Manchester Metropolitan University

Advanced Encryption Standard (AES)

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Introduction

- The **Advanced Encryption Standard** (AES), published by NIST in 2001.
- To replace DES due to security concerns over small key size and other considerations.
- AES designed to be more secure and fast.
- From 2008 on, chip manufacturers implement AES capabilities in low-level chip design.
- Now the most widely used cipher.

Polynomial arithmetic and the fields $GF(2^n)$

- Consider all polynomials

$$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 = \sum_{i=0}^{n-1} a_i x^i, \quad a_i \in \mathbb{Z}_2.$$

  of degree $n-1$ or less.
- Arithmetic follows the rules of $+$ and $\cdot$ for polynomials, with arithmetic of the coefficients a_i carried out in $\mathbb{Z}_2$, i.e. addition of coefficients is the same as XOR.
- If multiplication results in a polynomial of degree greater than $n-1$ then the product is reduced modulo a specified irreducible polynomial $m(x)$ of degree n, the modulos polynomial.

AES and $\mathrm{GF}(2^8)$.

- The Advanceed Encryption Standard (AES) uses such a field $\mathrm{GF}(2^8)$, consisting of polynomials of degree less than or equal to 7, with binary coefficients and polynomial operations carried out modulo the irreducible polynomial

$$m(x) = x^8 + x^4 + x^3 + x + 1.$$

- The figure on the right shows the calculation of an example product in $\mathrm{GF}(2^8)$.
- AES uses this since it is designed to operate on 8-bit bytes.
  - addition of bytes is just bit-wise XOR.
  - *multiplication* of bytes is defined as multiplication in the finite field $\mathrm{GF}(2^8)$, modulo the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$.

The Advanced Encryption Standard (AES) uses arithmetic in the finite field GF(2^8), with the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$. Consider the two polynomials $f(x) = x^6 + x^4 + x^2 + x + 1$ and $g(x) = x^7 + x + 1$. Then

$$\begin{aligned} f(x) + g(x) &= x^6 + x^4 + x^2 + x + 1 + x^7 + x + 1 \\ &= x^7 + x^6 + x^4 + x^2 \end{aligned}$$

$$\begin{aligned} f(x) \times g(x) &= x^{13} + x^{11} + x^9 + x^8 + x^7 \\ &\quad + x^7 + x^5 + x^3 + x^2 + x \\ &\quad + x^6 + x^4 + x^2 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \end{aligned}$$

$$\begin{array}{r} x^5 + x^3 \\ x^8 + x^4 + x^3 + x + 1 \overline{)\, x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1} \\ \underline{x^{13} + x^9 + x^8 + x^6 + x^5} \\ x^{11} + x^4 + x^3 \\ \underline{x^{11} + x^7 + x^6 + x^4 + x^3} \\ x^7 + x^6 + 1 \end{array}$$

Therefore, $f(x) \times g(x) \bmod m(x) = x^7 + x^6 + 1$.

The structure of AES

- Operates on plaintext message blocks of 16 bytes = 128 bits.
- Various key lengths allowed, 16, 24 or 32 bytes. Ciphers referred to as AES-128, AES-192 or AES-256, depending on how many bits used in key.
- Throughout encryption (and decryption) the message block is maintained as a 4×4 array of bytes. This is referred to as the **state**.
- First four bytes form the first column, next four bytes the second column, and so on.
- An initial transformation of the state is followed by N rounds. Where N depends on the ley length used.
    - $N = 10$ for 128 bit key.
    - $N = 12$ for 192 bit key.
    - $N = 14$ for 256 bit key.
- The key passes through a *key explansion* transformation to provide $N + 1$ sub-keys to be used in the initial transformation and N rounds.
- Each sub-key consists for four 4-byte **words**, which form the columns of the **round key matrix**.

