

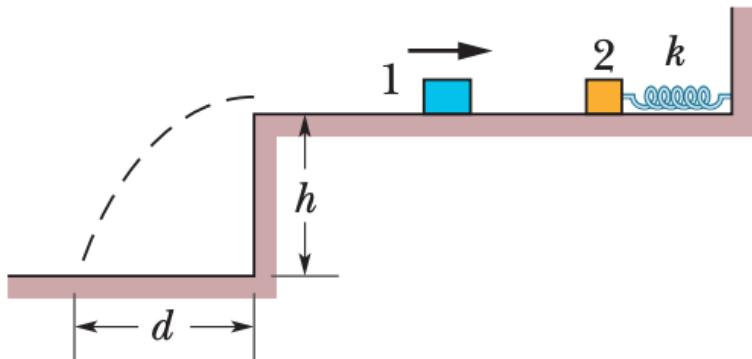
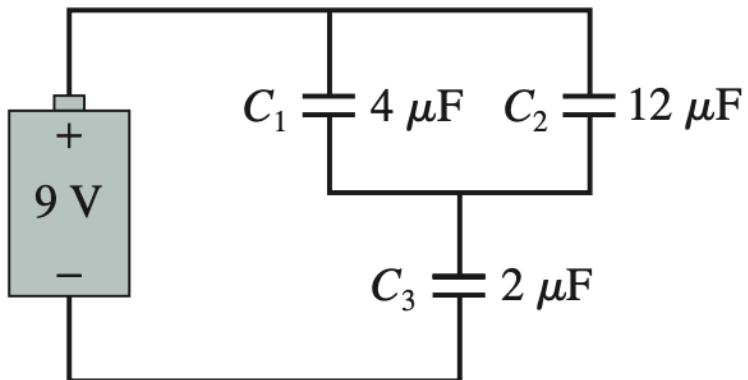
Question 1:

Figure shows block 1 of mass 0.200 kg sliding to the right over a frictionless elevated surface at a speed of 8.00 m/s . The block undergoes an elastic collision with stationary block 2, which is attached to a spring of spring constant 1208.5 N/m . (Assume that the spring does not affect the collision.) After the collision, block 2 oscillates in SHM with a period of 0.140 s , and block 1 slides off the opposite end of the elevated surface, landing a distance d from the base of that surface after falling height $h = 4.90 \text{ m}$. What is the value of d ?

Question 2:

What is the equal charge on the C_3 ? What is the equal charge on the C_3 ?

Q1

Step 1:

$$k = 1208.5 \text{ N/m}, T = 0.140 \text{ s} \Rightarrow m_2 = \frac{T^2 k}{4\pi^2} \approx 0.6 \text{ kg}$$

Step 2:

$$\text{Elastic collision} \Rightarrow v_f = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.2 - 0.6}{0.2} \cdot 2 = -4 \text{ m/s}$$

Step 3:

$$\text{block 1 falls } 4.9 \text{ m, } v_x = 4 \text{ m/s} \Rightarrow t = \sqrt{\frac{2h}{g}} = 1 \text{ s}$$
$$\Rightarrow d = v_x \cdot t = 4 \text{ m}$$

Q2:

Step 1:

C_1, C_2 are parallel ; in series with C_3

Step 2:

$$A. C_{12} = C_1 + C_2 = 16 \mu F$$

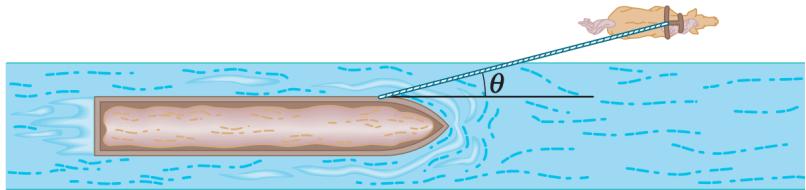
$$B. \frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} \Rightarrow C_{eq} = \frac{16}{9} \mu F$$

Step 3:

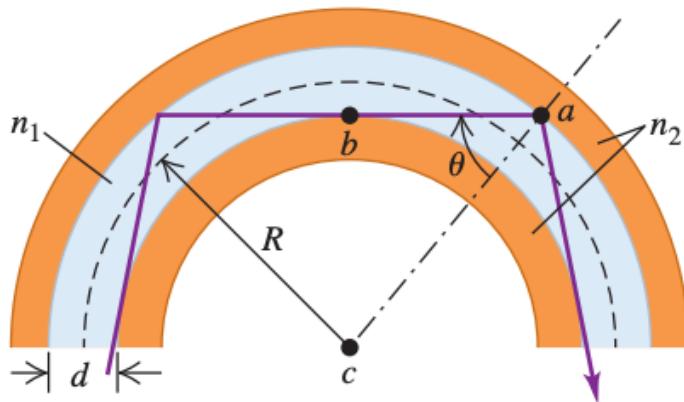
$$Q = C_{eq} \cdot V = \frac{16}{9} \cdot 9 = 16 \mu C$$

