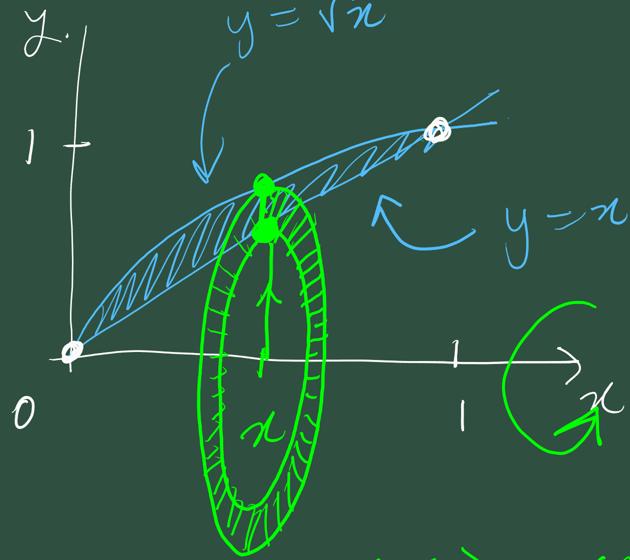
Q7



$A(x)$ = cross-sectional area through V at position x perpendicular to x-axis

$$A(x) = \pi R(x)^2 - \pi r(x)^2$$

where R, r are outer and inner radii as shown

$$R(x) = \sqrt{x}, \quad r(x) = x.$$

So justified by theorem 7.2.3

$$V = \int_0^1 A(x)\, dx.$$

$$= \pi \int_0^1 \left(\sqrt{x}^{\,2} - x^2 \right) dx$$

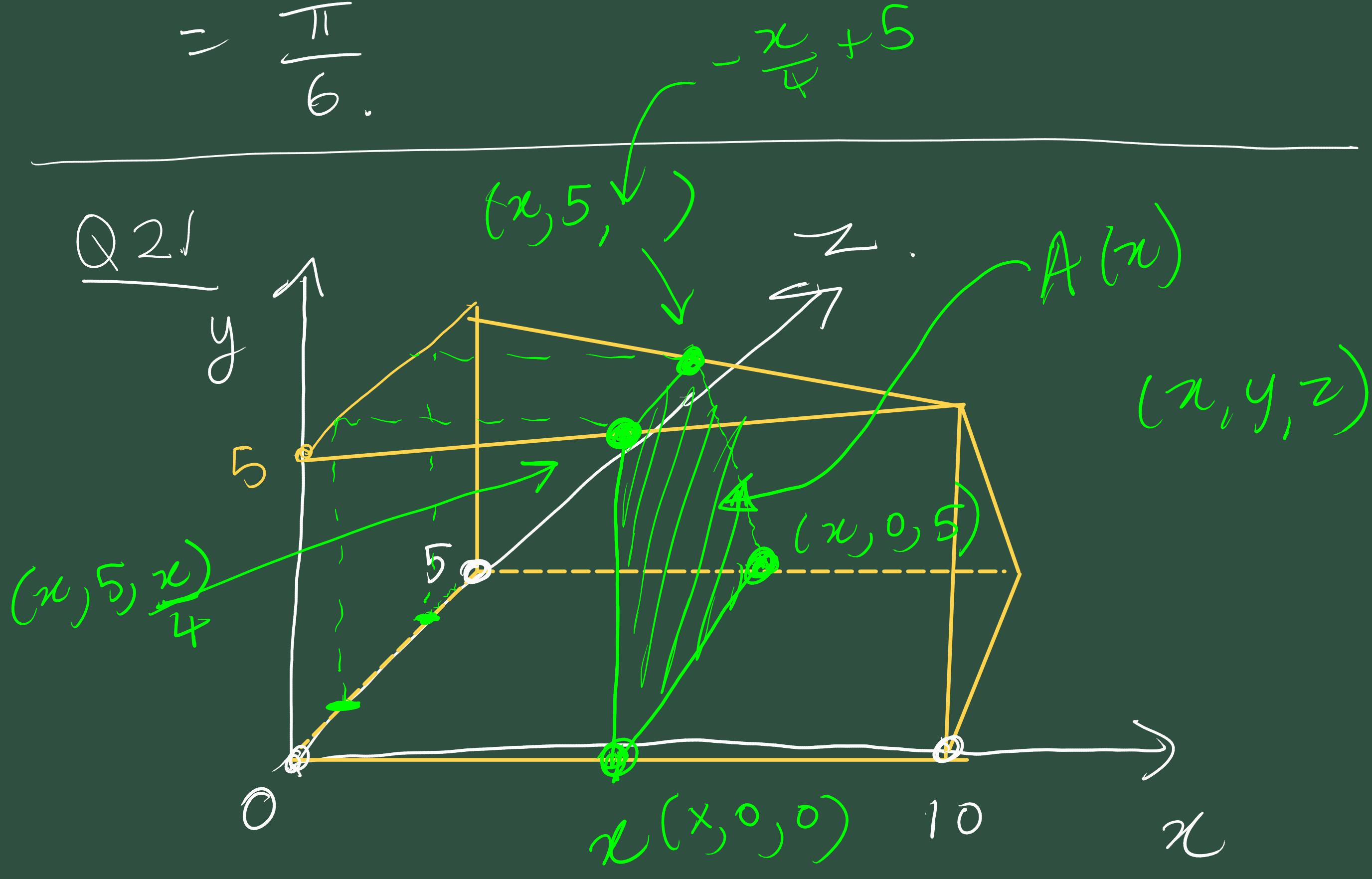
$$= \pi \int_0^1 (x - x^2)\, dx.$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