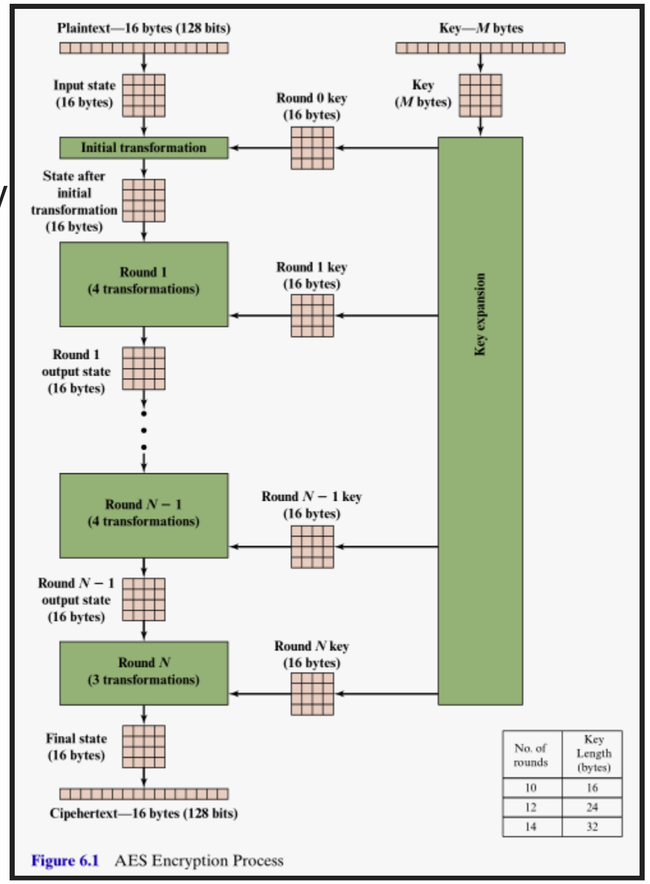


Figure 6.1 AES Encryption Process

AES detail

- This figure exposes the transformations within each round, for AES-128. The other schemes are similar.
- Note that is departs from the Feistel design. There is no notion of dividing the block into halves.
- Rounds 1 - 9 consists of four transformations
    - **Substitute bytes**: and S-box type permutation of the bytes of the state.
    - **Shift rows**: a simple permutation of the bytes within each row of the state.
    - **Mix columns**: a transformation that combines the bytes within each column of the state. This transformation utilies the $\mathrm{GF}(2^8)$ field.
    - **Add round key**: bit-wise XOR of the state with the appropriate round key matrix.
- The decryption algorithm reverses all the transformations. At each horizontal level, the intermediate states of the encryption and decryption algorithms are the same.

