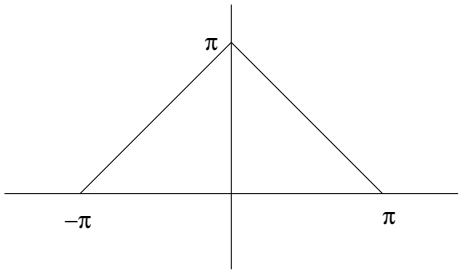
6G5Z3011 MULTI-VARIABLE CALCULUS AND ANALYTICAL METHODS

TUTORIAL SHEET 10

Qs 1 − 4 Further exercises on Fourier series.

(1) Find the Fourier series for the function whose graph is shown below.



Sketch the graph of the series over the range $(-4\pi, 4\pi)$ and use the series to evaluate the series

$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}.$$

(2) Using the formulae

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right),$$

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

find the Fourier series of the half way rectified wave f defined by

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ \sin x, & 0 < x \leq \pi \end{cases}.$$

Sketch the graph of the series over the interval $(-3\pi, 3\pi)$. Use the Fourier series to find the sum of the series

$$\sum_{m=1}^{\infty} \frac{1}{4m^2-1}.$$

(3) Find the cosine series for the sine function over the interval $(0, \pi)$. Use may be made of the formulae from question (2) above. Sketch the graph of the series over the interval $(-3\pi, 3\pi)$.

(4) Use the Fourier series for the function f defined by $f(x) = x$ to obtain Fourier series for the functions defined by $f(x) = x^2$ and $f(x) = x^3$. Hence evaluate the following series

(a)

$$\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2m-1}$$

(b)

$$\sum_{m=1}^{\infty} \frac{1}{m^2}$$

(c)

$$\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)^3}$$

(d)

$$\sum_{m=1}^{\infty} \frac{1}{m^4}$$