$$= \pi \left(\frac{1}{2} - \frac{1}{3} \right)$$

$$= \frac{\pi}{6}.$$

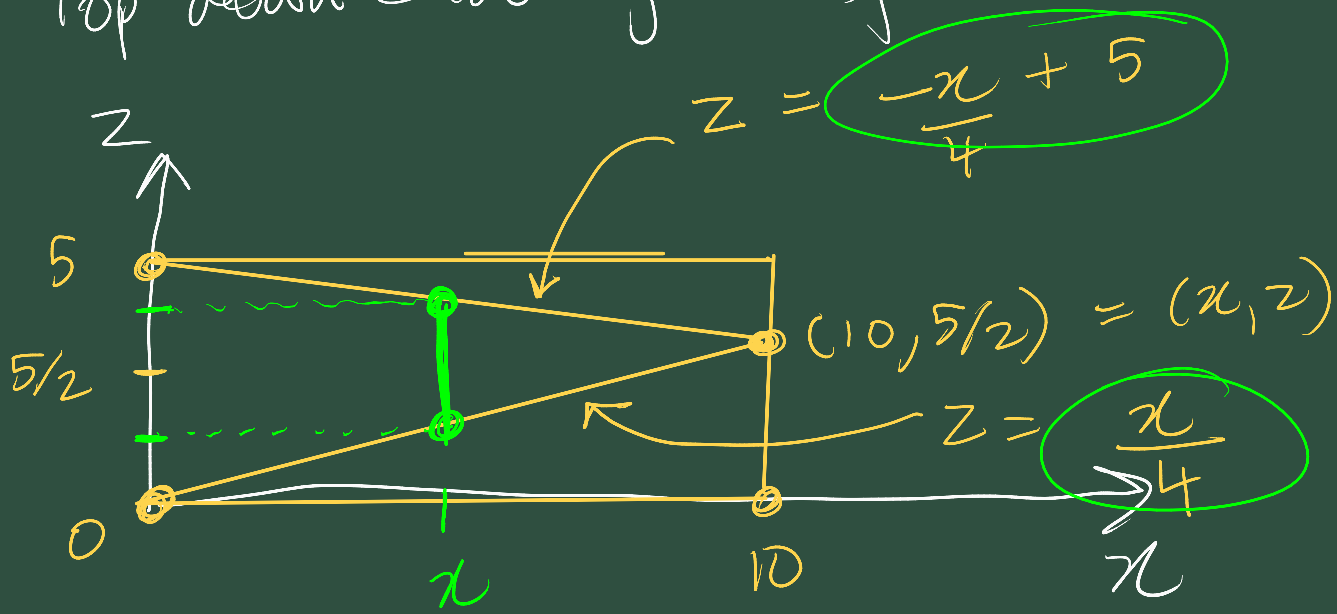
Q21



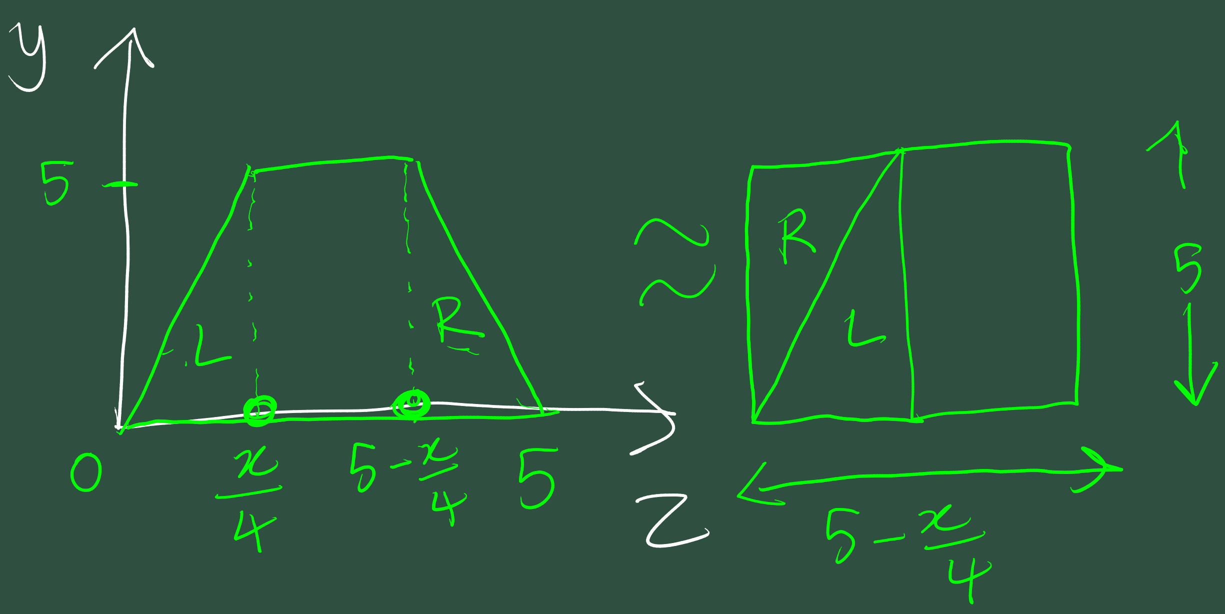
By theorem 7.2.3 we can express the volume V as

$$\boxed{V = \int_0^{10} A(x)\,dx.}$$

Top down – looking down from above.



Looking ~~from~~ leftwards towards the object from a position on the right we see the yz-plane at location x on X-axis



So $A(x) = 5\left(5 - \frac{x}{4}\right)$

$$= 25 - \frac{5x}{4}.$$

So the volume should be.

