Exercise 1.1

1. Let Z be the set of integers, and $*$ is a usual product, is $< Z,* >$ an algebraic system?

2. Let R be the set of real numbers, and $*$ is a usual division, is $*$ a commutative on R?

3. Let Z be the set of integers, for any $a, b \in Z$, $a * b = a^2 + b - a$, is the operation $*$ commutative or associative on Z?

4. Let $S = \{a, b, c\}$ Show the properties of the operation $*$

$*$	a	b	c
a	a	b	c
b	a	c	b
c	b	a	c

Exercise 1.2

1. Let $< Z,+>$ be an algebraic structure, where Z is the set of integers, and $*$ is a usual product, is $< Z,+>$ a semigroup?

2. Let $< R,+>$ be an algebraic structure, where R is the set of real numbers, and $+$ is a usual addition, is $< R,+>$ a commutative semigroup?

3. Let R be the set of all real numbers, for any $a,b,c \in R$, $a * b = a^2 + b - a$, which of the $< R,*>$ is a semigroup or monoid?

4. Let $S = \{a,b,c\}$, which of the following tables defines a semigroup?

$*$	a	b	c
a	b	b	a
b	b	c	b
c	b	a	c

Table1

$*$	a	b	c
a	b	b	b
b	b	b	b
c	b	b	b

Table2

5. Let $< S,*>$ and $< T,*'>$ be monoid with identities e and e', respectively. Let $f: S \rightarrow T$ be an homomorphism from $< S,*>$ to $< T,*'>$. Thus $f(e) = e'$

6. Let f be a homomorphism from a semigroup $< S,*>$ to semigroup $< T,*'>$ If S' is a subsequence of $< S,*>$, then $f(S') = \{t \in T | t = f(s) \; for \; some \; s \in S'\}$ the image of S'under f is a sub-semigroup of $< T,*'>$

Exercise 1.3

1. $< Z, +>$ is a group. Write the n-th power of every integer.

2. Let $< G,*>$ be a group, $a, b, e \in G$, where e is the identity element then prove:
   a) $(a^{-1})^{-1} = a$
   b) $(ab)^{-1} = b^{-1}a^{-1}$
   c) $a^n a^m = a^{n+m}$
   d) $(a^n)^m = a^{nm}$