AMSC 460 - HOMEWORK 2

(a) Write a MATLAB program to implement Newton's method for root finding.

```
clear all
syms x
f = input('Type your equation please: f = ');
x = input('The starting guess x0 = ');
xNew = x + 100;
fd = inline(diff(sym(f)));

iter = 0;
err = 100;

while err > 10^-8 %could be any number
    xNew = x - (f(x)./fd(x));
    err = abs(x-xNew);
    x = xNew;
    iter = iter + 1;
    fprintf('\tAfter %g steps, root = %.15g\n', iter, xNew)
end
```

(b) To compare root finding algorithms, we will approximate $\sqrt{2}$ using two methods: Newton and Bisection. Using the equation f(x) = x^2 _ 2 = 0, use your program from part (a) to ensure $\sqrt{2}$ is obtained. For Newton, use x0=2, and for Bisection use the starting bracket [1, 2]. In each case use 10^$_8$ for the error tolerance.

```
% Use Newton's method:
clear all
syms x
f = @(x) x^2-2;   % Given f(x) = x^2 # 2
x = 2;  % The starting guess x0 = 2
x1 = x + 100;
fd = inline(diff(sym(f)));

iter = 0;
err = 100;

while err > 10^-8
    x1 = x - (f(x)./fd(x));
    err = abs(x-x1);
    x = x1;
    iter = iter + 1;
    fprintf('\tAfter %g steps, root = %.15g\n', iter, x1)
end


% Use Bisection method
f = @(x) x^2-2; a=1; b=2;
fa = f(a); fb = f(b);
k=0;

while (b-a)/2 > 10^-8
  c = (a+b)/2;
```

```
  fc = f(c);
  k = k+1;
  fprintf('\tAfter %g steps, root = %.15g\n', k, c)

  if fc == 0
    break
  end
  if sign(fc)*sign(fa) < 0
    b = c; fb = fc;
  else
    a = c; fa = fc;
  end
end
xc = (a+b)/2;
```

```
 After 1 steps, root = 1.5
 After 2 steps, root = 1.41666666666667
 After 3 steps, root = 1.41421568627451
 After 4 steps, root = 1.41421356237469
 After 5 steps, root = 1.4142135623731
 After 1 steps, root = 1.5
 After 2 steps, root = 1.25
 After 3 steps, root = 1.375
 After 4 steps, root = 1.4375
 After 5 steps, root = 1.40625
 After 6 steps, root = 1.421875
 After 7 steps, root = 1.4140625
 After 8 steps, root = 1.41796875
 After 9 steps, root = 1.416015625
 After 10 steps, root = 1.4150390625
 After 11 steps, root = 1.41455078125
 After 12 steps, root = 1.414306640625
 After 13 steps, root = 1.4141845703125
 After 14 steps, root = 1.41424560546875
 After 15 steps, root = 1.41421508789063
 After 16 steps, root = 1.41419982910156
 After 17 steps, root = 1.41420745849609
 After 18 steps, root = 1.41421127319336
 After 19 steps, root = 1.41421318054199
 After 20 steps, root = 1.41421413421631
 After 21 steps, root = 1.41421365737915
 After 22 steps, root = 1.41421341896057
 After 23 steps, root = 1.41421353816986
 After 24 steps, root = 1.41421359777451
 After 25 steps, root = 1.41421356797218
 After 26 steps, root = 1.41421355307102
```

(c) Modify the algorithms to keep track of the absolute error en = `r - xn` at each iteration. Store these errors in a vector (for plotting purposes). Then plot the absolute errors on the same graph, and with a semilogarithmic y-axis (use semilogy in MATLAB). Which algorithm used the least steps to achieve the required error tolerance?

Modified method:

```
clear all
```

```
syms x
f = @(x) x^2-2;   % Given f(x) = x^2 # 2
x = 2;  % The starting guess x0 = 2
r = sqrt(2);      % Given root =  #2
xNew = x + 100;
fd = inline(diff(sym(f)));

iter = 0;
err = 100;
en = 0;

while err > 10^-8
    xNew = x - (f(x)./fd(x)); %absolute error en = |r - xn|%
    err = abs(x-xNew);
    x = xNew;
    en = abs(r-x);
    iter = iter + 1;
    N(iter) = en;
    fprintf(['\tAfter %g steps, root = %.15g,',...
        ' absolute error = %.15g\n'], iter, xNew, en)
end
```

