6G5Z3011 MULTI-VARIABLE CALCULUS AND ANALYTICAL METHODS

TUTORIAL SHEET 04 - SOLUTIONS

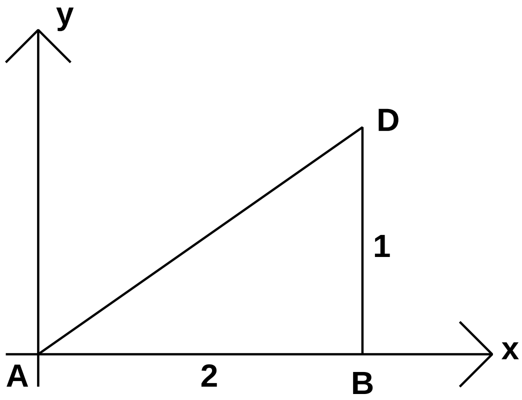
Solutions to questions 1 - 4 listed on the following pages under the heading of *Exercise 9*

BSc Mathematics/BSc Combined Honours

MA2101 Mathematical Methods

Partial Integration Worked Solutions

Exercise 9



Q1. a) AD is the line $y = \frac{1}{2}x$ with x ranging from 0 to 2

Then on AD $dy = \frac{1}{2}\,dx$

and
$$\int_C (x^2 + 2y + 4)dx + (x^2 + 2y + 4)dy$$

$$= \int_0^2 (x^2 + x + 4)dx + (x^2 + x + 4)\frac{1}{2}dx$$

$$= \int_0^2 \frac{3}{2}(x^2 + x + 4)dx = \frac{3}{2}\left[\frac{x^3}{3} + \frac{x^2}{2} + 4x\right]_0^2 = 19$$

b) On AB, y = 0, so dy = 0, and x changes from 0 to 2.
On BD, x = 2, so dx = 0, and y changes from 0 to 1.

So
$$\int_C (x^2+2y+4)dx+(x^2+2y+4)dy = \int_{AB} (x^2+2y+4)dx+(x^2+2y+4)dy + \int_{BD} (x^2+2y+4)dx+(x^2+2y+4)dy$$

$$= \int_0^2 (x^2+4)dx + \int_0^1 (2y + 8)dy$$

$$= 32/3 + 9 = 59/3$$

Q2. On C, $y = 2x+1$ from (1,3) to (3,7), so x ranges from 1 to 3 and $dy = 2dx$. Then

$$\int_C (x^2 + 2y)dx + (x + y^2)dy = \int_1^3 [(x^2 + 2(2x+1)]dx + [x + (2x+1)^2]2dx$$

$$= \int_1^3 (9x^2 + 14x + 4)dx = 142$$

Q3. On C x ranges from 0 to $\pi/2$.

a) If $y = \sin x$, $dy = \cos x \, dx$ so

$$\int_C x\,dy + (y + 1)dx = \int_0^{\pi/2} x \cos x \, dx + (\sin x + 1)dx$$

$$= \Big[x\sin x + \cos x - \cos x + x \Big]_0^{\pi/2} \quad \text{using integration by parts}$$

$$= \pi/2 + \pi/2 = \pi.$$

b) If $y = 2x/\pi$, $dy = 2/\pi \, dx$ so

$$\int_C x\,dy + (y + 1)dx = \int_0^{\pi/2} \frac{2}{\pi}x \, dx + \left(\frac{2}{\pi}x + 1\right)dx$$

$$= \left[\frac{2x^2}{\pi} + x \right]_0^{\pi/2}$$

$$= \pi/2 + \pi/2 = \pi.$$

c) The answer will again be π.

Q4. If $\underline{r}$ is the position vector, $\underline{i}x + \underline{j}y$ then $d\underline{r} = \underline{i}dx + \underline{j}dy$ and

the work done by force, $\underline{F}$ along C $= \displaystyle\int_C \underline{F}.d\underline{r} = \int_C (xy\underline{i} + y^2\underline{j}).(\underline{i}dx + \underline{j}dy)$

$$= \int_C xy\,dx + y^2dy$$

On C, t ranges from 0 to 1 and the path is $\underline{i}t + \underline{j}t^2$ so $x = t$ and $y = t^2$ and hence $dx = dt$ and $dy = 2tdt$.

Hence work done $= \displaystyle\int_0^1 t(t^2dt) + (t^2)^2 2tdt = \int_0^1 t^3 + 2t^5 \, dt$

$$= \left[\frac{t^4}{4} + \frac{t^6}{3} \right]_0^1 = \frac{7}{12}$$