$$\frac{x^2-12x+30}{x^2-7x+12}$$

$$x^2-12x+30 = (x^2-7x+12)+(-5x+18).$$

$$= \frac{(\quad\quad) \cancel{x} -5x+18}{(x^2-7x+12)}$$

$$= \frac{(\quad)}{(\quad)} + \frac{-5x+18}{x^2-7x+12}$$

$$= 1 + \frac{-5x+18}{x^2-7x+12}$$

$$\frac{1}{a}+\frac{1}{b}$$

$$\frac{\quad}{ab}$$

$$\frac{4x+29}{(x+7)^2} = \frac{A}{x+7} + \frac{B}{(x+7)^2}$$

$$= \frac{Ax+7A+B}{(x+7)^2}$$

$$I = \int^{19} \frac{1}{x^3+8x^2+19x}\,dx$$

$$= \int \frac{1}{x(\underbrace{x^2+8x+19}_{\text{irreducible}})}$$

$$\frac{A}{x} + \frac{Bx+C}{x^2+8x+19}$$

$$= \frac{A(x^2+8x+19)+Bx^2+Cx}{x(\quad \text{"} \quad)}$$

$$\Rightarrow \quad \begin{aligned} A+B&=0 \\ 8A+C&=0 \\ 19A&=1 \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} A&=1/19 \\ B&=-1/19 \\ C&=-8/19 \end{aligned}$$

So the integral I is

$$I = \int \frac{1/19}{x} + \frac{-1/19\,x - 8/19}{x^2+8x+19.}\,dx$$

$$= \frac{1}{19}\int x\,dx + \int \sim\sim\sim\;dx.$$

Note that $\frac{d}{dx}\left(\ln(x^2+8x+19)\cdot\right) = \frac{2x+8}{x^2+8x+19}$

So introduce the numerator
$2x+8$ by rewriting the numerator above

as
$$-\frac{1}{19}x - 8/19 = -\frac{1}{38}(2x+8) - \frac{8}{38}$$

So we can write.

$$\int \frac{-1/19\,x - 8/19}{x^2+8x+19}\,dx$$

$$= \int \frac{-\frac{1}{38}(2x+8)}{x^2+8x+19}\,dx - \frac{8}{38}\int \frac{1\,dx}{x^2+8x+19}$$

$$= -\frac{1}{38}\ln\left(x^2+8x+19\right) - \frac{8}{38}\int \frac{1}{x^2+8x+19}\,dx$$

Then deal with this last integral using atan formula.

Completing the square of the denominator

$$x^2+8x+19 = (x+4)^2+3.$$

So $$\frac{1}{x^2+8x+19} = \frac{1}{(x+4)^2+3}$$

$$= \frac{1}{3}\left(\frac{1}{\left(\frac{x}{\sqrt{3}}+\frac{4}{\sqrt{3}}\right)^2+1}\right)$$

$$= \frac{1}{3\sqrt{3}}\,\frac{\sqrt{3}}{\left(\frac{x}{\sqrt{3}}+\frac{4}{\sqrt{3}}\right)^2+1}$$