Contents

AMSC 460 - HW11

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clear all; format compact; close all; syms f(x) x y z
```

Problem 1

Suppose in designing a natural logarithm function for a calculator on the interval $[1, e]$, we are using a Chebyshev polynomial approximation. What is the smallest degree n of the polynomial that ensures an accuracy of 10^{-6} over the interval $[1, e]$?

From Chebyshev polynomial we have [a,b]=[1,e] and f(x)=lnx. The maximum will be(n-1)! where

because the maximum of 1/z on [1, e] occurs at z = 1. We now look for an integer d such that the error is strictly smaller than 10ˆ6

$$(\frac{1}{n * 2^(n-1)} * (\frac{e-1}{2})^n) < 10^(- a)$$

n = 10 error 1.7*10ˆ5

n = 13 error 1.04*10ˆ6

n = 14 error 4.17·10ˆ7

Thus n = 14 is required, the (n-1)th degree polynomial so 13.

Therefpre we need at least n = 15

Problem 2

Special functions appear in physics and applied mathematics, often as a solution to some ODE. The following function is in the \emph{Bessel} family (\url{https://en.wikipedia.org/wiki/Bessel_function})[J(x) = \frac{1}{\pi}\int_0ˆ\pi\!\cos(x\sin(s))\,\mathrm{d}s.\]

(a)Show that $|J(x)| \leq 1$, $|J'(x)| \leq 1$, $|J''(x)| \leq 1$, and in general that $|J^{(k)}(x)| \leq 1$ for any positive integer k.

n is from the even number above 0 that cos and sin both fluctuate bewteen 0 and 1 which meaning both sin and cosine will be bounded between 0 and 1, similarly for odd numbers that will also be bounded by 1 and 0

$|J(x)| \leq 1$

(b)Suppose we would like to approximate J with a Chebyshev interpolant. Determine how many interpolation points are required on the interval $[0, 10]$ so that the error (in the max-norm) is no more than 10^{-6}. [You don't have to write down the interpolant.]

Since $|J(x)| \leq 1$ Given values smaller than $10^(-6).for \frac{5^n}{(n!)(2^(n-1))}$

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for (n = 0; (5^n)/(factorial(n)(2^(n-1))) > 10^(-6); n++)
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f(x) =
   (exp(3*x)*sin(200*x^2))/(20*x^2 + 1)
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z =
   n =16
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