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Quantum Computing

Week 1

Why are finance companies turning towards quantum computing?

How quantum computing could change financial services?

- Financial institutions that can harness quantum computing are likely to see significant benefits. In particular, they will be able to more effectively analyze large or unstructured data sets. Sharper insights into these domains could help banks make better decisions and improve customer service, for example through timelier or more relevant offers.
- There are equally powerful use cases in capital markets, corporate finance, portfolio management, and encryption-related activities. In an increasingly commoditized environment, this can be a route to real competitive advantage. Quantum computers are particularly promising where algorithms are powered by live data streams, such as real-time equity prices, which carry a high level of random noise.
- The market changes in the last 2 decades required to develop AI based models. These models are real-time risk models. These models still work on classical computing.
- The arrival of quantum computing is potentially game changing, but there is a way to go before the technology can be rolled out at scale. Financial institutions are only just starting to get access to the necessary hardware and to develop the quantum algorithms they will need. Still, a rising number of initiatives suggest a tipping point is on the horizon. For banks yet to engage, and particularly those that rely on computing power to generate competitive edge, the time to act is now.

Some finance companies and their investment in quantum computing:

Goldman Sachs

A Goldman Sachs researcher in January 2020 said quantum has the potential to become a critical technology. Still, Goldman's efforts are at an early stage. In its early experiments, the bank found that Monte Carlo simulations, which require significant amounts of conventional computing power, cannot yet be parallelized on a quantum system. It therefore refocused on developing approaches to decrease the depth of quantum circuits required to do these calculations.

JPMorgan:
JPMorgan and Citigroup, meanwhile, have set up quantum computing initiatives and even bought stakes in computing start-ups. JPMorgan also experimented with Honeywell’s quantum computer in an effort to ease mathematical operations that involve Fibonacci numbers.

European banks:

- BBVA has formed a partnership to explore portfolio optimization and more efficient Monte Carlo modelling.
- Also in Spain, Caixa Bank is running a trial hybrid framework of quantum and conventional computing with the aim of better classifying credit risk profiles.
- In mid-2020, UK’s Standard Chartered revealed its exploration of quantum computing applications, such as portfolio simulation, in collaboration with US-based Universities Space Research Association.

Why these initiatives:

These initiatives make sense because they allow financial firms to test quantum algorithms on simulators or the cloud without acquiring full-scale quantum computers. This appears to be a sensible strategy as long as quantum computers remain subcritical for practical applications and there is no dominant design for scaling quantum capabilities.

What if we don't have any hardware right now?

Many organizations are patiently awaiting the actual arrival of quantum and the options that surround it. Others have learned from the evolution of classical computing: The time it takes to develop useful applications is wasted when the hardware is ready but you can’t actually use it. With the revolution of quantum computing, we have the advantage of being able to develop applications now, before the hardware is ready. And we know many areas that absolutely need attention.

Rather than wait 5-10 years, organizations can start now to develop new applications and test them for a Quantum world.

Courses followed: [Link]
Text book followed: Quantum Computing, A Gentle Introduction.
[Link of the textbook highlighted important passages and notes.]

This report is oriented towards showing which path to take to continue the work done till now. It also explains from scratch, how we did the quantum portfolio optimization.

- Watched the video lectures and read the textbook.
- Given below are the important highlights from each chapter of the book which are relevant to doing quantum portfolio optimization.

Chapter 1: Introduction

- Explains the motivation behind creation of such a diverse field of quantum computing.
- **Quantum Computing:** Combining the two of the most influential and revolutionary theories of the 20th century, information theory and quantum mechanics.
- It was found that the theoretical quantum computer cannot be simulated by a classical computer (traditional computer that you are using right now).
- This observation led to speculation that perhaps these quantum phenomena could be used to speed up computation in general.
- It also implies that quantum computers are more powerful than classical computers in some aspects.
- Classical computers compute on bits whereas quantum computers compute on qubits.
- Qubits will be explained further.

