Exercise 1

1. Find out every subgroup of $<\mathbf{Z}_6, +_6>$, then find out every left coset of each subgroup.

2. Let $<H, *>$ be a subgroup of $<G, *>$, if $A=\{x|x\in G, x*H*x^{-1}=H\}$, prove that $<A, *>$ is a subgroup of $<G, *>$.

Hint:

Theorem 7

- A necessary and sufficient condition for a nonempty subset H of a group $<G, *>$ to be a subgroup is that for $\forall a, b \in H \rightarrow a * b^{-1} \in H$.

3. Let $<G, *>$ be a group, R is a relation on G such that

$R=\{(a, b)|$there exists $x\in G$ such that $b=x*a*x^{-1}\}$

Show that R is an equivalence relation on G.

4. Let $<H, *>$ be a subgroup of $<G, *>$, show that among all cosets of H, there is only one coset A such that $<A, *>$ is a subgroup of $<G, *>$.