

Q 6

$$\text{claim } \forall n \geq 1 \quad \sum_{j=1}^n j^3 = \frac{1}{4} n^2 (n+1)^2$$

Base case:  $n = 1$ .

$$\sum_{j=1}^1 j^3 = 1^3 = 1 = \frac{4}{4} = \frac{1}{4} 1^2 (1+1)^2$$

✓

This is clearly true.

Assume result is true for some  $k \geq 1$

$$\text{i.e., } \sum_{j=1}^k j^3 = \frac{1}{4} k^2 (k+1)^2$$

The  $(k+1)$ th case of the result begins

as.

$$\begin{aligned} \sum_{j=1}^{k+1} j^3 &= \left( \sum_{j=1}^k j^3 \right) + (k+1)^3 \\ &= \frac{1}{4} k^2 (k+1)^2 + (k+1)^3, \end{aligned}$$

by assumption.

$$= \frac{1}{4} (k+1)^2 [k^2 + 4(k+1)]$$





Proof: Assume  $\frac{c|ab}{}$   
and that  $\gcd(c, a) = 1$

$$\Rightarrow \boxed{ab = bc}, \text{ from def. of divisibility.}$$

From theorem 2.3.  $b \in \mathbb{Z}$ .

we know  $\exists x, y \in \mathbb{Z}$   
such that  $\boxed{1 = xc + ya}$

$$1 = xc + ya$$

$$\Rightarrow b = bxc + bya$$

$$\Rightarrow b = bxc + ybc$$

$$\Rightarrow b = c \underbrace{(xb + ya)}_{\in \mathbb{Z}}$$

$\Rightarrow c|b$ , by definition.





So by the principle of induction

$$\forall n \geq 1 \quad 8 \mid 3^{2n} + 7.$$

Q2. Claim: If  $a \in \mathbb{Z}$  then one of  $a, a+2, a+4$  is divisible by 3.

We know

$$a = 3q + r$$

for some  $q, r \in \mathbb{Z}$   $r = 0, 1, 2$ .

Argue separately for cases  $r = 0, 1, 2$ .

If  $r=0$  then  $a=3q$  so result holds

If  $r=1$  then  $a+2 = 3q+1+2 = 3q+3 = 3(q+1)$

$$\Rightarrow \frac{a+2}{a+4} = \frac{3}{\in \mathbb{Z}}$$

If  $r=2$  then  $a+4 = 3q+2+4 = 3q+6 = 3(q+2)$