Contents

AMSC 460 - HW11

```
clear all; format compact; close all; syms f(x) x y z
```

Problem 1

Find a quartic Hermite polynomial that interpolates

$$p(0) = 1, \quad p'(0) = -1, \quad p(1) = -2, \quad p'(1) = 2, \quad p(2) = 2.$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-1)(x-2)}{(-1)(-2)} = \frac{1}{2}(x^2-3x+2)$$

now differentiate the equation above

$$L_0'(x) = (1/2)(x^2-3x+2) * \frac{d}{dx} \ = \frac{1}{2} * (2x-3)$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_0)} = \frac{(x-0)(x-2)}{(1)(-1)} = -x^2+2x$$

now differentiate the equation above

$$L_1'(x) == -2x-2$$

similarly, we can have

$$L_2(x) = \frac{1}{2}(x^2-2)$$

$$L_2'(x) = \frac{1}{2}(2x-1)$$

$$L_2(x_2) = \frac{1}{2}(2*2-1) = \frac{3}{2}$$

Now

$$p4(x) = f[0] + f_[0,0,1](x^2) + f[0,0,1,1](x^2)(x-1) + f[0,0,1,1,2]x^2(x-1)^2$$

$$= -10x^4 + 27x^3 - 19x^2 - x + 1$$

Problem 2 (

Consider the function

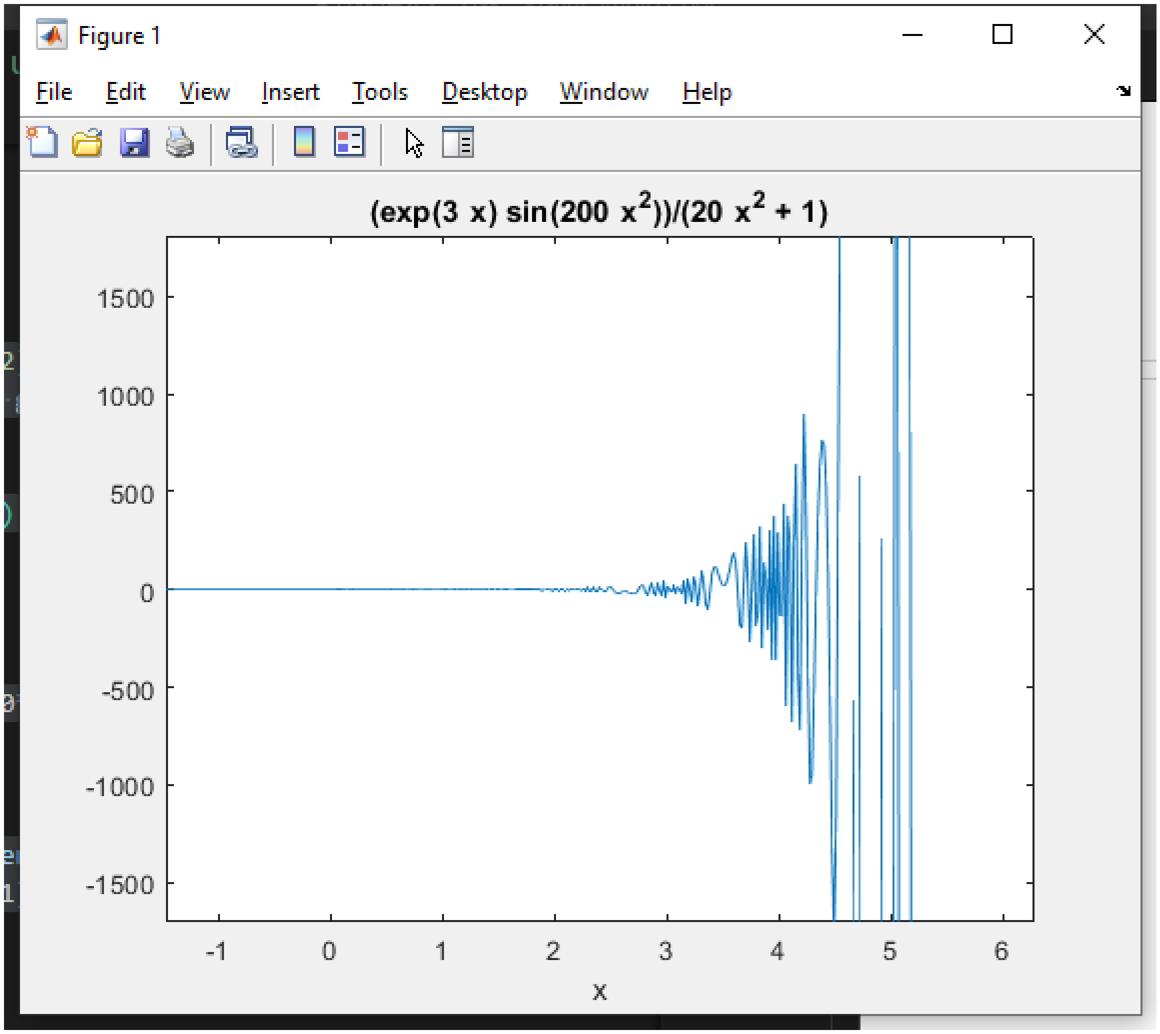
$$f(x) = \frac{e^{3x}\sin(200x^2)}{1 + 20x^2}$$

on the interval $0 \le x \le 1$. The goal of this problem is to observe the error reduction in cubic spline interpolation when increasing the number of nodes.

(a)Show that $|J(x)| \le 1$, $|J'(x)| \le 1$, $|J''(x)| \le 1$, and in general that $|J^{(k)}(x)| \le 1$ for any positive integer k.

```
syms x n
f(x) = (exp(3*x)*sin(200*x^2)/(1+20*x^2))
ezplot(f(x))
```

```
f(x) =
(exp(3*x)*sin(200*x^2))/(20*x^2 + 1)
```



(b)Write a short script using the MATLAB `spline` command, that interpolates $f(x)$ at equidistant points $x_i = i/n$ $(i = 0, 1, ..., n)$, where n is an arbitary fixed number of subintervals prescribed by the user.

```
z=6
test = 1: z
for k =1:z
    i = 0:test(k)
    xi = i/z
    spline(xi,fx)
end
```

```
z =
```

```
6
test =
1     2     3     4     5     6
i =
0     1
xi =
 0    0.1667
```

(c)For $n = 2^j$, $j = 4, 5, ..., 14$, run your script and record the maximum value of the error $e(x) := |f(x) - s(x)|$ at the points `x=0:0.001:1`, where $s(x)$ denotes the cubic spline interpolant. In other words, for each n, compute $\max_{x \in 0:0.001:1} e(x)$ and store this value. (For efficient coding, you shouldn't loop through all x. Vectorize f and use the `max` command to compute $\max_{x \in 0:0.001:1} |f(x) - s(x)|$ in one shot.)

This is asking for the each n max——f(x)- s(x)——

(d)Plot the errors against n on a loglog plot and make observations