Homework 3

- Textbook p426: 10
- Textbook p431: 6
- 1. Let *S*={1, 2, 3}

$$p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \qquad p_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

(1) Determine all the left cosets of $H = \{p_1, p_4, p_5\}$ in S_3.

(2) Determine all the left cosets of $H = \{p_1\}$ in S_3.

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- 2. Let $<A, +, \cdot>$ be a algebraic system, A are the following sets:

(1) $A=\{x|x = 2n, n \in \mathbf{Z}\}$

(2) $A=\{x|x = 2n + 1, n \in \mathbf{Z}\}$

(3) $A=\{x|x = a + b\sqrt{5}, a, b \in \mathbf{Q}\}$

Is $<A, +, \cdot>$ a ring? Is $<A, +, \cdot>$ a field?

- 3. Let $<A, +, \cdot>$ be a ring, show that if $a, b \in A$, then$(a + b)^2=a^2+a\cdot b+b\cdot a+b^2$, where $x^2=x \cdot x$.

- 4. Let $<A, +, \cdot>$ be a ring, and for $\forall a \in A$, $a \cdot a = a$. Show that:

(1) For $\forall a \in A$, $a + a = e$, where e is the identity of $<A, +>$.

(2) $<A, +, \cdot>$ is a communitive ring.

- 5. Let $<A, +, \cdot>$ be a field, $B_1 \subseteq A$, $B_2 \subseteq$ A. Show that $< B_1 \cap B_2, +, \cdot>$ is also a field.