e) let's do this analically:

$$w(x) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi x}{N}\right)$$

so $$W(k) = F(w(x)) = \sum_{x=0}^{N-1}\left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi x}{N}\right)\right)e^{-\frac{2\pi i x k}{N}}$$

$$= \frac{1}{2}\sum_{x=0}^{N-1} e^{-\frac{2\pi i k x}{N}} - \frac{1}{2}\sum_{x=0}^{N-1}\cos\left(\frac{2\pi x}{N}\right)e^{-\frac{2\pi x k}{N}}$$

$$= \sum_{x=0}^{N-1}\left(\frac{1}{2}e^{-\frac{2\pi i x k}{N}} - \frac{1}{4}\left[e^{-\frac{2\pi i(k-1)}{N}} + e^{-\frac{2\pi i(k+1)}{N}}\right]\right)$$

$$= \frac{1}{2}N\delta(k) - \frac{1}{4}N\,\delta(k-1) - \frac{1}{4}N\delta(k+1)$$

so $$W(k) = \begin{cases} \frac{N}{2} & k=0 \\ -\frac{N}{4} & k=\pm 1 \\ 0 & \text{OW} \end{cases}$$

Thus $W(k) = \left[\frac{N}{2}, -\frac{N}{4}, 0, \ldots, 0, -\frac{N}{4}\right]$ as because of circulant nature of DFT, -1 is the last term.

Consequently, $$F[y(x)w(x)](k) = \frac{1}{N}Y(k) * W(k)$$

$$= \frac{1}{N}Y(k) * \left(\frac{N}{2}\delta(k) - \frac{N}{4}\delta(k-1) \ldots\right.$$

$$- \frac{N}{4} \delta(k+1) \Big]$$

$$= \sum_{x=0}^{N-1} Y[(k-x) \bmod N] \left(\frac{1}{2}\delta(x) - \frac{1}{4}\delta(x-1) - \frac{1}{4}\delta(x+1) \right)$$

$$= \frac{1}{2} Y(k) - \frac{1}{4} Y[(k-1) \bmod N] - \frac{1}{4} Y[(k+1) \bmod N]$$