Justin's Guide to MATLAB in MATH 240 - Part 1

This "Justin's Guide" is an edited version of a guide by Justin Wyss-Gallifent. It is an instructional guide, before the actual Project 1, which will teach you some basic MATLAB commands. You do not need to turn in anything for this part. Project 1 starts a few pages down.

1. **Method**

   The way this guide is written is that it is assumed that you will sit down at a MATLAB terminal and start. The commands that you give to MATLAB are given but the output is not. The output is, however, talked about, with the understanding that you will put in the commands and see the output. Then you can read about it and keep going.

2. **Starting MATLAB**

   Find a system on campus and run MATLAB. The important window (there will probably be a couple) will have a bunch of subwindows in it but the major panel on the right has a `>>` in it. This is the prompt, it's where we tell MATLAB what we want it to do.

3. **Simple Calculation**

   For example, to do 4+5 we just type it in and hit "return"

   ```
   >> 4+5
   ```

4. **Putting in a Matrix**

   We can put in a matrix easily. To put in

   $$\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

   we do

   ```
   >> A=[2 3 ; 5 -1]
   ```

5. **Matrix Operations**

   Suppose now you put in a matrix and you wish to manually do row operations to it. MATLAB doesn't have any functions for doing row operations, instead we have to execute low-level commands to do so. This is easier than it seems.

   Here is an augmented matrix

   ```
   >> A=[1 -1 -3 7 ; -3 1 4 -16 ; 4 -3 -5 12]
   ```

   To get this to row echelon form, first we'll clean out the entries below the upper-left `1`. To do this we need to add 3 times row 1 to row 2. In MATLAB the code is

   ```
   >> A(2,:)=A(2,:)+3*A(1,:)
   ```

   Confused? I was! The expression `A(`*`rownumber`*`,:)` refers to the entire row, so basically this line is saying:

   $$\underbrace{\texttt{A(2,:)}}_{Row2} = \underbrace{\texttt{A(2,:)}}_{Row2} + \underbrace{3 * \texttt{A(1,:)}}_{3*Row1}$$

   Try it! The result will be given.

   Next we'll add -4 times row 1 to row 3.

   ```
   >> A(3,:)=A(3,:)+(-4)*A(1,:)
   ```

Now you'll see that we have 0s below the upper-left 1. Next we need to get a 0 in the bottom row where the 1 is. The easiest way to do this is first to interchange rows 2 and 3.

```
>> A([2 3],:)=A([3 2],:)
```

This is a confusing command but it does the job.

Now we add twice row 2 to row 3.

```
>> A(3,:)=A(3,:)+2*A(2,:)
```

The result you'll see is the matrix in row echelon form. Remember this means any rows of 0s are at the bottom (none in this case) and the leading entries go down and right.

To get to *reduced* row echelon form, we first have to get 1s for all our leading entries. The third row is the only problem, it needs to be multiplied by 1/9.

```
>> A(3,:)=1/9*A(3,:)
```

This means that row 3 is 1/9 times what it was.

Lastly we must clear out the entries above the leading 1s.

First above the rightmost leading 1.

```
>> A(1,:)=A(1,:)+3*A(3,:)
>> A(2,:)=A(2,:)+(-7)*A(3,:)
```

And then above the leading 1 to the left of that one.

```
>> A(1,:)=A(1,:)+1*A(2,:)
```

The result is now in reduced row echelon form. Since there are no free variables we can simply read off the solution: $x_1 = 3$, $x_2 = 5$ and $x_3 = -3$.

6. **Skipping to the Reduced Row Echelon Form**

Suppose now you want to solve a system of matrices by getting the augmented matrix in reduced row echelon form but you don't want to do all that work on the previous page. The `rref` command does this in MATLAB. For example if I put in the augmented matrix (corresponding to a system of linear equations) for the previous page's problem:

```
>> A=[1 -1 -3 7 ; -3 1 4 -16 ; 4 -3 -5 12]
```

and then reduce it

```
>> rref(A)
```

we see that MATLAB saves us time compared to all the work we needed to do on the previous page.

