## M202204W Discrete Mathematics

## Term 2 Coursework (I)

Deadline for submission: 8th May 2023, at 10:10

Please print the document single-sided on A3 paper, hand write it and submit on time.

This coursework is worth 20% of the final mark.

Please answer all questions. You must do this coursework completely by yourself. In this instance, it means you are not even allowed to discuss the coursework questions with other students. Your answer sheet will be checked for plagiarism.

You are allowed to ask the instructor about these questions. But instructor can only clarify the questions or give you suggestions if you get stuck, and instructor will try not to tell you whether you have the correct answer (to be fair to all students).

Name: _		 	
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- 1. Answer the following questions. (1 mark for total)
  - a) Let  $A = \{1, 2, 4, 8\}$ , and the binary operation \* on A is defined as:  $a * b = min\{a, b\}$ . < A, \*> is a monoid. In < A, \*>, the identity element is \_\_\_\_ and the zero element is \_\_\_\_.
  - b) Let  $\mathbb{Z}$  be the set of integers, if  $\forall a, b \in \mathbb{Z}$ , a \* b = a + b + 5, the inverse of a will be \_\_\_\_.
  - c) Define the operations  $\cup$  and  $\cap$  on negative integer set  $\mathbb{N}^-$  as follows:  $a \cup b = \max\{a, b\}$ ,  $a \cap b = \min\{a, b\}$ . Then
    - i.  $\langle N^-, U \rangle$  is a semigroup.
  - ii.  $\langle \mathbb{N}^-, \cap \rangle$  is not a semigroup.
  - iii. There is no identity element in  $\langle \mathbb{N}^-, \mathbb{U} \rangle$ .
  - v. There is no identity element in  $\langle \mathbb{N}^-, \cap \rangle$ .
  - d) Let the set  $A = \{ a + b\sqrt[4]{5} \mid a,b \in \mathbb{Q} \}$ , and the operations + and \* be the addition and multiplication of real numbers, then
    - i. The given set and operations can form a ring.
  - ii. The given set and operations can not form a field.
- 2. Answer the following questions. (1 mark for total)
  - a) Suppose that H is a subgroup of finite group G, |G| and |H| represent the number of elements in the G and H respectively. Then |G| must be divided exactly by |H|. (Y/N)
  - b) If a group is finite, the order of the element must be exactly divisible by the order of the finite group. (Y/N)\_\_\_\_\_
  - c) There can only be one generator in a cyclic group. (Y/N)\_\_\_\_
  - d) If the order of the group, denoted by |G|, is a prime number, then G has non-trivial subgroup. (Y/N)\_\_\_\_
- 3. Let \* be a binary operation on a set A, and suppose that \* satisfies the following properties for any a, b and c in A:
  - a) a = a \* a;
  - b) a \* b = b \* a;
  - c) a \* (b \* c) = (a \* b) \* c.

Define a relation R on A by a R b if and only if a = a \* b.

Show that  $(A, \mathbf{R})$  is a poset, and for all a, b in A, the greatest lower bound GLB(a, b) = a \* b. (2 marks)

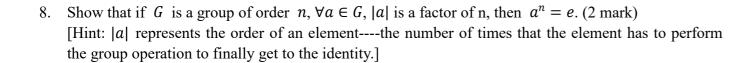
- 4. Let G be a group. Prove: A sufficient and necessary condition for G to be an Abelian group is that  $f(x) = x^{-1}$  is an isomorphism mapping of G. (2 marks)

  [Hint:  $x^{-1}$  means the inverse element of x]
- 6.  $\langle \mathbb{Z}_8, +_8 \rangle$  is an addition module 8 group, and  $\mathbb{Z}_8 = \{1, 2, 3, 4, 5, 6, 7, 0\}$ ,  $|\mathbb{Z}_8| = 8$ . Find all subgroups of  $\langle \mathbb{Z}_8, +_8 \rangle$  and their corresponding left cosets. (1 mark) [Hint:  $\langle \mathbb{Z}_n, +_m \rangle$ :  $\forall i, j \in \mathbb{Z}_n, i +_m j = (i + j) \mod m$ ]

5. Let < G, \*> be a finite semigroup, and satisfies the elimination law. Prove that G is a group. Then if  $\forall x \in G$ , x \* x = e, where e is an identity element, prove that < G, \*> is an Abelian group. (2 marks)

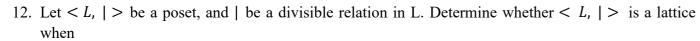
[Hint: the elimination law  $a * b = a * c \rightarrow b = c$   $a * c = b * c \rightarrow a = b$ ]

7. Prove that the  $\langle \mathbb{Z}, + \rangle$  is an infinite cyclic group where  $\mathbb{Z}$  is the set of integers. [Hint: + represent ordinary addition] (2 marks)



9. Let 
$$s = (18246573)$$
,  $t = (132)(5648)$ . Please find the inverse of them and the result of 11. Please prove that every homomorphism image of ring is also a ring. (1 mark) st and ts. (1 mark)

10. Prove that  $\langle A, \oplus, \otimes \rangle$  is a communitive ring with identity element, where  $\forall a, b \in A, a \oplus b = a + a \oplus b = a \oplus$ b-1,  $a \otimes b = a+b-ab$ . (1 mark)



 $L = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$ 

 $L = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$ 

and draw the Hasse diagrams respectively. (1 mark)

14. Let < L,  $\lor$ ,  $\land >$  be a lattice,  $\le$  is the corresponding partial order, a, b, c, d are any member of L. Prove that  $(a \land b) \lor (b \land c) \lor (c \land a) = (a \lor b) \land (b \lor c) \land (c \lor a)$ . (1 mark)

13. Let < L,  $\le$ > be a totally ordered set,  $|L| \ge 3$ . Prove that < L,  $\le$ > is a lattice, but it doesn't have a complement. (1 mark)

[Hint: If in a poset < A,  $\le$ >, for any a, b  $\in$  A, have a  $\le$  b or b  $\le$  a, then A will be a totally ordered set]

15. In bounded distributive lattices, prove that the elements with a complement will form a sub-lattice. (1 mark)