tinyurl.com/apexcalculus

Q3 Let $x, y \in \mathbb{R}$.

Constraint is $x+y=64$.

Maximise the product xy.

Rewrite constraint as, say,

$$y = 64 - x.$$

So the product function P can be written as

$$P(x) = x(64-x) = 64x - x^2$$

Find critical points of P by solving $P'(x)=0$

$$P'(x) = 64 - 2x = 0$$

$\iff x = 32$, and hence $y = 64 - x = 32$ too.

But is this a maximum? Check sign of P'' at $x=32$.

$$P''(x) = -2 < 0, \text{ everywhere.}$$

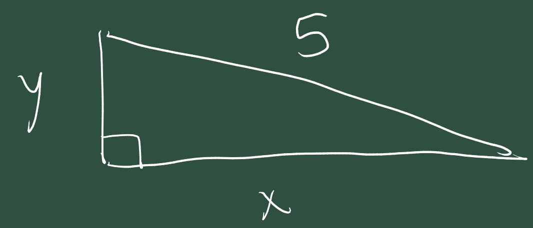
So yes, $x=y=32$ gives the maximum possible product of $32^2 = 1024$.

Q5) $\boxed{x \cdot y = 460}$

want $x+y$ huge

$x = 10^6$, $y = \frac{460}{10^6}$

Q7



Constraint on x, y is

$$x^2 + y^2 = 25 \iff y = \sqrt{25 - x^2}$$

$$\text{Area} = \frac{1}{2} x y$$

so $A(x) = \frac{1}{2} x \left(25 - x^2\right)^{1/2}$.

maximize A by solving $A'(x) = 0$.

(needs product rule and chain rule).

OR the area A will maximized at the same point where A^2 is maximised.

$$A^2 = \frac{1}{4} x^2 (25 - x^2)$$
$$= \frac{1}{4} (25 x^2 - x^4)$$

$$\frac{d}{dx}(A^2) = \frac{1}{4}(50x - 4x^3)$$

Solving $\frac{d}{dx}(A^2) = 0$

$$\Leftrightarrow \quad 50x - 4x^3 = 0$$
$$\Leftrightarrow \quad 4x^3 = 50x$$
$$\Leftrightarrow \quad 4x^2 = 50$$
$$\Leftrightarrow \quad x^2 = \frac{25}{2}$$
$$\Leftrightarrow \quad x = \frac{5}{\sqrt{2}}$$

Giving a local maximum at $x = \frac{5}{\sqrt{2}}$, and hence

$$y = \frac{5}{\sqrt{2}}$$