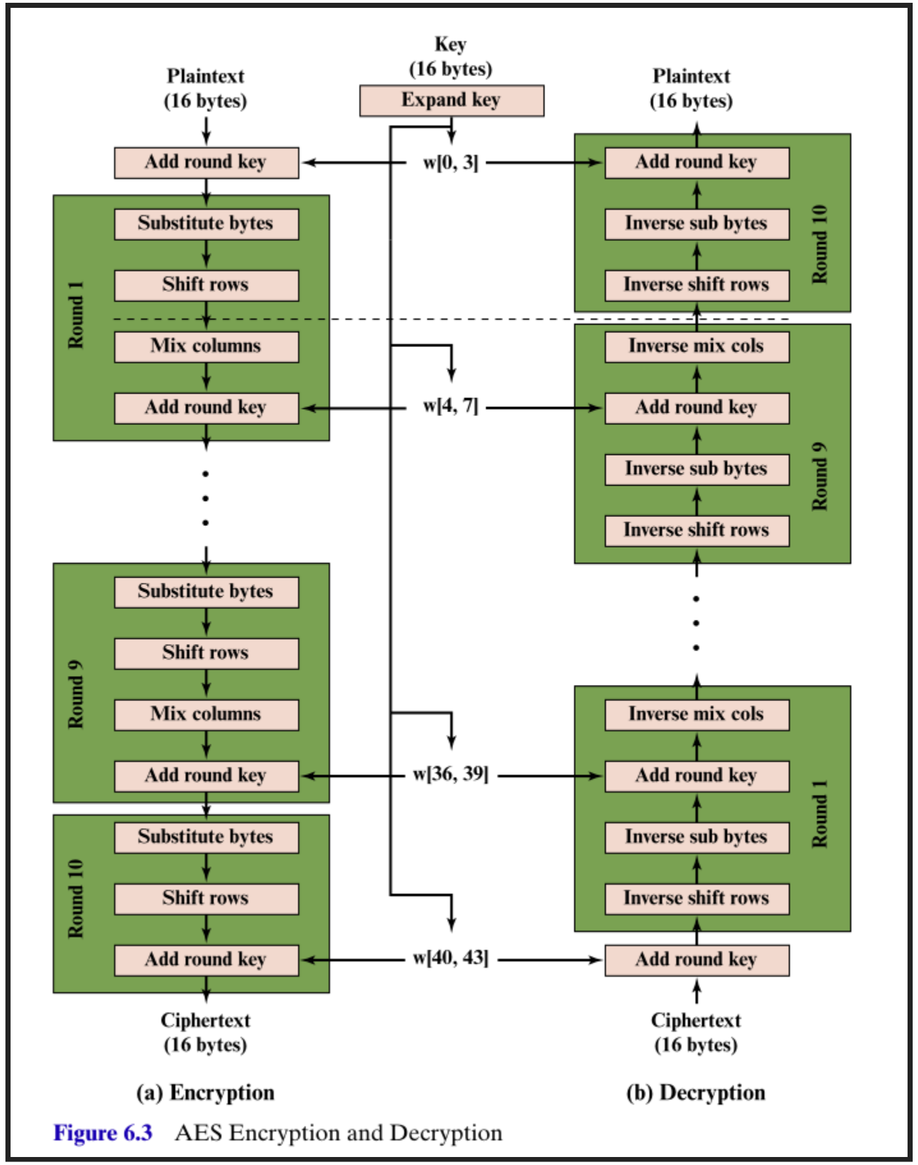


Figure 6.3 AES Encryption and Decryption

Visualizing a single round

- This figure visualizes the four transformations within a typical round.
- Here the state matrix is shown laid out as a row of 16 bytes.

