data points (x_n, y_n), which we assume are produced by a smoothly varying physical process.

a) What would be wrong with interpolating the 20 data points with a polynomial of degree 19? Why would you prefer to use a spline?

b) State the conditons which define a *complete* spline though the knots x_n. What information, in addition to the given data points, is needed for a complete spline?

c) Define what is meant by a *natural* spline.

5. Let $f(x) = x - e^{-x}$

a) Starting with $x_0 = 0$, use Newton's method to find a root of $f(x) = 0$. Calculate x_1, x_2 and x_3. What is the rate of convergence?

b) What would the rate of convergence of Newton's method be to the root of $x^3 - x^2 + x/4 = 0$ that lies in the interval $[.25, .75]$?

6. The one panel Simpson's rule for the interval $[a, b]$ is

$$S(f) = \frac{(b-a)}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)].$$

a) What is the degree of precision of Simpson's rule?

b) Is there another quadrature rule using 3 function evaluations which has a higher degree of precision? If so, what is it, and what is its degree of precision?

c) Derive the compound Simpson's rule.

7. Consider the initial value problem

$$y' = (x-1)y, \qquad y(0) = y_0 = 1.$$

a) Calculate two steps y_1 and y_2 using Euler's method with a step size $h = .1$.

b) Calculate two steps y_1 and y_2 using the backward Euler method with step size $h = .1$.

c) For what values of x is the equation stable, and for what values of x is it unstable?

d) For what values of x is the Euler method stable for this equation with a stepsize of $h = .1$?

c) If we were to calculate the solution on $[-1, 4]$ using the MATLAB code `ode45`, where would it take larger steps and where would it take smaller steps? Explain why.