Step 4:

$$\text{Charge on } C_3 \Rightarrow Q_{C_3} = 16 \mu C$$

Question 3:

In earlier days, horses pulled barges down canals in the manner shown in figure. Suppose the horse pulls on the rope with a force of 7900 N at an angle of $\theta = 18^\circ$ to the direction of motion of the barge, which is headed straight along the positive direction of an x axis. The mass of the barge is 9500 kg , and the magnitude of its acceleration is 0.12 m/s^2 . What is direction (relative to positive x) of the force on the barge from the water?

Question 4:

A fiber-optic cable consists of a thin cylindrical core with thickness d made of a material with index of refraction n_1 , surrounded by cladding made of a material with index of refraction $n_2 < n_1$. Light rays traveling within the core remain trapped in the core provided they do not strike the core-cladding interface at an angle larger than the critical angle for total internal reflection. How much longer would it take light to travel 1.00 km through the cable than 1.00 km in air?

Q3:

Step 1
 $\vec{F}_{\text{net}} = \vec{T} + \vec{F}_w \Rightarrow \vec{F}_w = \vec{F}_{\text{net}} - \vec{T}$

Step 2

$$\vec{F}_{\text{net}} = ma = 9500 \cdot 0.12 = 1140 \text{ N} : \quad T_x = 7900 \cdot \cos(18^\circ) = 7514 \text{ N}$$
$$T_y = 7900 \cdot \sin(18^\circ) \approx 2441 \text{ N}$$

$$\Rightarrow F_{w,x} = 1140 - 7514 = -6374 \text{ N}$$

$$F_{w,y} = 0 - 2441 \text{ N} = -2441 \text{ N}$$

Step 3

$$\theta = \tan^{-1} \left(\frac{2441}{6374} \right) \approx 20.9^\circ$$

$$\text{So: direction relative to } +x = 20.9^\circ + 18^\circ = 200.9^\circ$$

Q4:

Step 1:

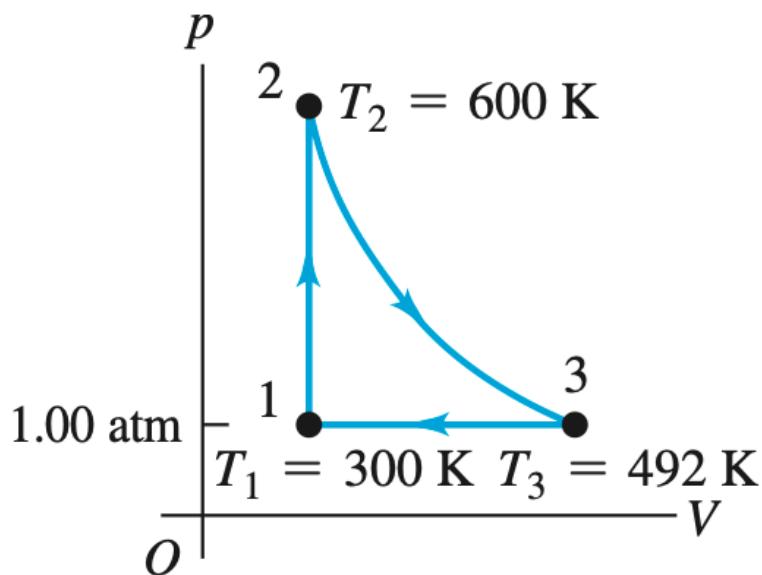
$$\text{Time to travel in air } t_{\text{air}} = \frac{L}{v_{\text{air}}} = \frac{1 \times 10^3}{3 \times 10^2} = 3.33 \times 10^{-6} \text{ s}$$

Step 2:

$$\text{Time to travel in fabric core } t_{\text{fiber}} = \frac{L/\cos\theta}{v_{\text{core}}} = \frac{L u_1}{c \cos\theta}$$

$$\Delta t = t_{\text{fiber}} - t_{\text{air}} = \frac{L}{c} \left(\frac{u_1}{\cos\theta} - 1 \right)$$

$$= (3.33 \times 10^{-6} \text{ s}) \left(\frac{u_1}{\cos\theta} - 1 \right)$$

Question 5:

A heat engine takes 0.350 mol of a diatomic ideal gas around the cycle shown in the pV -diagram of figure. Process $1 \rightarrow 2$ is at constant volume, process $2 \rightarrow 3$ is adiabatic, and process $3 \rightarrow 1$ is at a constant pressure of 1.00 atm. The value of γ for this gas is 1.40. Find the net work done by the gas in the cycle.

