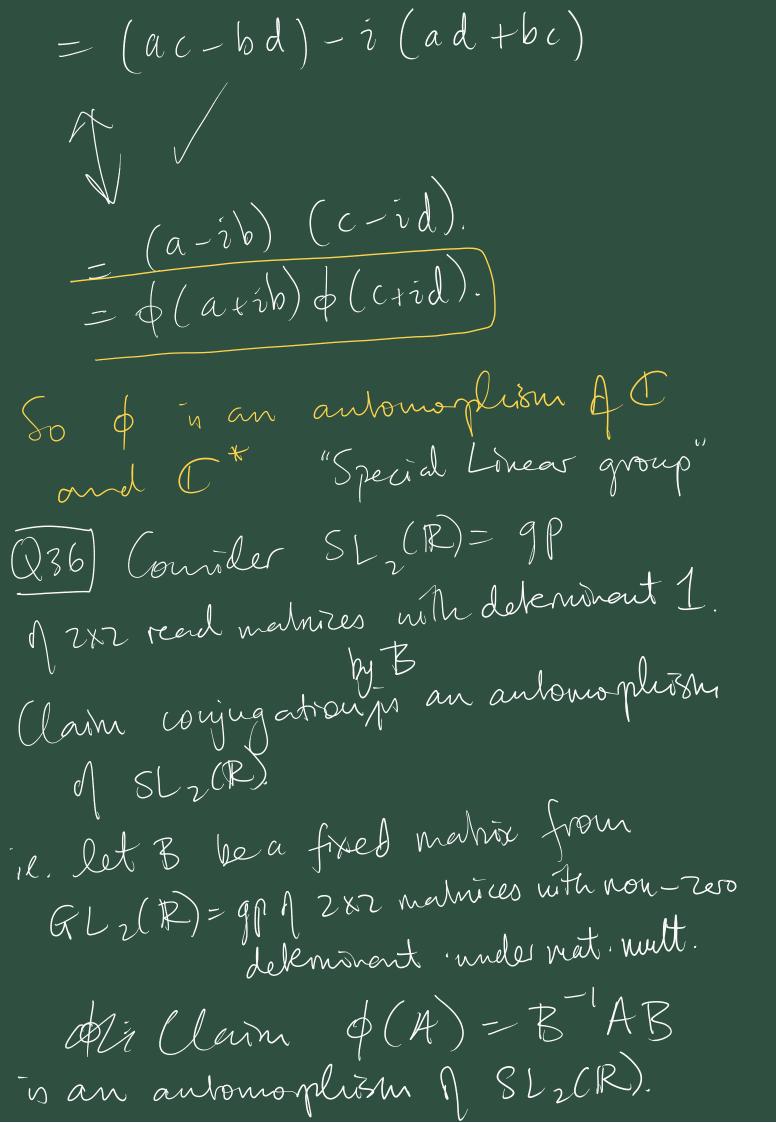
Q9, $G = R \setminus \{-1\}$. Define & operation & on G by $a \times b = a + b + ab$ 75 a*b & G? What if a*b = a+b+ab = -1? $\langle = \rangle$ a(1+b) = -b-1.(=) a = - (b+1) mue b+1. / b+-1 (=) a = -1. So for a, b e G, a x b e G So x v a closed binary aperation on G. ls * arrocialire on G. a*(b*c) = a*(b*c*bc). = a*b*c*bc + a(b*c*bc).= a + b + c+bc + ab + ac + abc. = a + b + ab + c + c (a + b + ab) = (a + b + ab) * C =(a*b)*c.* is arrowable on G.

The identity element for * on G. e=0 vine $a \times 0 = a + 0 + a \cdot 0 = a$ And given $a \in G$., a should be elment salvetyng $\alpha \times \alpha^{-1} = 2$. $a + a^{-1} + aa^{-1} = 0$ $() \quad \alpha^{\prime\prime}(\mid + \alpha) = -\alpha.$ rive a +-1 and note _a l + a group. So (G, K) ~ a the group of non zero reads under regular multiplication. Claim GZR, Proof.

We need to define an isomophism. Lepred by $\phi(a) =$ and then more that $\phi(axb) = \phi(a)\phi(b)$ Note that this of does map 0 -> 1 Widn's eg- > ept / Prove the homomorphism property $\phi(a*b) = \phi(a+b)$ = a + b + ab + 1 = (a+i)(b+i)= φ(a)φ(b) φ in Tromophism

and of GZIR* Automorphisms na aversonphisms na aversonptisms na (learly (id: (T -> G)) g H is an automorphism, but it's known as the trivial one. What is interlying to whether or not there are non-trivial automorphisms (T ->) G. il. Symmetries & G' n a mer Complex conjugation $\phi: \mathbb{C} \longrightarrow \mathbb{C}$. $\left(\mathcal{C}_{j}+\right)$ $\phi(a+ib) = a - ib$ Claim of is an automorphism (I -> (I Q35 Claim 11 " () () ()

