Q3

(a). Consider $\{a_n\}_{n=0}^{\infty}$

where $a_n = \dfrac{3n}{n^2+1}$.

From plotting it seems $\{a_m\}_{m=1}^{\infty}$ is a decreasing sequence.

To prove this consider the ratio of consecutive terms.

$$\frac{a_{n+1}}{a_n} = \frac{3(n+1)/((n+1)^2+1)}{3n/n^2+1}$$

$$= \frac{3n+3}{3n} \frac{n^2+1}{(n+1)^2+1}$$

$$= \frac{n+1}{n} \frac{n^2+1}{n^2+2n+2}$$

$$= \frac{n^3+n^2+n+1}{n^3+2n^2+2n}$$

$$= \frac{(n^3+2n^2+2n)-n^2-n+1}{n^3+2n^2+2n}$$

$$= \underbrace{1}_{} + \boxed{\frac{1-n^2-n}{n^3+2n^2+2n}}$$

$$\left(\begin{array}{l} \text{for all } n \geq 1 \quad 1-n^2-n < 0 \\ \text{and } n^3+2n^2+2n > 0 \end{array} \right)$$

< 1

So $\frac{a_{n+1}}{a_n} < 1$ for all $n \geq 1$

ie. $a_{n+1} < a_n$

So this is a decreasing

Sequence.

$$a_{n+1} - a_n$$

$$= \frac{3n+3}{(n+1)^2+1} - \frac{3n}{n^2+1}$$

$$= \frac{3n+3}{n^2+2n+2} - \frac{3n}{n^2+1}$$

$$= \frac{(3n+3)(n^2+1) - 3n(n^2+2n+2)}{(n^2+2n+2)(n^2+1)}$$

$$= \frac{-3n^2 - 3n + 3}{(\quad)(\quad)}$$

$$< 0$$

So $a_{n+1} < a_n$