HW3

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1 Question 1

In three dimensions write down the translation matrix which shifts +1 in the x-direction, 2 in the y-direction and -7 in the z-direction. Apply this matrix to the points (1; 2; 0) and (1; 4;-3).

We identify the $\mathbb{R}^3$ with $\mathbb{R}^4$ as (a,b,c,1)

$$\text{and then we put } \vec{u} = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix} \text{ for the transformation which we can get } \vec{x} = \vec{x} + \vec{u}$$

$$T_{\vec{u}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{\vec{u}} * \vec{x} =$$

$$T_{\vec{u}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x+1 \\ y+2 \\ z-7 \\ 1 \end{bmatrix} = \vec{x} = \vec{x} + \vec{u} \text{ where } \vec{x} + \vec{u} \in \mathrm{R}^3$$

$$T(1, 2-7) \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -7 \\ 1 \end{bmatrix} => (2, 4, -7)$$

$$T(1, 4-3) \begin{bmatrix} 1 \\ 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -8 \\ 1 \end{bmatrix} => (2, 6, -8)$$

2 Question 2

In three dimensions find the image of the three points(1; 2; 3); (-2; 3; 1); (3; 2; 2) under rotation around the y-axis by $\pi = 4$.

$$R_Y(\pi/4) = \begin{bmatrix} \cos(\pi/4) & 0 & \sin(\pi/4) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/4) & 0 & \cos(\pi/4) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$R_Y(\tfrac{\pi}{4}) \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.828 \\ 2 \\ 1.414 \\ 1 \end{bmatrix} => (2.828, 2, 1.414)$$

$$R_Y(\tfrac{\pi}{4}) \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.71 \\ 3 \\ 2.12 \\ 1 \end{bmatrix} => (-0.71, 2.12, 1)$$

$$R_Y(\tfrac{\pi}{4}) \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.54 \\ 2 \\ -0.71 \\ 1 \end{bmatrix} => (3.54, 2, -0.71)$$

3 Question 3

In three dimensions find the image of the three points(1; 2; 3); (; 3; 1); (3; 2; 2) under the perspective projection with center of perspective at z = 10.

$$P(10)A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 1.4286 \\ 2.8571 \\ 0 \\ 1 \end{bmatrix}$$

The point will be $(1.4286, 2.8571, 0)$

$$P(10)A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ 0.9 \end{bmatrix} = \begin{bmatrix} -2.2222 \\ 3.3333 \\ 0 \\ 1 \end{bmatrix}$$

The point will be $(-2.2222, 3.3333, 0)$

$$P(10)A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 3.75 \\ 2.5 \\ 0 \\ 1 \end{bmatrix}$$

The point will be $(3.75, 2.5, 0)$

4 Question 4

2.20 In three dimensions we can rotate around an axis parallel to the y-axis by translating the desired axis so that it's on top of the y-axis, rotating, and then translating back. Using this method find the rotation matrix which will rotate around the line x = -2, z = 4 with direction opposite to the y-axis by 2π=3 radians.

$$T(2,0,4)P_Y(\tfrac{2\pi}{3})T(-2,0,4) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\frac{2\pi}{3}) & 0 & sin(\frac{2\pi}{3}) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\frac{2\pi}{3}) & 0 & cos(\frac{2\pi}{3}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 0 & \frac{\sqrt{3}}{2} & 6.46 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -0.5 & -4.2679 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5 Question 5

2.25 Projection onto one of the other two coordinate planes (the xz- plane and the yz-plane) can easily be accomplished by rotation. For example if we wish to project onto the xz-plane what we do is first rotate the y-axis to the z-axis (which is a rotation around the x-axis), then project, then rotate back. (a) Write down the projection matrix which does this. (b) Use this to project the three points (1; 2; 3); (4;-1; 0); (5; 2; 3) with center of perspective at y = 10.

a)

$$\text{(a) } R_Z\left(\frac{\pi}{4}\right) R_X\left(\frac{\pi}{6}\right) \mathrm{R_Z}\left(-\frac{\pi}{4}\right)$$

$$= \begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) & 0 \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\pi}{b}\right) & -\sin\left(\frac{\pi}{6}\right) \\ 0 & \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix} \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) & 0 \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{4} + \frac{1}{2} & \frac{1}{2} - \frac{\sqrt{3}}{4} & \frac{\sqrt{2}}{4} \\ \frac{1}{2} - \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{4} + \frac{1}{2} & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

b)

$$R_x\left(-\frac{\pi}{6}\right) R_z\left(\frac{\pi}{3}\right) R_x\left(\frac{\pi}{b}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(-\frac{\pi}{6}\right) & -\sin\left(-\frac{\pi}{6}\right) \\ 0 & \sin\left(-\frac{\pi}{6}\right) & \cos\left(-\frac{\pi}{6}\right) \end{bmatrix} \begin{bmatrix} \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) & 0 \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) \\ 0 & \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{3}{4} & \frac{5}{8} & \frac{\sqrt{3}}{8} \\ -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{8} & \frac{7}{8} \end{bmatrix}$$

c)

$$R_Z\left(\frac{\pi}{3}\right) R_Y\left(\frac{\pi}{6}\right) R_Z\left(\frac{3}{4}\pi\right) R_Y\left(-\frac{\pi}{6}\right) R_Z\left(-\frac{\pi}{3}\right) = \begin{bmatrix} \frac{1}{16} - \frac{15\sqrt{2}}{32} & \frac{\sqrt{3}}{16} - \frac{7\pi}{32} & \frac{\sqrt{3}}{8} + \frac{3\sqrt{6}}{16} \\ \frac{\sqrt{3}}{16} + \frac{9\sqrt{6}}{32} & \frac{3}{16} - \frac{13\sqrt{2}}{32} & \frac{\sqrt{2}}{16} + \frac{3}{8} \\ \frac{\sqrt{3}}{8} - \frac{\sqrt{b}}{16} & \frac{5\sqrt{2}}{16} + \frac{3}{8} & \frac{3}{4} - \frac{\sqrt{2}}{8} \end{bmatrix} \tag{4}$$

6 Question 6

Projection onto one of the other two coordinate planes (the xz- plane and the yz-plane) can easily be accomplished by rotation. For example if we wish to project onto the xz-plane what we do is first rotate the y-axis to the z-axis (which is a rotation around the x-axis), then project, then rotate back. (a) Write down the projection matrix which does this. (b) Use this to project the three points (1; 2; 3); (4;-1; 0); (5; 2; 3) with center of perspective at y = 10.

a)

$$R_x(-\pi/2)P(Y)R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(-\pi/2) & -sin(\pi/2) & 0 \\ 0 & sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/y & 1 \end{bmatrix}$$
$$\begin{bmatrix} [1 & 0 & 0 & 0 \\ 0 & cos(\pi/2) & -sin(-\pi/2) & 0 \\ 0 & sin(\pi/2) & cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1] \end{bmatrix}$$

b)

$$R_x(-\pi/2)P(Y)R_x(\pi/2) \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 1.2 \end{bmatrix}$$
$$R_x(-\pi/2)P(Y)R_x(\pi/2) \begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0.9 \end{bmatrix}$$
$$R_x(-\pi/2)P(Y)R_x(\pi/2) \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1.2 \end{bmatrix}$$

7 Question 7

8 Question 8

It's possible to do a 2D version of projection, where projection is done with the COP at y = d and projection is onto the x-axis. Develop this. Specifically, what would the projection matrix look like, how would it work, would post-processing be necessary and so on