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# Comparison of logistic functions for modeling wind turbine power curves

Daniel Villanueva*, Andrés Feijóo

*Departamento de Enxeñería Eléctrica, Universidade de Vigo, EEI, Campus de Lagoas, 36310 Vigo, Spain*

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ABSTRACT

In recent years logistic functions have been used to model wind turbine power curves. Generally speaking, it can be said that the results provided by the logistic functions are good enough to choose them over other options considering its continuity and adaptability. However, there are some logistic functions that have never been used to model wind turbine power curves although their use can be adequate. Comparing all logistic functions can help definitely to decide which are the best options.

In this paper, the most known logistic functions are presented and tested to model wind turbine power curves, included those already used. Moreover, a comparison is made among them, after which two logistic functions are eventually recommended and some other are definitively discarded.



## 1. Introduction

A wind turbine (WT) power curve relates the wind speed through the area swept by the blades of a WT and the electric power provided in their output terminals. Usually, manufacturers provide the power curve of a WT by means of a graph or as a set of pairs of points given as a table. That is an empirical approximation of the power curve, but it should be taken into account that some aspects as weather, terrain conditions and, mainly, aging, affect to the power curve. As times go by, aging changes the behaviour of the wind turbine inevitably. Referring to a new WT or to a not new one, for a long time there have been attempts to model the power curve by means of mathematical functions [1], in order to implement them in an application program or to derive more complex relationships. Formerly, linear [2], quadratic [3] or cubic [4] models were the options more usually chosen to model power curves. They can be generally considered as good options because they are very simple and it is very easy to work with them. However, the results they provide are far from a good approximation, at least, in the area of the rated wind speed. Other alternatives are the splines [5] which give very good results but include a lot of parameters and are piecewise defined, which is an obstacle when they are combined with other expressions in order to derive more complex ones. Lately, logistic functions have been used to model WT power curves because they provide good results, need few parameters and they are easy to deal with because they are continuous functions. The applicability of continuous functions is mainly to combine the power curve with other expressions, such as the wind speed distributions, in order to obtain combinations of them, such as wind power distributions.

A logistic function, also called ‘s’ shape function or sigmoid function appears in models of population growth and spread of epidemic diseases, using four [6,7] or five parameters [8]. In these types of behaviours the function relates the size of a set with respect to the time. In all of them, the growth is exponential at the beginning, then some kind of competition appears among the members of the set so the growth decreases and finally the size of the set reaches its limit. At a glance, it does not seem to have anything to do with WT power curves, however the shape of the curve is exactly the same as in those cases. More in depth, both behaviours coincide, considering the wind speed equivalent to the time and also in the limit, which in the case of the WT is the rated power. In fact, some kind of analogy can be established between the population growth and the power curve, just considering that the wind speed is increased gradually to obtain the power curve, so it is equivalent to the time, and the size of the population represents the output power.

As it has been said, over the last years, some logistic functions have been successfully applied to model WT power curves, using powers [9–12], square roots [13], exponentials [14–16], and simplification of them [17–19]. However, there are other functions of the same type that have not been applied to that assignment as

* Corresponding author.
*E-mail addresses:* dvillanueva@uvigo.es (D. Villanueva), afeijoo@uvigo.es (A. Feijóo).

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the Gompertz [20,21], the Bass [22], and others [21,23,24]. Therefore, checking if these functions are valid or not and comparing all of them seem to be necessary and this is the proposal of this paper. Besides, some functions, used or not, may be derived from other, so those curves derived from other are discarded in order to avoid duplications. And, as all logistic functions have some kind of relationship among them, each relationship is explained.

Another aspect to be taken into account regarding functions to model WT power curves is the number of parameters. This number can rise up to six in the case of logistic functions. However, when dealing with fewer than three, the adaptability of the function to model the WT power curve is almost null. Therefore, the number of parameters considered in this survey ranges from three to six.

In order to obtain the values of the parameters of the logistic functions, there are two types of processes: deterministic and optimization approaches. In the first case, the results obtained are worse than in the second one but the process to obtain the parameters is simpler: it consists of just performing some operations using the WT parameters. This is true in the case of three or four parameters [17] but in case of a higher number, the process becomes more complicated. Therefore, here and in order to compare all the logistic functions with the same indicators, optimization processes are used [9].

As it has been said, the comparison is made regardless the number of parameters of the logistic function, but a low number of them is preferred, so a weighted comparison is performed, too.

Therefore, the focus of this paper is to find a logistic function with the minimum number of parameters and the best performance when modeling WT power curves. As two objectives are pursued, there may be one or two logistic functions proposed, according to the intended use.

This paper is organized as follows: in Section 2, the logistic functions and related information to them are presented. In Section 3, the results of applying the logistic functions to several WTs are provided. Finally, Section 4 states the conclusions.

