

补充材料

Cyclic permutation, Transposition

DEFINITION 2 Let σ be a n order permutation of $S=\{1,2,\dots,n\}$. If $\sigma(i_1)=i_2, \sigma(i_2)=i_3, \dots, \sigma(i_{k-1})=i_k, \sigma(i_k)=i_1$, and keep others unchanged, then σ is called *k-order cyclic transposition*, denoted by $(i_1 i_2 \dots i_k)$.

σ is called a *transposition*, if $k=2$.

Example

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}, \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 5 \end{pmatrix}$$

4-order cyclic transposition: $\sigma=(1\ 2\ 3\ 4)$,

2-order cyclic transposition and transposition: $\tau=(1\ 3)$.

置换：

$$\sigma = (1\ 4\ 2\ 3), \quad \tau = (1\ 2\ 3\ 4)$$

$$\sigma\tau = (1\ 3\ 2)$$

$$\tau\sigma = (2\ 4\ 3)$$

先用右边的置换，再用左边的置换

求逆，

按照正常方法求逆后，化简表示即可

群的阶和元素的阶

DEFINITION 6 The number of elements in a finite group G is called the *order of the group* and is denoted by $|G|$.

Example 12 Let us consider the group $G = \{a, e\}$, then $|G| = 2$. G is a group of order 2.

DEFINITION 7 Let G be a group and $a \in G$, then the *order of an element* a is the least positive integer n such that $a^n = e$, and is denoted by $|a|$.

If there exists no such n , then the order of a is infinity or zero. (denoted by: $a^0 = e$)

Example 13 Let $G = \{a, e\}$, e is identity element, $a*a=e$, then $\langle G, * \rangle$ is a finite group. The order of a is 2, and order of e is 1.