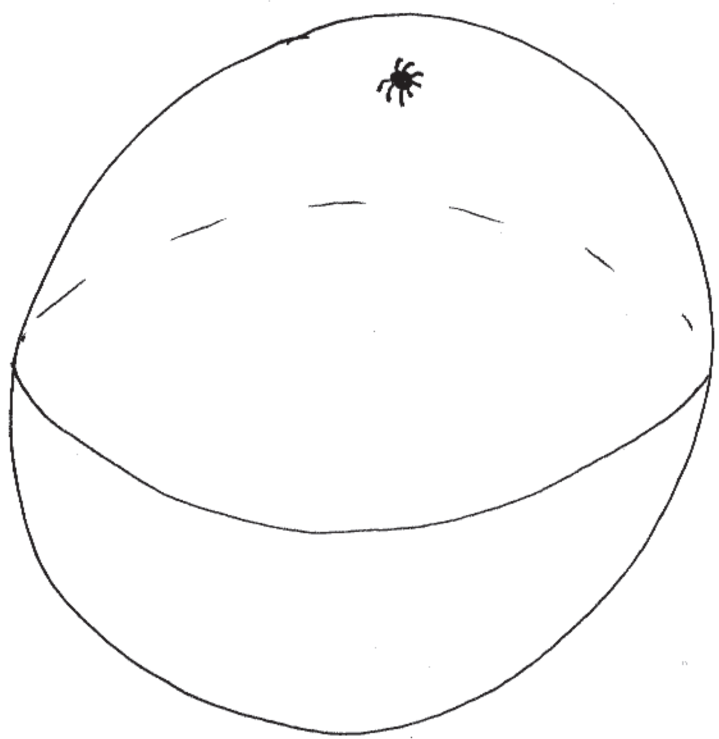
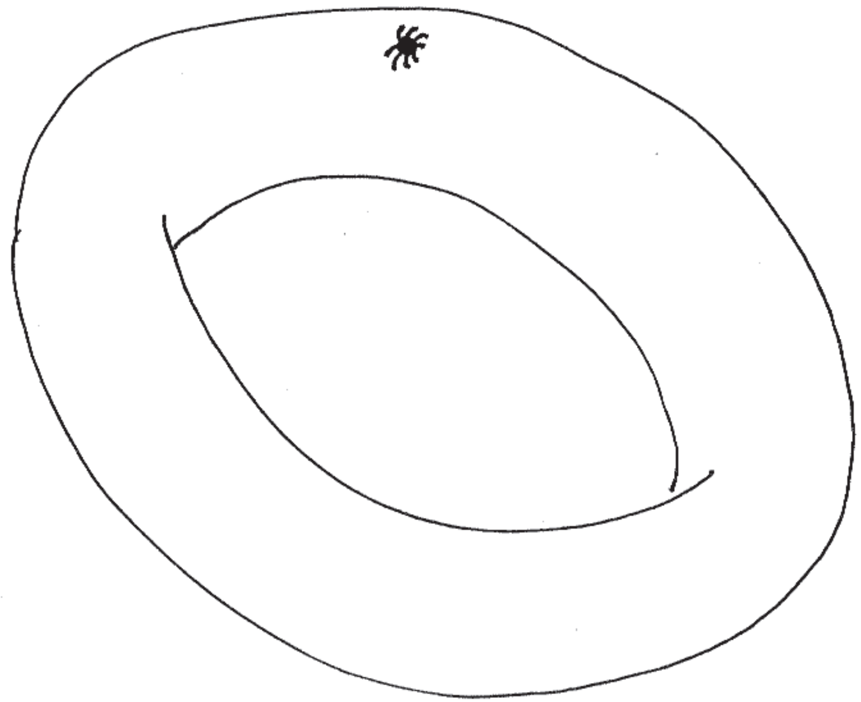
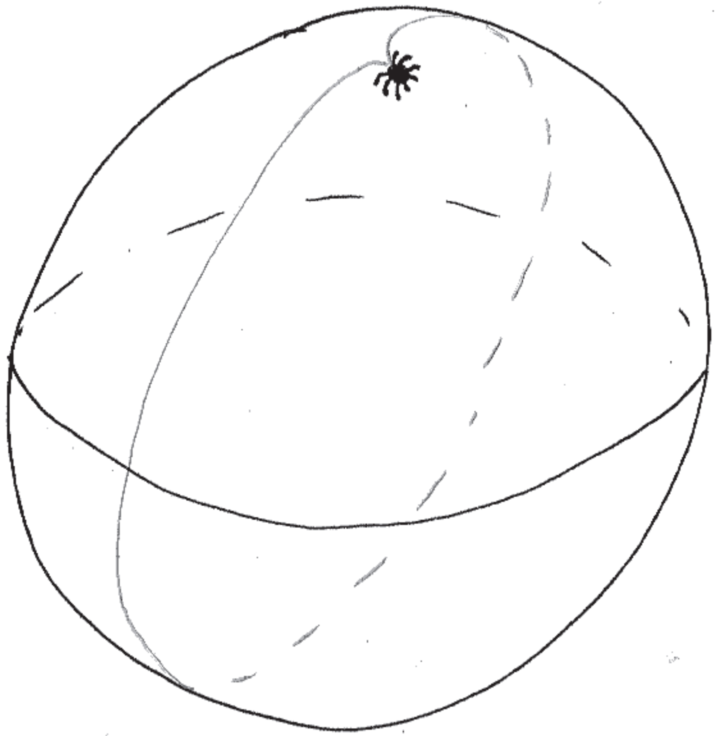
Topological xenoscope