## 2. Logistic functions

As it was mentioned, logistic functions have been applied to model the WT power curves over the last years in an effective way because they are continuous functions and the errors involved are very low.

The functions are classified depending on the number of parameters, from three to six. Some of the functions are related among them, and the relationships are explained just after one of the functions involved.

In some cases there is available information about the technical meaning of the parameters and can be provided, but not always.

### 2.1. 3-Parameter logistic function

It is generally known that the most used 3-parameter logistic function is the 3PLE, shown in Eq. (1) [17].

$$P(v) = \frac{\alpha}{1 + \exp\left(-\beta \cdot (v - v_0)\right)} \tag{1}$$

where P is the output power, v is the wind speed, $\alpha$ is the curve's maximum value, $v_0$ is the value of the midpoint and $\beta$ is the slope of the curve. Notice that the term $\exp\left(-\beta \cdot (v - v_0)\right)$ can be converted into $Q \cdot \exp\left(-\beta \cdot v\right)$ where $Q = \exp\left(\beta \cdot v_0\right)$ providing a different expression for the same relationship. Eq. (1) has been used in Ref. [17] to model the WT power curve based on the parameters of the WT, as it can be seen in Eq. (2).

$$P(v) = \frac{P_r}{1 + \exp\left(4s\left(v_{ip} - v\right)/P_r\right)} \tag{2}$$

where $P_r$ is the rated power of the WT, $v_{ip}$ its wind speed inflection point and s is the slope at the inflection point. In fact, Eqs. (1) and (2) coincide when equalling $\alpha = P_r$, $v_0 = v_{ip}$ and $\beta = 4s/P_r$. The only difference is that in Eq. (2) the parameters can be obtained directly from the data provided by the manufacturer of the WT while in other case, they have to be calculated by an optimization procedure.

Mbamalu et al. introduced Eq. (3) in Ref. [13] saying it is a typical symmetrical "s" curve with horizontal asymptotes at both A and C, and proposing it as a hypothetical mathematical model for WT power curves.

$$P(v) = \frac{A \cdot v}{\sqrt{B - v^2}} + C \tag{3}$$

There is no relationship between Eqs. (1) and (3) but it seems to be a function with the same shape and it has been used to model a WT power curve.

It is also well known the Gompertz (GPTZ) function [20], which is shown in Eq. (4) and it was not applied to WT power curves, yes.

$$P(v) = D \cdot \exp\left(-G \cdot \exp\left(-K \cdot v\right)\right) \tag{4}$$

where D is an asymptote, $G > 0$ sets the displacement along the horizontal axis (translates the graph to the left or right) and $K > 0$ sets the vertical scaling. Besides, in Ref. [20], it is proposed some kind of equivalence between the parameters of the Gompertz function and the logistic function in Eq. (1): $D = \alpha$, $G = \exp\left(\beta \cdot v_0\right)$, $K = \beta$.

Another model is the Bass one [22], which can be seen in Eq. (5).

$$P(v) = S\frac{1 - \exp\left(-(P + Q)v\right)}{1 + \frac{P}{Q}\exp\left(-(P + Q)v\right)} \tag{5}$$

where S is the maximum value but the other two parameters have no specific meaning, at least when applied to WT power curves. This model has been used to know the degree of diffusion of wind power.

Finally, there is another 3-parameter model [15] with the shape shown in Eq. (6).

$$P(v) = \frac{k \cdot y_0 \cdot \exp\left(r \cdot v\right)}{k + y_0 \cdot \exp\left(r \cdot v\right)} \tag{6}$$

where r is the rate of increase, k is the maximum value and $y_0$ has no specific meaning.

Eq. (6) can be easily converted into Eq. (1) just making the following changes ($\alpha = k$, $\beta = r$ and $v_0 = \log\left(k/y_0\right)/r$).

### 2.2. 4-Parameter logistic function

The 4-parameter logistic function most applied to power curve modeling is the one shown in Eq. (7) [1] named here 4PLEE.

$$P(v) = L\frac{1 + m \cdot \exp\left(-v/\tau\right)}{1 + n \cdot \exp\left(-v/\tau\right)} \tag{7}$$

which can be related with the Bass one $\left(L = S, m = -1, n = P/Q, \tau = 1/(P + Q)\right)$ and with the 3PLE $\left(L = \alpha, m = 0, \tau = 1/\beta, n = \exp\left(v_0/\tau\right)\right)$.