$$V = \int_0^{10} \left(25 - \frac{5x}{4}\right) dx.$$

$$= \left[25x - \frac{5x^2}{8} \right]_0^{10}$$

$$= 250 - \frac{500}{8}$$

$$= 250 - \frac{125}{2}$$

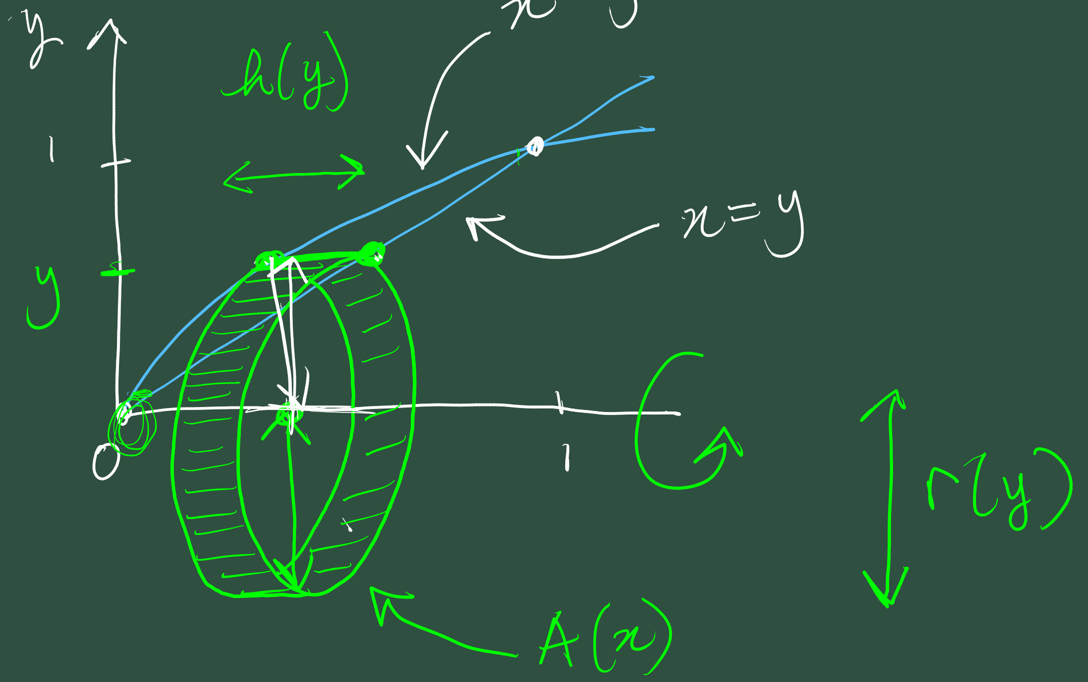
$$= \frac{375}{2}$$

$$= 187.5$$

7.3 Shell Method.

