Weeks 0 & 1 | Feedback on submission problems Dr Killian O'Brien
6GZ3012 Group Theory 14/10/17

Individual feedback has been given to you on your submissions. The problems served as a nice way to reacquaint ourselves with proof-by-induction – though some of the details of handling the Fibonacci sequence can be tricky. Here are some comments on the parts of question 17.

The Fibonacci numbers are

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots.$$

We can define them inductively by $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for $n \in \mathbb{N}$.

(a) Prove that $f_n < 2^n$.
(b) Prove that $f_{n+1}f_{n-1} = f_n^2 + (-1)^n$, $n \geq 2$.
(c) Prove that $f_n = [(1 + \sqrt{5}\,)^n - (1 - \sqrt{5}\,)^n]/2^n\sqrt{5}$.
(d) Show that $\lim_{n\to\infty} f_n/f_{n+1} = (\sqrt{5} - 1)/2$.
(e) Prove that f_n and f_{n+1} are relatively prime.

(a)