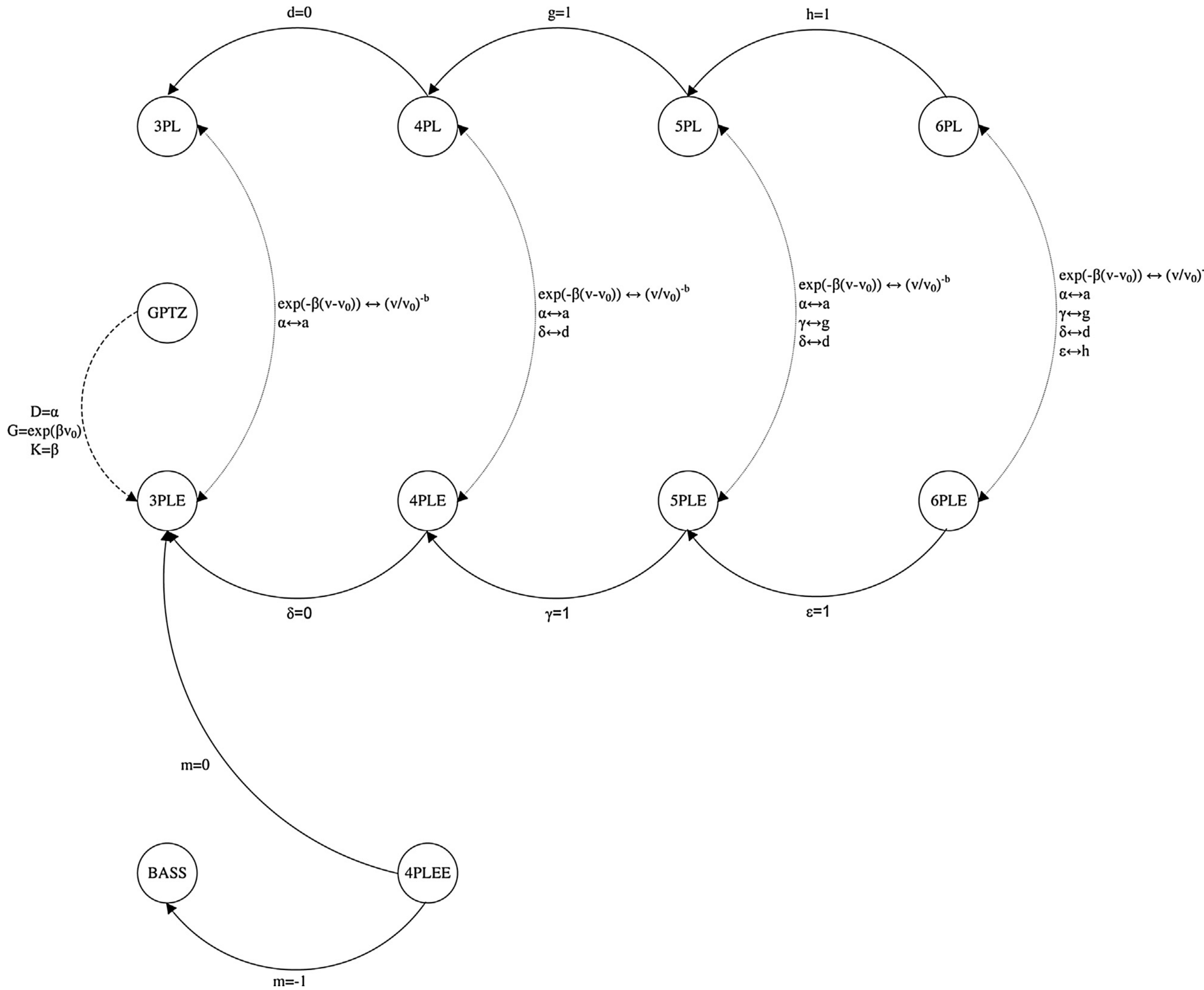


**Fig. 1.** Relationships among the different logistic functions.

### 2.3. 5-Parameter logistic function

The 5PL function appears in Ref. [1] and is expressed in Eq. (8)

$$P(v) = d + \frac{a - d}{\left(1 + (v/v_0)^{-b}\right)^{1/g}} \tag{8}$$

where a is the minimum value that can be obtained, d is the maximum one, $c > 0$ is the point of inflection, b is the slope of the curve at point c and $g > 0$ is the asymmetry factor (if $g = 1$ it is a symmetrical curve around the inflection point).

Other logistic functions can be introduced here, such as: 4PL by setting $g = 1$ in Eq. (8) and 3PL by setting also $d = 0$ in the same equation, following the trend of the 3PLE function.

### 2.4. 6-Parameter logistic function

The generalized logistic function [25] can be considered the reference for the logistic functions, expressed in Eq. (9)

$$P(v) = \delta + \frac{\alpha - \delta}{\left(\varepsilon + \exp(-\beta \cdot (v - v_0))\right)^{1/\gamma}} \tag{9}$$

where $\delta$ is the lower asymptote, $\alpha$ is the upper asymptote, $\beta$ is the growth rate, $\gamma$ determines the closest asymptote to the maximum growth part of the curve, $v_0$ is related to the value P(0) and $\varepsilon$ takes a value around 1.