Q5 :

Step 1

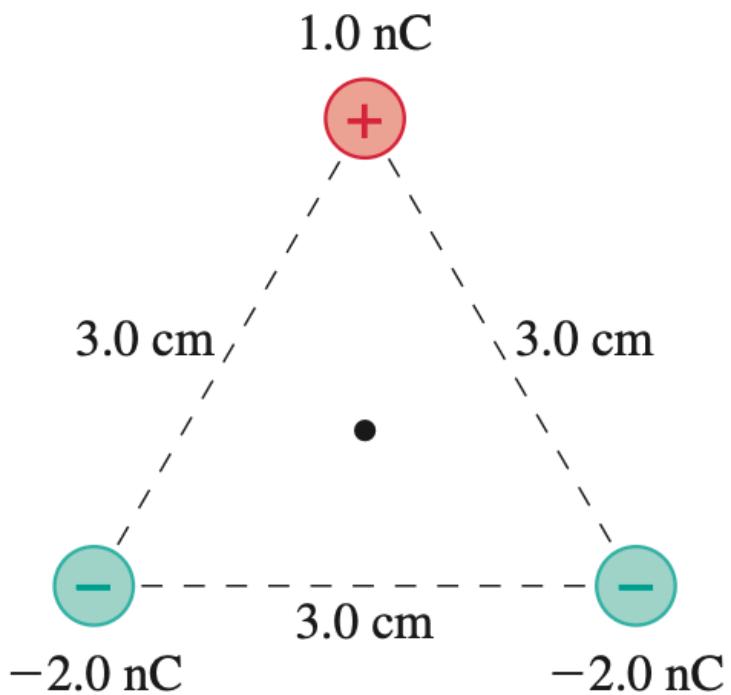
$$W_{3 \rightarrow 1} = -P\Delta V = -nR(T_1 - T_3) = -(0.35)(8.314)(300 - 492) \approx 559 \text{ J}$$

Step 2

$$W_{2 \rightarrow 3} = \frac{nR(T_3 - T_2)}{\gamma - 1} = \frac{(0.35)(8.314)(492 - 600)}{1.4 - 1} \approx -786 \text{ J}$$

Step 3

$$W_{\text{net}} = W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = -786 + 559 = -227 \text{ J}$$

Question 6:

What is the electric potential at the point indicated with the dot? What is the electric potential at the point indicated with the dot?

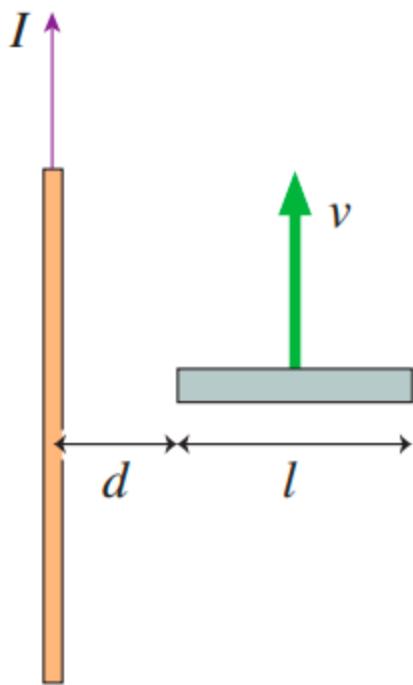
Step 1:

$$\text{Electric potential } V = \frac{kq}{r} ; \quad r = \frac{s}{\sqrt{3}} = 0.01732 \text{ m}$$

Step 2:

$$V = \frac{k}{r} (q_1 + q_2 + q_3) = \frac{8.99 \times 10^9}{0.01732} (1 - 2 - 2) \times 10^{-9}$$

$$\approx -1557 \text{ V}$$

Question 7:

The metal wire in figure moves with speed v parallel to a straight wire that is carrying current I . The distance between the two wires is d . Find an expression for the potential difference between the two ends of the moving wire.

A. $(\mu v_0 I / \pi) \ln[(d + l)/d]$

B. $(\mu v_0 I / 3\pi) \ln[(d + l)/d]$

C. $(\mu v_0 I / 2\pi) \ln[(d + l)/d]$

D. $(\mu v_0 I / 6\pi) \ln[(d + l)/d]$

Step 1:

