6G5Z3011 Multi-variable calculus and analytical methods

Tutorial Sheet 05

Qs 1 – 5 on **integrals over rectangular regions**, Qs 6 - 8 on **integrals over non-rectangular regions and transforming coordinates**

1. Evaluate the following double integral

$$\int_1^2 \int_0^3 x^2y + y^2x \;\; dx\, dy.$$

   What does this integral represent?

2. Sketch the region over which the double integral

below is taken and hence evaluate it.

$$\int_{-1}^{1}\int_{-2}^{2} 4xy + \sin x + \cos y \;\; dx\, dy.$$

3. Sketch the regions of integration for the following two integrals and hence rewrite the sum as a single integral and evaluate it.

$$\int_{0}^{3}\int_{1}^{3} 9x^2y^2+4xy+5 \;\; dx\, dy+\int_{0}^{1}\int_{0}^{3} 9x^2y^2+4xy+5 \;\; dy\, d$$

4. Evaluate the following double integrals by first reversing the order of integration.

   (a)
   $$\int_{0}^{1}\int_{0}^{0.5} ye^{xy} \;\; dx\, dy$$

   (b)
   $$\int_{0}^{1}\int_{0}^{0.5} x\sin(xy) \;\; dx\, dy$$

5. Find the average height of the surface defined by $z = x^2 + y^2$ that lies above the square bounded by the lines $x = 1$, $x = -1$, $y = 1$ and $y = -1$.

6. Sketch the region over which the following double integral is taken and hence evaluate it.

$$\int_1^2 \int_{-x}^{x} \frac{y}{x} + 1 \;\; dy\, dx$$

7. Evaluate the following integrals by first reversing the order of integration.

   (a)
   $$\int_0^1 \int_0^{\cos^{-1} y} \sec x \;\; dx\, dy$$

   (b)
   $$\int_0^2 \int_{\frac{y}{2}}^{1} e^{x^2} \;\; dx\, dy,$$

   (Hint: make use of the substitution $u = x^2$ towards the end)

8. Evaluate the following double integrals by first transforming to polar coordinates.

(a)

$$\int_0^2 \int_0^{\left(4-y^2\right)^{\frac{1}{2}}} \left(x^2+y^2\right)^{\frac{5}{2}} \tan^{-1}\left(\frac{y}{x}\right) \; dx\, dy$$

(b)

$$\int_0^\infty \int_0^\infty \frac{1}{\left(x^2+y^2+1\right)^2} \; dx\, dy$$