Eq. (9) can be named as 6PLE in order to keep a certain coherence with the names of similar logistic functions.

Some other logistic functions can be derived from the 6PLE function in order to follow a path that arrives at the 3PLE function. The 6PLE can be converted into the 5PLE by setting $\varepsilon = 1$, into the 4PLE also by setting $\gamma = 1$ and into the 3PLE (Eq. (1)) by setting also $\delta = 0$.

From Eq. (9) and considering 5PL, it can be obtained the 6PL following the trend of 6PLE, as in Eq. (10).

$$P(v) = d + \frac{a - d}{\left(h + (v/v_0)^{-b}\right)^{1/g}} \tag{10}$$

where the parameter h appears besides the parameters of 5PL, and has a value around 1 (in 5PL, $h = 1$).

Eq. (3) can be derived from Eq. (10) by making the following changes: $d = C$, $a = A + C$, $h = -1$, $b = 2$, $g = 2$ and $v_0 = B^{-1/2}$.

### 2.5. Relationships among the logistic functions

Two main types of logistic functions have been presented. Those that can be obtained from Eq. (9), named 6PLE, 5PLE, 4PLE and 3PLE and those that can be derived from Eq. (10), named 6PL, 5PL, 4PL and 3PL. The relationship between both groups is mainly the function applied directly to the variable, which in the first case is $\exp\left(-\beta \cdot (v - v_0)\right)$ and in the second one is $\left(v/v_0\right)^{-b}$.

There is also another small group of functions obtained from Eq. (7), named 4PLEE and BASS (The Bass one may be named as 3PLEE but is already known by that name). The Bass one is the 4PLEE where $m = -1$. The relationship between the group of the 4PLEE and the group of the 6PLE is that the 4PLEE can be converted into the 3PLE by setting $m = 0$, as has been said.

The relationship between the GPTZ and the other groups is due to the equivalence of the parameters between the GPTZ and the 3PLE.

All these relationships can be seen in Fig. 1. Notice that a solid line indicates a direct transformation of parameters, a dotted line shows a change of the main sub-function and a dashed line expresses an equivalence of parameters.

## 3. Application to wind turbine power curves

WT power curves are provided by the manufacturers by means of pairs of data in intervals of 1 m/s or as a graph. The most accurate way to approximate the behaviour of a WT is by using splines [1]. The splines model each interval of 1 m/s by a 3-degree polynomial and take into account the continuity of the piecewise function amid intervals. The splines can be considered as a reference in order to assess the performance of other models. However, the splines are just valid to implement the power curve in an application program due to its complexity.

There are two types of processes to obtain the parameters of the logistic functions. The optimization process provides the best option for the parameters considering an objective function. The deterministic process takes into account the technical meaning of the parameters. As can be seen in Ref. [17] the first one provides better results from the point of view of the error made and, therefore, this one has been chosen to compare the logistic functions.

The WTs used to check the performance of the proposed functions are a Vestas V80 (2000 kW), an Enercon E82 (2050 kW), a Siemens S82 SWT-2.3 82 (2300 kW), a Repower MM82 (2050 kW), a Nordex N90 (2300 kW), a Siemens SWT-3.6 107 (3600 kW) and a Vestas V164 (6995 kW). The parameters of these WTs are given in Table 1.

The error is measured by using the following indicators: Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE) [1]. The results given here for each logistic function include the values of the parameters and these three indications for each WT checked.

Taking into account all the relationships among the functions, the analysis can be reduced to Eqs. (4), (7), (9) and (10). It means that the functions to be checked are: GPTZ, BASS, 3PLE, 3PL, 4PLEE, 4PLE, 4PL, 5PLE, 5PL, 6PLE and 6PL. Those functions that have not been included here can be derived from these.

### 3.1. Results for 3-parameter functions

The 3-parameter functions checked are the GPTZ (Table 2), the BASS (Table 3), the 3PLE (Table 4) and the 3PL. The results for this last one are not provided because they do not reflect the values and errors corresponding to a model for a WT power curve, they are far from expected.

Even though the results obtained using the GPTZ function can model a WT power curve, the other two functions are considerably better than the GPTZ one. In fact, the BASS function and the 3PLE functions provide similar values for the errors. Besides, it can be pointed out that the S-parameter of the BASS function and the a-parameter of the 3PLE function are equal and also approximately equal to the rated power of the WT. The same happens between the P-parameter and the b-parameter and, in this case, this value is related to the slope of the curve at the inflection point. Moreover, the $v_0$-parameter has a value close to the wind speed inflection point, as expected. The values of the parameters obtained are not exactly the values of the parameters of the WTs due to the optimization procedure which provides a better global result for the power curve.

### 3.2. Results for 4-parameter functions

The performance of three 4-parameter functions has been checked, the 4PLEE (Table 5), the 4PLE (Table 6) and the 4PL. As in the former case, the results for the 4PL are not close to model the WT power curves, so they are not provided.