Chapter 2: Single Qubits Systems

- Quantum bits are the fundamental units of information in quantum information processing in much the same way that bits are the fundamental units of information for classical processing.
- It is quite involved mathematically and theoretically to explain how the qubits are realized and their measurements. One can directly read from the textbook to fully understand how qubits are created and used in quantum computers.
- The important topics to read:
    - 2.1: The Quantum Mechanics of Photon Polarization
        - 2.1.1 A Simple Experiment
        - 2.1.2 A Quantum Explanation
    - 2.2 Single Quantum Bits
    - 2.3 Single-Qubit Measurement
    - 2.5 The State Space of a Single-Qubit System
        - 2.5.1 Relative Phases versus Global Phases

        - 2.5.2 Geometric Views of the State Space of a Single Qubit
        - 2.5.3 Comments on General Quantum State Spaces
    - 2.7 Exercises

Week 2

Chapter 3: Multiple Qubit Systems:

- Only a single qubit system cannot be used to simulate larger tasks. Quantum properties like entanglement works only with multiple qubits.
- Multiple qubits provide the exponential state space (basis vectors) to quantum systems.
- To understand these concepts: Read
    - 3.1 Quantum State Spaces
        - 3.1.1 Direct Sums of Vector Space
        - 3.1.2 Tensor Products of Vector Spaces
        - 3.1.3 The State Space of an n-Qubit System
    - 3.2 Entangled States
    - 3.3 Basics of Multi-Qubit Measurement
    - 3.6 Exercises
- Two of the most important concepts in quantum computing are:
    - Superposition:
        - Classical bits can only have values 0 and 1.
        - Whereas, quantum bits can have values as probability of being 0 or 1.
        - Here, 0 and 1 are said to be superimposed.
    - Entanglement:
        - There are uncountably more quantum states than the one represented by individual qubits.
        - When the tensor product of qubits cannot be decomposed into individual qubits, we say that these sets of qubits are entangled.
        - In other words, qubits when measured collapse to one of the states, so if two qubits are entangled, then if we measure one of them, then we know what is the value of the other qubit.

Chapter 4: Measurement of Multiple Qubits:

- The chapter explains how to do measurement of qubits in order to get the results out of quantum systems.
- This chapter is significantly tough and mathematical. The starting parts are easy though to those who have taken advanced linear algebra courses.
- The important topic to read are given below:

- 4.1 Dirac's Bra/Ket Notation for Linear Transformations
- 4.2 Projection Operators for Measurement
- 4.3 Hermitian Operator Formalism for Measurement
- 4.4 EPR Paradox and Bell's Theorem (only read the starting part)
- 4.6 Exercises

Week 3

Classical Algorithm for Portfolio Optimization

- We used Markowitz's Portfolio Optimization technique as our classical algorithm. We understood the theory behind it from this Medium Article.

Chapter 5: Quantum State Transformation

- Here, some of the important concepts including quantum gates and circuits are explained and it's quite tough to understand from this single source for the first time. Watching NPTEL course lectures and reading the chapter multiple times can only help you grasp the concepts.
- If one wants to change the state of a quantum system, then for each qubit, quantum transformations are applied to achieve the required quantum state.
- These quantum transformations are nothing but a matrix being applied to quantum states.
- The chapter is quite large and most of the topics are important.
- The list of topics to be read are given below:
    - 5.1 Unitary Transformations
        - 5.1.1 Impossible Transformations: The No-Cloning Principle
    - 5.2 Some Simple Quantum Gates
        - 5.2.1 The Pauli Transformations
        - 5.2.2 The Hadamard Transformation
        - 5.2.3 Multiple-Qubit Transformations from Single-Qubit Transformations
        - 5.2.4 The Controlled-NOT and Other Singly Controlled Gates
    - 5.3 Applications of Simple Gates
        - 5.3.1 Dense Coding
        - 5.3.2 Quantum Teleportation
    - 5.4 Realizing Unitary Transformations as Quantum Circuits
        - 5.4.1 Decomposition of Single-Qubit Transformations
        - 5.4.2 Singly-Controlled Single-Qubit Transformations
        - 5.4.3 Multiply-Controlled Single-Qubit Transformations (tough to understand)
        - 5.4.4 General Unitary Transformations (tough to understand)
    - 5.5 A Universally Approximating Set of Gates
    - 5.6 The Standard Circuit Model

