Overhanging block stacks

Enlivening Mathematics

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The challenge

Make a tower of bricks ...

... with as much of an overhang as you can.

An optimal solution for simple stacking

A couple of basic principles

- A body balances on a flat surface if its centre of gravity lies over the ‘footprint’.
- Determine the (horizontal) location of a stack’s centre of gravity by ‘averaging’ over the locations of centres of the blocks.

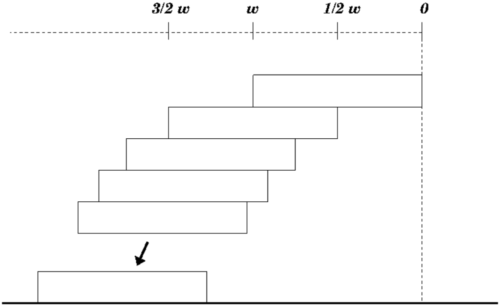
Iterate the following construction

- Build by adding to the bottom of the pile
- When adding a block, position its edge directly under the centre of gravity of the existing stack.

Analyzing the solution

Defining h_n

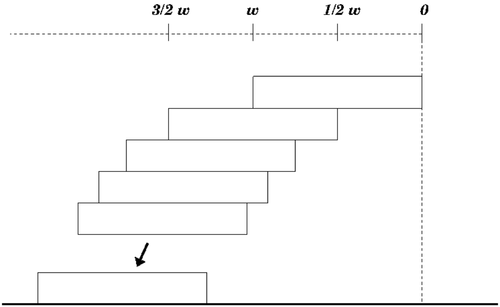
Let h_n denote the overhang of this optimal simple stack of n blocks, each of width w.



$$h_n = \text{ centre of gravity of the upper } n-1 \text{ blocks}$$

Analyzing the solution

A recurrence formula for h_n



$$h_1 = 0,$$

$$h_n = h_{n-1} + \frac{1}{n-1}\frac{w}{2}$$

Analyzing the solution

$$h_n = h_{n-1} + \frac{1}{n-1}\frac{w}{2}$$

Developing h_1, h_2, $\ldots$

$$\begin{aligned}
h_1 &= 0, \\
h_2 &= \frac{w}{2}, \\
h_3 &= \frac{w}{2} + \frac{1}{2}\frac{w}{2} = \frac{w}{2}\left(1 + \frac{1}{2}\right), \\
h_4 &= \frac{w}{2}\left(1 + \frac{1}{2}\right) + \frac{1}{3}\frac{w}{2} = \frac{w}{2}\left(1 + \frac{1}{2} + \frac{1}{3}\right) \\
&\vdots \\
h_n &= \frac{w}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}\right)
\end{aligned}$$

A nice formula for h_n involving the harmonic series

But how does it behave as n increases?

$$h_n = \frac{w}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}\right)$$

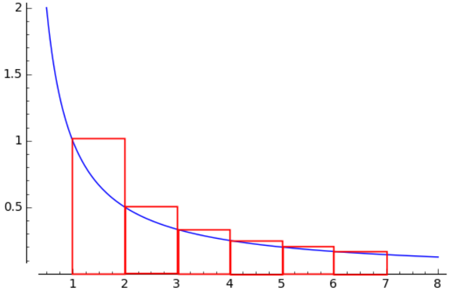
In fact, $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}$ increases without any upper bound. Proof of this based on grouping the terms together in groups of size $2, 4, 8, 16, \ldots$

$$\begin{aligned}
&1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \cdots + \frac{1}{8}\right) + \left(\frac{1}{9} + \cdots + \frac{1}{16}\right) + \ldots \\
>&1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \frac{8}{16} + \ldots \\
=&1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots
\end{aligned}$$

Conclusion: Given enough blocks one can achieve any overhang whatsoever!

Evaluating h_n

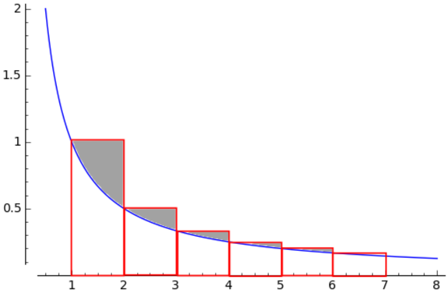
$$h_n = \frac{w}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}\right)$$



$$h_n \approx \frac{w}{2}\int_1^n \frac{1}{x}\,dx$$

Evaluating h_n

$$h_n \approx \frac{w}{2} \int_1^n \frac{1}{x}\, dx$$



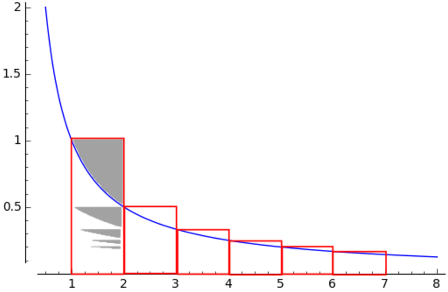
$$h_n = \int_1^n \frac{1}{x}\, dx + \gamma_n,$$

where

$$\gamma_n = \text{ area of shaded regions}$$

Evaluating h_n

$$h_n = \frac{w}{2}\left(\int_1^n \frac{1}{x}\,dx + \gamma_n\right),$$



$$\lim_{n\to\infty} \gamma_n = \gamma \approx 0.6$$

γ is the Euler–Mascheroni constant.

Evaluating h_n

So for large n a very good approximation is

$$h_n \approx \frac{w}{2}\left(\int_1^n \frac{1}{x}\,dx + 0.6\right),$$
$$= \frac{w}{2}\Big(\log(n) + 0.6\Big).$$

An actual stack of Jenga blocks

$$h_n \approx \frac{w}{2}\Big(\log(n) + 0.6\Big)$$

How tall is a simple optimal stack with a 3 metre overhang?

The Jenga block is 7.5cm wide and 1.5cm tall.

$$3 = \frac{0.075}{2}\Big(\log(n) + 0.6\Big)$$

Solve this to find number, n, of blocks required.

$$\begin{aligned} n &= e^{\left(\frac{3}{0.0375} - 0.6\right)} \\ &\approx 3.0 \times 10^{34}, \text{ a LOT of bricks} \end{aligned}$$

Such a stack would be $3.0 \times 10^{34} \times 0.015 \approx 4.6 \times 10^{32}$ metres tall.

Now that's tall!

4.6×10^{32} metres is approximately 520 000 times the diameter of the observable universe.