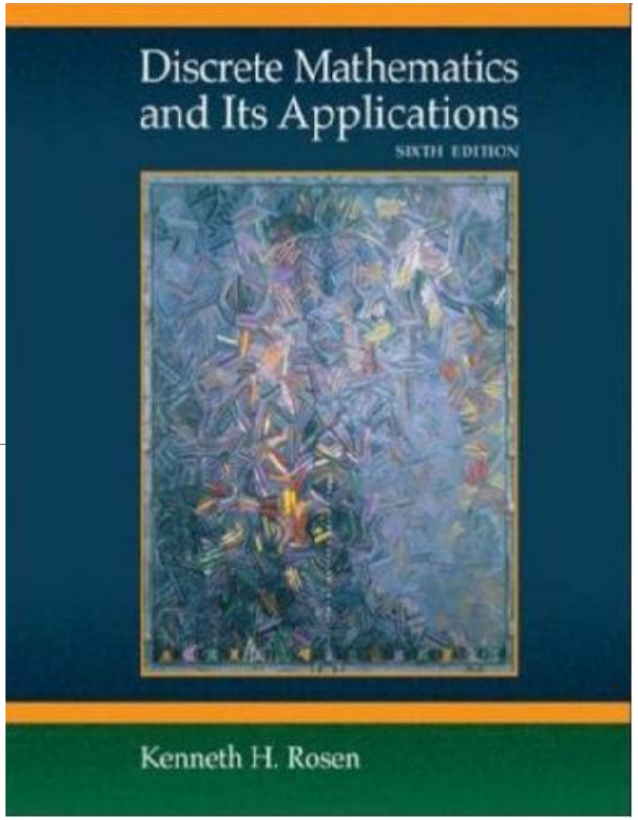
Discrete Mathematics



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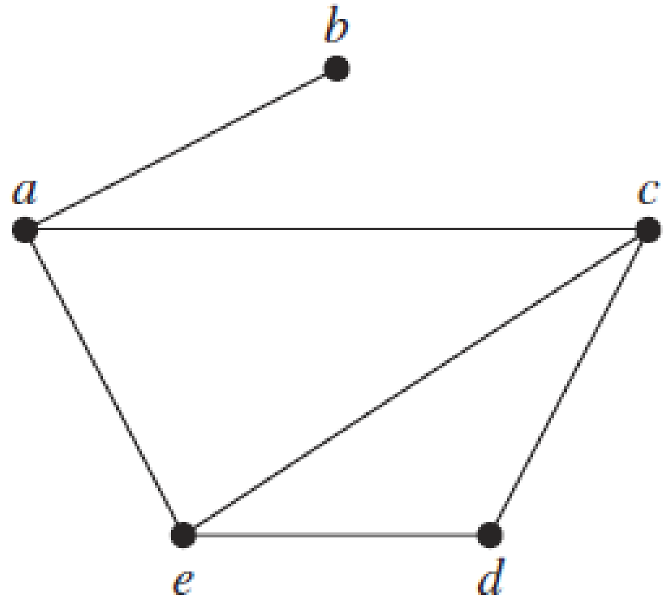
Represent a graph

- List all the edges of this graph
- **Adjacency Lists**
- **Adjacency Matrices**
- **Incidence Matrices**

