6G5Z3011 Multi-variable calculus and analytical methods

Tutorial Sheet 04

Qs 1 – 4 on **Line integrals**

1. A, B and D are the points $(0, 0)$, $(2, 0)$ and $(2, 1)$ respectively. Evaluate the path integral

$$\int_C (x^2 + 2y + 4)\, dx + (x^2 + 2y + 4)\, dy$$

when (a) C is the straight line segment AD and (b) when C is the path made up of the straight line segments AB and BD.

2. Evaluate the path integral

$$\int_C (x^2 + 2y)\,dx + (x + y^2)\,dy$$

where C is the segment of the line $y = 2x + 1$ from $(1, 3)$ to $(3, 7)$.

3. Evaluate the path integral

$$\int_C x\,dy + (y + 1)\,dx$$

where C is

   (a) the segment of the curve $y = \sin x$ from $(0, 0)$ to $(\frac{\pi}{2}, 1)$,
   (b) the segment of the line $y = \frac{2x}{\pi}$ from $(0, 0)$ to $(\frac{\pi}{2}, 1)$,
   (c) any other path from $(0, 0)$ to $(\frac{\pi}{2}, 1)$.

4. When a force $\mathbf{F}$ moves along a path C in the plane then the total workis given by

$$\int_C \mathbf{F}.\,d\mathbf{r}$$

where $\mathbf{r}(x, y)$ is the position vector $x\mathbf{i} + y\mathbf{j}$.

Show that when the force $\mathbf{F}$, given by $\mathbf{F}(x, y) = xy\mathbf{i} + y^2\mathbf{j}$, moves along the path C, defined by $t\mathbf{i} + t^2\mathbf{j}$ where $0 \leq t \leq 1$, the work done by the force is $\frac{7}{12}$.