$x = y^2 \iff y = \sqrt{x}$

Q12.



$$V = 2\pi \int_0^1 r(y)\, h(y)\, dy.$$

$$= 2\pi \int_0^1 y\left(y - y^2\right) dy$$

$$= 2\pi \int_0^1 (y^2 - y^3)\, dy$$

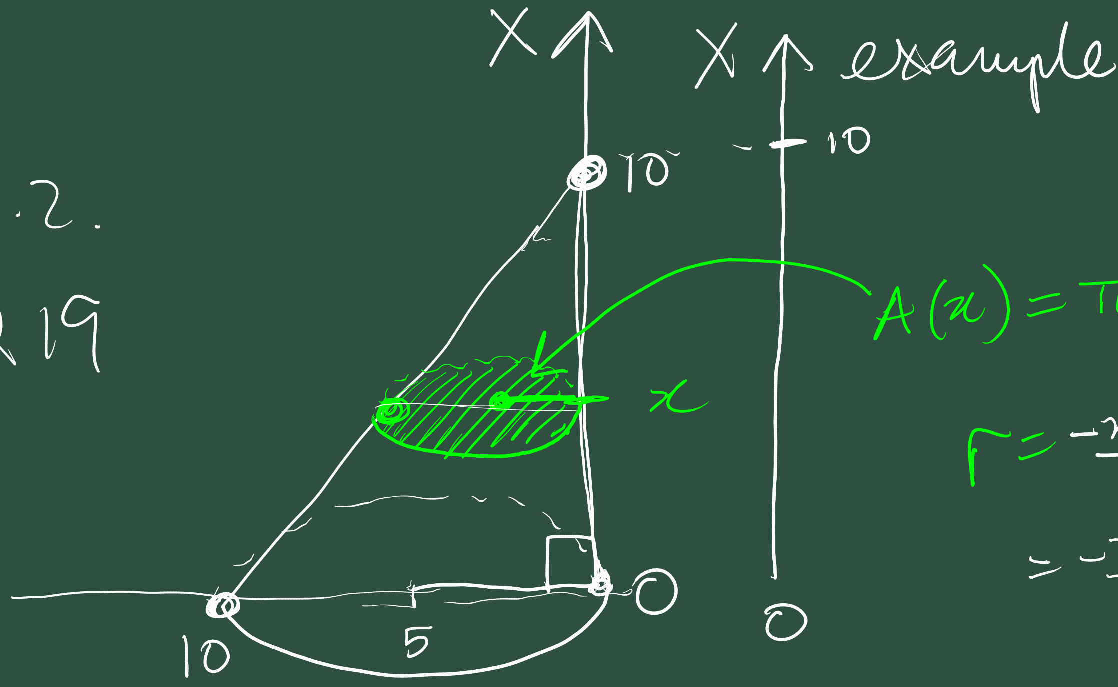
$$= 2\pi \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$

