tinyurl.com/apexcalculus

Additional Question

Given the product rule, and the derivate of $f(x) = \frac{1}{x}$ is the function $f'(x) = \frac{-1}{x^2}$.

Can you use these to quickly establish derivatives of other reciprocal powers.

eg. $g(x) = \frac{1}{x^2}$, $h(x) = \frac{1}{x^3}$, ...

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$
$$(uv)'(x) = u'(x)v(x) + u(x)v'(x)$$

For $\boxed{g(x) = \frac{1}{x^2}}$, $g'(x) = ?$

$$= \frac{1}{x} \cdot \frac{1}{x}$$

Can then use derivatives of $\frac{1}{x}$ and product rule to obtain.

$\boxed{g'(x)} = \left(\frac{1}{x}\right)' \cdot \frac{1}{x} + \frac{1}{x} \cdot \left(\frac{1}{x}\right)'$, by product rule.

$$= \frac{-1}{x^2} \cdot \frac{1}{x} + \frac{1}{x}\left(-\frac{1}{x^2}\right)$$
$$= -\frac{1}{x^2} - \frac{1}{x^3} \quad \boxed{= \frac{-2}{x^3}}$$

For $\boxed{h(x) = \frac{1}{x^3}} = \frac{1}{x^2} \cdot \frac{1}{x}$

by prod. rule.

$$\boxed{h'(x)} = \left(\frac{1}{x^2}\right)' \cdot \frac{1}{x} + \frac{1}{x^2} \cdot \left(\frac{1}{x}\right)'$$

$$= \frac{-2}{x^3} \frac{1}{x} + \frac{1}{x^2}\left(-\frac{1}{x^2}\right)$$

$$= -\frac{2}{x^4} - \frac{1}{x^4} = \boxed{-\frac{3}{x^4}}$$

And so on .....