

Discrete Maths : 1  
Workshop Unit 1 : Sets

SCC120 Fundamentals of Computer Science

*Exercise 1*

Determine whether each of the following pairs of sets is equal

- a)  $\{1, 3, 5\}$  and  $\{5, 3, 1\}$
- b)  $\{1, 3, 5\}$  and  $\{5, 1, 6\}$

*Exercise 2*

Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{0, 3, 6\}$ . Find

- a)  $A \cap B$
- b)  $A \cup B$
- c)  $A - B$
- d)  $B - A$

*Exercise 3*

Let  $A = \{0, 2, 4, 6, 8, 10\}$ ,  $B = \{0, 1, 2, 3, 4, 5, 6\}$  and  $C = \{4, 5, 6, 7, 8, 9, 10\}$ . Find

- a)  $A \cap B \cap C$
- b)  $A \cup B \cup C$
- c)  $(A \cup B) \cap C$
- d)  $(A \cap B) \cup C$

*Exercise 4*

If  $A = \{a, b, c, y\}$ ,  $B = \{a, b, c, d, e\}$ ,  $C = \{x, y\}$  evaluate:

- a)  $A \cup (B \cap C)$
- b)  $(A \cup B) \cap C$
- c)  $C - A$
- d)  $(A - B) - C$
- e)  $A - (B - C)$
- f)  $(A \cap C) \cup B$
- g)  $A \cap (C \cup B)$

*Exercise 5*

Let  $A$  be the set of students who live within one mile of school and  $B$  the set of students who walk to classes. Describe the students in each of the following sets, in English.

- a)  $A \cap B$
- b)  $A \cup B$
- c)  $A - B$
- d)  $B - A$

*Exercise 6*

Let  $A = \{a, b, c\}$  and  $B = \{y, z\}$ . Find

- a)  $A \times B$
- b)  $B \times A$

*Exercise 7*

How many different elements does  $A \times B$  have if  $A$  has  $m$  elements and  $B$  has  $n$  elements?

*Exercise 8*

List the members of the following sets

- a)  $\{x \mid x \text{ is a positive integer less than } 12\}$
- b)  $\{x \mid x \text{ is the square of an integer and } x < 100\}$

*Exercise 9*

For each of the following sets, determine if 2 is an element of that set

- a)  $\{x \in \mathbb{R} \mid x \text{ is an integer greater than } 1\}$
- b)  $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\}$

Discrete Maths : 1  
Workshop Unit 2 : Relations and Functions  
SCC120 Fundamentals of Computer Science

**Part 1. Relations**

For the exercises in this part, we have the following relations on  $A = \{1, 2, 3, 4\}$ :

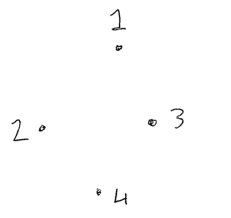
$$R1 = \{ \langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 4 \rangle \}$$

$$R3 = \{ \langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 1 \rangle, \langle 4, 4 \rangle \}$$

$$R4 = \{ \langle 2, 1 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 4, 1 \rangle, \langle 4, 2 \rangle, \langle 4, 3 \rangle \}$$

*Exercise 1*

Draw diagrams of the 3 relations, using this as your starting point.



*Exercise 2*

Which of these relations are reflexive? The ones that are not reflexive – why are they not reflexive?

*Exercise 3*

Which of these relations are symmetric?

*Exercise 4*

Which of these relations are transitive? What makes the non-transitive ones non-transitive?

**Part 2. Functions**

*Exercise 1*

Let  $A = \{a, b, c, d, e\}$  and  $B = \{1, 2, 3, 4\}$  with  
 $f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 1$  and  $f(e) = 1$ .

- (a) What is the domain of this function?
- (b) What is the co-domain?
- (c) What is the range of this function?

*Exercise 2*

Let  $f_1$  and  $f_2$  be two functions from A to B such that

$$f_1(x) = x^2 \text{ and } f_2(x) = x - x^2.$$

- a) What is the function  $f_1 + f_2$ ?
- b) What is the function  $f_1 \times f_2$ ?

*Exercise 3*

Let f and g be two functions from the set of integers to the set of integers defined by

$$f(x) = 2x + 3 \text{ and } g(x) = 3x + 2$$

- a) What is the composition of f and g?
- b) What is the composition of g and f?

*Exercise 4*

Let  $f(x) = x^2 + 1$  and  $g(x) = x + 2$  be two functions from A to B. Find the following functions

- a)  $f + g$
- b)  $fg$  (or  $f \times g$ )
- c)  $f \circ g$
- d)  $g \circ f$

*Exercise 5*

Let the function  $f(x) = ax + 5$ . Find its inverse

*Exercise 6*

Find the inverse function of  $f(x) = x^3 + 1$ , where  $x \in \mathbb{N}$ .

*Exercise 7*

For each of the following relations defined on the positive integers:

$$>, <, =, \geq, \leq$$

justify whether the relation is: (a) reflexive (b) symmetric (c) transitive

*Hint*

Build the 5 sets required where  $R \subseteq A \times A$  and  $A = \{1, 2, 3, 4, 5\}$ .

E for equal, L for less than, G for greater than, LE for less than or equal, GE for greater than or equal.

Then test each set for the 3 qualities.

Discrete Maths  
Workshop Unit 3 : Recursion

SCC120 Fundamentals of Computer Science

**Recursive Functions**

*Exercise 1 : use recursive function*

Suppose that  $f$  is defined recursively by

$$\begin{aligned}f(0) &= 3 \\ f(n) &= 2 f(n-1) + 3\end{aligned}$$

Find  $f(1)$ ,  $f(2)$ ,  $f(3)$ , and  $f(4)$ .

*Exercise 2 : define the following function recursively, using the formula  $f(n) = f(n-1) + B$  :  $f(n) = 3n - 4$ .*

*Solve using algebra rather than numerically. Also, work out the value of the base clause,  $f(0)$ .*