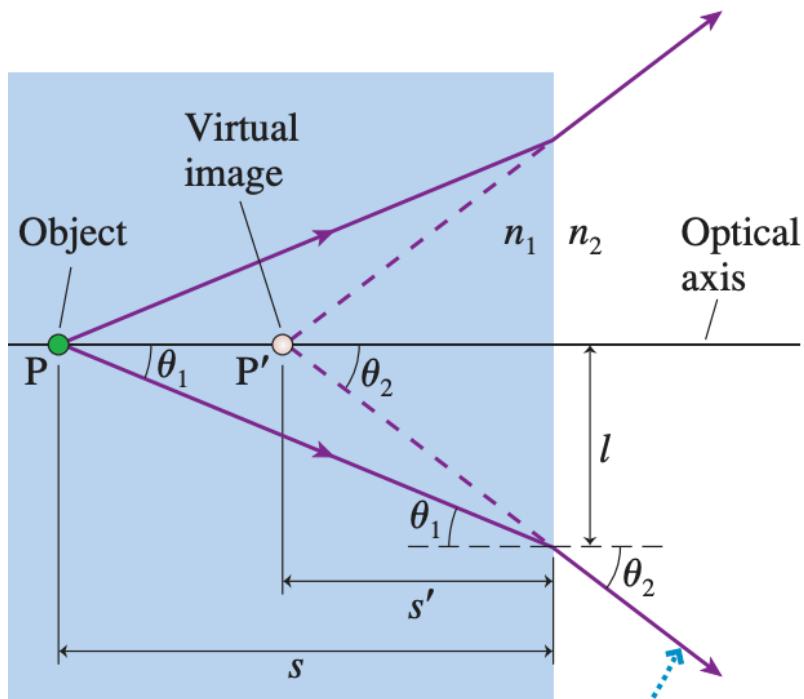
$$\text{Magnetic field } B(r) = \frac{\mu_0 I}{2\pi r}$$

Step 2:

$$\text{EMF of the segment } dV = -v B(r) dr$$

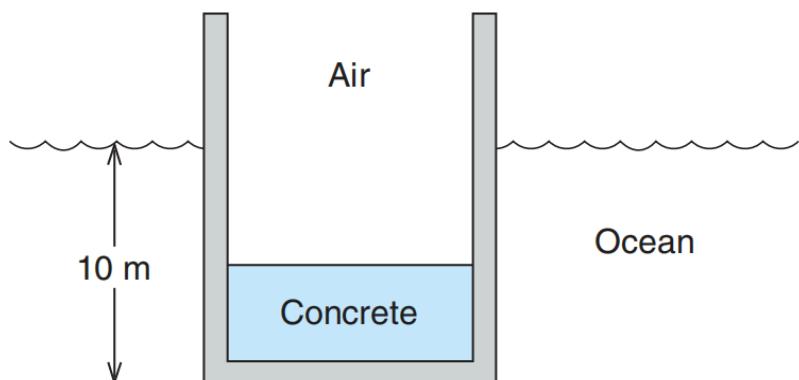
$$V = \int_d^{d+l} -v B(r) dr$$

$$= \frac{\mu_0 I v}{2\pi} \ln\left(\frac{d+l}{d}\right)$$

Question 8:

Rays diverge from the virtual image at P' .

A fish and a sailor look at each other through a 5.0 cm -thick glass porthole in a submarine. There happens to be an air bubble right in the center of the glass. How far behind the surface of the glass does the air bubble appear to the fish?

Question 9:

A steel tank of cross-sectional area 3 m^2 and height 16 m weighs $10\,000\text{ kg}$ and is open at the top, as shown in figure. We want to float it in the ocean so that it is positioned 10 m straight down by pouring concrete into its bottom. How much concrete should we use?

Q8:

Step 1:

Given $l = 5 \text{ cm}$, $S = 2.5 \text{ cm}$

We know $n_{\text{glass}} = 1.5$; $n_{\text{water}} = 1.33$

Step 2:

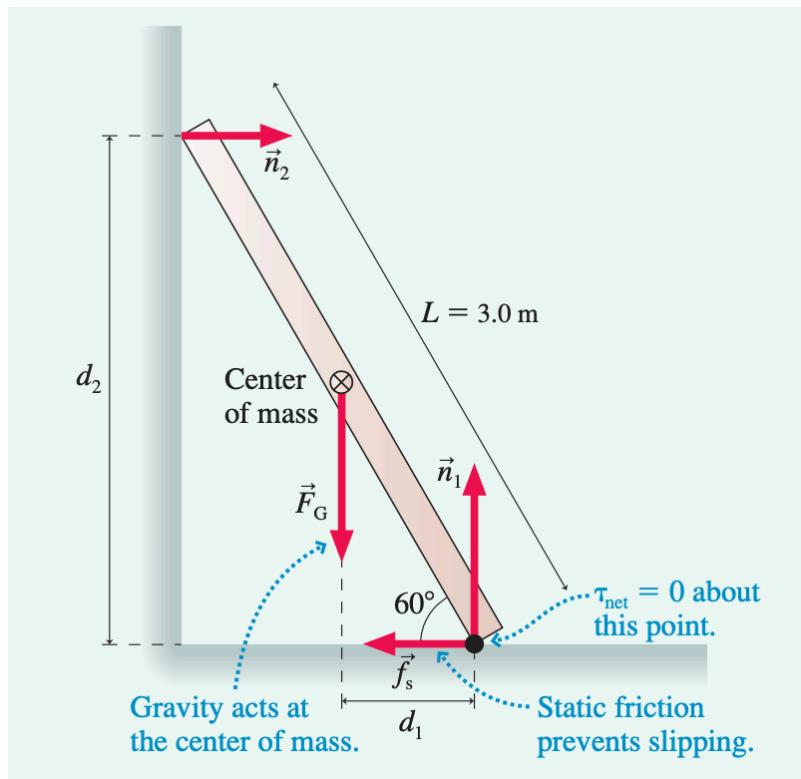
$$S' = \frac{n_2}{n_1} S = \frac{1.33}{1.50} \cdot 2.5 \approx 2.22 \text{ cm}$$