The same as with the 3-parameter functions happens, 4PLEE and 4PLE model the WT power curves with the same accuracy. It can be pointed out that the error values are similar to the ones made in the 3-parameter cases (BASS and 3PLE). It can also be said that the L-parameter of the 4PLEE and the a-parameter of the 4PLE are respectively equal and approximately equal to the rated power of each WT.

### 3.3. Results for 5-parameter functions

The 5-parameter functions checked are just the 5PLE and the 5PL. The values of their parameters and their errors are provided respectively in Tables 7 and 8.

The 5-parameter function 5PLE provides results that have half the errors of those provided by BASS, 3PLE, 4PLEE and 4PLE functions, so this function models better the power curves but needs a bigger number of parameters. In the 5PL case the results are not as good and, even though the function provides a model of the power curve, the results are worse than in cases with less parameters. Again, a-parameter in 5PLE and a-parameter in 5PL are close to the rated power value, however, the rest of parameters do not have a direct relationship with the parameters of the WTs.

### 3.4. Results for 6-parameter functions

The functions checked with six parameters are the 6PLE (Table 9) and the 6PL (Table 10).

In the case of six parameters, both functions provide good results, however, the 6PLE case outperforms considerably the 6PL. It can be pointed out that the 6PLE provides results with errors that are almost similar to those provided by the 5PLE function. Besides, in the 6-parameter case none of the parameters can be directly related with the parameters of the WTs.

### 3.5. Results comparison

As it has been said, the best options to model the WT power curves are those of the type XPLE, being X the number of parameters, between three and six. Others like BASS and 4PLEE functions present similar errors as their corresponding XPLE one. Moreover, by comparison, functions as the GPTZ, 5PL and 6PL can be discarded to model WT power curves.

Considering the mean MAPE of the WTs checked, a comparison among all functions, considering the number of parameters, is provided in Fig. 2. In this figure it can be seen that BASS, 3PLE, 4PLEE and 4PLE functions make approximately the same level of errors. The same happens with 5PLE and 6PLE.

In Fig. 3, a graph is provided where 3PLE and 5PLE models for a Vestas V80 are shown. It can be seen that the performance of 5PLE is better than the one corresponding to 3PLE as the MAPE values predicted.

**Table 1**
Parameters of the WTs.

| | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| $P_r$ (kW) | 2000 | 2050 | 2300 | 2050 | 2300 | 3600 | 6995 |
| $u_{ci}$ (m/s) | 4 | 2 | 4 | 4 | 4 | 4 | 4 |
| $u_r$ (m/s) | 16 | 13 | 18 | 15 | 13 | 17 | 15 |
| $u_{ip}$ (m/s) | 9.37 | 8.57 | 9.53 | 8.87 | 8.94 | 9.18 | 8.86 |

$P_r$ is the rated power, $u_{ci}$ is the cut-in wind speed, $u_r$ is the rated wind speed and $u_{ip}$ is the wind speed at the inflection point of the power curve.

**Table 2**
Parameter and error values for several WTs using the GPTZ function.

| | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| D | 2034.7000 | 2079.3000 | 2340.7000 | 2082.4000 | 2337.7000 | 3658.1000 | 7095.9000 |
| G | 41.4747 | 49.6854 | 38.1412 | 37.1716 | 46.5980 | 44.9352 | 58.9351 |
| K | 0.4616 | 0.5141 | 0.4421 | 0.4659 | 0.4964 | 0.4757 | 0.5181 |
| MAPE | 0.0223 | 0.0220 | 0.0204 | 0.0201 | 0.0238 | 0.0219 | 0.0207 |
| MAE | 44.6104 | 45.0627 | 46.9288 | 41.1291 | 54.7649 | 78.8874 | 144.9291 |
| RMSE | 57.0571 | 59.6498 | 59.4956 | 53.4347 | 72.6748 | 100.8521 | 193.0473 |

**Table 3**
Parameter and error values for several WTs using the BASS function.

| | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| S | 2014.5000 | 2063.0000 | 2314.8000 | 2062.3000 | 2318.0000 | 3623.5000 | 7039.0000 |
| P | 0.6797 | 0.7486 | 0.6527 | 0.6846 | 0.7312 | 0.7001 | 0.7581 |
| Q | 0.0014 | 0.0013 | 0.0016 | 0.0017 | 0.0013 | 0.0013 | 0.0010 |
| MAPE | 0.0095 | 0.0096 | 0.0081 | 0.0078 | 0.0128 | 0.0089 | 0.0090 |
| MAE | 19.0999 | 19.7188 | 18.6647 | 15.8886 | 29.4143 | 32.0833 | 63.2570 |
| RMSE | 25.0311 | 25.2714 | 23.4489 | 20.5955 | 39.0720 | 42.9873 | 84.5190 |