How does the spider know what surface it's on?

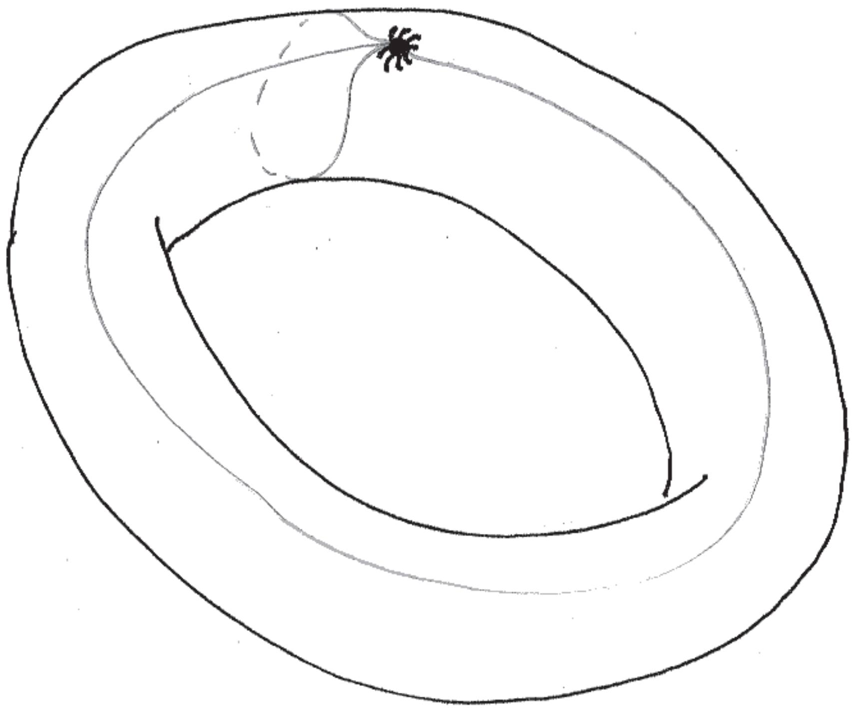




How does the spider know what surface it's on?



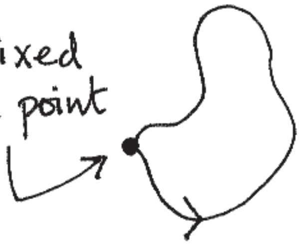
Sphere: all closed loops can be shrunk to a point.