Q9:

Step 1

Since buoyance = tank weight + concrete weight

$$\Rightarrow m_{\text{concrete}} = \rho_{\text{water}} Ah - m_{\text{tank}} = (1025)(3)(10) - 1000 \\ = 20750 \text{ kg}$$

Question 10:

A 3.0 m-long ladder leans against a frictionless wall at an angle of 60° . The coefficient of static friction with the ground, that prevents the ladder from slipping. What is the minimum value of μ_s ?

Step 1:

$$f_s = n_2 ; \quad n_1 = W$$

Step 2:

$$\text{Torque due to weight (clockwise)} : \tau_w = W \cdot \frac{L}{2} \cos \theta$$

$$\text{Torque due to wall force (counter-clockwise)} : T_{N_2} = N_2 \cdot L \sin \theta$$

$$\tau_w = T_{N_2} \Rightarrow N_2 = \frac{W}{2} \cot \theta$$

Step 3:

$$\text{To prevent slipping} \Rightarrow f_s \leq \mu_s N_1 \Rightarrow \frac{W}{2} \cot \theta \leq \mu_s W \Rightarrow \mu_s \geq \frac{1}{2} \cot \theta$$

$$\theta = 60^\circ \Rightarrow \underline{\mu_s \geq 0.289}$$

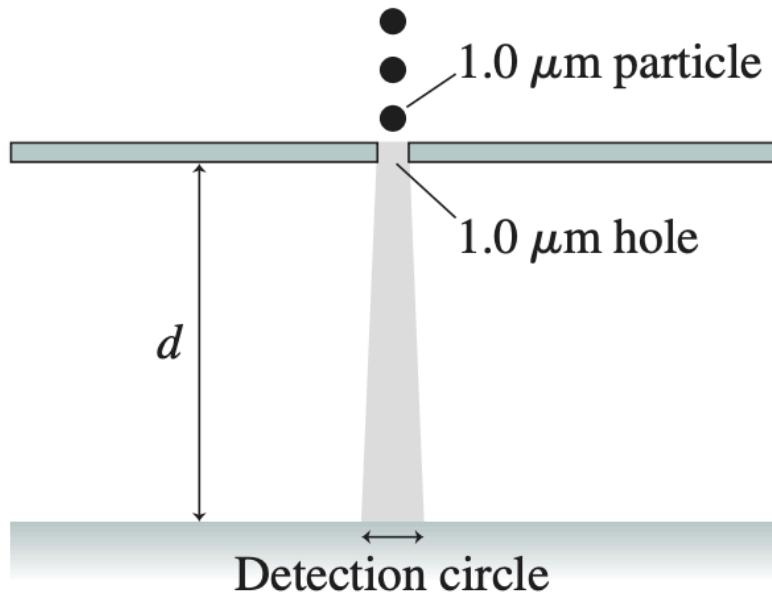
Question 11:

Figure shows 1.0- μm -diameter dust particles ($m = 1.0 \times 10^{-15} \text{ kg}$) in a vacuum chamber. The dust particles are released from rest above a 1.0- μm -diameter hole, fall through the hole (there's just barely room for the particles to go through), and land on a detector at distance d below.

Quantum effects would be noticeable if the detection-circle diameter increased by 10% to 1.1 μm . At what distance d would the detector need to be placed to observe this increase in the diameter?

