2017 EXAMINATIONS

Lancaster University

Part I (First Year)

SCHOOL OF COMPUTING AND COMMUNICATIONS

SCC.120 Fundamentals of Computer Science (1 hour & 30 Minutes)

- **Answer any <u>THREE</u> out of the four questions.**
- **Use a <u>separate</u> answer book for <u>each</u> question.**

Question 1

1.a

i. Given the sets A={1,2} and B={3,4}, perform and show the result of A · B.

[4 marks]

ii. Given the sets A={1,2,3} and B={a,1,c}, show the result of performing A U B.

[1 mark]

iii. Given the sets A={1,2,3} and B={a,1,c}, show the result of performing A ∩ B.

[1 mark]

iv. The set A={1,2,3} is a subset of B={1,2,3}, but is not a proper subset. Explain why.

[1 mark]

1.b

i. Explain what we mean by the "domain" and "codomain" of a function.

[2 marks]

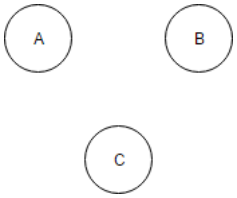
ii. The function f(x)=3x+2x, has the codomain {5,20,40,80}. Derive it's inverse function in terms g(x)=?. Use this to determine the domain. Show any working.

[4 marks]

iii. Given the domain {1,2,3} calculate the function f(x)=? that results in the codomain {6,10,14}. Show any working.

[3 marks]

1.c For all of the remaining questions in this section, start with the following nodes as the start of your graphs:



i. Draw a graph that demonstrates the transitive property. You do not have to use all the states.

[3 marks]

Question 1 continues on next page…

Question 1 continued.

ii. Draw a graph that demonstrates the reflexive property. You do not have to use all the states.

[3 marks]

iii. Draw a graph that demonstrates the symmetric property. You do not have to use all the states.

[3 marks]

[Total 25 marks]

Question 2

2.a What's the decimal equivalence of 8-bit two's complement binary 11100110?

[2 marks]

2.b Consider the following C code which takes an array of integers, the size of the array, and a target number as arguments.

```
int mystery (int* array, int size, int target) {
            int low = 0;
            int high = size - 1;
            while(low <= high) {
                    int mid = (low + high)/2;
                    if (array[mid] == target)
                             return mid;
                    else if (array[mid] < target)
                             low = mid + 1;
                    else high = mid - 1;
           }
           return -1;
}
```

Here we have an array int a[] = {-3, 4, 5, 7, 8, 12, 13, 20};

Give the results of two function calls, (1) mystery (a, 8, 5) and
(2) mystery (a, 5, 12).

[4 marks]

Question 2 continues on the next page…

2.c We have a structure type called node which is defined as follows:

```
typedef struct _node {
        char token[4];
        struct _node* next;
}node;
```

Consider the following C code:

```
1  node* pt = NULL;
2  unsigned int size;
3  node* foo(const char* s) {
4          size = sizeof(node);
5          node *p = malloc(size);
6          strcmp(p->token, s);
7          p->next = pt;
8          pt = p;
9          return p;
10 }
11
12 int main() {
13         foo("SCC");
14         foo("120");
15         foo("Paper");
16         return 0;
17 }
```

i. Assume each character occupies one byte, the word size of the target machine is 4 bytes and the target machine uses 32 bits for memory addressing. What is the value stored in variable size after calling function foo()?

[2 marks]

ii. This program has a bug which may lead to a program crash.

Which line of code in the main() function may crash the program?

[2 marks]

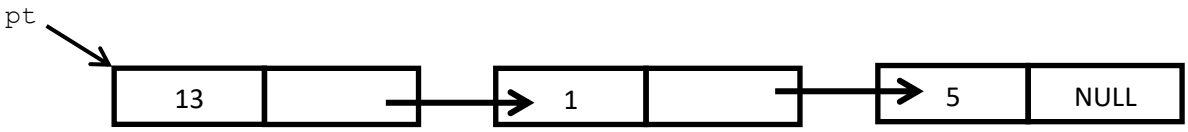
How will you change the definition of the node structure to make this example bug free? Write down your definition of node.

[3 marks]

Question 2 continues on the next page…

Question 2 continued.

iii. Assuming the bug of Q2 c.ii has been fixed. Using "box-and-arrow" notation (with an example shown below) to illustrate what the linked list (that **pt** points to) looks like after executing the main() function.



A general example for "box-and-arrow" diagrams. For question 2.c.ii, your answer would have three boxes with string values.

[4 marks]

2.d. Choose one of the following possible notations for each of the sub-questions.

1 $\log N$ $\sqrt{N}$ N $N \log N$ N^2

i. What is the worst-case running time for adding an item to the beginning of a singly linked list?

[2 marks]

ii. What is the worst-case running time for returning the item at a given position in a singly linked list?

[2 marks]

2.e

i. Convert an array with values [22, 12, 2, -10, 30, 22, 35, 6] into a min-heap. Draw the final result on the answer sheet.