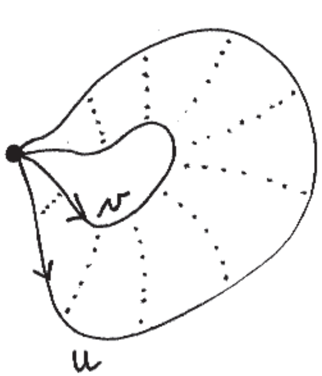
Torus: some loops can't be shrunk to a point.

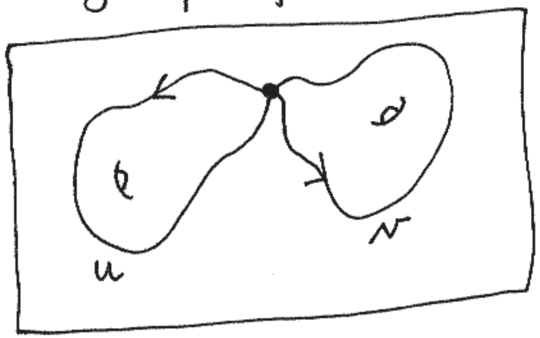
$\pi_1(X)$, the fundamental group of X.

fixed base point



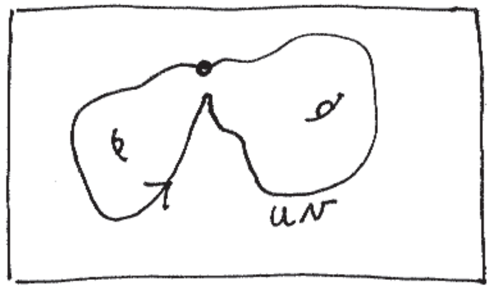
closed loop.





loops u and v

concatenation



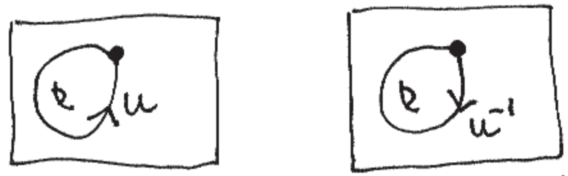
the loop uv

loop u is homotopic to v if u can be 'continuously deformed' into v in the space X.

homotopy is an equivalence relation.

$\pi_1(X)$ is the group of (homotopy classes of) closed loops based at the base point. The group operation is concatenation of loops.

identity element is the null loop

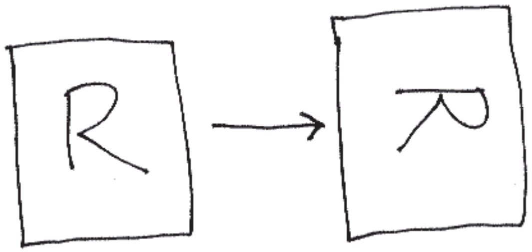


$SO(3)$, the 'special orthogonal group', is the group of all rotations in three dimensions (of a body about an axis through its centre of gravity).

operation is composition of rotations

Appreciate the difference between 'rotation' and 'rotational motion'.

<u>rotation</u>



before after

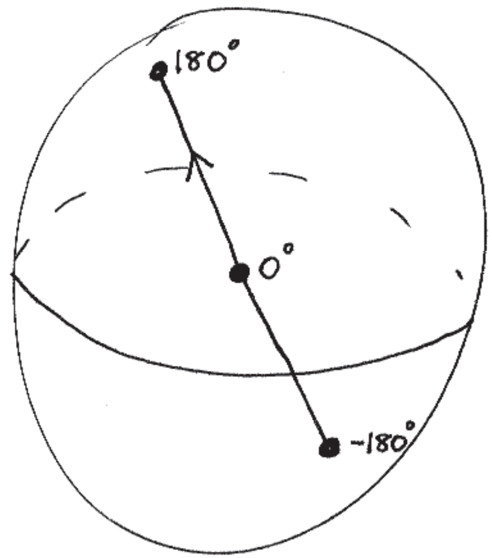
<u>rotational motion</u>

see 'spinning student' video.