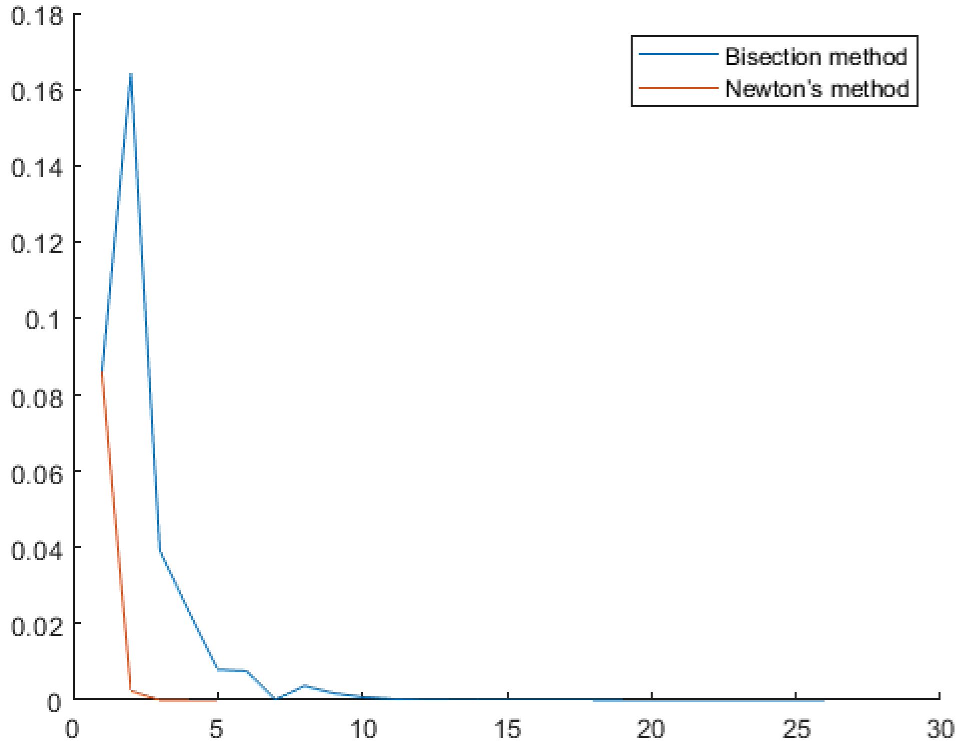
```
 After 1 steps, root = 1.5, absolute error = 0.0857864376269049
 After 2 steps, root = 1.41666666666667, absolute error =
 0.00245310429357116
 After 3 steps, root = 1.41421568627451, absolute error =
 2.12390141474117e-06
 After 4 steps, root = 1.41421356237469, absolute error =
 1.59472435257157e-12
 After 5 steps, root = 1.4142135623731, absolute error = 0
```

Modified Bisection method:

```
f = @(x) x^2-2; a=1; b=2;
fa = f(a); fb = f(b);
k=0; Ben = 0; r=sqrt(2);

while (b-a)/2 > 10^-8
  c = (a+b)/2;
  Ben = abs(r-c); %absolute error en = |r - xn|
  fc = f(c);
  k = k+1;
  B(k) = Ben;
  fprintf(['\tAfter %g steps, root = %.15g,',...
        ' absolute error = %.15g\n'], k, c, Ben)
  if fc == 0
    break
  end
  if sign(fc)*sign(fa) < 0
    b = c; fb = fc;
  else
    a = c; fa = fc;
  end
end
```

```
xc = (a+b)/2;

 After 1 steps, root = 1.5, absolute error = 0.0857864376269049
 After 2 steps, root = 1.25, absolute error = 0.164213562373095
 After 3 steps, root = 1.375, absolute error = 0.0392135623730951
 After 4 steps, root = 1.4375, absolute error = 0.0232864376269049
 After 5 steps, root = 1.40625, absolute error = 0.00796356237309515
 After 6 steps, root = 1.421875, absolute error = 0.00766143762690485
 After 7 steps, root = 1.4140625, absolute error =
 0.000151062373095145
 After 8 steps, root = 1.41796875, absolute error =
 0.00375518762690485
 After 9 steps, root = 1.416015625, absolute error =
 0.00180206262690485
 After 10 steps, root = 1.4150390625, absolute error =
 0.000825500126904855
 After 11 steps, root = 1.41455078125, absolute error =
 0.000337218876904855
 After 12 steps, root = 1.414306640625, absolute error =
 9.30782519048545e-05
 After 13 steps, root = 1.4141845703125, absolute error =
 2.89920605951455e-05
 After 14 steps, root = 1.41424560546875, absolute error =
 3.20430956548545e-05
 After 15 steps, root = 1.41421508789063, absolute error =
 1.52551752985453e-06
 After 16 steps, root = 1.41419982910156, absolute error =
 1.37332715326455e-05
 After 17 steps, root = 1.41420745849609, absolute error =
 6.10387700139547e-06
 After 18 steps, root = 1.41421127319336, absolute error =
 2.28917973577047e-06
 After 19 steps, root = 1.41421318054199, absolute error =
 3.81831102957975e-07
 After 20 steps, root = 1.41421413421631, absolute error =
 5.71843213448275e-07
 After 21 steps, root = 1.41421365737915, absolute error =
 9.50060552451504e-08
 After 22 steps, root = 1.41421341896057, absolute error =
 1.43412523856412e-07
 After 23 steps, root = 1.41421353816986, absolute error =
 2.42032343056309e-08
 After 24 steps, root = 1.41421359777451, absolute error =
 3.54014104697598e-08
 After 25 steps, root = 1.41421356797218, absolute error =
 5.59908808206444e-09
 After 26 steps, root = 1.41421355307102, absolute error =
 9.30207311178322e-09

hold on;
semilogy(B);
semilogy(N);
legend({'Bisection method','Newton’s method'});
hold off;
```

```
%The Newton's method used the least steps to achieve the required
 error tolerance.
```



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