2019 EXAMINATIONS

Lancaster University

Part I (First Year)

SCHOOL OF COMPUTING AND COMMUNICATIONS

SCC.120 Fundamentals of Computer Science (1 hour & 30 minutes)

- **Answer any <u>THREE</u> out of the four questions.**
- **Use a <u>separate</u> answer book for <u>each</u> question.**

QUESTION 1 (25 Marks)

1.a Let A = {a, b, c} and B = {a, y, z}. Find

i. $A \cup B$
ii. $A \cap B$
iii. $(A - B) \cap (B - A)$
iv. A x B

[2 marks]

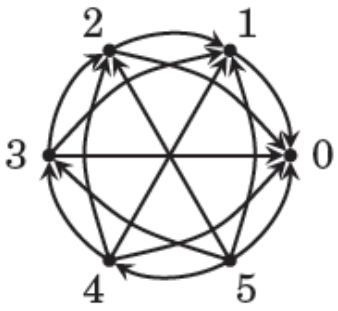
1.b Given sets A and B defined above, define the following two sets:

i. C of cardinality 1, such that C is a proper subset of A
ii. D of cardinality 3, such as D is a superset of B

[2 marks]

1.c Specify the sets A and relation R from A to A, where A is the set on which the relation is represented in the diagraph below:

[1.5 marks]



1.d Let A = {a, b, c, d} and R = {<a, a>, <b, b>, <c, c>, <d, d>}.
Is relation R:

i. reflexive
ii. symmetric, or
iii. transitive?

For each property, explain your answer.

[1.5 marks]

1.e Let f(x) = 2x and g(x) = 3x + 2 be two functions on the set of real numbers.
Compute the following:

i. sum function: f + g
ii. product function: f x g
iii. composite function: $f \circ g$

[3 marks]

Question 1 continues on next page…

Question 1 continued.

1.f Show that function g(x) defined above in 1.e is

i. injective
ii. surjective
iii. and compute its inverse

[6 marks]

1.g. Build the truth table to show the following equivalence:
$P \rightarrow Q$ is equivalent to $\sim P \vee Q$

[2 marks]

1.h Let's start with five propositions:
P "Tomorrow will be sunny"
Q "Tomorrow will be colder than today"
R "We will go to the beach"
S "We will play football"
T "We will go home early"

Translate the following four sentences into propositional logic, using the propositions above.

i. Tomorrow will not be sunny and will be colder than today.
ii. We will go to the beach only if it is sunny.
iii. If we do not go to the beach, we will play football.
iv. If we play football, we will go home early.

[2 marks]

1.i What conclusion can be inferred from the four premises given in 1.h?

[1 marks]

1.j Show the validity of this conclusion (your answer to 1.i) by describing one by one the steps involved in the chain of reasoning leading from the premises to the conclusion. Here there are some basic rules of inferences. It is up to you decide which ones are useful for this problem.

- $(P \wedge (P \rightarrow Q)) \Rightarrow Q$ modus ponens
- $((P \rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$ modus tollens
- $P \Rightarrow (P \vee Q)$ addition
- $(P \wedge Q) \Rightarrow P$ simplification
- $((P \rightarrow Q) \wedge (Q \rightarrow R)) \Rightarrow (P \rightarrow R)$ hypothetical syllogism
- $((P \vee Q) \wedge \sim P) \Rightarrow Q$ disjunctive syllogism
- $(P \rightarrow Q) \Rightarrow P \rightarrow (P \wedge Q)$ absorption

[4 marks]

[Total 25 Marks]

QUESTION 2 (25 Marks)

2.a Briefly outline the main differences between a one-dimensional array and a record.

[2 marks]

2.b Given the following declarations and code:

```
char x = ‘a’;     // line A

void stuff()
{
     int y = 4;      // line B
     int *z = malloc(sizeof(int));
    *z = 5;   // line C
}
```

Where would the following values be stored (in terms of areas of memory, NOT the variables they’ve been assigned to).

i. the value ‘a’ in line A
ii. the value 4 in line B
iii. the value 5 in line C

[3 Marks]

2.c Given the following declarations :

```
char one[] = "Bye!";
char two[] = { 'B', 'y', 'e', '!'};
```

i. What does sizeof(one) return? What does sizeof(two) return?

[2 marks]

ii. Consider the following line of code. What value would you expect “x” to hold after the following function call?

```
int x = strlen(one)
```

[2 marks]

iii. Consider the following line of code. What value would you expect “x” to hold after the following function call?

```
int x = strlen(two)
```

[2 marks]

Question 2 continues on next page…

Question 2 continued.

For the next two questions, consider the following declarations.

```
typedef struct Node {
      int data;
      struct Node *next;
} Node;

1. int length(Node *head)
2. {
3.    int count = 0;
4.    Node *pt = head;
5.    while (pt != NULL)
6.    {
7.          count++;
8.    };
9.    return count;
10. };

bool sameValues(Node *x, Node *y)
{
      if ((x == NULL) & (y == NULL)) return true; // point A
      if ((x == NULL) | (y == NULL)) return false; // point B
      bool same = true;
      Node *xp = x, *yp = y; // scan pointers
      while ((xp != NULL) & (same == true)) // point C
      {
            if (xp->data != yp->data) same = false;
            xp = xp->next;
            yp = yp->next;
            if (((xp == NULL) & (yp != NULL)) // point D
                  | ((xp != NULL) & (yp == NULL)))
                        same = false;
      };
      return same;
};
```

Question 2 continues on next page…

2.d Consider the "length" function.

i. Unfortunately there is a missing line in this function. How would this function behave when it is executed?

