$\int_a^b f(x)dx$ — ①

$y = f(x)$, let $y_1, y_2, y_3, \cdots y_n$ be values of $f(x)$ at

$x_1, x_2, x_3, \cdots x_n$

$x_0 = a \quad x_n = b = x_0 + nh$, let $t = \frac{x - x_0}{h}$, $x - x_0 = ht$ so $x - x_0 = ht$

$\Downarrow$

when $t = \frac{x - x_0}{h} = 0$ $\quad dx = h\,dt$

$$I = \int_a^b f(x)dx = \int_{x_0}^{x_n} f(x)dx = h\int_0^n f(x_0 + ht)dt = h\int_0 F(t)dt$$

$$= h\int_0^n [y_0 + \frac{\Delta y_0}{1!} t + \frac{\Delta y_0^2}{2!} t^2 + \frac{\Delta y_0^3}{3!} t^3 + \cdots + \frac{\Delta y_0^n}{n!} t^n + h^{n+1} t^{n+1} \frac{y^{n+1}}{(n+1)!}]$$

$$t^n = t(t-1)(t-2)\cdots(t-n+1)$$

$$\Rightarrow \int_a^b f(x)dx = (b-a)\frac{f(a)+f(b)}{2} + \frac{1}{2}f''(\xi)\int_a^b (x-a)(x-b)dx$$

$$\Rightarrow \int_a^b f(x)dx = \underbrace{(b-a)\frac{f(a)+f(b)}{2}}_{\text{trapezoid}} - \frac{1}{12}(b-a)^3 f''(\xi)$$