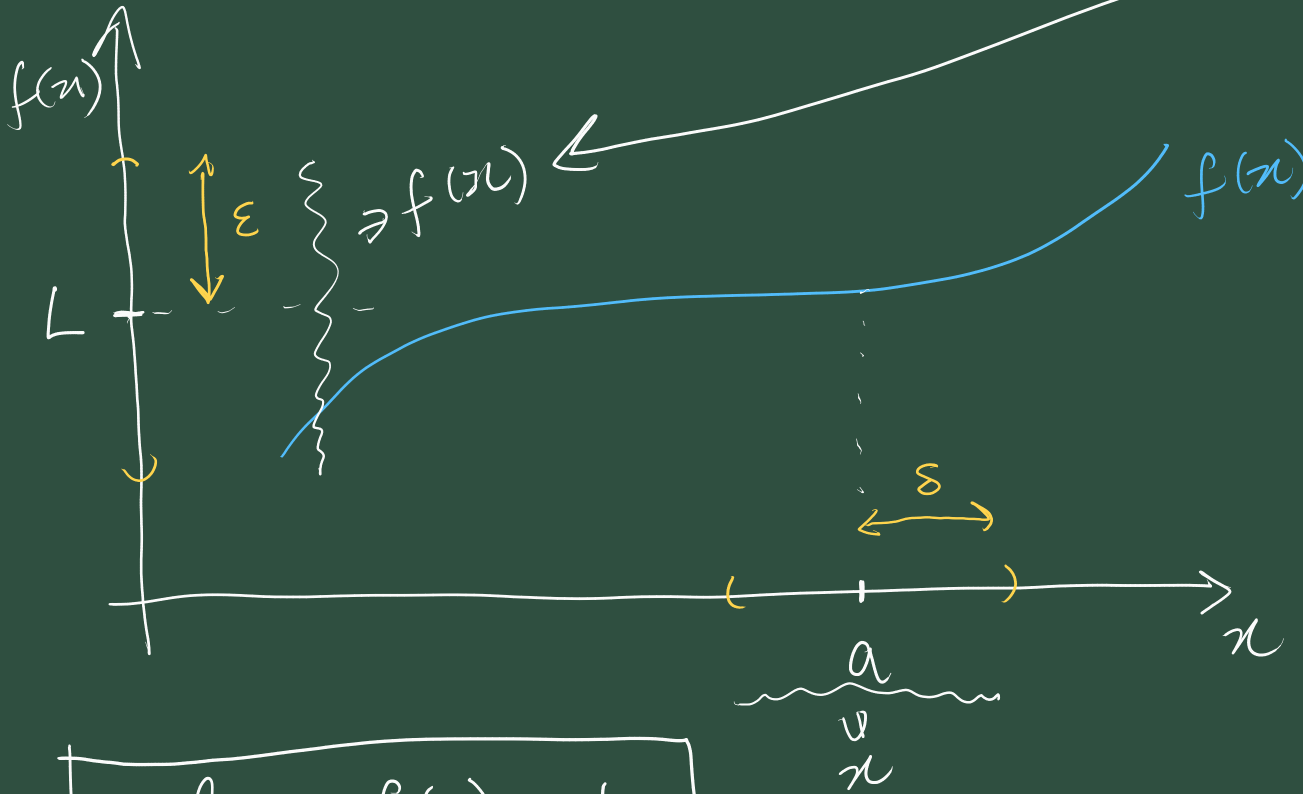
tinyurl.com/apexcalculus



$$\boxed{\lim_{x\to a} f(x) = L}$$

$$\forall \varepsilon > 0 \quad \exists \delta > 0 \quad \underbrace{|x-a| < \delta}_{} \Rightarrow \underbrace{|f(x)-L| < \varepsilon}_{}$$

Q 10

$$\lim_{x\to 2_{c}} (\underbrace{x^3 - 1}_{f(x)}) = \underset{L}{7}.$$

$\boxed{\text{Let } \varepsilon > 0}$ be given.

Consider the final inequality.

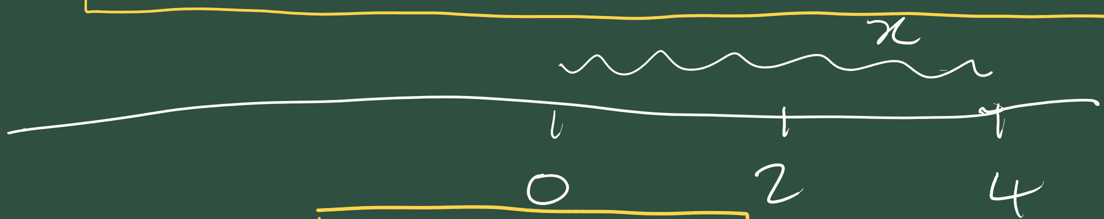
$$|x^3 - 1 - 7| < \varepsilon$$

$$\iff |x^3 - 8| < \varepsilon$$

$$\iff |(x-2)(x^2+2x+4)| < \varepsilon$$

$$\iff |x-2|\,|x^2+2x+4| < \varepsilon$$

$$\iff \boxed{|x-2| < \frac{\varepsilon}{|x^2+2x+4|}}$$



Let's assume $\boxed{|x-2| < 2.}$ 1

i.e. $0 < x < 4$

$$\implies 0 < x^2 < 16$$

and $0 < 2x < 8$

so $0 < x^2 + 2x + 4 < 16 + 8 + 4 = \underline{\underline{28}}$

$$\implies \frac{1}{28} < \frac{1}{|x^2+2x+4|}$$

$$\implies \boxed{\frac{\varepsilon}{28} < \frac{\varepsilon}{|x^2+2x+4|}}$$ 2

If ① is true, then ② follows.

so if we make $|x-2| < \frac{\varepsilon}{28}$

we will $|x-2| < \frac{\varepsilon}{|x^2+2x+4|}$

So now let $\boxed{\delta = \min\left(2, \frac{\varepsilon}{28}\right)}$

then $\boxed{\text{if } |x-2| < \delta}$, then

$|x-2| < 2$ AND $|x-2| < \frac{\varepsilon}{28}$

$$\Rightarrow |x-2| < \frac{\varepsilon}{|x^2+2x+4|}$$

$$\Rightarrow \boxed{\begin{array}{l} |(x^3-1) - 7| < \varepsilon \\ |f(x) \quad - L| < \varepsilon \end{array}}$$

which is the definition of $\lim_{x\to 2}(x^3-1) = 7$.

To prove a limit statement is

not true. you show the def is false.

$$\neg\left[\forall \varepsilon > 0\ \exists \delta > 0\quad |x-c|<\delta \Rightarrow |f(x)-L|<\varepsilon\right]$$

$$\equiv \underbrace{\exists \varepsilon > 0}_{\text{counterexample}}\ \forall \delta > 0\quad |x-c|<\delta \text{ AND } |f(x)-L| \geq \varepsilon$$

Q5

$$|2x+5-13|<\varepsilon$$

$$\Leftrightarrow \quad |2x-8|<\varepsilon$$

$$\Leftrightarrow \quad 2|x-4|<\varepsilon$$

$$\Leftrightarrow \quad |x-4|<\frac{\varepsilon}{2}.$$

So just set $\delta = \frac{\varepsilon}{2}$.

So if $|x-4| < \delta$

then $|(2x+5) - 13| < \varepsilon$