



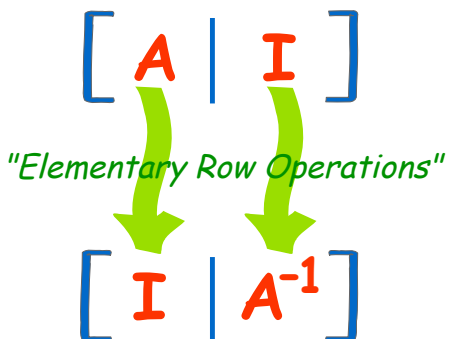
Inverse of a Matrix

using Elementary Row Operations

Also called the Gauss-Jordan method.

This is a fun way to find the Inverse of a Matrix:

Play around with the rows (adding, multiplying or swapping) until we make Matrix **A** into the Identity Matrix **I**



And by ALSO doing the changes to an Identity Matrix it magically turns into the Inverse!

The "**Elementary Row Operations**" are simple things like adding rows, multiplying and swapping ... but let's see with an example:

Example: find the Inverse of "A":

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

We start with the matrix **A**, and write it down with an Identity Matrix **I** next to it:

$$\begin{bmatrix} 3 & 0 & 2 & | & 1 & 0 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

(This is called the "Augmented Matrix")

Identity Matrix

The "Identity Matrix" is the matrix equivalent of the number "1":

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A 3x3 Identity Matrix

- It is "square" (has same number of rows as columns),
- It has **1s** on the diagonal and **0s** everywhere else.
- It's symbol is the capital letter **I**.

Now we do our best to turn "A" (the Matrix on the left) into an Identity Matrix. The goal is to make Matrix A have **1s** on the diagonal and **0s** elsewhere (an Identity Matrix) ... and the right hand side comes along for the ride, with every operation being done on it as well.

But we can only do these "**Elementary Row Operations**":

- **swap** rows
- **multiply** or divide each element in a row by a constant
- replace a row by **adding** or subtracting a multiple of another row to it

And we must do it to the **whole row**, like this:

$$\begin{bmatrix} 3 & 0 & 2 & | & 1 & 0 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Start with **A** next to **I**

$$\begin{bmatrix} 5 & 0 & 0 & | & 1 & 1 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Add

Add row 2 to row 1,

$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Divide by 5

then divide row 1 by 5,

$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 0 & 0 & -2 & | & -0.4 & 0.6 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Subtract x 2

Then take 2 times the first row, and subtract it from the second row,

$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 0 & 0 & 1 & | & 0.2 & -0.3 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Multiply by -1/2

Multiply second row by -1/2,

$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \\ 0 & 0 & 1 & | & 0.2 & -0.3 & 0 \end{bmatrix}$$

Swap

Now swap the second and third row,

$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & | & -0.2 & 0.3 & 1 \\ 0 & 0 & 1 & | & 0.2 & -0.3 & 0 \end{bmatrix}$$

Subtract

Last, subtract the third row from the second row,

And we are done!

I **A⁻¹**

And matrix **A** has been made into an

Identity Matrix ...

... and at the same time an Identity Matrix got made into **A⁻¹**

$$A^{-1} = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}$$

DONE! Like magic, and just as fun as solving any puzzle.

And note: there is no "right way" to do this, just keep playing around until we succeed!

(Compare this answer with the one we got on [Inverse of a Matrix using Minors, Cofactors and Adjugate](#) . Is it the same? Which method do you prefer?)

Larger Matrices

We can do this with larger matrices, for example, try this 4x4 matrix:

$$B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Start Like this:

$$\begin{bmatrix} 4 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & | & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

See if you can do it yourself (I would begin by dividing the first row by 4, but you do it your way).

You can check your answer using the [Matrix Calculator](#) (use the "inv(A)" button).

Why it Works

I like to think of it this way:

- when we turn "8" into "1" by dividing by 8,
- and do the same thing to "1", it turns into "1/8"

$$\begin{bmatrix} 8 & | & 1 \\ 1 & | & \frac{1}{8} \end{bmatrix}$$

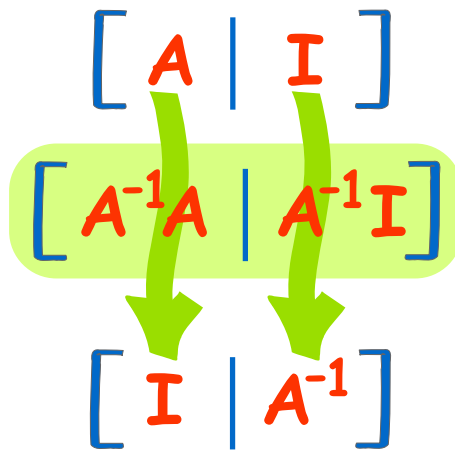
And "1/8" is the (multiplicative) **inverse of 8**

Or, more technically:

The **total effect of all the row operations** is the same as **multiplying by A⁻¹**

So **A** becomes **I** (because **A⁻¹A = I**)

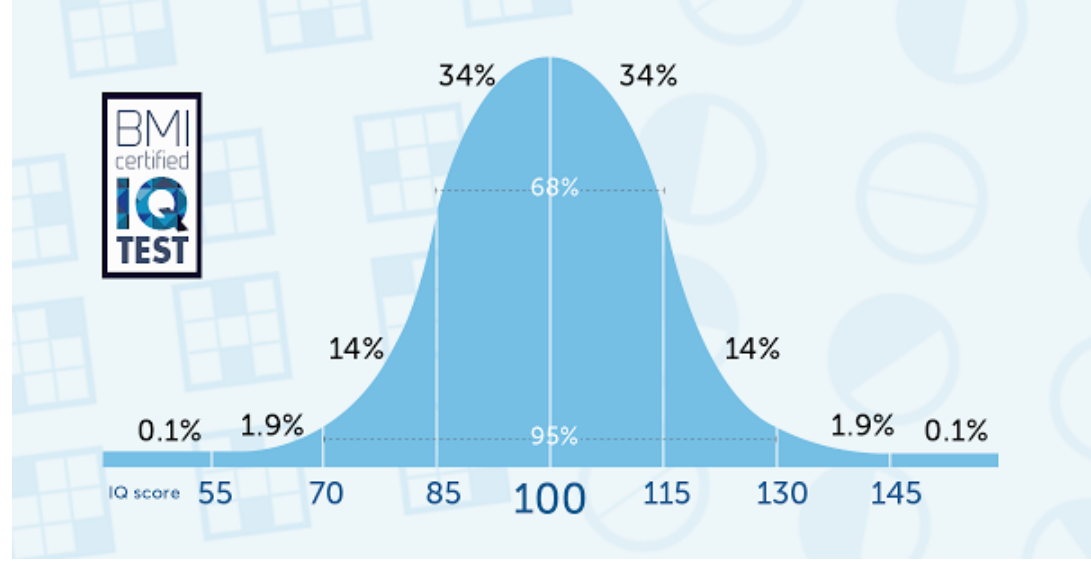
And **I** becomes **A⁻¹** (because **A⁻¹I = A⁻¹**)



[Question 1](#) [Question 2](#) [Question 3](#) [Question 4](#) [Question 5](#)
[Question 6](#) [Question 7](#) [Question 8](#) [Question 9](#) [Question 10](#)

here's
more:

- [Multiplying Matrices](#)
- [Determinant of a Matrix](#)
- [Matrix Calculator](#)
- [Inverse of a Matrix using Elementary Row Operations](#)
- [Algebra Index](#)



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