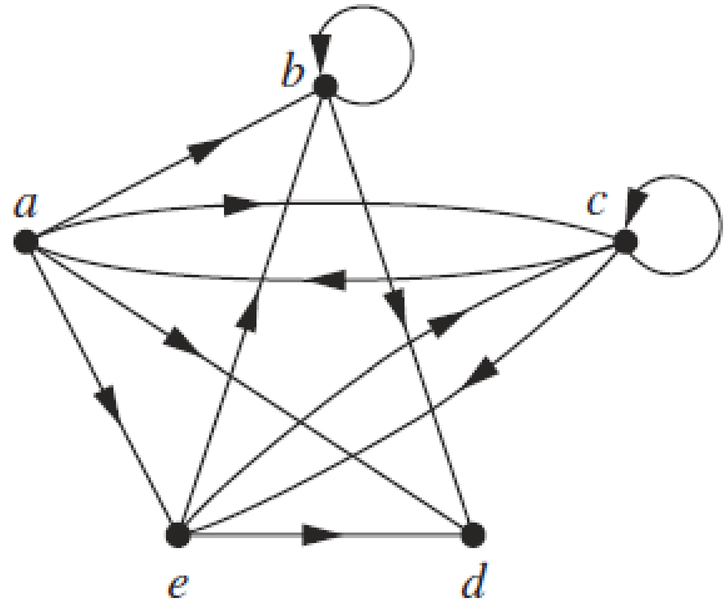
Example 1

- Use adjacency lists to describe the simple graph.



Vertex	*Adjacent Vertices*
a	*b, c, e*
b	*a*
c	*a, d, e*
d	*c, e*
e	*a, c, d*

Example 2



Initial Vertex	*Terminal Vertices*
a	*b, c, d, e*
b	*b, d*
c	*a, c, e*
d	
e	*b, c, d*

Adjacency Matrices

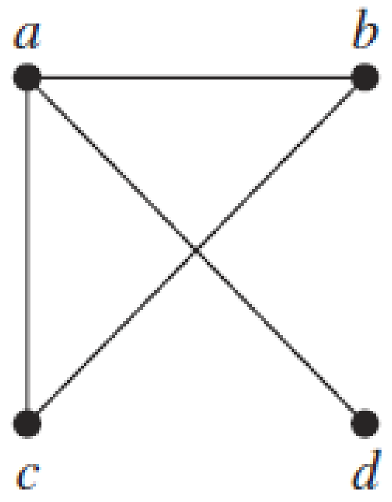
- Suppose that $G = (V, E)$ is a simple graph where $|V| = n$. Suppose that the vertices of G are listed arbitrarily as $v_1, v_2, \ldots, v_n$. The **adjacency matrix** A (or A_G) of G, with respect to this listing of the vertices, is the n x n zero–one matrix $A = [a_{ij}]$,

$$a_{ij} = \begin{cases} 1 & if \{v_i, v_j\} \text{ is an edge of } G \\ 0 & otherwise \end{cases}$$

- The adjacency matrix of a simple graph is **symmetric**.
- Because a simple graph has no loops, each entry a_{ii}, $i = 1, 2, 3, \ldots, n$, is 0.

Example 3

- Use an adjacency matrix to represent the graph. Order the vertices as *a, b, c, d*.



$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Example 4

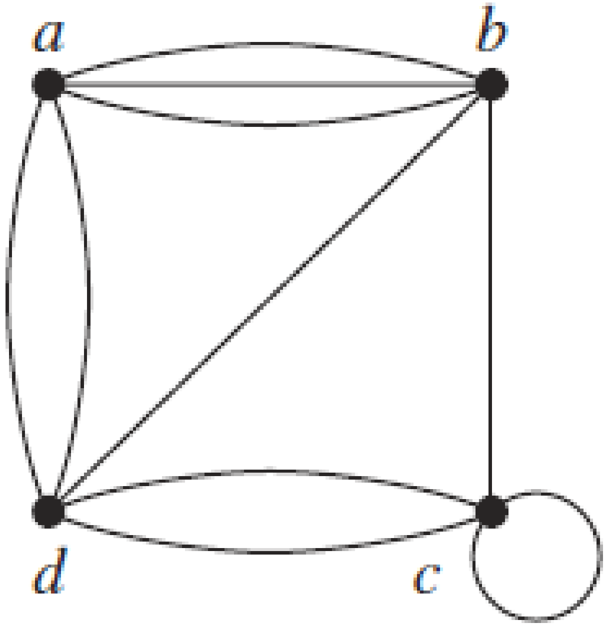
- Draw a graph with the adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Example 5

- Use an adjacency matrix to represent the **pseudograph**.