[1 mark]

ii. Are there any circumstances under which the function would deliver a correct result?

[1 mark]

iii. Provide the missing line. State the number of the line it follows in the "length" function.

[2 marks]

2.e Consider the "sameValues" function.

i. Briefly describe, at a high level, the purpose of this function.

[2 marks]

ii. What is the purpose of the statement at point A?

[1 mark]

iii. What is the purpose of the statement at point B?

[1 mark]

iv. Consider the loop control expression at point C. If this was changed to

```
while (xp != NULL)
```

would the function still perform correctly? If so, what is the purpose of the "(same == true)" part of the expression?

[4 marks]

v. Finally, consider the Boolean expression of the "if" statement at point D. This is quite a complex expression. Briefly describe, at a high level, what is being tested for at this point in the code.

[2 marks]

[Total 25 Marks]

QUESTION 3 (25 Marks)

3.a Consider the *sortSearch* algorithm below that takes as input an array of integers and an integer. Assume that *insertionSort* and *binarySearch* are exactly as given in the Appendix following this question.

```
sortSearch(int[] A, int s) {
      insertionSort(A);
      if (binarySearch(A, s))
            return true;
      else
            return false;
}
```

Assume the length of the array is *n*. Give the following time complexities.

i. Best case runtime in Θ notation.
ii. Worst case runtime in Θ notation.

[6 marks]

3.b Cleversort is an algorithm for sorting an array of integers. It has worst case complexity Θ(n log n) and best case complexity Θ(n), where n is the size of the array being sorted.

For each statement below, indicate whether the statement is true or false.

i. Cleversort is O(n log n)
ii. Cleversort is O(n log n) in worst case
iii. Cleversort is Ω(n)

[6 marks]

3.c Refer to the algorithm for **insertion sort** in the Appendix following this question (it is no different from what you studied in class). Suppose the algorithm were run on the array **[18, 5, 3, 6, 19, 6, 8]**. Referring to the loop counters *i* and *j* in the algorithm, what would the contents of the array be when we are about to enter the inner loop ***i=6*** and ***j=4***.

[5 marks]

Question 3 continues on next page…

3.d The problem *Searching* is defined as follows: Given an array of integers A and an integer s, determine whether *s* is in A.

For each of the following statements, indicate whether it is true or false.

i. Searching is a decision problem.
ii. Searching is in NP.
iii. If a polynomial-time algorithm for Propositional Satisfiability is found, then a polynomial-time algorithm exists for Searching.
iv. If a polynomial-time algorithm for Searching is found, then a polynomial-time algorithm exists for Propositional Satisfiability.

Caveat: No marks if you get any of the first two incorrectly.

[8 marks]

[Total 25 Marks]

Appendix to Question 3

Insertion sort

```
void insertionSort (int[] A) {
      for (int i=1; i<A.length; i++) {
            int x = A[i];
            int j;
            for (j=i-1; j>=0 && A[j]>x; j--) {
                  A[j+1] = A[j];
            }
            A[j+1] = x;
      }
}
```

Binary Search

```
boolean binarySearch(int[] A, int s) {
     int lo = 0;
     int hi = A.length - 1;
     int mid;

     while (hi >= lo) {
            //round to higher integer
            mid = (lo + hi)/2;
            if (A[mid] == s)
               return true;
            else if (A[mid] < s)
               lo = mid + 1;
            else
               hi = mid - 1;
     }
     return false;
}
```

QUESTION 4 (25 Marks)

4.a You are asked to write a navigation system which can select the best driving routes between cities that are connected by roads. Starting in any city, a user can request the fastest route to any other city, and is then given detailed directions.

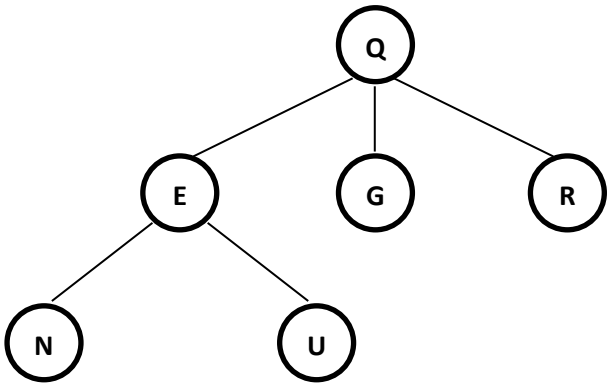
i. What Abstract Data Type would you use to represent the cities and roads? Justify your choice.

[3 marks]

ii. With the help of a diagram, explain in words an algorithm which would produce a set of detailed directions to the user, based on the data structure that you have chosen.

[3 marks]

4.b Consider the tree T, illustrated below.



i. Draw a representation of this tree as an adjacency matrix.

[2 marks]

ii. Describe three different methods of navigating this tree, and provide the corresponding orderings with which each node is visited in each method.

[6 marks]

Question 4 continues on next page…

4.c i. Consider an unsorted list of integers, for example [1, 4, 7, 1, 3, 9, 4, 7]. Describe, in words, the quicksort algorithm to sort this list into ascending order, and demonstrate how the list becomes sorted by showing intermediate steps.

[5 marks]

ii. Quicksort can be implemented as a recursive algorithm. Which data structure is crucial to implementing recursion, and why?

[2 marks]

4.d For the "Set" Abstract Data Type, describe in words how you would add a new item if the Set is implemented with an array. How would you add an item for a Set implemented as a linked structure? Your answer should cover all possible conditions.

[4 marks]

[Total 25 Marks]

---End of Paper---