6G5Z3011 MULTI-VARIABLE CALCULUS AND ANALYTICAL METHODS

TUTORIAL SHEET 04

Qs 1 – 4 on **Line integrals**

(1) A, B and D are the points $(0,0)$, $(2,0)$ and $(2,1)$ respectively. Evaluate the path integral

$$\int_C (x^2+2y+4)\,dx + (x^2+2y+4)\,dy$$

when (a) C is the straight line segment AD and (b) when C is the path made up of the straight line segments AB and BD.

(2) Evaluate the path integral

$$\int_C (x^2+2y)\,dx + (x+y^2)\,dy$$

where C is the segment of the line $y = 2x+1$ from $(1,3)$ to $(3,7)$.

(3) Evaluate the path integral

$$\int_C x\,dy + (y+1)\,dx$$

where C is

(a) the segment of the curve $y = \sin x$ from $(0,0)$ to $(\frac{\pi}{2},1)$,

(b) the segment of the line $y = \frac{2x}{\pi}$ from $(0,0)$ to $(\frac{\pi}{2},1)$,

(c) any other path from $(0,0)$ to $(\frac{\pi}{2},1)$.

(4) When a force $\mathbf{F}$ moves along a path C in the plane then the total workis given by

$$\int_C \mathbf{F}.\,d\mathbf{r}$$

where $\mathbf{r}(x,y)$ is the position vector $x\mathbf{i}+y\mathbf{j}$.

Show that when the force $\mathbf{F}$, given by $\mathbf{F}(x,y) = xy\mathbf{i}+y^2\mathbf{j}$, moves along the path C, defined by $t\mathbf{i}+t^2\mathbf{j}$ where $0 \le t \le 1$, the work done by the force is $\frac{7}{12}$.