Q3 Second part.

$a(x) = x^4 + 8x^3 + 7x + 8$, $b(x) = 2x^3 + 9x^2 + 10x + 1$

as polys. with coeffs in $GF(11)$, i.e. $\mathbb{Z}_{11} = \{0, 1, \dots, 10\}$
with $+, \times$ done modulo 11.

Execute the E.A. to find $\gcd(a(x), b(x))$.

First division we need is

$$a(x) = q(x)\, b(x) + r(x), \quad \deg(r(x)) < \deg(b(x))$$

Might needs inverse mod 11

x	1	2	3	4	5	6	7	8	9	10
x^{-1}	1	6	4	3	9	2	8	7	5	10

$$\boxed{a(x)} = 6x\, b(x) + r(x)$$
$$= (x^4 + 10x^3 + 5x^2 + 6x) + (-2x^3 - 5x^2 + x + 8)$$
$$= 6x\, b(x) + 9x^3 + 6x^2 + x + 8$$
$$= 6x\, b(x) + 10\, b(x) + r(x)$$
$$= 6x\, b(x) + 9x^3 + 2x^2 + x + 10 + r(x)$$
$$= \underbrace{(6x + 10)}_{q(x)}\, b(x) + \underbrace{4x^2 - 2}_{r(x)}$$

1st division.

$$2x^3 + 9x^2 + 10x + 1$$
$$= \boxed{b(x)} = q_1(x)\, r(x) + r_2(x)$$
$$= 6x\, r(x) + r_2(x)$$
$$= (2x^3 + 10x) + 9x^2 + 1$$

$$= 6x\,r(x) + 5\,r(x) + r_2(x)$$

$$= 6x\,r(x) + 9x^2+1 + 0$$

$$\boxed{= (6x+5)\,r(x) + 0}$$

So E.A. stops. And reports the last non-zero remainder as the gcd.

So finally,

$$\gcd(a(x), b(x)) = r(x) = 4x^2-2.$$

When working over $GF(2)$ we can be quicker.

$$a(x) = x^3+1$$

$$b(x) = x^2+x+1$$

$$\boxed{a(x)} = q(x)\,b(x) + r(x)$$

$$= x\,b(x) + r(x)$$

$$= (x^3+x^2+x) + x^2+x+1$$

$$= x\,b(x) + b(x)$$

$$\boxed{= (x+1)\,b(x) + 0}$$

we have exact divisibility here

and $\gcd(a(x), b(x)) = b(x)$