

CS61C Spring 2017 Discussion 8

1. Analyzing C Code

```
#define NUM_INTS 8192
int A[NUM_INTS]; /* A lives at 0x10000 */
int i, total = 0;
for (i = 0; i < NUM_INTS; i += 128) { A[i] = i; } /* Line 1 */
for (i = 0; i < NUM_INTS; i += 128) { total += A[i]; } /* Line 2 */
```

Let's say you have a byte-addressed computer with a total memory of 1MiB. It features a 16KiB CPU cache with 1KiB blocks.

1. How many bits make up a memory address on this computer? **20**
2. What is the T:I:O breakdown? tag bits: **6** index bits: **4** offset bits: **10**
3. Calculate the cache hit rate for the line marked Line 1: **50%**

The integer accesses are $4 \times 128 = 512$ bytes apart, which means there are 2 accesses per block. The first accesses in each block is a cache miss, but the second is a hit because $A[i]$ and $A[i + 128]$ are in the same cache block.

4. Calculate the cache hit rate for the line marked Line 2: **50%**

The size of A is $8192 \times 4 = 2^{15}$ bytes. This is exactly twice the size of our cache. At the end of line 1, we have the second half of A inside the cache, while in line 2 we start accesses from the beginning of the array. Thus we cannot reuse any of the content of A and we get the same hit rate as before. Note that we do not have to consider cache hits for total, since the compiler will probably leave it in a register.

2. Floating Point

The IEEE 754 standard defines a binary representation for floating point values using three fields:

- The *sign* determines the sign of the number (0 for positive, 1 for negative)
- The *exponent* is in **biased notation** with a bias of 127
- The *significand* is akin to unsigned, but used to store a fraction instead of an integer.

The below table shows the bit breakdown for the single precision (32-bit) representation:

| Sign | Exponent | Significand |
|-------|----------|-------------|
| 1 bit | 8 bits | 23 bits |

There is also a double precision encoding format that uses 64 bits. This behaves the same as the single precision but uses 11 bits for the exponent (and thus a bias of 1023) and 52 bits for the significand.

How a float is interpreted depends on the values in the exponent and significand fields:

For normalized floats:

$$\text{Value} = (-1)^{\text{Sign}} \times 2^{(\text{Exponent} - \text{Bias})} \times 1.\text{mantissa}_2$$

For denormalized floats:

$$\text{Value} = (-1)^{\text{Sign}} \times 2^{(\text{Exponent} - \text{Bias} + 1)} \times 0.\text{mantissa}_2$$

| Exponent | Significand | Meaning |
|----------|-------------|----------|
| 0 | Anything | Denorm |
| 1-254 | Anything | Normal |
| 255 | 0 | Infinity |
| 255 | Nonzero | NaN |

Exercises

1. How many zeroes can be represented using a float?
2.
2. What is the largest finite positive value that can be stored using a single precision float?
 $0x7F7FFFFF = (2 - 2^{-23}) \times 2^{127}$
3. What is the smallest positive value that can be stored using a single precision float?
 $0x00000001 = 2^{-23} \times 2^{-126}$
4. What is the smallest positive normalized value that can be stored using a single precision float?
 $0x00800000 = 2^{-126}$

5. Convert the following numbers from binary to decimal or from decimal to binary:

| | | | | | |
|--|------|------------|---------|------------|-----------|
| 0x00000000 | 8.25 | 0x00000F00 | 39.5625 | 0xFF94BEEF | $-\infty$ |
| $0x00000000 = 0$ | | | | | |
| $8.25 = 0x41040000$ | | | | | |
| $0x00000F00 = (2^{-12} + 2^{-13} + 2^{-14} + 2^{-15}) \times 2^{-126}$ | | | | | |
| $39.5625 = 0x421E4000$ | | | | | |
| $0xFF94BEEF = \text{NaN}$ | | | | | |
| $-\infty = 0xFF800000$ | | | | | |

3. AMAT

AMAT is the average (expected) time it takes for memory access. It can be calculated using this formula:

$$\text{AMAT} = \text{hit time} + \text{miss rate} \times \text{miss penalty}$$

Miss rates can be given in terms of either local miss rates or global miss rates. The *local miss rate* of a cache is the percentage of accesses into the particular cache that miss at the cache, while the *global miss rate* is the percentage of all accesses that miss at the cache.

Exercises

Suppose your system consists of:

- A L1\$ that hits in 2 cycles and has a local miss rate of 20%
- A L2\$ that hits in 15 cycles and has a global miss rate of 5%
- Main memory hits in 100 cycles

1. What is the local miss rate of L2\$?
 $\text{Local miss rate} = 5\% / 20\% = 0.25 = 25\%$
2. What is the AMAT of the system?
 $\text{AMAT} = 2 + 20\% \times 15 + 5\% \times 100 = 10$ (using global miss rates)
Alternatively, $\text{AMAT} = 2 + 20\% \times (15 + 25\% \times 100) = 10$
3. Suppose we want to reduce the AMAT of the system to 8 or lower by adding in a L3\$. If the L3\$ has a local miss rate of 30%, what is the largest hit time that the L3\$ can have?
Let H = hit time of the cache. Using the AMAT equation, we can write:
 $2 + 20\% \times (15 + 25\% \times (H + 30\% \times 100)) \leq 8$
Solving for H , we find that $H \leq 30$. So the largest hit time is 30 cycles.

4. Flynn Taxonomy

1. Explain SISD and give an example if available.

Single Instruction Single Data; each instruction is executed in order, acting on a single stream of data. For example, traditional computer programs.

2. Explain SIMD and give an example if available.

Single Instruction Multiple Data; each instruction is executed in order, acting on multiple streams of data. For example, the SSE Intrinsics.

3. Explain MISD and give an example if available.

Multiple Instruction Single Data; multiple instructions are executed simultaneously, acting on a single stream of data. There are no good modern examples.

4. Explain MIMD and give an example if available.

Multiple Instruction Multiple Data; multiple instructions are executed simultaneously, acting on multiple streams of data. For example, map reduce or multithreaded programs.