Step 1:

$$\text{de Broglie wavelength } \lambda = \frac{h}{p} = \frac{h}{m\sqrt{2g\ell}}$$

Step 2:

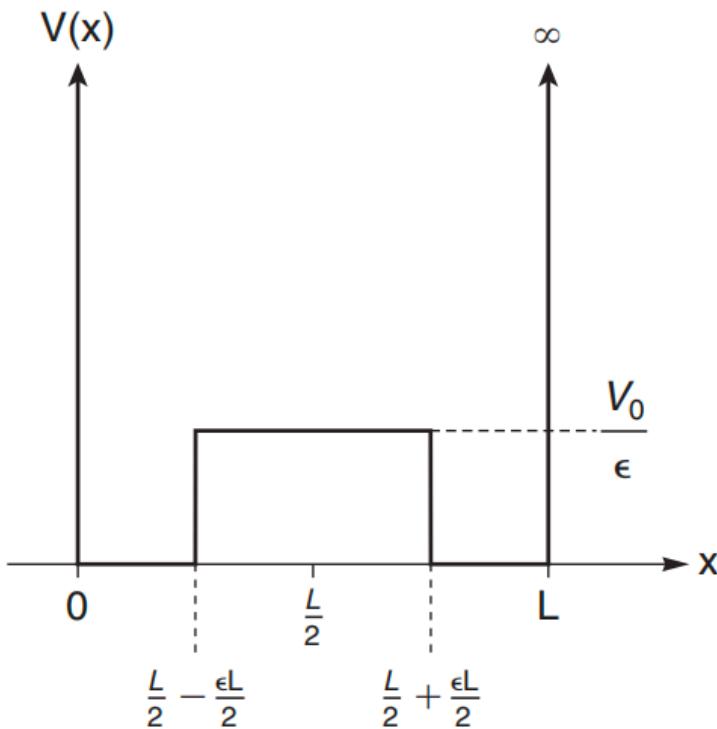
$$1.1 \times 10^{-6} = 2d \cdot \frac{h}{am\sqrt{2g\ell}} \Rightarrow \frac{1.1 \times 10^{-6}}{2} = d \cdot \frac{h}{am\sqrt{2g\ell}}$$

$$\frac{1.1 \times 10^{-6}}{2} = \frac{h\sqrt{2g}}{am\sqrt{2g}} \Rightarrow \sqrt{d} = \frac{1.1 \times 10^{-6}}{2} \cdot \frac{am\sqrt{2g}}{h}$$

Step 3:

$$h = 6.626 \times 10^{-34}, \quad a = 1 \times 10^{-6} \text{ m}, \quad m = 1 \times 10^{-15} \text{ kg}$$

$$\sqrt{d} = \frac{0.55 \times 10^{-6} \cdot 1 \times 10^{-6} \cdot 1 \times 10^{-15} \cdot \sqrt{2 \times 9.8}}{6.626 \times 10^{-34}} \approx 1.35 \times 10^{13} \text{ m}$$

Question 12:

Consider an infinite square well potential with walls at $x = 0$ and $x = L$; that is, $V(x) = 0$ for $0 < x < L$; $V(x) = \infty$ otherwise. Now impose a perturbation on this potential of the form $H' = LV_0\delta(x - L/2)$, where $\delta(x)$ is the Dirac delta function. Now consider the case where we impose a perturbation on the infinite square well potential as shown in figure, with ϵ a small number. Calculate the first-order correction to the energy of the ground state of the infinite well.

- A. 0
- B. V_0
- C. $3V_0$
- D. $2V_0$

First order correction to the energy due to perturbation

$$E_1^{(n)} = \int_0^L |\psi_n(x)|^2 V(x) dx ; \quad \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$= \int_{\frac{L}{2}-\frac{\epsilon L}{2}}^{\frac{L}{2}+\frac{\epsilon L}{2}} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \right)^2 V_0 dx = \frac{2V_0}{L} \int_{-\frac{\epsilon L}{2}}^{\frac{\epsilon L}{2}} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$x = \frac{L}{2} + y, \quad y \in \left[-\frac{\epsilon L}{2}, \frac{\epsilon L}{2}\right] \Rightarrow E_1^{(n)} = \frac{2V_0}{L} \int_{-\frac{\epsilon L}{2}}^{\frac{\epsilon L}{2}} \cos^2\left(\frac{n\pi y}{L}\right) dy = \epsilon V_0$$

So, the answer is V_0

Question 13:

A heat engine's high temperature T_H could be ambient temperature, because liquid nitrogen at 77 K could be T_L and is cheap. The Carnot engine made use of heat transferred from air at room temperature (293 K) to the liquid nitrogen fuel (figure). What would be the efficiency of a Carnot engine?

