tinyurl.com/apexcalculus.

<u>Q3</u> Let $x, y \in \mathbb{R}$. $\longrightarrow$ Applying the constraint to reduces the situation to 1-variable

$$\boxed{P(x,y) = xy}$$

constrained by $x + y = 64$.

$$\iff y = 64 - x$$

$$\Rightarrow P(x,y) = xy$$
$$= x(64-x)$$
$$\Rightarrow P(x) = 64x - x^2$$

$$P'(x) = 64 - 2x$$

$$P'(x) = 0$$
$$\iff 64 - 2x = 0$$
$$\iff x = 32 \text{ hence } y = 32$$

$P''(x) = -2 < 0$ so any critical point will be a maximum.

So maximum such product is $P(32) = 32^2$
$$= 1024$$

Q5 A maximum sum??

$$1000000 + \frac{460}{1000000}$$

Q4 Let $x, y \in \mathbb{R}^+$

The constraint is $xy = 440$

$$\Leftrightarrow \quad y = \frac{440}{x}$$

So we want to minimize

$$S(x, y) = x + y.$$

$$\Rightarrow \quad S(x) = x + \frac{440}{x}$$

Diff. and solve $S'(x) = 0$

$$S'(x) = 1 - \frac{440}{x^2}$$

$$S'(x) = 0$$

$$\iff 1 - \frac{440}{x^2} = 0$$

$$\iff x^2 = 440$$

$$\iff x = \sqrt{440}$$
$$= 2\sqrt{110}$$

and the associated y value is also $y = 2\sqrt{110}$

So check $S''(x) = +\frac{880}{x^3} > 0$

so any critical point is a local minimum.

So the minimum possible sum is $4\sqrt{110}$

$0 < x, y < 350$.

$$\Rightarrow y = \frac{350}{x}$$

$$y = \frac{490}{x} \leq 350$$

$$\frac{1}{x} \leq \frac{350}{490}$$

$$x \geq \frac{490}{350}$$