Q1 (iv) $\sum_{n=1}^{\infty} \underbrace{\left(\sum_{i=1}^{n} i^{-2} \right)}_{x_n}$

$= x_1 + x_2 + x_3 + \dots$

$= 1 + \left(1 + \frac{1}{4}\right) + \left(1 + \frac{1}{4} + \frac{1}{9}\right) + \dots$

Ex 9.2.4.

Q 15 Consider $\sum_{n=1}^{\infty} \frac{3n^2}{n(n+2)}$

Consider the term

$$a_n = \frac{3n^2}{n(n+2)} = \frac{3n^2}{n^2+2n}$$

Consider $\lim_{n\to\infty} a_n = ?$

$$\lim_{n\to\infty} \left(\frac{3n^2}{n^2+2n} \right) \cdot \frac{1/n^2}{1/n^2}$$

$$= \lim_{n\to\infty} \left(\frac{3}{1+2/n} \right)$$

$$= \frac{\lim_{n\to\infty} 3}{\lim_{n\to\infty}(1+2/n)}$$

by the algebra of limits theorem

$$= \frac{3}{1+0}, \text{ since } \frac{2}{n} \to 0 \text{ as } n\to\infty.$$

$$= 3$$

So $\lim_{n\to\infty} a_n = 3$

So we conclude, from theorem 9.2.23, that $\sum_{n=1}^{\infty} a_n$ <u>diverges</u>.

Q18. $a_n = \dfrac{5^n - n^5}{5^n + n^5}$

eg. $a_{1000} = \dfrac{5^{1000} - 1000^5}{5^{1000} + 1000^5} \simeq 1.$

It seems $a_n \to 1$ as $n\to\infty$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5^n - n^5}{5^n + n^5} \cdot \frac{1/5^n}{1/5^n}$$

$$= \lim_{n\to\infty} \left(\frac{1 - n^5/5^n}{1 + n^5/5^n} \right)$$

$$= \lim_{n\to\infty} \left(\frac{1 - \left(\frac{n}{5}\right)^5 \frac{1}{5^{n-5}}}{1 + \left(n/5\right)^5 \frac{1}{5^{n-5}}} \right)$$

$\left(\frac{n}{5}\right)^5 \frac{1}{5^{n-5}} \to 0$ as $n \to \infty$, $\left(n/5\right)^5 \frac{1}{5^{n-5}} \to 0$

Observation.

as n increases 5^{n-5} increases to very large quantities compared to $\left(\frac{n}{5}\right)^5$

So $\lim_{n\to\infty} a_n = 1$

Therefore the series diverges.

From the tutorial seet.

Q6 (i). Geometric series. $\sum_{n=1}^{\infty} ar^{n-1}$

$$\sum_{n=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$$

$$= \sum_{n=1}^{\infty} \frac{2}{3}\frac{1}{9}\left(\frac{1}{3}\right)^{n-1}$$

$$= \sum_{n=1}^{\infty} \frac{2}{27}\left(\frac{1}{3}\right)^{n-1}$$

$a \quad r^{n-1}$

This is a geometric series $\sum_{n=1}^{\infty} ar^{n-1}$

with $a = 2/27$ and $r = 1/3$

$|r| < 1$. So this series does converge and

$$\sum_{n=1}^{\infty} \frac{2}{27}\left(\frac{1}{3}\right)^{n-1} = \frac{2/27}{1 - 1/3}$$

$$= \frac{2/27}{2/3}$$

$$= \frac{3}{27} = \frac{1}{9}$$

Q7

$$a = 0.\underset{\sim\sim}{1234}\,\underline{1234}\,1234\ldots$$

$$= \frac{1234}{10^4} + \frac{1234}{10^8} + \frac{1234}{10^{12}}$$

$$= \frac{1234}{10^4}\left(1 + \frac{1}{10^4} + \frac{1}{10^8} + \ldots\right.$$

$$= \frac{1234}{10^4}\left(1 + \frac{1}{10^4} + \left(\frac{1}{10^4}\right)^2 + \left(\frac{1}{10^4}\right)^3 + \ldots\right.$$

$$= \frac{1234}{10^4}\sum_{n=1}^{\infty}\left(\frac{1}{10^4}\right)^{n-1}$$

$$= \frac{1234}{10^4}\frac{1}{1-1/10^4}. = \frac{1234}{10^4}\frac{1}{\frac{9999}{10^4}}.$$

$$= \frac{1234}{9999} = a$$