[2 marks]

ii. Give the array representation of the min-heap that you have constructed in Q2.e.i. Note that the array indices start from 1 in this question.

Index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Value								

[2 marks]

[Total 25 marks]

Question 3

3.a What is the worst case complexity of wellDone? Its input is an array of integers A.

```
wellDone(int A[]) {
     for (int i=1; i<A.length; i++) {
          for (j=i-1; j>=0 && A[j]> A[i]; j--) {
               A[j+1] = A[j];
          }
          A[j+1] = A[i];
     }
}
```

[4 marks]

3.b What is the worst case complexity class of the following code fragment, where n is the size of the input array? (2 marks) If input is limited to arrays of size less than 100, what is the complexity then? (2 marks)

```
for (j=10; j<n; j=10j) {
     print j
}
```

[4 marks]

3.c What is the worst case complexity of the following code fragment?

```
int a(int n) {
     if(n is 0 or n is 1)
          return n
     else
          return a(n-1) + a(n-2)
}
```

[5 marks]

3.d For each of the following pairs of functions, state which one grows faster asymptotically for input of size n, in other words, is worse in terms of efficiency.

i. $f(n) = 1000n + 53$ and $g(n) = \log_{10}(n)$ **[2 marks]**

ii. $f(n) = n!$ and $g(n) = 2^n + n$ **[2 marks]**

iii. $f(n) = n^3$ and $g(n) = 2^n + n$ **[2 marks]**

iv. $f(n) = \log_2(n)$ and $g(n) =$ square-root(n) **[2 marks]**

Question 3 continues on next page…

Question 3 continued.

3.e Indicate **all** the statements that are true about propositional satisfiability? (Note: More than one statement may be true. No points awarded if you fail to indicate all true statements.)

i. Truth tables is a straightforward way to solve it
ii. It is intractable
iii. It can be reduced to subset-sum in polynomial time
iv. It is well-known that it is not in the complexity class P

[2 marks]

3.f If a problem, say X, is in the complexity class NP, then which of the following statements are true of X?

i. X can be verified in polynomial time
ii. X is a decision problem
iii. If an NP-complete problem can be solved in polynomial time, then X can be solved in polynomial time
iv. X cannot be solved in polynomial time

Indicate **all** true statements (Note: More than one statement may be true. No points awarded if you fail to indicate all true statements.)

[2 marks]

[Total 25 marks]

QUESTION 4 (25 marks)

4.a

For the "Set" abstract data type, what is the O() complexity to add an element and the O() complexity to remove an element? Explain your answers.

[4 marks]

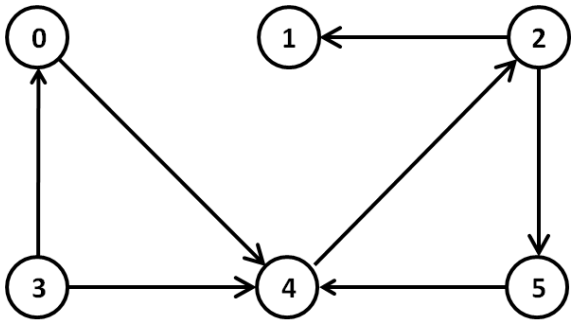
4.b

You are given four numbers (n_1, n_2, n_3, and n_4). They are added in this order into a stack. Then you remove all four of them from the stack. You then take the same four numbers (in the original order) and add them to a queue. Then you remove all four of them from the queue. Give one example of four numbers (i.e. with actual values) where the two ordered sequences (the one removed from the stack and the one removed from the queue) give the same values.

[4 marks]

4.c

Consider the directed graph **G** shown below:



i. State whether **G** is non-connected, weakly connected, or strongly connected. Explain your answer.

[2 marks]

ii. How many nodes of **G** have the same in-degree and out-degree? Explain your answer.

[2 marks]

iii. What is the density of **G**? Explain your answer.

[2 marks]

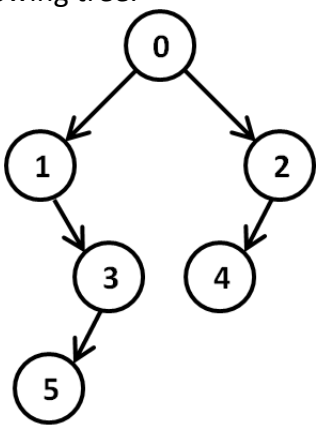
iv. Write down the order in which the nodes are visited during depth-first traversal, starting at node 0. Include the symbol ↑ wherever the traversal is forced to back up to a previous node. You can assume that adjacent nodes are selected in increasing order of their value.

[4 marks]

Question 4 continues on the next page…

4.d

This question is about the following tree.



i. What is the sequence of nodes visited by the in-order traversal?

[2 marks]

ii. How many values of "1" are there in the array for the "parent vector" representation of this tree?

[2 marks]

iii. Give a tree with a post-order traversal sequence of nodes that is the same as the sequence you have from part (i).

[3 marks]

[Total 25 marks]

---End of Paper---