$$= 2\pi \left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= 2\pi \frac{1}{12} = \frac{\pi}{6}$$, as expected from previous

7.2.

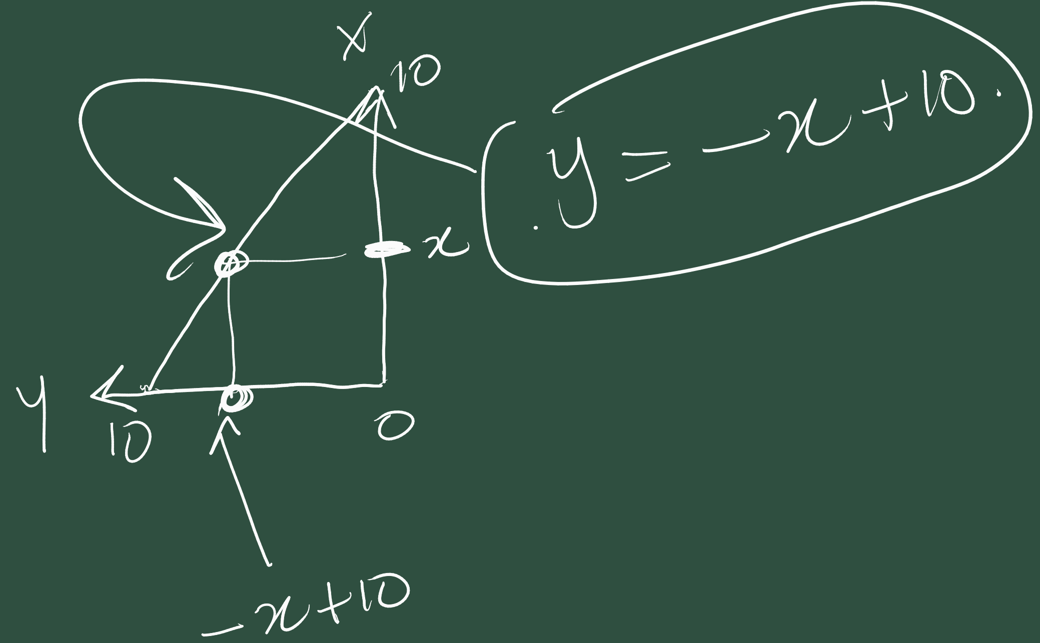
Q19



$A(x) = \pi r^2$

$r = \frac{-x+10}{2}$

$= -\frac{x}{2} + 5.$



$y = -x+10.$

So by Theorem 7.23

$$V = \int_0^{10} A(x)\, dx$$

$$= \pi \int_0^{10} \left(5 - \frac{x}{2}\right)^2 dx.$$

$$= \pi \left[\frac{-2\left(5 - \frac{x}{2}\right)^3}{3} \right]_0^{10}$$

$$= -\pi\left[\frac{-2(5^3)}{3}\right]$$

$$= \frac{2\pi}{3}\,125$$

$$= \frac{250\pi}{3}.$$