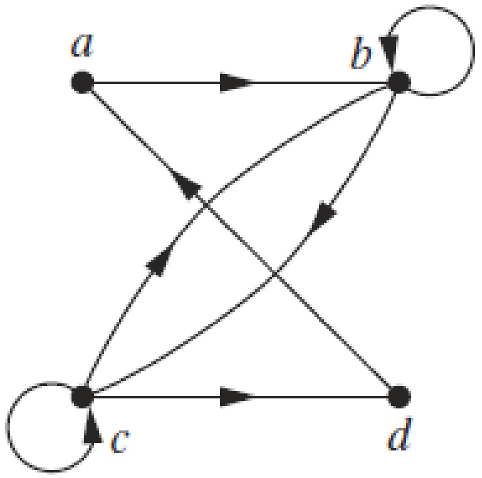
$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

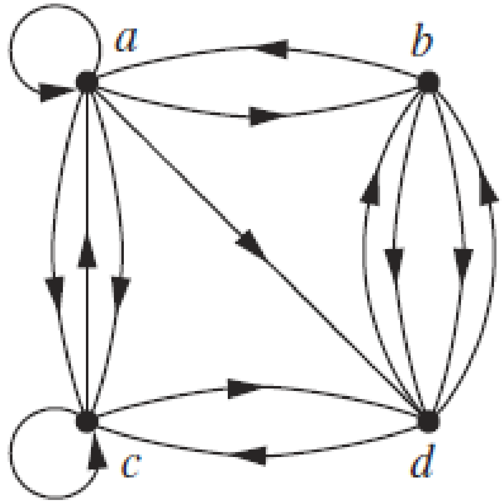


- The adjacency matrix is no longer a **zero–one** matrix.
- All undirected graphs have **symmetric** adjacency matrices.

Exercise 10.3(19, 21)

- Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.





- The adjacency matrix for a directed graph does not have to be symmetric.

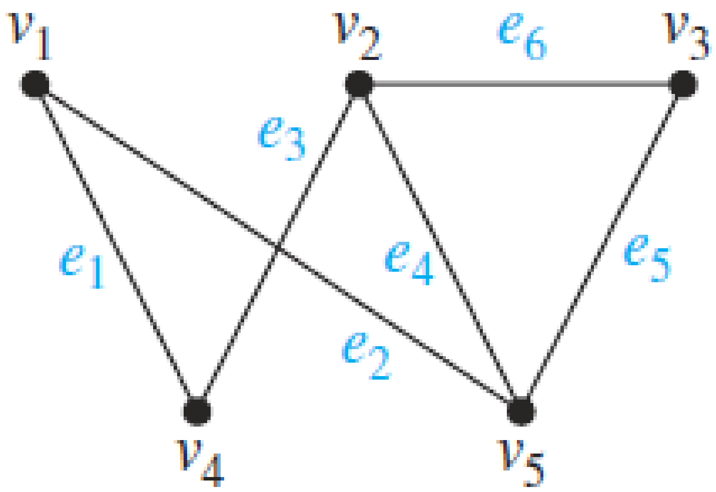
Incidence Matrices

- Let $G = (V, E)$ be an undirected graph. Suppose that $v_1, v_2, \ldots, v_n$ are the vertices and $e_1, e_2, \ldots, e_m$ are the edges of G. Then the incidence matrix with respect to this ordering of V and E is the $n \times m$ matrix $M = [m_{ij}]$, where

- $m_{ij} = \begin{cases} 1 & when\ edge\ e_j\ is\ incident\ with\ v_i \\ 0 & otherwise \end{cases}$

Example 6

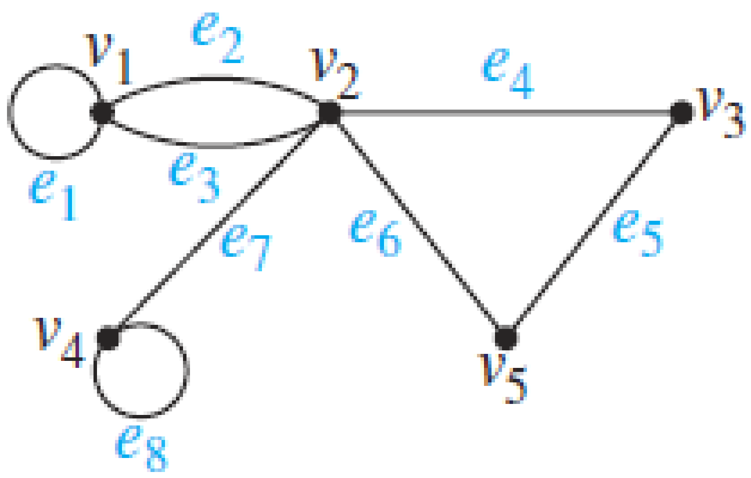
- Represent the graph with an incidence matrix.