(= gp Non-zero complex runhers under mutt. Proof trally \$: C > C Here are holl hjedious. Hom. prop for + φ ((a + ib) + (c+id)) $=\phi((a+c)+i(b+d))$ = (a+c) - i(b+d) = (a-ib) + (c-id.) $=\phi(a+ib)+\phi(c+id)$ Smilarly, for mult. ϕ (a +ib) (c+id) $=\phi(ac-bd)+i(ad+bc)$



Proof So we need to show two (D) Given AESLICR), $\phi(A) = B^{T}ABESLICR)$ 2) For A, Azeslzer $\phi(A_1A_2) = \phi(A_1)\phi(A_2).$ 1), Assume (AESLLIP), Let (A)=1.

and BEGL2(P), Let (B) + 0. $|s|\phi(A)=B^{-1}ABESL_2(R)^{?}$ let (\$\phi(A)) = det (B^-1AB) = let(B-1) let(A) let(B) (sme dot in multiplicative ie.)

det(XY) = det(X) det(Y).) - det(B) det(B). = dit(A) = 1

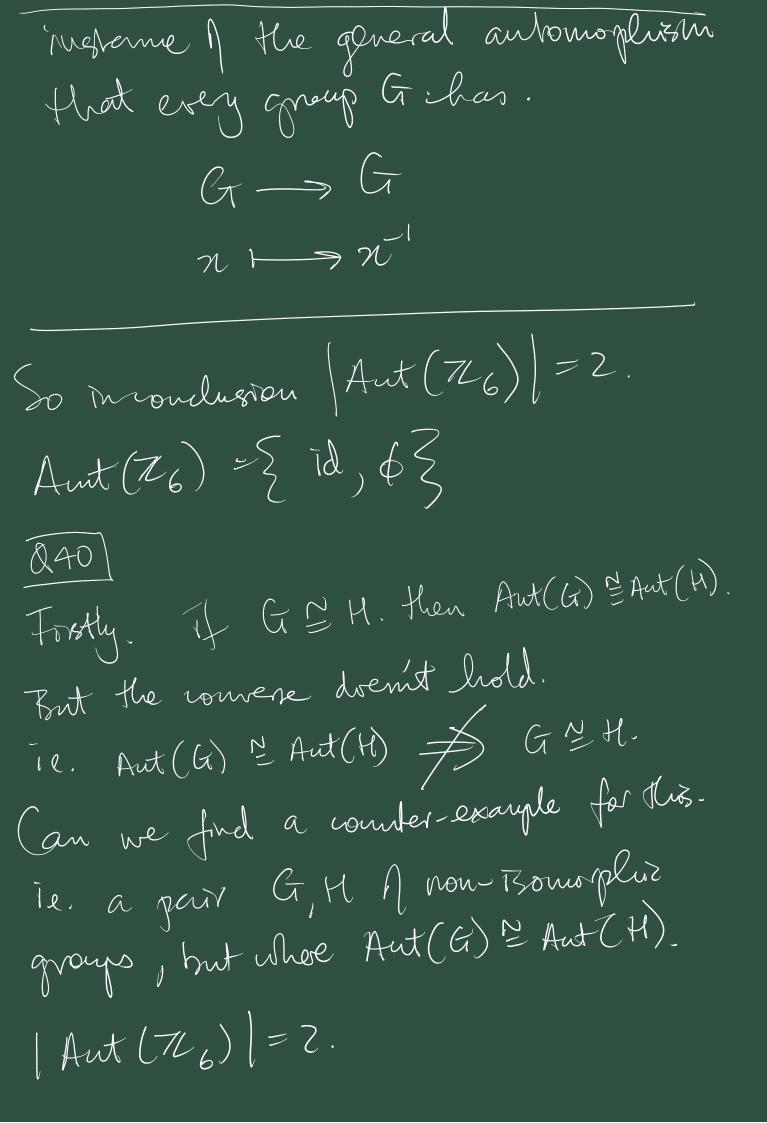
 \Rightarrow $\phi(R) \in SL_2(R)$. So o to a map SU(R)->SL2(R). ls plana hijetion. 1-17. onto? Assuming $\phi(A_1) = \phi(A_2)$. => B-1 A, B = B-1 A, B => B(B'A,B)B' = B(B'A2B)B' \Rightarrow $A_1 = A_2$. So o N 1-1 /. To prove outo. Let YESL2(R). $\phi(BYB) = B'BYB'B = Y$ det = 4 det (Y) \$ B outo.

So d: S(r(R) -> SL2(R) is a bijection. Finally to show of satisfies the liver prop. $\phi(A_1A_2) = B^{-1}A_1A_2B$ TABBIA2B $=\phi(A_1)\phi(A_2)$ So & dres sentisfy the how. Trop. composition. Pf: Composition of mays is always and if \$1:4-36, \$7:6-36 are Bouwrphisms. In lesture we

proved $\phi_z \circ \phi_1 : G \rightarrow G$ is an Bomonflusm too. So Aut (4) is closed under op. of composition The identity element of Aut (G) is the automorphism given by the identity map id: (-> G. (se above). and ϕ ord = id $\phi = \phi$. We've already proved the result about juines il. goven au Bourglosm d: G-7G the more map d'; G-5G is also an thomosphism, ie. am antonoplism. and $\phi \circ \phi = id.$ So Aut (G) - 10 a group (groperation).