**Table 4**
Parameter and error values for several WTs using the 3PLE function.

| | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| a | 2014.2000 | 2062.9000 | 2314.3000 | 2061.9000 | 2317.8000 | 3623.0000 | 7038.5000 |
| b | 0.6846 | 0.7524 | 0.6582 | 0.6907 | 0.7355 | 0.7045 | 0.7615 |
| g | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| d | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| e | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $v_0$ | 9.0431 | 8.4754 | 9.2433 | 8.7228 | 8.6602 | 8.9449 | 8.7436 |
| MAPE | 0.0095 | 0.0096 | 0.0080 | 0.0077 | 0.0127 | 0.0088 | 0.0090 |
| MAE | 19.0033 | 19.6265 | 18.4743 | 15.7185 | 29.2203 | 31.8537 | 62.7644 |
| RMSE | 24.7896 | 24.9829 | 23.3146 | 20.3315 | 38.8933 | 42.5799 | 84.0552 |

**Table 5**
Parameter and error values for several WTs using the 4PLEE function.

| 4PLEE | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| L | 2014.5000 | 2062.1000 | 2315.3000 | 2032.6000 | 2310.4000 | 3623.4000 | 7039.0000 |
| m | −0.9863 | 4.0810 | −2.3324 | −1.9312 | 1.7140 | −0.8671 | −0.8580 |
| n | 470.4139 | 660.2752 | 397.1661 | 379.4427 | 667.8390 | 529.8410 | 762.6253 |
| t | 1.4681 | 1.3091 | 1.5403 | 1.4652 | 1.3320 | 1.4249 | 1.3168 |
| MAPE | 0.0095 | 0.0094 | 0.0082 | 0.0078 | 0.0118 | 0.0089 | 0.0090 |
| MAE | 19.0982 | 19.1926 | 18.9460 | 16.0599 | 27.1972 | 32.0523 | 63.1893 |
| RMSE | 25.0273 | 24.3133 | 23.7463 | 20.9098 | 39.1501 | 42.9289 | 84.4487 |

**Table 6**
Parameter and error values for several WTs using the 4PLE function.

| 4PLE | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| alfa | 2014.2000 | 2062.1000 | 2314.3000 | 2061.9000 | 2317.8000 | 3623.0000 | 7038.5000 |
| beta | 0.6846 | 0.7639 | 0.6582 | 0.6907 | 0.7355 | 0.7045 | 0.7615 |
| gamma | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| delta | 0.0000 | 12.7449 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| epsilon | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $v_0$ | 9.0431 | 8.4996 | 9.2433 | 8.7228 | 8.6602 | 8.9449 | 8.7436 |
| MAPE | 0.0095 | 0.0094 | 0.0080 | 0.0077 | 0.0127 | 0.0088 | 0.0090 |
| MAE | 19.0033 | 19.1927 | 18.4743 | 15.7185 | 29.2203 | 31.8537 | 62.7644 |
| RMSE | 24.7896 | 24.3133 | 23.3146 | 20.3315 | 38.8933 | 42.5799 | 84.0552 |

**Table 7**
Parameter and error values for several WTs using the 5PLE function.

| 5PLE | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| alfa | 2004.2000 | 2053.3000 | 2305.4000 | 2054.6000 | 2306.2000 | 3606.0000 | 7011.7000 |
| beta | 0.9517 | 1.0621 | 0.8243 | 0.8736 | 1.1172 | 0.9699 | 1.0168 |
| gamma | 2.4166 | 2.3832 | 1.9681 | 2.0242 | 2.8632 | 2.3466 | 2.1836 |
| delta | −81.9551 | −60.5035 | −80.1789 | −75.3702 | −110.8954 | −134.9048 | −214.3071 |
| epsilon | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $v_0$ | 10.5739 | 9.8598 | 10.5101 | 9.9757 | 10.3055 | 10.3936 | 10.0009 |
| MAPE | 0.0041 | 0.0035 | 0.0041 | 0.0038 | 0.0059 | 0.0036 | 0.0045 |
| MAE | 8.2598 | 7.1612 | 9.4684 | 7.8114 | 13.6332 | 12.8688 | 31.6375 |
| RMSE | 11.4650 | 9.9892 | 13.0473 | 10.4179 | 20.8145 | 18.7156 | 47.1771 |

