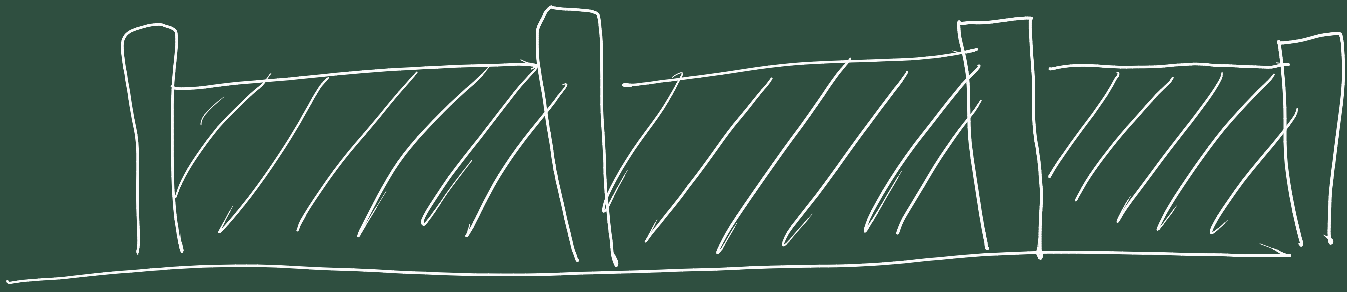
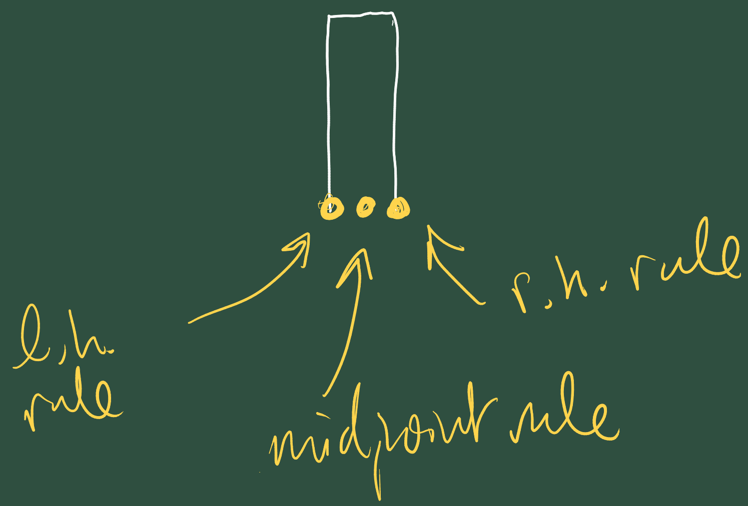
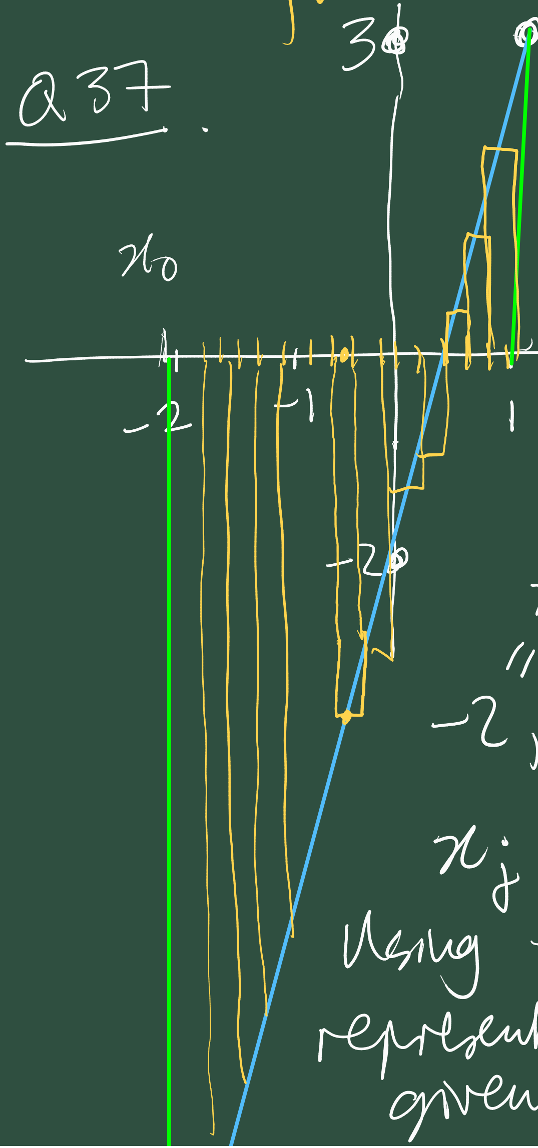




