6G5Z3011 MULTI-VARIABLE CALCULUS AND ANALYTICAL METHODS

TUTORIAL SHEET 06

(1) Evaluate the integral,

$$\oint_C 2xy\ dx + \left(3x^2+1\right)\ dy,$$

where C is the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

(2) Evalaute the integral,

$$\oint_C e^{\frac{y}{x}}\ dx + yx^3\ dy,$$

where C is the path from $(0,0)$ to $(1,1)$ along the curve given by $y = x^2$ and then back along the line given by $y = x$.

(3) Verify that Green's theorem holds for the integral

$$\oint_C \sin y\ dx + \cos x\ dy,$$

where C is the square with sides given by $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$, $x = \frac{\pi}{2}$ and $y = \frac{\pi}{2}$.

(4) Verify that Green's theorem holds for the integral

$$\oint_C \left(x^3y + xy^3\right)\ dx + (x+y+1)\ dy,$$

where C is the triangle with vertices $(0,1)$, $(1,0)$ and $(1,1)$.

(5) Give two reasons why Green's theorem cannot be applied to the double integral

$$\iint_R \ln(xy+1) + x^2 + y^2\ dx\, dy,$$

where R is the region bounded by the circles given by $x^2+y^2 = 1$ and $x^2+y^2 = 3$.

(6) Using Green's theorem or otherwise, evaluate the integral

$$\oint_C \left(x^2y\cos x + 2xy\sin x - y^2e^x\right)\ dx + \left(x^2\sin x - 2ye^x\right)\ dy,$$

where C is the curve given by $x^4 + y^4 = 1$.

(7) Using a path integral and suitable parametric equations for x and y show that the area of the circle of radius r is πr^2.

(8) Using a path integral find the area A enclosed by the curve $x = 4 - y^2$ and the y-axis.

(9) Show that the integral

$$\int_{\mathrm{AB}} \left(2xy - y^4 + 3\right)\ dx + \left(x^2 - 4xy^3\right)\ dy,$$

where A is $(0,1)$ and B is $(2,3)$, is independent of the path joining A to B. Hence evaluate such an integral.

(10) Evaluate the path integral

$$\int_{\mathrm{AB}} \left(3x^2 + ye^y\right)\ dx + x(1+y)e^y\, dy,$$

along the curve path given by $y = \sin x$ from the point $\mathrm{A}(0,0)$ to the point $\mathrm{B}(\pi,0)$.