Q7. (c)

$$\lim_{n\to\infty}\left(\frac{5n^2-2}{3n^2-n}\right).$$

$$= \cancel{\frac{\lim_{n\to\infty}(5n^2-2)}{\lim_{n\to\infty}(3n^2-n)}}$$, but this is not valid as neither numerator nor the denominator is convergent.

$$= \lim_{n\to\infty}\left(\frac{5-2/n^2}{3-1/n}\right)$$, dividing above and below by n^2.

$$= \frac{\lim_{n\to\infty}(5-2/n^2)}{\lim_{n\to\infty}(3-1/n)}$$, Quotient form of the algebra of limits theorem.

$$= \frac{\lim_{n\to\infty}(5) - 2\lim_{n\to\infty}(1/n^2)}{}$$, algebra of limits

$$\lim_{n\to\infty}(3) - \lim_{n\to\infty}(1/n)$$ theorem.

$$= \frac{5}{3},$$ since $1/n^2$, $1/n \to 0$ as $n \to \infty$.

Q3 (b).

$$\{b_n\}_{n=1}^{\infty}, \quad b_n = \frac{2^n}{2+2^n}.$$

Seems to be increasing.

To prove this.

Consider

$$\frac{b_{n+1}}{b_n} = \frac{2^{n+1}/(2+2^{n+1})}{2^n/(2+2^n)}$$

$$= \frac{2^{n+1}}{2^n} \frac{2+2^n}{2+2^{n+1}}$$

$$= 2\,\frac{2+2^n+2^n-2^n}{2+2^{n+1}}$$

$$= 2\,\frac{2+2\cdot 2^n-2^n}{2+2^{n+1}}$$

$$= 2\cdot\frac{(2+2^{n+1})-2^n}{2+2^{n+1}}$$

$$= 2\left(1-\frac{2^n}{2+2^{n+1}}\right)$$

$$= 2\left(\frac{2+2^n}{2+2^{n+1}}\right)$$

$$2\cdot 2^n = 2^{n+1}$$

$$= \frac{4 + 2^{n+1}}{2 + 2^{n+1}}$$

$$= \frac{(2 + 2^{n+1}) + 2}{2 + 2^{n+1}}$$

$$= 1 + \frac{2}{2 + 2^{n+1}}$$

$$> 1, \text{ since } \frac{2}{2 + 2^{n+1}} > 0$$

So $\frac{b_{n+1}}{b_n} > 1$ for all n

$\Rightarrow b_{n+1} > b_n$ for all n

i.e. $\{b_n\}$ is increasing.

Q3 (a).

$$a_n = \frac{3n}{n^2+1}$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{3}(n+1)}{(n+1)^2+1} \cdot \frac{n^2+1}{\cancel{3}n}$$

$$= \frac{n+1}{n^2+2n+2} \quad \frac{n^2+1}{n}.$$

$$= \frac{n^3+n^2+n+1}{n^3+2n^2+2n}$$

$$= \frac{n^3+2n^2+2n-n^2-n+1}{n^3+2n^2+2n}$$

$$= 1 + \frac{1-n^2-n}{n^3+2n^2+2n}.$$

(Remember $n \geq 1$

so actually $1 - n^2 - n < 0$ for all $n \geq 1$.

and $n^3 + 2n^2 + 2n > 0$)

< 1

So $\dfrac{a_{n+1}}{a_n} < 1$ for all $n \geq 1$

So $a_{n+1} < a_n$

So the ~~de~~ sequence is decreasing for all $n \geq 1$.