Step 1:

$$\text{Carnot efficiency formula : } \eta = 1 - \frac{T_L}{T_H} = 1 - \frac{77}{293} = \underline{\underline{0.77}}$$

Question 14:

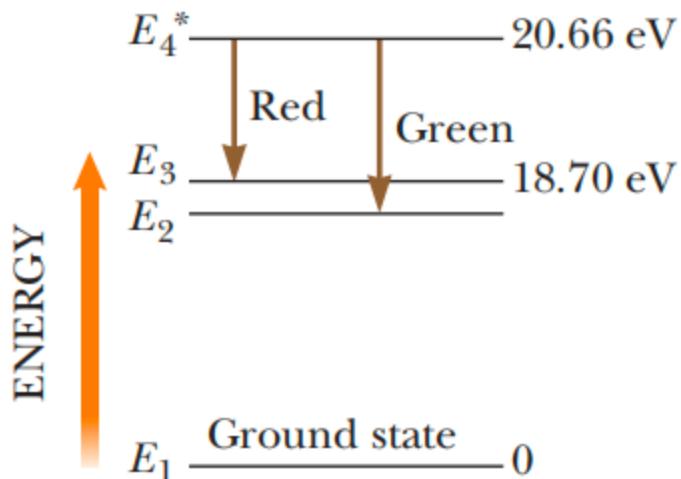
A common device for entertaining a toddler is a jump seat that hangs from the horizontal portion of a doorframe via elastic cords. Assume that only one cord is on each side in spite of the more realistic arrangement shown. When a child is placed in the seat, they both descend by a distance d_s as the cords stretch (treat them as springs). Then the seat is pulled down an extra distance d_m and released, so that the child oscillates vertically, like a block on the end of a spring. Suppose you are the safety engineer for the manufacturer of the seat. You do not want the magnitude of the child's acceleration to exceed $0.20g$ for fear of hurting the child's neck. If $d_m = 10\text{ cm}$, what value of d_s corresponds to that acceleration magnitude?

Step 1 :

$$k d_s = mg \Rightarrow k = \frac{mg}{d_s}$$

Step 2 :

$$\begin{aligned} \text{Max acceleration formula } a_{\max} &= \omega^2 A = (\sqrt{\frac{k}{m}})^2 A = \frac{k}{m} A \\ &= \frac{mg}{md_s} A = \frac{g}{d_s} A = 0.2g \\ \Rightarrow d_s &= 5A \end{aligned}$$

Question 15:

A helium-neon laser can produce a green laser beam instead of a red one. Figure shows the transitions involved to form the red beam and the green beam. After a population inversion is established, neon atoms make a variety of downward transitions in falling from the state labeled E_4^* down eventually to level E_1 (arbitrarily assigned the energy $E_1 = 0$). The atoms emit both red light with a wavelength of 632.8 nm in a transition $E_4^* - E_3$ and green light with a wavelength of 543 nm in a competing transition $E_4^* - E_2$. What is the energy E_2 ? Assume the atoms are in a cavity between mirrors designed to reflect the green light with high efficiency but to allow the red light to leave the cavity immediately. Then stimulated emission can lead to the buildup of a collimated beam of green light between the mirrors having a greater intensity than that of the red light. To constitute the radiated laser beam, a small fraction of the green light is permitted to escape by transmission through one mirror. The mirrors forming the resonant cavity can be made of layers of silicon dioxide (index of refraction $n = 1.458$) and titanium dioxide (index of refraction varies between 1.9 and 2.6).

- A. 24.16 eV
- B. 22.01 eV
- C. 15.31 eV
- D. 18.37 eV

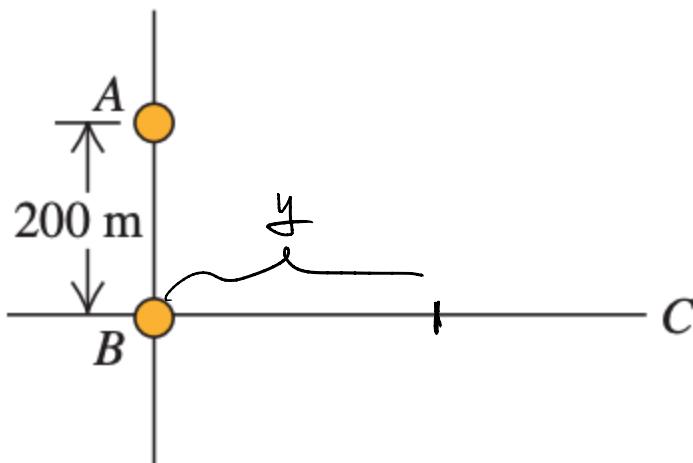
Step 1:

Photo energy formula :

$$E = \frac{hc}{\lambda} = \frac{(4.1357 \times 10^{-15})(3 \times 10^8)}{543 \times 10^{-9}} \approx 2.285 \text{ eV}$$

Step 2:

$$E_4^* - E_2 = 2.285 \text{ eV} \Rightarrow E_2 = 20.66 - 2.285 \\ = 18.37 \text{ eV}$$

Question 16:

Step 1:

$$\text{Wave length } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{5.8 \times 10^6} \approx 51.7 \text{ m}$$

Step 2:

