6G5Z3011 MULTI-VARIABLE CALCULUS AND ANALYTICAL METHODS

TUTORIAL SHEET 7

Qs 1 − 5 on working with the Laplace transform and its inverse.
Qs 6 − 7 on solving ODEs and systems of ODEs using the Laplace transform method.

(1) Find the general solution of the equation
$$\frac{d^3y}{dx^3} = 6.$$
What does the solution become if we are given the conditions
$$y(0) = 0, \quad y(1) = 0, \quad y(2) = 1?$$

(2) Using the fact that
$$\sinh t = \frac{1}{2}\left(e^t - e^{-t}\right)$$
and the definition of the Laplace transform show that
$$\mathcal{L}\{\sinh at\} = \frac{a}{s^2 - a^2}.$$

(3) Without using the tables (i.e. by working the integral definition of the transform) show that
$$\mathcal{L}\{t^2\} = \frac{2}{s^3}.$$

(4) Using the table of standard transforms and transform properties find the Laplace transforms of the functions defined by

(a)
$$1 + 2t + 3t^2,$$
(b)
$$5\cos t,$$
(c)
$$4e^{2t}\sin 3t.$$

(5) Using the table find the inverse Laplace transforms of

(a)
$$\frac{7}{s+4},$$
(b)
$$\frac{4s+3}{s^2},$$
(c)
$$\frac{2s+8}{(s+4)^2+100},$$
(d)
$$\frac{3}{(s+1)(s-2)}.$$

(6) Use the Laplace transform method to solve the following differential equations subject to the given initial conditions

(a)
$$\frac{dy}{dt} + y = 1, \quad y(0) = 0.$$

(b)
$$\frac{dy}{dt} - 2y = 4e^{-2t}, \quad y(0) = 2.$$
(c)
$$\frac{d^2y}{dt^2} + 2y = 3t, \quad y(0) = 1,\ y'(0) = 7.$$
(d)
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3, \quad y(0) = 1,\ y'(0) = 4.$$

(7) Use the Laplace transform method to solve the following pairs of coupled differential equations.

(a)
$$x + \frac{dy}{dt} = 2$$
$$\frac{dx}{dt} - 6x - 13y = 1$$
with the initial conditions $x(0) = -10$ and $y(0) = 3$.

(b)
$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} + y = 0$$
$$\frac{dy}{dt} - \frac{dx}{dt} - 2y + 2x = \sin t.$$
with the initial conditions $x(0) = 0$, $y(0) = 0$ and $y'(0) = 0$.