Here is another, this one has free variables at the end. We will see that after `rref` because there are non-pivot columns corresponding to variables.

```
>> A=[1 3 4 2;1 -3 0 1]
```

and then reduce it

```
>> rref(A)
```

we get the reduced form. It's in decimals but clearly it corresponds to the augmented matrix

$$\begin{bmatrix} 1 & 0 & 2 & | & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & | & \frac{1}{6} \end{bmatrix}$$

Note that the third column is not a pivot column so x_3 is free. Hence we have the solution

$$\begin{aligned} x_1 &= \frac{3}{2} - 2x_3 \\ x_2 &= \frac{1}{6} - \frac{2}{3}x_3 \\ x_3 &= \text{free} \end{aligned}$$

7. Lastly, note that we can deal with matrices with unknown constants in them but we have to explicitly tell MATLAB that they are unknown constants. For example, suppose we wish to enter the matrix

$$\begin{bmatrix} -2 & 3 & | & h \\ 5 & -1 & | & k \end{bmatrix}$$

First we must tell MATLAB that h and k are to be dealt with symbolically. We do this with the `syms` command

```
>> syms h
```

```
>> syms k
```

And now we can do

```
>> A=[-2 3 h ; 5 -1 k]
```

```
>> rref(A)
```

MATH 240 – Fall 2020 – MATLAB Project 1 – due Tuesday, September 22.

- **What to turn in:** You will need to write a script (.m file) and then "publish" it in MATLAB as a pdf in order to turn it in on Gradescope.
- **What is a script?** A script is just a sequence of MATLAB commands, with each command on its own line. If you click "Run", MATLAB will execute all of the commands in the script, in order. The output will be produced in the Command Window, as if you entered each command individually. Your script should include all the necessary commands to solve the problems in the assignment. Scripts are useful because you can easily make edits to the commands if you make a mistake, or if you want to change something. **There is a useful template on ELMS under** `Files>Matlab`.
- **What is publishing?** When you "publish" a script, MATLAB will produce a nice-looking document that displays all of the code in your script **as well as** the output produced by the commands after they were executed. As a general rule, we need to be able to see **all of your commands as well as the output produced by your commands**. Do NOT end your lines of code with a semicolon ; Doing so suppresses the output of that line from appearing. Here are some other important points about publishing.
  - You may need to go into the options in MATLAB to change the format of the published document from html to pdf. Please be sure to submit a pdf document.
  - In the published document, MATLAB displays ALL of the commands and then ALL of the output afterwards. For a long script, this results in something that is very difficult to read. Here is how you separate your code problem-by-problem in order to make it better: Before each numbered problem, type %% in the script to make a heading. This signifies to MATLAB that it is a new portion of code, and MATLAB separates it accordingly when published. Any text after the %% reads as a heading; remember to put a space between it and any text!

Do this	Not this
%% Problem 1	%%Problem 1

    After you publish, it should look like
    * Code for Problem 1
    * Output for Problem 1
    * Code for Problem 2
    * Output for Problem 2
    * ⋮
  - You will need to include some answers to questions that are not MATLAB commands (e.g. you may have to explain why something happened). These questions are marked with a ⋆, and they require you to type an answer that is not just a MATLAB command. These answers will need to be typed as *comments* in your script, which is done by typing a single % sign, followed by your comment, like this:

    `% This is a comment!`

    This % tells MATLAB to not treat what follows as code. A detailed example and instructions are here: `https://www.mathworks.com/help/matlab/matlab_prog/publishing-matlab-code.html`.
- **Format:** The command `format rat` will cause your numerical output to appear as fractions. You might find that convenient or necessary at times. The command `format short` changes back to the decimal format. The command `format long` produces decimal output with more decimal places. Sometimes, certain problems will insist that you use certain formats. Be mindful of the fact that `format rat` sometimes does not give exact answers, but instead gives rational approximations. If you are dealing with messy numbers, especially irrational numbers, you should probably NOT use `format rat`. For example, if you type `pi` into the command window while in `format rat`, MATLAB will

return `355/113`. As another example, if you enter `10100/104729`, MATLAB will return `669/6937`, despite the fact that the first fraction cannot be reduced. (Note these two fractions are *approximately the same*.) Generally, you should avoid `format rat` when the output stops looking "nice".

- **Rules for working in groups:** You are encouraged to collaborate on this project, if it is a true collaboration: no slackers allowed. You can't learn a programming language without typing it yourself. Everyone will turn in their own project on Gradescope.
- **But first:** Before you begin the project, go through "Justin's Guide", if you haven't done so already. Be sure to type along in MATLAB as you read. That should be enough to get you through this project. Don't turn in anything from what you did in "Justin's Guide".

The project problems begin on the next page.