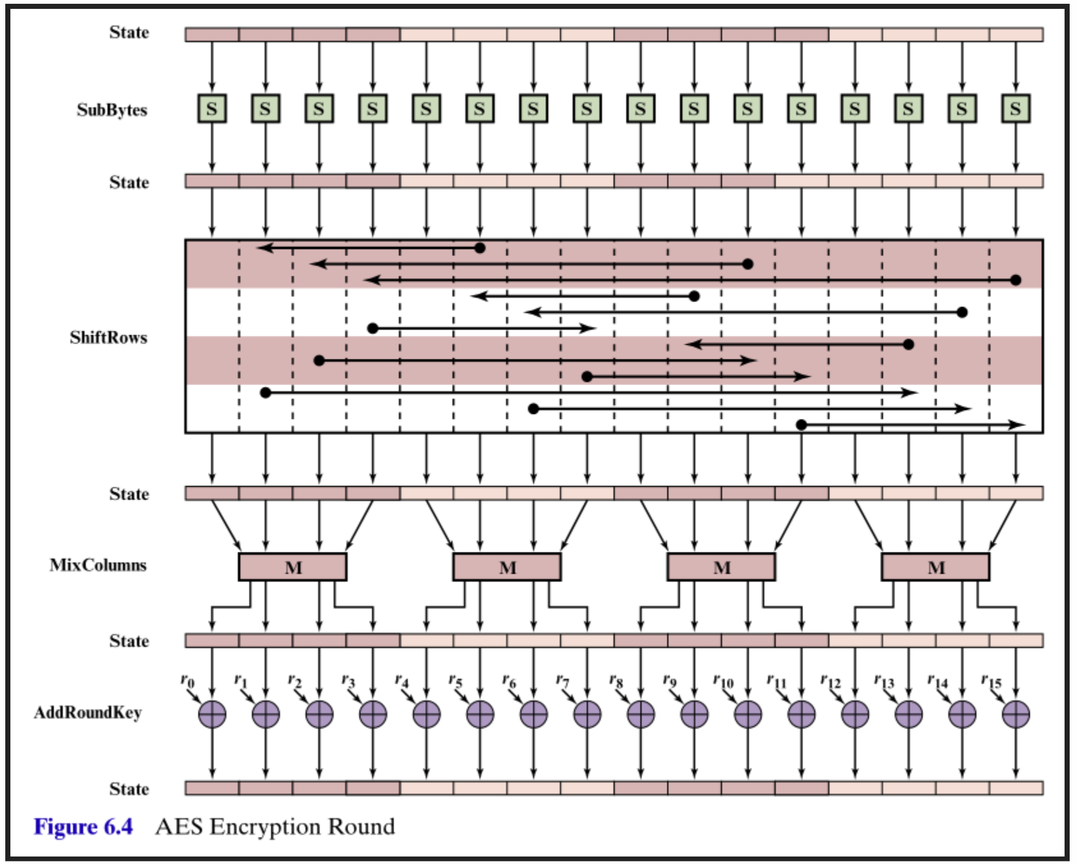
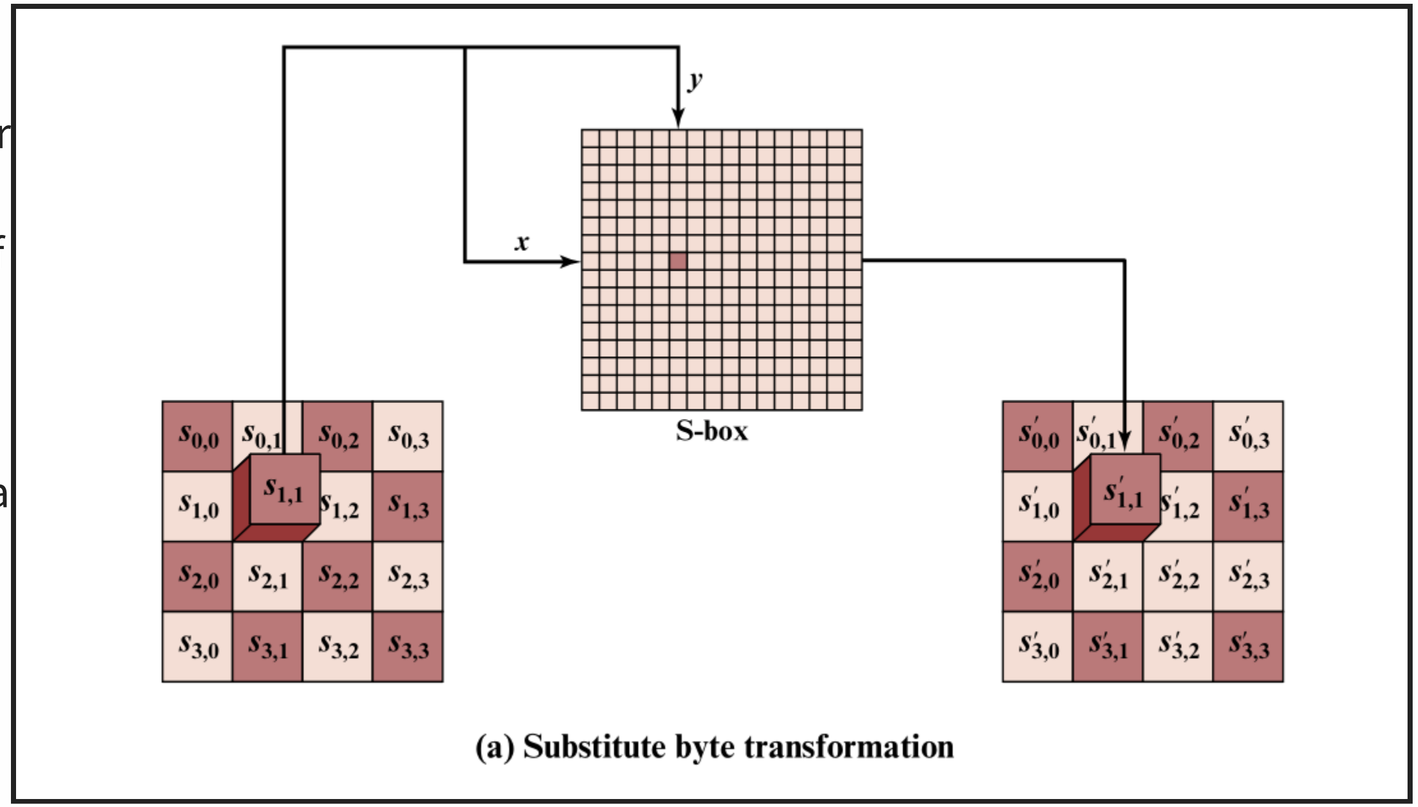


Figure 6.4 AES Encryption Round

Substitute bytes

- This figure shows the how the substitute bytes transformation is defined.
- For each entry of the incoming state matrix, i.e. for each byte
    - the first four bits denote the row index x of the S-box
    - the second four bites denote the column index y of the S-box
    - the S-box entry at that row and column is a byte that replaces the original byte of the state.
- After replacing each entry of the incoming state matrix, we get the outgoing state matrix.



