Question n°4 =

$$E(r') = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(r')}{r^2} \hat{r}\, da$$

$$dq = \sigma\, da = \sigma R^2 \sin\theta\, d\theta\, d\phi$$

$$r^2 = R^2 + z^2 - 2Rz\cos\theta \quad [\text{Cosine Law}]$$

$$\cos\psi = \frac{z - R\cos\theta}{r}$$

Also $E(r') = E_z \hat{z}$, $E_z = |E(r')|\cos\psi$ and $\int d\phi = 2\pi$

Thus $$E_z = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma R^2 \sin\theta\, d\theta\, d\phi\, (z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}$$

$$= \left(\frac{2\pi R^2 \sigma}{4\pi\varepsilon_0}\right) \int_0^{\pi} \frac{(z - R\cos\theta)\sin\theta}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}\, d\theta$$