HOMEWORK 10

Due Date: Wednesday, June 16

1. Consider the following system of equations.

$$f_1(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$
$$f_2(x, y, z) = 2x^2 + y^2 - 2z = 0$$
$$f_3(x, y, z) = 3x^2 - 4y + z^2 = 0$$

This system can be concisely represented as $\mathbf{F}(\mathbf{x}) = \mathbf{0}$, where $\mathbf{F}(\mathbf{x}) = (f_1, f_2, f_3)^T$, $\mathbf{x} = (x, y, z)^T$ and $\mathbf{0} = (0, 0, 0)^T$.

   (i) Find the Jacobian matrix $D\mathbf{F}(\mathbf{x})$.

   (ii) (MATLAB) Starting with the initial condition $\mathbf{x}_0 = (0.5, 0.5, 0.5)^T$, implement 5 steps of the multivariable Newton method to find the approximation $\mathbf{x}_5$.

   (iii) Compute the absolute backward error of the approximate solution $\mathbf{x}_5$ in the 2-norm.

2. (Optional, not graded) Let $S_r(x, y, z)$ denote a sphere with center (x, y, z) and radius $r > 0$. The spheres $S_{\sqrt{8}}(1, 0, 1)$, $S_{\sqrt{2}}(0, 2, 2)$, and $S_{\sqrt{2}}(0, 3, 3)$ intersect in a single point. Find this point.