0. Reread all directions on the previous page that are typed in boldface.

1. Consider the system of equations

$$\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ -2x_1 - 7x_2 - 2x_3 = -14 \\ 3x_1 - 2x_2 + 5x_3 = 32 \end{cases}$$

   (a) Enter it into MATLAB as an (augmented) matrix named `A`.
   (b) Use elementary row operations (as in part 5 of the guide) to reduce it to row echelon form.
   (c) Continue using elementary row operations to get to reduced row echelon form.
   (d) $\star$ Give the solution of the system.

2. Consider the system of equations

$$\begin{cases} x_1 - 2x_2 + 4x_4 - x_5 = 4 \\ -3x_1 + 3x_2 + 12x_3 - 17x_4 + 5x_5 = -9 \\ 2x_1 - 4x_2 + 11x_4 + 4x_5 = 17 \\ x_1 - 20x_2 + 72x_3 - 32x_4 - x_5 = 4 \end{cases}$$

   (a) Enter it into MATLAB as an (augmented) matrix named `B`.
   (b) Use elementary row operations (as in part 5 of the guide) to reduce it to row echelon form.
   (c) Continue using elementary row operations to get to reduced row echelon form.
   (d) Re-enter the original matrix into MATLAB and use the `rref` command to instantly get the reduced row echelon form. Make sure it is the same as your previous answer. (From this point onward, we will make extensive use of the `rref` command in MATLAB. You do not need to do row reduction in MATLAB one elementary row operation at a time anymore.)
   (e) $\star$ Give the solution of the system in parametric vector form.

3. Start this problem in `format short`. Consider the linear system whose augmented matrix is

$$A = \begin{bmatrix} 8 & 3 & -3 & 3 \\ 5 & -3 & -7 & 2 \\ 5 & 6 & -5 & -6 \end{bmatrix}$$

   (a) Use the `rref` command to give the reduced row echelon form of A. Make sure your result in presented in decimals, not fractions.
   (b) $\star$ Write the solution of the linear system which corresponds to A in decimals, not fractions. (Your "solution" is actually a decimal approximation to the solution.)
   (c) Give the reduced row echelon form of A in fractions rather than decimals.
   (d) $\star$ Write the solution of the linear system which corresponds to A in fractions, not decimals.

4. Do problem #3 on p. 91 of the textbook (the "Mac and Cheese" problem). For each part, be sure to explictly give the appropriate system of equations (as a comment) before entering the appropriate matrices into MATLAB. Show all of your necessary MATLAB computations. (If you have the 5th edition of the textbook, it is #3 on p. 87)

5. Suppose you wish to determine whether

$$\begin{bmatrix} 24.1 \\ 17.9 \\ 24.7 \end{bmatrix} \quad \text{is in the span of} \quad \left\{ \begin{bmatrix} 25.1 \\ 20.9 \\ 27.7 \end{bmatrix}, \begin{bmatrix} 26.1 \\ 15.9 \\ 23.7 \end{bmatrix} \right\}.$$

   (a) With this goal in mind, use `rref` on an appropriate matrix.

   (b) $\star$ Is the given vector in the span of the other two? Explain how your computation justifies your answer.

   (c) $\star$ Are the three vectors above linearly independent or linearly dependent? Justify your answer.

6. Suppose you wish to show that $\begin{bmatrix} a \\ b \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 8 \\ 4 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \end{bmatrix} \right\}$ for any a and b.

   (a) First declare a and b as symbolic variables.

   (b) Enter a matrix A which can help you show this and use `rref` on it.

   (c) $\star$ Explicitly give the weights w_1 and w_2 such that $\begin{bmatrix} a \\ b \end{bmatrix} = w_1 \begin{bmatrix} 8 \\ 4 \end{bmatrix} + w_2 \begin{bmatrix} -5 \\ 1 \end{bmatrix}$. (They should be given in terms of a and b.)

7. Suppose you wish to show that the set of vectors

$$\left\{ \begin{bmatrix} 7 \\ 1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 11 \\ 20 \\ 11 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 4 \\ -5 \end{bmatrix} \right\}$$

   is linearly dependent.

   (a) Enter a matrix A which can help you show this and use `rref` on it.

   (b) $\star$ Explain how you can tell from the `rref` that the vectors are linearly dependent.

   (c) $\star$ Give a nontrivial linear combination of the vectors which yields $\mathbf{0}$. (Hint: Make a free variable nonzero.)

   (d) $\star$ Find a theorem in §1.7 which allows us to conclude, without doing any of the previous computations, that this set must be linearly dependent.

   (e) $\star$ Does this set of vectors span $\mathbb{R}^4$? Justify your answer.