SO(3) is a group, but is also a three dimensional space itself.

<u>A model for SO(3):</u>

a rotation is determined by an axis and a rotation angle



Let B denote the ball of radius 180 in $\mathbb{R}^3$. Each point in B represents a rotation (axis, angle).

NB: rotation by 180° = rotation by -180°

SO(3) is obtained from B by identifying (glueing) antipodal points of the surface of B

$SO(3) = B$ with antipodal surface points identified.

points in $SO(3) \leftrightsquigarrow$ rotations of $\mathbb{R}^3$.

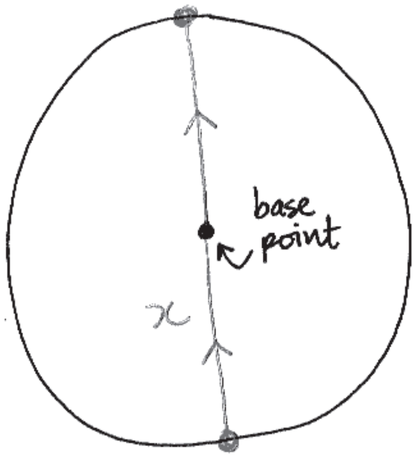
paths in $SO(3) \leftrightsquigarrow$ rotational motions in $\mathbb{R}^3$.

What would it be like to live in $SO(3)$?

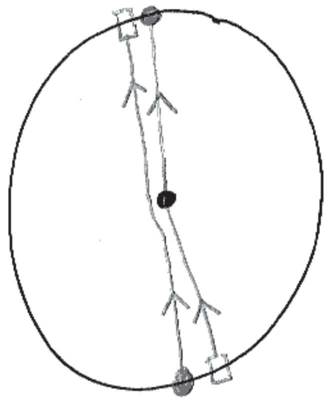
Locally, $SO(3)$ looks like $\mathbb{R}^3$. But globally, it has a different topology.

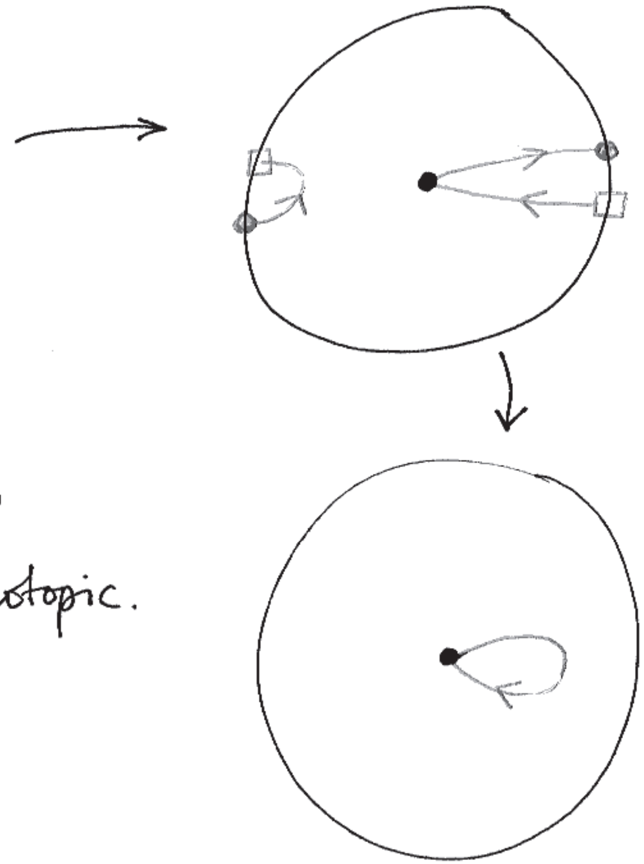
$\pi_1(\mathbb{R}^3) = \{e\}$

What is $\pi_1(SO(3))$?



x, a closed loop that is not null homotopic.





x^2, a closed loop that __is__ null homotopic.