Computational Methods Summer 2021 I

HOMEWORK 1

Due Date: Thursday, June 3

Homework should be handed in *individually*, though you may work with others and collaboration is encouraged. For MATLAB problems please follow the guidelines specified in ELMS (in particular see the file "Formatting MATLAB assignments")

1. Let $f(x) = e^x + x^2 - 5x$.

   (a) The bracket $[1.5, 2]$ contains a root. Explain why using the Intermediate Value Theorem. For this bracket, estimate the number of iterations N that would be needed to compute the root to an accuracy of 10^{-4}.

   (b) The bracket given in (a) contains a root, but there is another root. Find a bracket for it. Then use the bisection method to find the two roots to an accuracy of 10^{-4}.

2. Consider the cubic $f(x) = x^3 - x - 1$.

   (a) Use the MATLAB command `fzero` to find a root in the interval $[1, 2]$.

   (b) Show that $f(x) = 0$ can be rewritten as a fixed point problem for both the functions (i) $g_1(x) = x^3 - 1$, and (ii) $g_2(x) = (1 + x)^{1/3}$.

   (c) Which of the functions g_1 and g_2 is a contraction mapping near the root r from part (a)? Which of g_1 or g_2 will be successful in making the iteration $x_{i+1} = g(x_i)$ converge locally to the root r?

   (d) Write a script in MATLAB to carry out 10 steps of the fixed point iteration (use a `for` or `while` loop to do this) for both g_1 and g_2, each starting with the guess $x_0 = 0$. What approximate root does your algorithm give for g_1? For g_2? Are your results consistent with the analysis from part (c)?