for choosing x_i^*

Q37



We'll use the equally spaced partition

$$x_0, x_1, x_2, \ldots, x_n$$

$x_0 = -2$, $x_n = 1$

$$x_j = -2 + j\frac{3}{n}, \quad j = 0, \ldots, n$$

Using the midpoint rule the representative points are x_j^* given by

$$x_j^* = -2 + j\frac{3}{n} + \frac{3}{2n} \quad , j = 1, \dots, n$$
$$= -2 + \frac{3(2j+1)}{2n}$$

$$\Delta x_j = \frac{3}{n} \qquad f(x) = 5x - 2.$$

height width

$$\int_{-2}^{1} (5x-2)\,dx = $$
$$= \lim_{n\to\infty} \left(\sum_{j=1}^{n} f(x_j^*) \cdot \frac{3}{n} \right)$$
$$= \lim_{n\to\infty} \left(\sum_{j=1}^{n} \left(5\left(-2 + \frac{3(2j+1)}{2n}\right) - 2\right) \frac{3}{n} \right)$$
$$= \lim_{n\to\infty} \left(\frac{3}{n} \sum_{j=1}^{n} \left(-10 + \frac{15(2j+1)}{2n} - 2\right)\right)$$
$$= \lim_{n\to\infty} \left(\frac{3}{n} \left[\left(\sum_{j=1}^{n} (-12) \right) + \frac{15}{2n} \sum_{j=1}^{n} (2j+1) \right]\right)$$
$$= \lim_{n\to\infty} \left(\frac{3}{n} \left[(-12n) + \frac{15}{2n} \left(2\left(\sum_{j=1}^{n} j\right) + n \right) \right]\right)$$
$$= \lim_{n\to\infty} \left(-36 + \frac{45}{2n} + \frac{45}{n^2} \frac{n(n+1)}{2} \right)$$
$$= \lim_{n\to\infty} \left(-36 + \frac{45}{2n} + \frac{45n^2 + 45n}{2n^2} \right)$$

$$= \lim_{n\to\infty}\left(-36 + \frac{45}{2n} + \frac{45}{2} + \frac{45}{2n}\right)$$

$$= \left(-36 + \frac{45}{2}\right), \quad \frac{45}{2n}), \frac{90}{2n} \to 0$$

as $n \to \infty$

$$= \frac{-72+45}{2}$$

$$= -\frac{27}{2}$$

Q3 $\int_{-2}^{1} 3x^2\,dx$, using partition as above and left hand rule.

$$= \lim_{n\to\infty}\left(\sum_{j=1}^{n} 3\left(-2+j\frac{3}{n}\right)^2\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{9}{n}\sum_{j=1}^{n}\left(4 - \frac{12j}{n} + \frac{9}{n^2}j^2\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{9}{n}\left(4n - \frac{12}{n}\sum_{j=1}^{n} j + \frac{9}{n^2}\sum_{j=1}^{n} j^2\right)\right)$$

$$= \lim_{n\to\infty}\left(36 - \frac{108}{n^2}\frac{n(n+1)}{2} + \frac{81}{n^3}\frac{n(n+1)(2n+1)}{6}\right.$$

(anything with factors of n in their denominators will $\longrightarrow 0$ as $n \longrightarrow \infty$)

$$= 36 - 54 + \frac{81}{3}$$

$$= 9 \quad \checkmark$$

$$\sum_{j=1}^{n} j^2$$

$$= 1^2 + 2^2 + 3^2 + \dots + n^2$$