Exercise 1.3-2

Let G be Abelian group, then $(ab)^n = a^n b^n$.

$< Z, + >$ is a infinite cyclic group. Please show that generator of Z.

If $|a| = n$, then $G = \{e, a, a^2, \cdots, a^{n-1}\}$ is a cyclic group of order n, $|G| = |a|$

$< Z_{12}, +_{12} >$ is a addition module 12 group, write the order of the group and the order of all elements of Z_{12}

Let G be a group, if $H, K \leq G$, then $H \cup K \leq G \Leftrightarrow H \subseteq K \vee K \subseteq H$

Please write all generated subgroups of group $< Z_{12}, +_{12} >$

Let G be a group, $B \subseteq G$, then $< B > = \cap \{H | H \leq G, B \subseteq H\}$ is a subgroup of G.

Let $S = \{a, b, c, d\}$, $f(a) = b, f(b) = c, f(c) = d, f(d) = a$, $F = \{f^0, f^1, f^2, f^3\}$, then $< F, \circ >$ is Abelian group.

Let G is a group, $H < G$, then

(1) $a, b \in G, Ha \cap Hb = \emptyset$ or $Ha = Hb$ and $\cup Ha = G$

(2) $H \approx Ha$ (equipotential)

(Remark: According to theorem 6, and the theorem about equivalence relation.)

Exercise 1.3-3

Please write the cyclic transposition of permutation.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 2 & 8 & 6 & 4 & 7 & 5 \end{pmatrix}$$

Show that $(i_1 i_2 \cdots i_k) = (i_1 i_2)(i_1 i_3)(i_1 i_4) \cdots (i_1 i_k)$.

Please write the product of the following cyclic transpositions.

a.$(1)(12)$ b.$(23)(12)$ c.$(123)(123)$

The group $< Z_{12}, +_{12} >$, $B = \{2,3\}$: $< B >=?$

The Klein 4-ary group, $B = \{a, b\}$: $< B >=?$

Exercise 1.4

Please show that the following algebraic systems are rings.

$< Z, +, \times >, < Q, +, \times >, < Z_E, +, \times >$, and $< C, +, \times >$

(The Q is the set of all rational numbers, the Z_E is the set of all even numbers, and the C is the set of all complex numbers.)

Please show that $< Z_n, \oplus, \otimes >$ is a ring. ($\oplus$ is addition module n, and $\otimes$ is multiplication module n.)

Please show that the following algebraic systems are fields.

$< Q+,*>, < R, +,*>, < C, +,*>$

Please show that $< Z, +, * >$ is not field.

Please show that $< Z_p, +, * >$ is finite field. (p is prime number).

($+$ is addition module n, and $*$ is multiplication module n.)

Every finite integral ring is field.

Every homomorphism image of ring is a ring.