HOMEWORK 13

Due Date: Wednesday, June 23

1. Suppose in designing a natural logarithm function for a calculator on the interval $[1, e]$, we are using a Chebyshev polynomial approximation. What is the smallest degree n of the polynomial that ensures an accuracy of 10^{-6} over the interval $[1, e]$?

2. Special functions appear in physics and applied mathematics, often as a solution to some ODE. The following function is in the *Bessel* family (https://en.wikipedia.org/wiki/Bessel_function)

$$J(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin(s)) \, \mathrm{d}s.$$

   (a) Show that $|J(x)| \le 1$, $|J'(x)| \le 1$, $|J''(x)| \le 1$, and in general that $|J^{(k)}(x)| \le 1$ for any positive integer k.

   (b) Suppose we would like to approximate J with a Chebyshev interpolant. Determine how many interpolation points are required on the interval $[0, 10]$ so that the error (in the max-norm) is no more than 10^{-6}. [You don't have to write down the interpolant.]