Week 4

Chapter 6: Quantum Versions of Classical Computations

- This chapter explains how to convert a classical circuit into a quantum circuit efficiently.
- First, we make a reversible classical circuit, then we replace the parts of the circuit with the quantum gates. It could be done more efficiently and is explained in the chapter.
- The important sections from the chapter are:
    - 6.1 From Reversible Classical Computations to Quantum Computations
        - 6.1.1 Reversible and Quantum Versions of Simple Classical Gates
    - 6.2 Reversible Implementations of Classical Circuits
        - 6.2.1 A Naive Reversible Implementation
        - 6.2.2 A General Construction
    - 6.3 A Language for Quantum Implementations
    - 6.4 Some Example Programs for Arithmetic Operations

There are multiple implementations of quantum computers. Whatever we studied in these four weeks was a gate based approach. It is the most famous and powerful approach as it can solve a large class of problems.

Research Paper: Solving systems of linear equations on a quantum computer
- This is quite a complicated research paper which uses the photon based concept of quantum computing. The overall implementation is gate based.

Research Paper: https://arxiv.org/pdf/2007.01430.pdf
- This is the paper where we got the right path to which we were finally able to develop the first quantum algorithm.
- It uses D'Wave technology. The term might look like it is related to physics, but it is a quantum annealer computer provider company.
- Quantum annealing is another approach to do quantum computing.

Week 5

Quantum Annealing:
- It is a physics concept which is used to solve real world optimization problems.
- Any physical system wants to achieve minimum energy configuration.

- If we can map the solution of our optimization problem to the minimum energy state of a quantum system, then we can use quantum annealing to solve the problem.
- Quantum annealing, the state of the art computers support 5000 qubits, which is 100 times more than state of the art computers of gate based approach.
- Currently, if we want to make something which actually does portfolio optimization, then quantum annealing is the best approach.

Some videos to understand these things deeply:
1. What is Quantum Annealing?
2. How The Quantum Annealing Process Works
3. Physics of Quantum Annealing - Hamiltonian and Eigenspectrum
4. Measuring Quantum Physics in a Quantum Annealer
5. Quantum Computing Tutorial Part 1: Quantum annealing, QUBOs and more

The most important video which teaches how to solve any problem related to optimization by giving examples:
Quantum Programming 101: Solving a Problem From End to End | D-Wave Webinar

Research Paper: https://arxiv.org/pdf/2008.08669.pdf
- This paper is a more advanced version of the previous paper as more stocks are involved in the portfolio.
- Here also, D'Wave quantum annealer is used.

Research Paper: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2278729
- It explains all the different models supported by D'Wave quantum annealer and which models would be good to program for quantum portfolio optimization.

Week 6

- We solved simple problems related to quadratic unconstrained binary optimization.
- Some of the links to these problems are given below:
    - https://github.com/dwave-examples/job-shop-scheduling
    - https://github.com/dwave-examples/n-queens
    - For more tutorials:
        - 35 repositories of DWave: https://github.com/dwave-examples.
- We developed the understanding of how to formulate a QUBO which can convert our classical constraint optimization problem into a QUBO problem.

Research Paper: https://arxiv.org/pdf/2012.01121.pdf

- It is the best research paper which actually guided us to the basic version of QUBO for portfolio optimization.

Binary portfolio optimization:

- It is explained in the document for practical implementation, section 1.1: Implementation_Instructions_PO.pdf

Week 7

Increasing precision of the portfolio.

- Here the weights can be fractional.
- It is explained in the document for practical implementation, section 1.2: Implementation_Instructions_PO.pdf

Week 8

Backtesting:

- Intensive testing of the quantum portfolio optimization algorithm was done.
- Backtesting is a tool which measures the performance of the portfolio over a time period.
- We have given the results of backtesting in the document for practical implementation, section 3. Implementation_Instructions_PO.pdf

Code:

Entire codebase is explained in section 2 of the document for practical implementation.
Implementation_Instructions_PO.pdf

Future Work: section 4 of the document for practical implementation.
Implementation_Instructions_PO.pdf