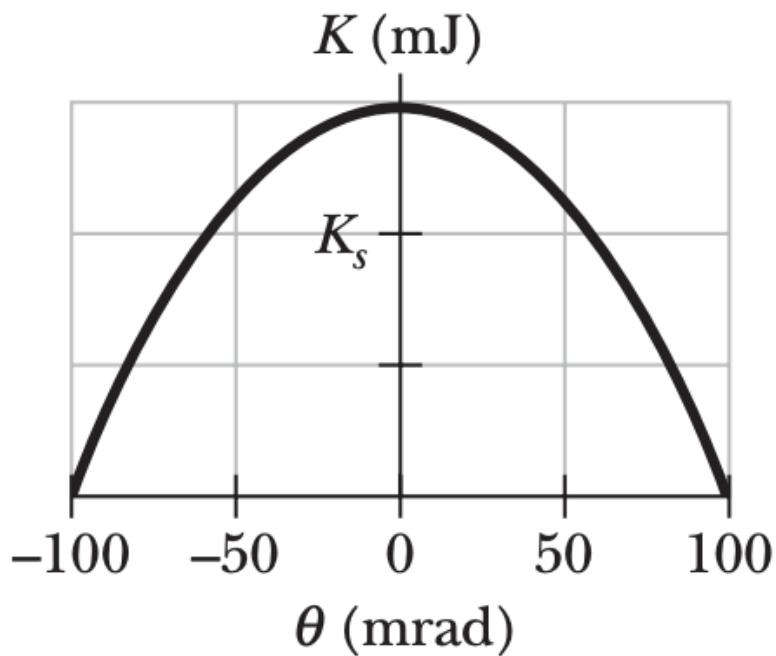
$$\text{Destructive} \Rightarrow \Delta r = (m + \frac{1}{2}) \lambda \\ = \sqrt{y^2 + 200^2} - y$$

Step 3:

$$\text{When } m=0$$

$$\frac{1}{2} \cdot 51.7 = \sqrt{y^2 + 200^2} - y \\ \Rightarrow y = 760 \text{ m}$$

Two radio antennas radiating in phase are located at points A and B, 200 m apart (figure). The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point B along a line perpendicular to the line connecting A and B (line BC shown in figure). At what distances from B will there be destructive interference?

Question 17:

Step 1:

$$\text{Total mechanical energy } E = 0.01 \text{ J}$$

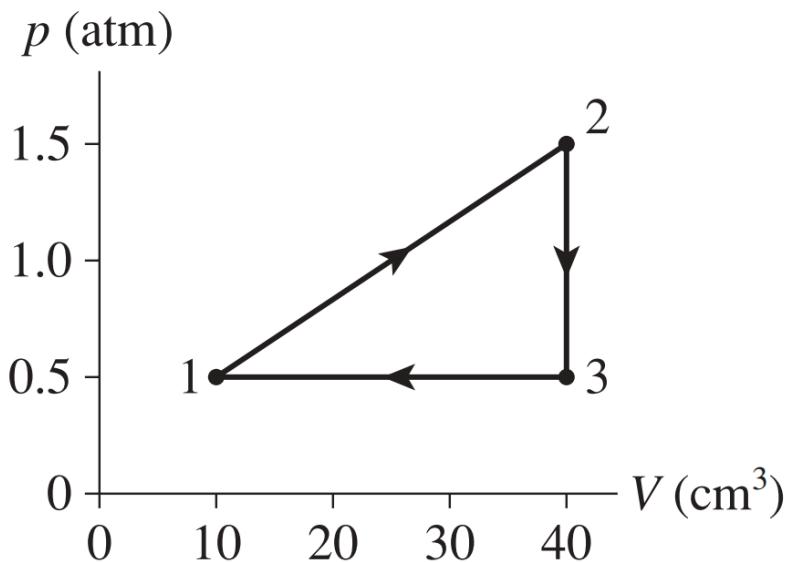
Step 2:

$$\text{Vertical height change } h \approx \frac{1}{2} L \theta^2$$

$$\Rightarrow E = 0.01 = mg \cdot \frac{1}{2} L \theta^2 \\ = 0.2 \times 9.8 \times \frac{1}{2} L (0.1)^2$$

$$\Rightarrow L \approx 1.02 \text{ m}$$

Figure shows the kinetic energy K of a simple pendulum versus its angle θ from the vertical. The vertical axis scale is set by $K_s = 10.0 \text{ mJ}$. The pendulum bob has mass 0.200 kg . What is the length of the pendulum?

Question 18:

A heat engine using a diatomic gas follows the cycle shown in figure. Its temperature at point 1 is 20 °C. What is the power output of the engine if it runs at 500 rpm?

Step 1:

$$\text{net work } W = \frac{1}{2} \cdot 30 \times 10^{-6} \cdot 1.013 \times 10^5 = 1.52 \text{ J}$$

Step 2

$$P = W \cdot f = 1.52 \cdot \frac{500}{60} \approx 12.7 \text{ W}$$