f

$f(x) = x^2 - 5$, $f'(x) = 2x$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$\boxed{= x_n - \frac{x_n^2 - 5}{2x_n}}$$

line: $y = mx + c$ tangent line at $x = 3$

gradient, from derivative of f.

$m = f'(3) = 6$

$y = 6x + c$

at $x = 3$ $f(x) = x^2 - 5 = 9 - 5 = 4$.

so $y = 6x + c = 4$ at $x = 3$

$$\Rightarrow 6\cdot 3 + c = 4$$

$$\Rightarrow 18 + c = 4$$

$$\Rightarrow c = -14$$

So this tangent line has equation $y = 6x - 14$.

and a root at.

$$y = 0 \Rightarrow 6x - 14 = 0$$

$$\Rightarrow x = \frac{14}{6} = \frac{7}{3}$$

$$x_1 = x_0 - \frac{x_0^2 - 5}{2\cdot x_0}$$

$$= 3 - \frac{9-5}{6}$$

$$= 3 - \frac{4}{6}$$

$$= 3 - 2/3 = \frac{9}{3} - \frac{2}{3} = \frac{7}{3}$$

$$\boxed{x_{n+1}} = x_n - \frac{x_n^2 - 5}{2x_n}$$

$$= \frac{1}{2x_n}\left(2x_n^2 - x_n^2 + 5\right)$$

$$\boxed{= \frac{1}{2x_n}\left(x_n^2 + 5\right)}$$

- If we know $x_n > 0$ what about x_{n+1}?

$\Rightarrow x_{n+1} > 0$

But $x_0 = 3 > 0$

so $x_1 > 0$

$\Rightarrow x_2 > 0 \Rightarrow x_3 > 0 \Rightarrow x_4 > 0$

"Proof by induction" says that

$\forall\ m \geq 1 \quad x_m > 0$

$$x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n}$$

$$\Rightarrow \quad x_{n+1}^2 = x_n^2 - 2 \cdot x_n \frac{(x_n^2 - 5)}{2x_n} + \left(\frac{x_n^2-5}{2x_n}\right)^2$$

$$= 5 + \left(\frac{x_n^2-5}{2x_n}\right)^2$$

$$\geqslant 5$$

$$\Rightarrow \quad x_{n+1} \geqslant \sqrt{5} \quad \text{or} \leqslant -\sqrt{5}$$

$$\Rightarrow \quad x_{n+1} \geqslant \sqrt{5}, \text{ since } x_{n+1} > 0$$