Machine Learning for Example Generation in Pure Math

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Abstract

Determining whether a set of matrices generates a group can be a difficult task. Moreover, if a set of matrices does generate a group, the minimum number of generator applications needed to generate any given element of the group is not always known. In this project, we applied supervised and unsupervised machine learning methods to understand how they manage these tasks. We simplified the problem to learning the optimal next generator matrix to apply in order to reach identity in as few transformations as possible. Using this approach, we first trained simple neural networks and an unsupervised deep Monte Carlo Tree Search to generate elements of $\mathbb{Z}^2$ given a point on the x, y-axes using Euclidean algorithm transformations as bases. We then trained variations of tabular Q-learning models to optimally generate elements of the Heisenberg Group—which is an open problem—with 97% accuracy. Finally, we used Q-learning once more to determine whether two generators can generate $SL_3(\mathbb{Z})$, which is an open problem.