$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}.
$$

Example 7

- Represent the pseudograph using an incidence matrix.



$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}.
$$

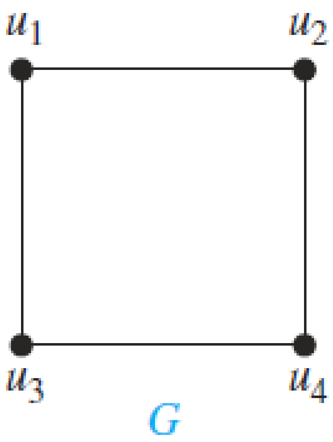
Isomorphism of Graphs

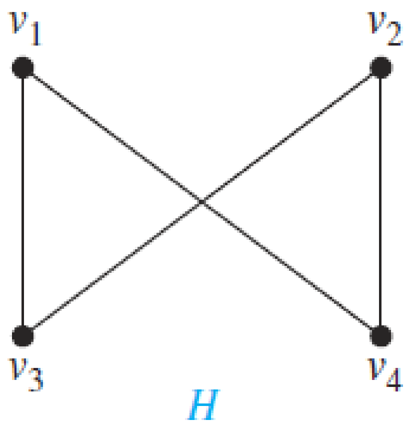
- The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic** if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an **isomorphism**.

- In other words, when two simple graphs are isomorphic, there is a one-to-one correspondence between vertices of the two graphs that preserves the adjacency relationship.

Example 8

- Show that the graphs *G* and *H* are isomorphic.





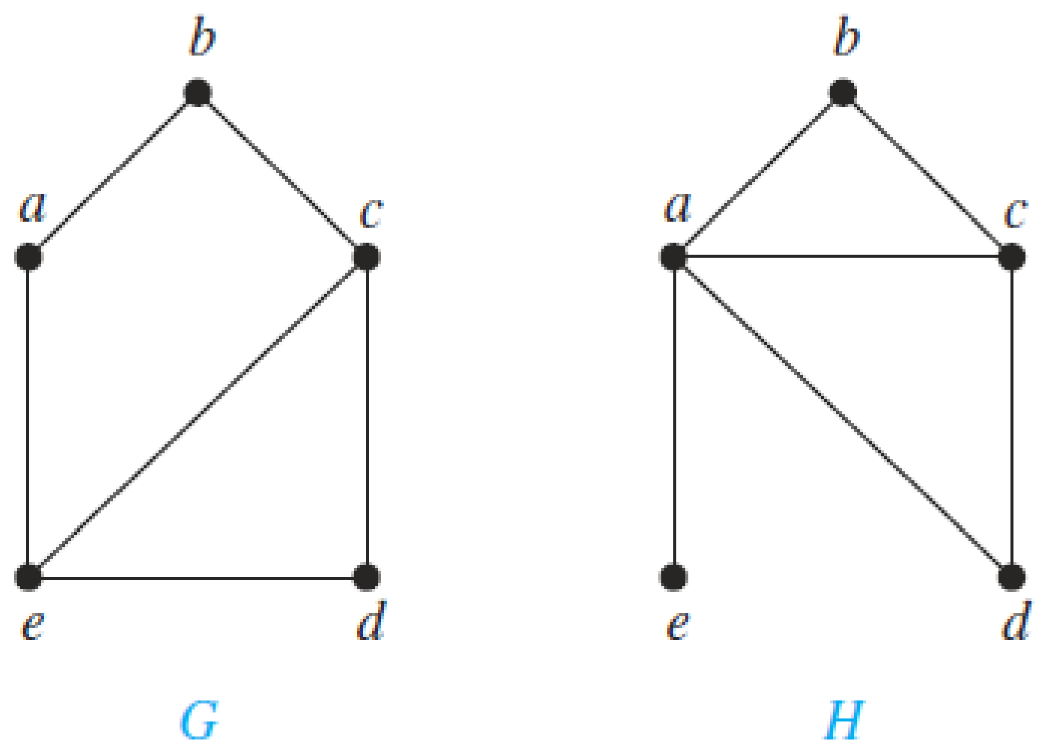
$f(u_1) = v_1$, $f(u_2) = v_4$, $f(u_3) = v_3$, and $f(u_4) = v_2$

