6G5Z3011 Multi-variable calculus and analytical methods

Tutorial Sheet 06

1. Evaluate the integral,

$$\oint_C 2xy \; dx + \left(3x^2 + 1\right) \; dy,$$

where C is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$.

2. Evalaute the integral,

$$\oint_C e^{\frac{y}{x}} \; dx + yx^3 \; dy,$$

where C is the path from $(0, 0)$ to $(1, 1)$ along the curve given by $y = x^2$ and then back along the line given by $y = x$.

3. Verify that Green's theorem holds for the integral

$$\oint_C \sin y \; dx + \cos x \; dy,$$

where C is the square with sides given by $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$, $x = \frac{\pi}{2}$ and $y = \frac{\pi}{2}$.

4. Verify that Green's theorem holds for the integral

$$\oint_C \left(x^3 y + x y^3\right) \; dx + (x + y + 1) \; dy,$$

where C is the triangle with vertices $(0,1)$, $(1,0)$ and $(1,1)$.

5. Give two reasons why Green's theorem cannot be applied to the double integral

$$\iint\limits_R \ln(xy+1) + x^2 + y^2 \;\; dx\, dy,$$

where R is the region bounded by the circles given by $x^2 + y^2 = 1$ and $x^2 + y^2 = 3$.

6. Using Green's theorem or otherwise, evaluate the integral

$$\oint_C \left(x^2 y \cos x + 2xy \sin x - y^2 e^x\right) \; dx + \left(x^2 \sin x - 2y e^x\right)$$

where C is the curve given by $x^4 + y^4 = 1$.

7. Using a path integral and suitable parametric equations for x and y show that the area of the circle of radius r is πr^2.

8. Using a path integral find the area A enclosed by the curve $x = 4 - y^2$ and the y-axis.

9. Show that the integral

$$\int_{\mathrm{AB}} \left(2xy - y^4 + 3\right)\, dx + \left(x^2 - 4xy^3\right)\, dy,$$

where A is $(0, 1)$ and B is $(2, 3)$, is independent of the path joining A to B. Hence evaluate such an integral.

10. Evaluate the path integral

$$\int_{\mathrm{AB}} \left(3x^2 + ye^y\right)\, dx + x(1 + y)e^y\, dy,$$

along the curve path given by $y = \sin x$ from the point $\mathrm{A}(0, 0)$ to the point $\mathrm{B}(\pi, 0)$.