(a) Substitute byte transformation

The S-box itself

- This figure shows the S-box.
- Remember
  - a four bit block is denoted by a hexadecimal digit `0,1,...9,a,b,c,d,e,f`.
  - a singel byte (i.e. a 8-bit block) is denoted by a two-digit hexadecimal number.
- A corresponding inverse S-box table is used in the decryption algorithm.
- Lots of detail in Stallings on the contruction of this S-box table.
  - Designed like this to minimize any correlation between incoming and outgoing bits.

		y														
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

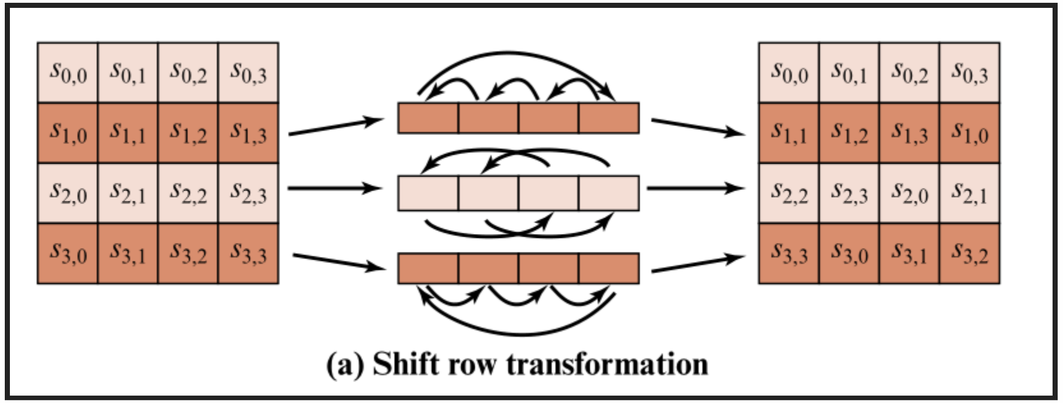
(a) S-box

- An example substitute bytes transformation is shown here

EA	04	65	85			87	F2	4D	97
83	45	5D	96			EC	6E	4C	90
5C	33	98	B0	→		4A	C3	46	E7
F0	2D	AD	C5			8C	D8	95	A6

The shift rows transformation

- In each of the second, third and fourth rows of state, permute the bytes in each row as shown.
- This has a significant effect on the original positions of the bits within the 128 bit message block.



(a) Shift row transformation

Mix columns transformation

- The equation shows how the incoming state matrix $s_{i,j}$ is multiplied by the matrix of constants to get the outgoing state matrix $s'_{i,j}$, shown on the right of the equation.

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \quad \textbf{(6.3)}$$

- This results in the following transformations within the j^{th} column of the state

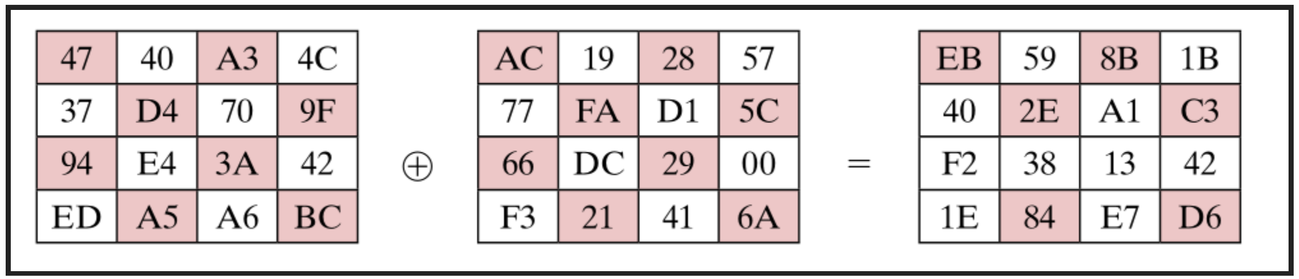
$$\begin{aligned} s'_{0,j} &= (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\ s'_{1,j} &= s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j} \\ s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j}) \\ s'_{3,j} &= (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j}) \end{aligned}$$

- The operations $\oplus$ and $\cdot$ shown here are the operations from $\mathrm{GF}(2^8)$, carried out on the entries of the state, i.e. on the bytes, i.e. on the 8-bit blocks which are interpreted as the coefficients of degree 7 polynomials.
- So $\oplus$ is bitwise XOR and $\cdot$ is the multiplication obtained from the multiplication of these polynomials, modulo the polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$.
- The design of this Mix Columns transformation ensures good mixing of the bytes within a column, and the use of the constants `01`, `02` and `03` results in efficient implementation of the encryption algorithm.

