AMSC/CMSC 460 Dr. Wolfe FINAL EXAM July 25, 2003

1. (5 points) Suppose we run the following MATLAB script:

   $x = 1;$
   while $x + 1 > 1$
   $y = x;$
   $x = x/2;$
   end
   y

   The result will be $y = 2.2204\mathrm{e}-16$. Explain this result.

2. (5 points) A radio news reader announces "The stock market in Quito, Equador went up 100 points yesterday."
   (a) Explain why this information is meaningless to someone unfamiliar with the Quito stock market.
   (b) What is a more meaningful way to present this information ?
   (c) How does this relate to AMSC/CMSC 460 ?

3. (15 points) Consider the system

$$\begin{aligned} 4x - 2y &= 6 \\ 2x + 5y &= 9 \end{aligned}$$

   (a) Solve the system by using the LU factorization, foward elimination and back substitution.
   (b) Set up an iterative method for solving the system. Compute three iterations starting at $(x_0, y_0) = (0, 0)$. Does it look like your iterations are converging to the solution ? Can you prove that the iterates converge ?

4. (15 points) Given the data points $(0, 2), (1, 1)$, find the following:
   (a) The straight line interpolating this data.
   (b) The function $f(x) = a + be^x$ interpolating this data.
   (c) The function $f(x) = a/(b + x)$ interpolaing this data.

5. (8 points)
   (a) Given $x_1 < x_2 < \cdots < x_N$, define what it means to say that $S(x)$ is a *cubic spline* with knots at $\{x_1, x_2, \ldots, x_N\}$.
   (b) In fitting a large number of data points, why is it generally preferable to use cubic spline interpolation rather than interpolation by a single polynomial ?

6. (15 points) Let

$$I = \int_1^2 \ln x \, dx = .3862943611$$

   (a) Compute T_4, the 4-panel trapezoid rule approximation to I. Compare your answer with the exact value of I.

(b) Compute CT_4, the 4-panel corrected trapeziod rule approximation to I. Compare your answer with the exact value of I.

(c) How many panels would you need to compute I with an error of $< 10^{-6}$ using the trapezoid rule ?

7. (7 points) Suppose f is a smooth function such that

$$I = \int_2^3 f(x)\,dx = 7.$$

Suppose the result of applying Simpson's rule with 10 panels to approximate I is $S_{10} = 6.984$. Assuming roundoff error is not a factor, approximately what result would you expect for the 20 panel Simpson's rule, S_{20} ?

8. (15 points) The iteration $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ will converge to $\alpha = 1$ for some values of c (provided x_0 is chosen sufficiently close to α). Find the values of c for which convergence occurs. For what values of c, if any, will the convergence be quadratic ?

9. (15 points) Consider the initial value problem

$$\frac{dy}{dt} = ty^2, \qquad y(1) = 2.$$

(a) Verify that the solution is $Y(t) = \frac{2}{2-t^2}$.

Find approximations to $Y(1.2)$ by using

(b) two steps of the Euler method with $h = .1$.

(c) one step of the Improved Euler method with $h = .2$.

In (b) and (c) compare your answers with the exact solution.