Term 2 Coursework (I) for M202204W Discrete Mathematics

Submission Deadline: APRIL 24, 2024, by 10:10 AM

Instructions:

1. Please ensure that the document is printed **single-sided on A3 paper.**
2. **Handwrite** your responses neatly and legibly.
3. Submit your completed coursework on time.

Academic Integrity:

1. This coursework must be completed independently.
2. Discussion of the coursework questions with other students is strictly prohibited.
3. Your answer sheet will be thoroughly checked for plagiarism.

Guidance:

1. You may seek clarification on the questions from the instructor.
2. The instructor can provide clarification or suggestions if you encounter difficulties.
3. However, the instructor will refrain from confirming the correctness of your answers to maintain fairness among all students.

Please ensure strict adherence to these guidelines.

This coursework constitutes 20% of your final mark.

Name: ____________________

BJTU ID: ________________________

Total score: __________________/20

1. Answer the following questions. (1 mark for total)
   a) Let $A = \{-1, -2, -4, -8\}$, and the binary operation $*$ on A is defined as: $a * b = \min\{a, b\}$.

      $\langle A, * \rangle$ is a monoid. In $\langle A, * \rangle$, the identity element is ____ and the zero element is ____.

   b) Let $\mathbb{Z}$ be the set of integers，if $\forall a, b \in \mathbb{Z}, a * b = a + b + 5$，the inverse of a will be____.

2. Answer the following questions. (1 mark for total)
   a) If the order of the group, denoted by $|G|$, is a prime number, then G has non-trivial subgroup. (T/F)____________
   b) Suppose that H is a subgroup of finite group G, $|G|$ and $|H|$ represent the number of elements in the G and H respectively. Then $|G|$ must be divided exactly by $|H|$. (T/F)____

3. Find generators of the following cyclic group. (2 marks)

   Let $G = \{[1], [2], [3], [4], [5], [6]\}$, and the binary operation $*$ on G is defined as:

*	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]
[2]	[2]	[4]	[6]	[1]	[3]	[5]
[3]	[3]	[6]	[2]	[5]	[1]	[4]
[4]	[4]	[1]	[5]	[2]	[6]	[3]
[5]	[5]	[3]	[1]	[6]	[4]	[2]
[6]	[6]	[5]	[4]	[3]	[2]	[1]

4. Let $*$ be a binary operation on a set A, and suppose that $*$ satisfies the following properties for any a,b and c in A :

a) $a = a * a$
b) $a * b = b * a$
c) $a * (b * c) = (a * b) * c$

Define relation $\mathbf{R}$ on set A where $a\mathbf{R}b$ implies $a = a * b$.

Show that $(A, \mathbf{R})$ is a poset, and for all a,b in A, the greatest lower bound $\mathrm{GLB}(a,b) = a * b$. (2 marks)

5. Let $\langle L, | \rangle$ be a poset, and $|$ be a divisible relation in L. Determine whether $\langle L, | \rangle$ is a lattice when

$L = \{1, 2, 3, 5, 6, 10, 15, 30\}$

and draw the Hasse diagram. (2 marks)

6. Let $s = (14826573)$, $t = (132)(548)(67)$. Find the inverse of them and results of st and ts (2 marks)

7. Please prove that every homomorphism image of ring is also a ring. (2 marks)

8. Let $\langle L,\vee,\wedge\rangle$ be a distributed lattice, a,b,c are any member of L. Prove that

$(a\wedge b)\vee(b\wedge c)\vee(c\wedge a)=(a\vee b)\wedge(b\vee c)\wedge(c\vee a)$ (2 marks)

9. Show that if G is a group of order n, $\forall a\in G$, $|a|$ is a factor of n, then $a^n=e$. (2 marks)

[Hint: $|a|$ represents the order of an element----the number of times that the element has to perform the group operation to finally get to the identity.]

10. Let $\langle A,\vee,\wedge,-\rangle$ is a boolean algebra, $x,y\in S$, prove $x\preccurlyeq y$ if and only if $\bar{y}\preccurlyeq\bar{x}$ (2 marks)

11. Let G be a group. Prove: A sufficient and necessary condition for G to be an Abelian group is that $f(x)=x^{-1}$ is an isomorphism mapping of G. (2 marks)
[Hint: x^{-1} means the inverse element of x]