Add round key transformation

- Perhaps the most straightforward, this is just bitwise XOR amongst the bit entries of the state and round key matrices.
- Consider the examples shown here of

$$\text{incoming state} \oplus \text{round key} = \text{outgoing state}$$



47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

$\oplus$

AC	19	28	57
77	FA	D1	5C
66	DC	29	00
F3	21	41	6A

$=$

EB	59	8B	1B
40	2E	A1	C3
F2	38	13	42
1E	84	E7	D6

A summary flowchart for AES encryption

- This figure summarizes a typical encryption round.
- We still need to describe the key expansion process for the derivation of the round keys from the original key.

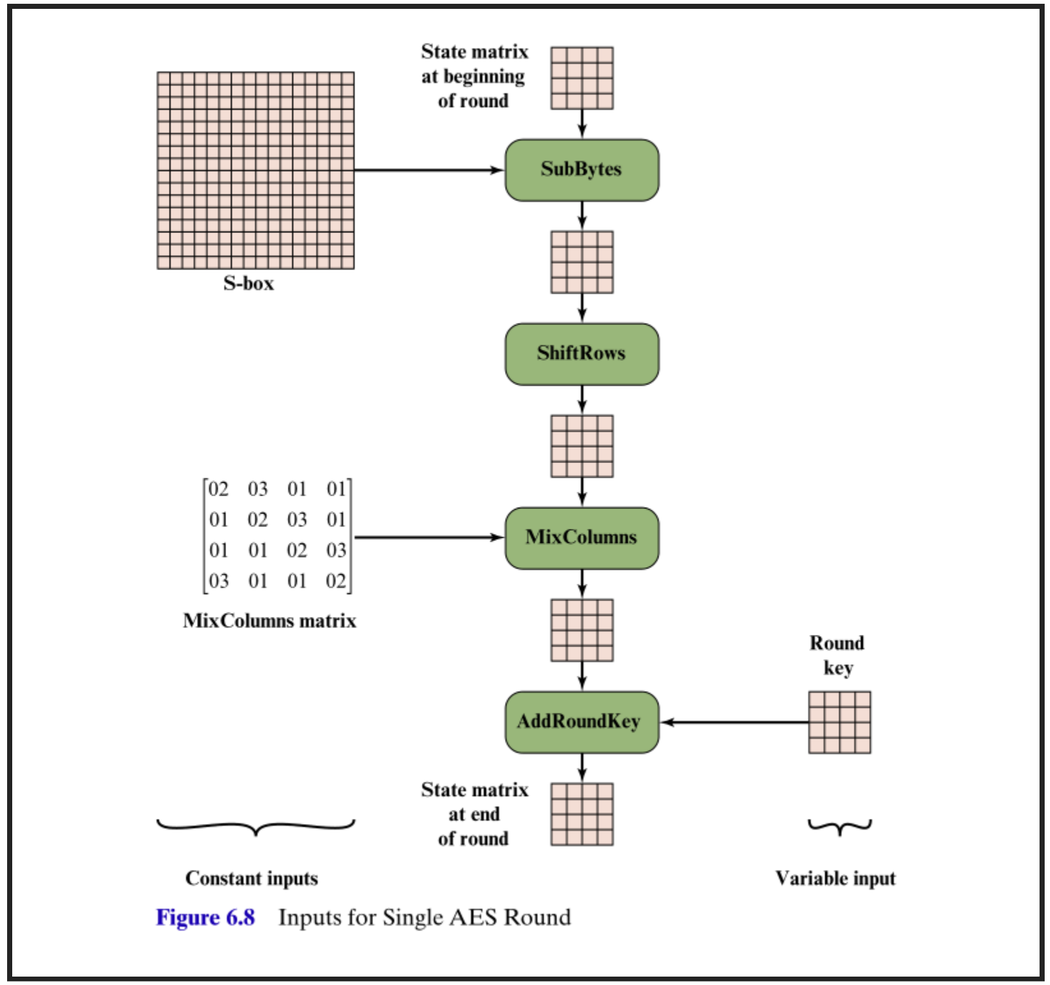


Figure 6.8 Inputs for Single AES Round

// reveal.js plugins