**Table 8**
Parameter and error values for several WTs using the 5PL function.

| 5PL | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| a | 2062.5000 | 2097.2000 | 2379.8000 | 2111.5000 | 2363.6000 | 3704.2000 | 7119.9000 |
| b | −4.5431 | −4.9218 | −4.3605 | −4.4129 | −4.7164 | 4.6390 | 6.3842 |
| g | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0003 | 0.0004 | 1.0154 |
| d | 74.7708 | 90.7273 | 79.1060 | 75.9578 | 84.0167 | 132.1640 | 74.3104 |
| h | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $v_0$ | 1.2410 | 1.4208 | 1.0405 | 1.1848 | 1.3920 | 1.4837 | 8.7059 |
| MAPE | 0.0264 | 0.0267 | 0.0255 | 0.0251 | 0.0293 | 0.0261 | 0.0173 |
| MAE | 52.7504 | 54.7286 | 58.7194 | 51.4200 | 63.4755 | 93.9686 | 120.6837 |
| RMSE | 69.6292 | 70.7188 | 76.0712 | 67.3456 | 88.0213 | 122.9346 | 156.2219 |

**Table 9**
Parameter and error values for several WTs using the 6PLE function.

| 6PLE | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| alfa | 942.4714 | 806.2447 | 1923.5000 | 1682.0000 | 547.0978 | 1637.6000 | 2870.7000 |
| beta | 0.9517 | 1.0621 | 0.8243 | 0.8737 | 1.1986 | 0.9700 | 1.0168 |
| gamma | 2.4169 | 2.3833 | 1.9683 | 2.0244 | 3.2302 | 2.3470 | 2.1839 |
| delta | −81.9660 | −60.5077 | −80.1915 | −75.3820 | −124.5824 | −134.9357 | −214.3527 |
| epsilon | 0.1793 | 0.1195 | 0.7094 | 0.6776 | 0.0157 | 0.1733 | 0.1559 |
| $v_0$ | 8.7680 | 7.8598 | 10.0943 | 9.5302 | 6.9714 | 8.5864 | 8.1734 |
| MAPE | 0.0041 | 0.0035 | 0.0041 | 0.0038 | 0.0055 | 0.0036 | 0.0045 |
| MAE | 8.2588 | 7.1610 | 9.4675 | 7.8103 | 12.5604 | 12.8665 | 31.6355 |
| RMSE | 11.4645 | 9.9891 | 13.0466 | 10.4173 | 19.9800 | 18.7138 | 47.1750 |

**Table 10**
Parameter and error values for several WTs using the 6PL function.

| 6PL | V80 | E82 | S82 | R82 | N90 | S107 | V164 |
|---|---|---|---|---|---|---|---|
| a | 29.7426 | 31.1370 | 29.6373 | 32.4198 | 33.7156 | 40.6413 | 30.9488 |
| b | 9.0352 | 9.2403 | 9.3558 | 9.4921 | 9.2467 | 9.3790 | 9.7900 |
| g | 2.3733 | 2.3267 | 2.4901 | 2.6643 | 2.4478 | 2.5538 | 2.0487 |
| d | 8.3755 | 9.5244 | 9.5777 | 7.1799 | 6.7574 | 1.5350 | 9.1693 |
| h | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $v_0$ | 3.2311 | 3.1481 | 3.1038 | 3.0537 | 3.1456 | 3.0951 | 2.9518 |
| MAPE | 0.0085 | 0.0087 | 0.0065 | 0.0056 | 0.0115 | 0.0073 | 0.0102 |
| MAE | 17.0420 | 17.9231 | 14.9352 | 11.5005 | 26.4589 | 26.3518 | 71.2818 |
| RMSE | 21.7682 | 22.2776 | 18.8917 | 14.8074 | 34.8328 | 34.4803 | 104.3302 |

Moreover, in Fig. 4 a graph where BASS and 4PLEE models for the same WT is provided. It can be checked than the performance of both models is very similar, as the MAPE values are.

## 4. Conclusions

The logistic functions have been recently used to model WT power curves. In this paper a comparison has been presented among some of these generally used curves and some new curves that have not been so commonly used and have been tested here. Considering the mean MAPE as the indicator to compare their performances, the following can be concluded:

1) The 6PLE is the best option to model a WT power curve. However, dealing with six parameters is cumbersome.
2) The results provided by the 5PLE are very similar to those obtained with the 6PLE, so the 5PLE can be considered better than the 6PLE as it needs only 5 parameters.
3) The 4PLE gives worse results than the 5PLE. Instead, its advantage is the use of only 4 parameters.
4) The errors made by the 3PLE are approximately the same as those made by the 4PLE, so 3PLE can be considered better than 4PLE.
5) Other options as BASS or 4PLEE give good results but a bit worse than the 3PLE and 4PLE.
6) XPL type functions and the GPTZ one can be discarded, where X is the number of parameters and between three and six.