Aut (G) is the "Symmetry group of G" R38] Find Aut (7/6) We should be able to list the automorphisms un Aut (TL6). Mb = int. under addition modulo 6. Clearly there is the identity autonomorphism. id: 76 -> 76 $n \mapsto n$. What other automorphisms might there Note 76 ha a cylic group, generated $M_{6} = \langle 1 \rangle$ From what we're proved about 1305 already we know that if $6: 16 \Rightarrow 16$ is an 130 then $\phi(i)$ must also be a generalor. generatos. So that are the options for $\phi(i)$.

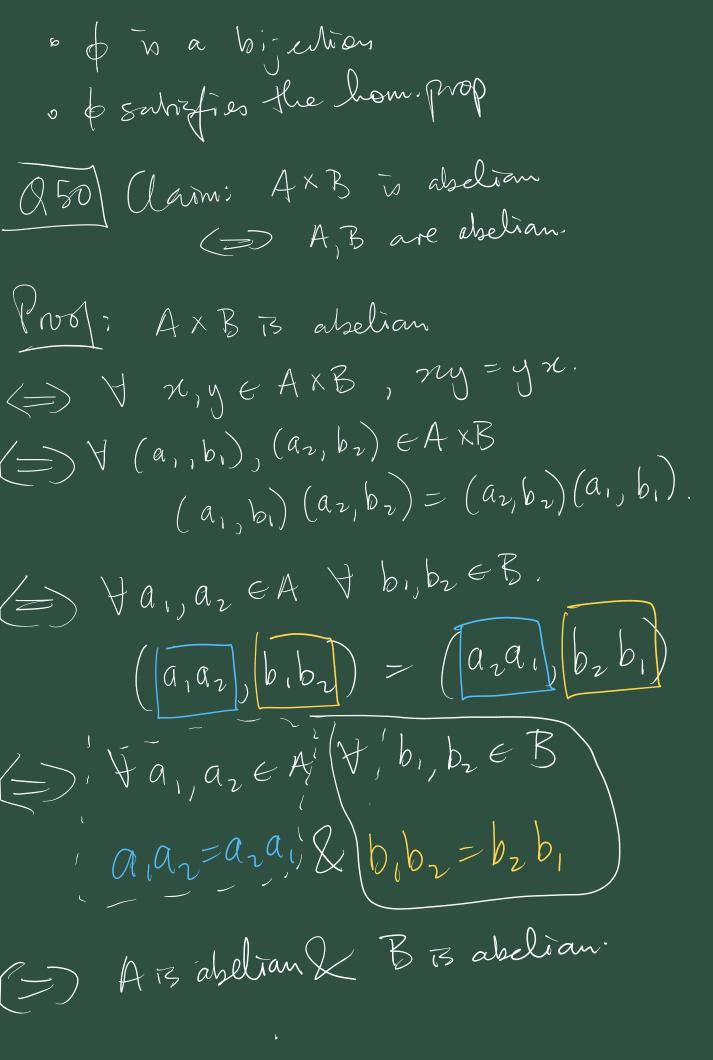
What offer generations does \$76 have? $\mathcal{I}_{6} = \langle 1 \rangle = \langle 5 \rangle$ But 2,3,4 are not generalors of 76 (22)=20,2,43 = <4>,<3>=20,33 (co. floorer, 1) See theorem 4.13for $m \in 116$ $|m| = \frac{6}{d}$, $d = g \cdot d \cdot (m_1 b)$. $\{0,5,4,3,2,3=(5)=16.$ So there is only other emborrophism of 76, while is the map of sursfy veg 5=-1 modb 4=-2 modb $\langle (2) = 4 \rangle$ 3=-3 mud6 $\sqrt{(3)} = 3$ 6(4)=2 (b (5) = 1 So achilly fliss & is an



Not all groups order 2 are Bounghiz. e e n n n e (an we produce another group H Hut also has | Aut(H) |= 2. Manh 1 7/4 = <1> = <3> + <2> So the same analyts as before opreomrorgh for Ma. Ma has only Jus antonophisms. 6. 1/4 -> The id:724 > 14 1-33 2 -> 2 -> 1.

 $So |Aut(Z_A)| = 2$ $So Aut(Z_A) \stackrel{\vee}{\rightarrow} Aut(Z_6)$

But La & No. |12/=9 + 6 = 116| QAT If GZG and HZH Hen claim: GXH Z GXH Proof: What's the Bomophism.
So there exist two Bomophisms.
M: G > G, N: H > H O: GXH -> GXH defrued hy. $\phi((g,h)) = (\mu(g),\lambda(h))$ of is a hijection since both proud 2 are (detents as an exercise). Does & sahrfy the how. prop?



(Euler's) theorem. Nementres of qcd(a,m)=1(m)) (mod m) $\frac{0}{0}$ So this means the cycle length of Hirs divides M.