Isomorphism of Graphs

- Necessary condition
  - ✓Same number of vertices
  - ✓Same number of edges
  - ✓A vertex v of degree d in G must correspond to a vertex $f(v)$ of degree d in H
- It is often difficult to determine whether two simple graphs are isomorphic.
- Sometimes it is not hard to show that two graphs are not isomorphic.

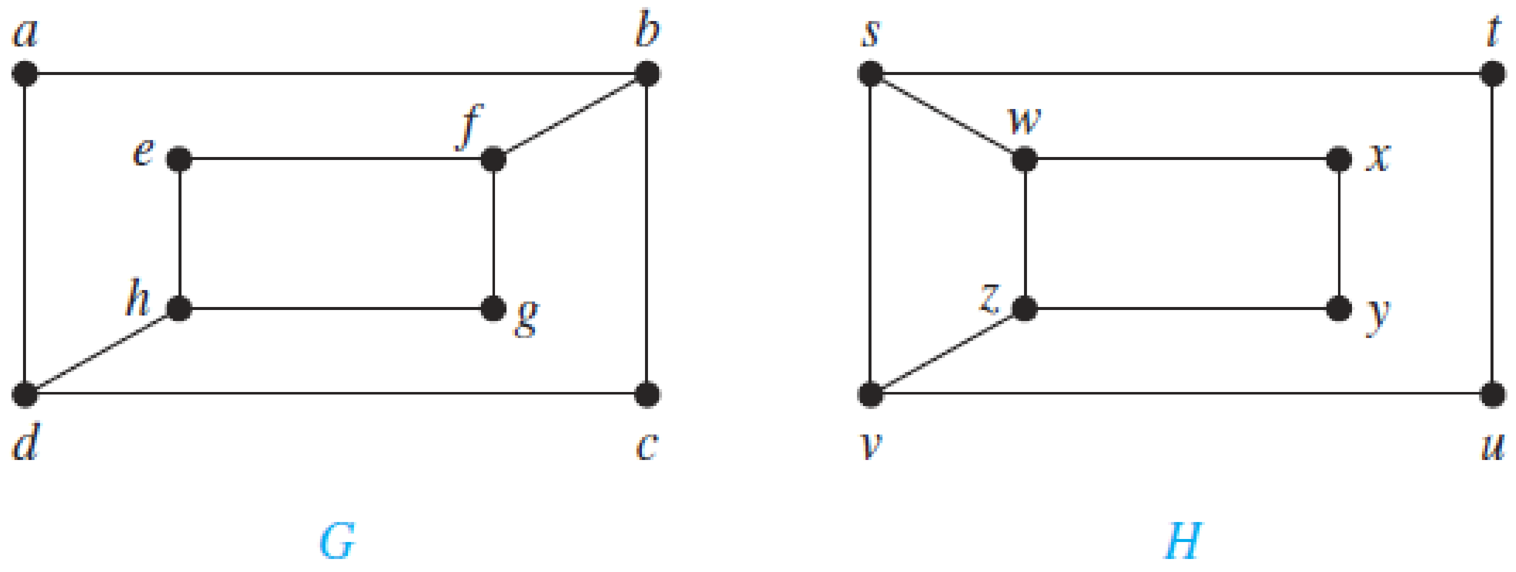
Example 9

- Show that the graphs are not isomorphic.



Example 10

- Determine whether the graphs are isomorphic.



Example 11

- Determine whether the graphs *G* and *H* are isomorphic.