Therefore, as a result, a strong recommendation can be given about the use of the 5PLE or the 3PLE functions to model WT power curves. In order to choose between these two models, good criteria

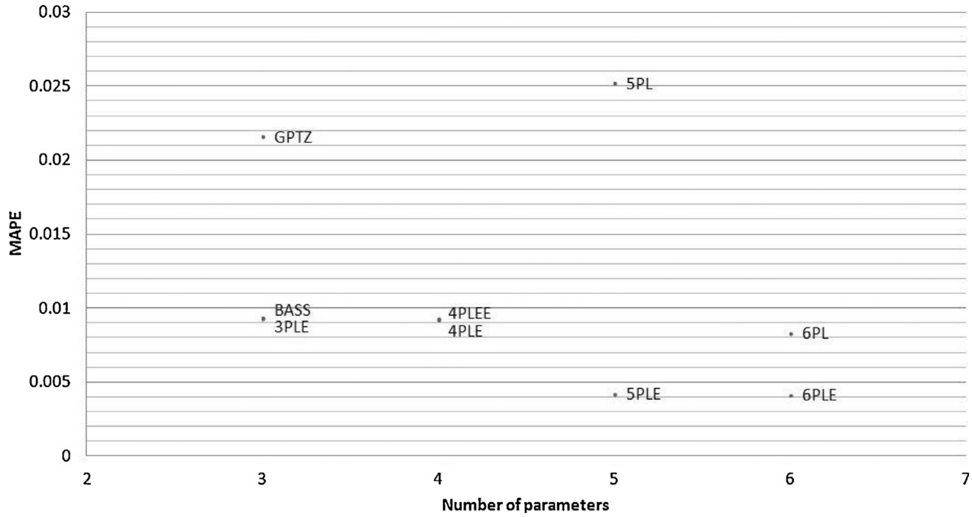


**Fig. 2.** Mean MAPE values for different logistic functions.

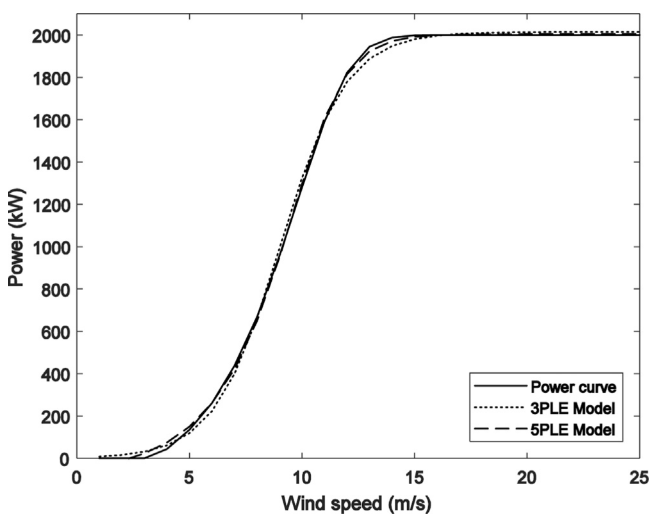


**Fig. 3.** Power curve for a Vestas V80 using 3PLE model or 5PLE model.

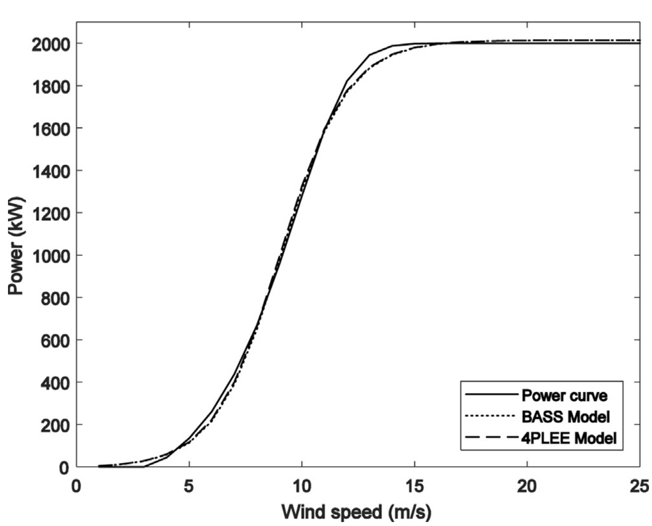


**Fig. 4.** Power curve for a Vestas V80 using BASS model or 4PLEE model.

can be the number of parameters (five and three), if the technical meaning is valuable (3PLE) and the error made (5PLE half the error than 3PLE).

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**Daniel Villanueva** obtained his Ph.D. degree in Electrical Engineering from the *Departamento de Enxeñería Eléctrica, Universidade de Vigo*, Spain, in 2012, where he is currently working on the impact of wind energy in power systems, including simulation of wind speeds, analysis of wind power and probabilistic analysis of power systems including wind power generation.

**Andrés Feijóo** obtained his Ph.D. degree in Electrical Engineering from the *Departamento de Enxeñería Eléctrica, Universidade de Vigo*, Spain, in 1998. He is now in the same department and is interested in wind energy and its impact in electrical power networks, in particular wind speed simulation and steady-state and dynamic modelling and simulation of electrical machines in wind parks.