

Overhanging block stacks

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September 2025

The challenge

Make a tower of bricks ...

... with as much of an overhang as you can.

An optimal solution for simple stacking

A couple of basic principles

- ▶ A body balances on a flat surface if its centre of gravity lies over the ‘footprint’.
- ▶ Determine the (horizontal) location of a stack’s centre of gravity by ‘averaging’ over the locations of centres of the blocks.

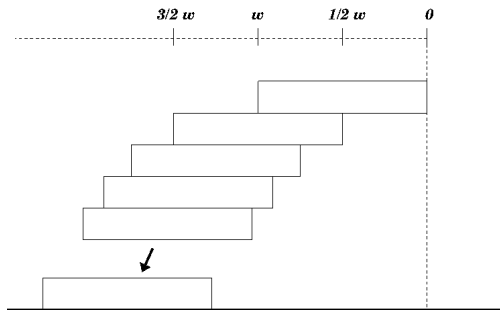
Iterate the following construction

- ▶ Build by adding to the bottom of the pile
- ▶ When adding a block, position its edge directly under the centre of gravity of the existing stack.

Analyzing the solution

Defining h_n

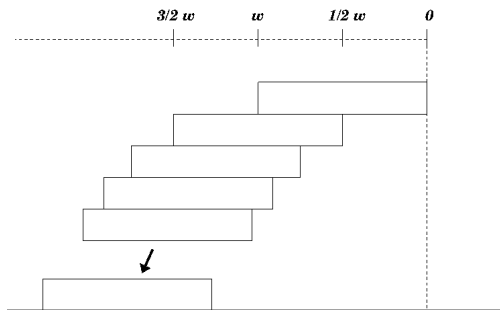
Let h_n denote the overhang of this optimal simple stack of n blocks, each of width w .



$h_n =$ centre of gravity of the upper $n - 1$ blocks

Analyzing the solution

A recurrence formula for h_n



$$h_1 = 0,$$

$$h_n = h_{n-1} + \frac{1}{n-1} \frac{w}{2}$$

Analyzing the solution

$$h_n = h_{n-1} + \frac{1}{n-1} \frac{w}{2}$$

Developing h_1, h_2, \dots

$$h_1 = 0,$$

$$h_2 = \frac{w}{2},$$

$$h_3 = \frac{w}{2} + \frac{1}{2} \frac{w}{2} = \frac{w}{2} \left(1 + \frac{1}{2}\right),$$

$$h_4 = \frac{w}{2} \left(1 + \frac{1}{2}\right) + \frac{1}{3} \frac{w}{2} = \frac{w}{2} \left(1 + \frac{1}{2} + \frac{1}{3}\right)$$

\vdots

$$h_n = \frac{w}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}\right)$$

A nice formula for h_n involving the harmonic series

But how does it behave as n increases?

$$h_n = \frac{w}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} \right)$$

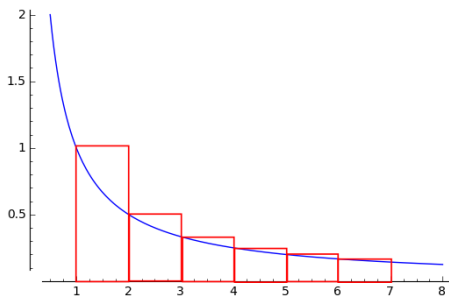
In fact, $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}$ increases without any upper bound. Proof of this based on grouping the terms together in groups of size 2, 4, 8, 16, ...

$$\begin{aligned} & 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4} \right) + \left(\frac{1}{5} + \cdots + \frac{1}{8} \right) + \left(\frac{1}{9} + \cdots + \frac{1}{16} \right) + \cdots \\ & > 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \frac{8}{16} + \cdots \\ & = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots \end{aligned}$$

Conclusion: Given enough blocks one can achieve any overhang whatsoever!

Evaluating h_n

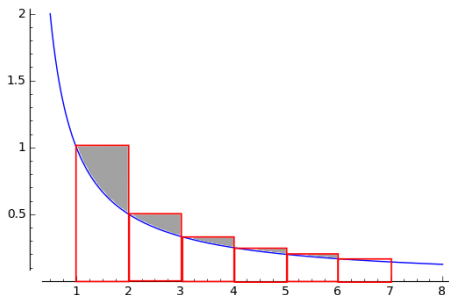
$$h_n = \frac{w}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} \right)$$



$$h_n \approx \frac{w}{2} \int_1^n \frac{1}{x} dx$$

Evaluating h_n

$$h_n \approx \frac{w}{2} \int_1^n \frac{1}{x} dx$$



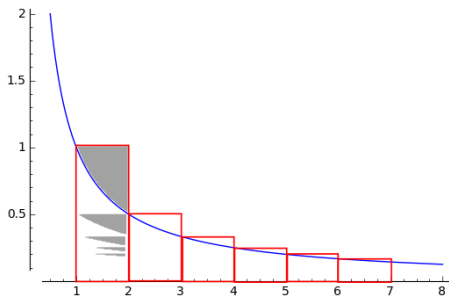
$$h_n = \int_1^n \frac{1}{x} dx + \gamma_n,$$

where

$\gamma_n =$ area of shaded regions

Evaluating h_n

$$h_n = \frac{w}{2} \left(\int_1^n \frac{1}{x} dx + \gamma_n \right),$$



$$\lim_{n \rightarrow \infty} \gamma_n = \gamma \approx 0.6$$

γ is the Euler–Mascheroni constant.

Evaluating h_n

So for large n a very good approximation is

$$\begin{aligned} h_n &\approx \frac{w}{2} \left(\int_1^n \frac{1}{x} dx + 0.6 \right), \\ &= \frac{w}{2} (\log(n) + 0.6). \end{aligned}$$

An actual stack of Jenga blocks

$$h_n \approx \frac{w}{2} \left(\log(n) + 0.6 \right)$$

How tall is a simple optimal stack with a 3 metre overhang?

The Jenga block is 7.5cm wide and 1.5cm tall.

$$3 = \frac{0.075}{2} \left(\log(n) + 0.6 \right)$$

Solve this to find number, n , of blocks required.

$$\begin{aligned} n &= e^{\left(\frac{3}{0.0375} - 0.6\right)} \\ &\approx 3.0 \times 10^{34}, \text{ a LOT of bricks} \end{aligned}$$

Such a stack would be $3.0 \times 10^{34} \times 0.015 \approx 4.6 \times 10^{32}$ metres tall.

Now that's tall!

4.6×10^{32} metres is approximately 520 000 times the diameter of the observable universe.