

Computational Methods Summer 2021
HOMEWORK 19

Due Date: Friday, July 2

Homework should be handed in *individually*, though you may work with others and collaboration is encouraged. For MATLAB problems please follow the guidelines specified in ELMS.

1. Using the method of undetermined coefficients, develop the most accurate difference formula for approximating $f''(x)$ that uses the data $f(x)$, $f(x+h)$, and $f(x+2h)$, and $f(x+3h)$. [MATLAB may be useful for solving the resulting system of equations, or you can do it by hand.]
2. The forward-difference formula for the first derivative (which is an $O(h)$ method) can be written in the following form:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

Apply Richardson extrapolation twice to obtain an $O(h^3)$ formula for $f'(x_0)$.

3. (Optional, not graded) In class we showed the 2-point forward difference formula for $f'(x)$ has error that is $O(h)$. Prove the refined estimate

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2}f''(x) - \frac{h^2}{6}f'''(\xi).$$

4. (Optional, not graded) Develop a finite difference formula for approximating $f''(x)$ that uses the data values $f(x)$, $f'(x)$, and $f(x+h)$.