$\frac{\partial g}{\partial x} = \dots$

$\frac{\partial g}{\partial y} = \dots$

} a critical pt. (a,b) is one a sol. of this simultaneous pair of equations

i.e. $\frac{\partial g}{\partial x}(a,b) = 0$

$\frac{\partial g}{\partial y}(a,b) = 0.$

Let's take $\frac{\partial g}{\partial x} = 0$ first.

Solve this

↳ conditions on x, y that make $\frac{\partial g}{\partial x} = 0$.

take each condition and impose it on $\frac{\partial g}{\partial y} = 0$ and then solve this.

and then put conditions together.

24. 2.9 in CLP3

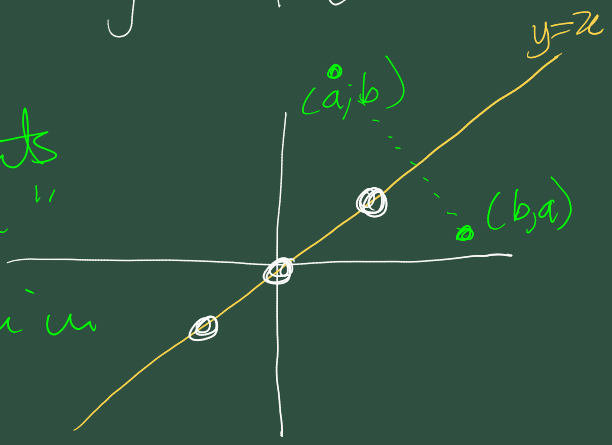
$$f(x,y) = x^4 + y^4 - 4xy.$$

Consider the system

$$\frac{\partial f}{\partial x} = 0 \quad \& \quad \frac{\partial f}{\partial y} = 0$$

$$\iff \quad 4x^3 - 4y = 0 \quad \& \quad 4y^3 - 4x = 0$$

Note: critical points must be "paired" under reflection in $y = x$.



Consider $\frac{\partial f}{\partial x} = 0$ first

$$\iff 4x^3 - 4y = 0$$

$$\iff x^3 - y = 0$$

$$\iff y = x^3$$ a single condition.

Now impose this on $\frac{\partial f}{\partial y} = 0$.

$$\iff 4y^3 - 4x = 0$$

$$\iff 4x^9 - 4x = 0$$

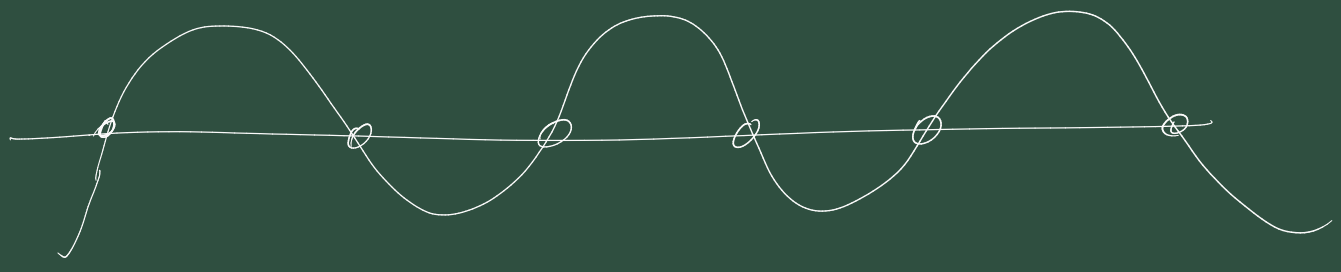
$\Leftrightarrow \quad x^9 - x = 0$

$\Leftrightarrow \quad x(x^8 - 1) = 0.$

$\Leftrightarrow \quad x = 0 \quad \text{or} \quad x^8 = 1$

$\Leftrightarrow \quad x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = +1$

So this means that we have 3 critical points $(0,0)$, $(-1,-1)$ $(1,1)$; and no others.



Q.6 (a) tutorial sheet

$$g(x,y) = \underbrace{e^{x+y}}\ \underbrace{(x^2 - xy + y^2)}$$

$$\frac{\partial g}{\partial x} = e^{x+y}(x^2 - xy + y^2) + e^{x+y}(2x - y)$$

$$\frac{\partial g}{\partial y} = e^{x+y}(x^2 - xy + y^2) + e^{x+y}(-x + 2y).$$

Lets consider $\frac{\partial g}{\partial y} = 0$ first.

$$\Leftrightarrow e^{x+y}(x^2 - xy + y^2) + e^{x+y}(-x+2y) = 0$$

$$\Leftrightarrow x^2 - xy + y^2 - x + 2y = 0.$$

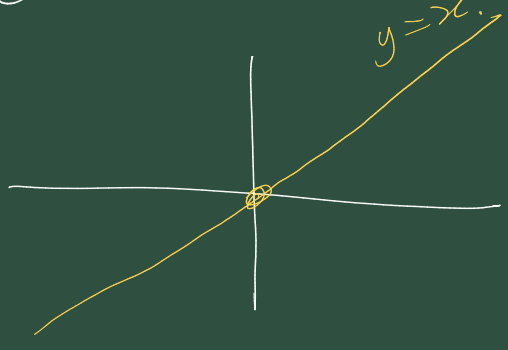
$$\frac{\partial g}{\partial x} = 0 \Leftrightarrow x^2 - xy + y^2 - y + 2x = 0$$

A critical point must satisfy.

$$-x + 2y = -y + 2x$$

$$\Leftrightarrow 3y = 3x$$

$$\Leftrightarrow x = y.$$



~~go back to~~ Under this condition.

$$\frac{\partial g}{\partial x} = 0 \Leftrightarrow \frac{\partial g}{\partial y} = 0$$

$$\Leftrightarrow x^2 - x^2 + x^2 - x + 2x = 0$$

$$\Leftrightarrow x^2 + x = 0$$

$$\Leftrightarrow x(x+1) = 0$$

$$\Leftrightarrow x = 0 \quad \text{or} \quad x = -1.$$

Gives two critical points $(0,0)$, $(-1,-1)$

Q3