2018 EXAMINATIONS

Lancaster University

Part I (First Year)

SCHOOL OF COMPUTING AND COMMUNICATIONS

SCC.120 Fundamentals of Computer Science (1 hour & 30 Minutes)

- **Answer any <u>THREE</u> out of the four questions.**
- **Use a <u>separate</u> answer book for <u>each</u> question.**

Question 1

a) Let A = { a, d, e ,i, m, n, p, r, s}, B = { a, e, m, n, p, r, s, u}, C = {m, n, o } and D = {22, 42}
Calculate:

i. $A \cup B$ **[1 mark]**
ii. $A \cap B$ **[1 mark]**
iii. $A - B$ **[1 mark]**
iv. $C \times D$ **[2 marks]**

b) Let $f(x) = 3x + 1$ and $g(x) = 2x - 4$. Calculate:

i. $f + g$ **[1 mark]**
ii. $f * g$ **[2 marks]**
iii. $f \circ g$ **[2 marks]**
iv. f^{-1} **[2 marks]**
v. Define f(x) recursively **[3 marks]**

c) Prove that $(A \wedge B) \rightarrow B$ is a tautology by performing transformations. Show every step of the transformation and name the equivalence you are applying. Here are some equivalences.

Implication Law. $P \rightarrow Q \Leftrightarrow \sim P \vee Q$
De Morgan's Law. $\sim(P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$
Associative Law. $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
Complement Law. $P \vee \sim P \Leftrightarrow$ True
Disjunction Identity Element Law. $P \vee T \Leftrightarrow$ True

[6 marks]

d) i. Convert the following English statement into a proposition:

"Diane will eat if she is hungry"

Where A = "Diane will eat" and B = "she is hungry".

[1 mark]

ii. Let L(x) be the statement "x has no experience in public speaking", where the universe of discourse for x is the set of people. Express the following logical predicates as English statements:

$\exists x$ L(x)
$\exists x$ ~L(x)
$\forall x$ ~L(x)

[3 marks]
[Total 25 Marks]

Question 2

a) Suppose we have a linear array A, where each element in the array occupies one byte and each byte has its own memory address. We wish to access element i, i.e. A[i], using the following method (pseudo-code):

```
if (i < P or i > Q) {
    return a runtime error;
} else{
       access the memory location whose address is R + i
}
```

i. What names are used for the quantities P and Q? (4 marks)

ii. What name is used for quantity R? (2 marks)

[6 marks]

b) i. What is the worst-case running time for returning an item at a given position in an array of **N** elements? **[2 marks]**

ii. What is the worst-case running time for returning an item at a given position in a singly linked list of **N** elements? **[2 marks]**

c) Convert an array with values [6, 10, 0, 3, 12, 3] into a balanced binary search tree. Draw the result on the answer sheet.

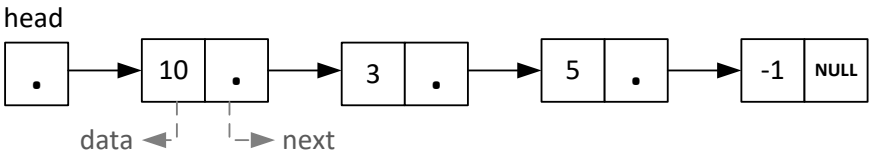
[5 marks]

Question 2 continues on next page...

d) We have a structure type called `node` which is defined as follows:

```
typedef struct _node {
        int data;
        struct _node* next;
}node;
```

Consider a linked list represented by the following diagram:



The following C code intends to add a new node to the above list:

```
1  node *newNode;
2  node *prev = head->next->next;
3  newNode->data = 2;
4  newNode->next = prev->next;
5  prev->next = newNode;
```

i. The above code has a bug which can crash the program. Which line of the code can lead to a program crash? **[2 marks]**

ii. How will you fix the problem identified in question 2.d.i? **[3 marks]**

iii. Assume the bug has been fixed. Draw a diagram of the resulting list after executing the corrected code on the above list. **[5 marks]**

[Total 25 Marks]

Question 3

a) Give the worst case complexity of the following code fragment in the Big Θ notation. The algorithm's input is an array of characters **A** of size **n**. What does the algorithm compute?

```
for(int i=0; i<n; i++) {
     if(i == n-1-i)
          return true;
     if (A[i] == A[n-1-i])
          continue;
     else
          return false;
}
return true;
```

[5 marks]

b) For each of the following pairs of functions, state which one grows faster asymptotically for input of size n, in other words, is worse in terms of efficiency.

i. $f(n) = n\log_2 n + 53$ and $k(n) = 10\log_2 n + 53$ **[2 marks]**

ii. $f(n) = n^n$ and $k(n) = n^5 + n^4 + 68$ **[2 marks]**

iii. $f(n) = 2^n$ and $k(n) = n^2 + 100n + 2$ **[2 marks]**

iv. $f(n)$ = square-root(n) and $k(n) = n^2 + \log_2(n)$ **[2 marks]**

c) For each problem listed in the left-most column of the table and for each of the properties listed in the other columns, do the following:

Put **T** if the property is true for the problem and put **F** if the property is false. Be careful to reproduce this table clearly in your answer book.

Problem	**Is Decision Problem**	**Is Tractable**	**Is in NP**	**Is in NP-Complete**
Shortest road between any two cities in England				
Is given number in the array				
Sorting array of integers				
Propositional satisfiability				

[8 marks]

Question 3 continues on next page...

d) Imagine an algorithm that searches for an integer in an array of integers by first using insertion-sort to sort the array and then using binary-search to search for the number. State the best case and worst case complexities of this algorithm in the Big Θ notation.

[4 marks]

[Total 25 Marks]

Question 4

a) Draw an example of a queue and with reference to the diagram, describe removing an element from the queue (code or pseudocode can be used).

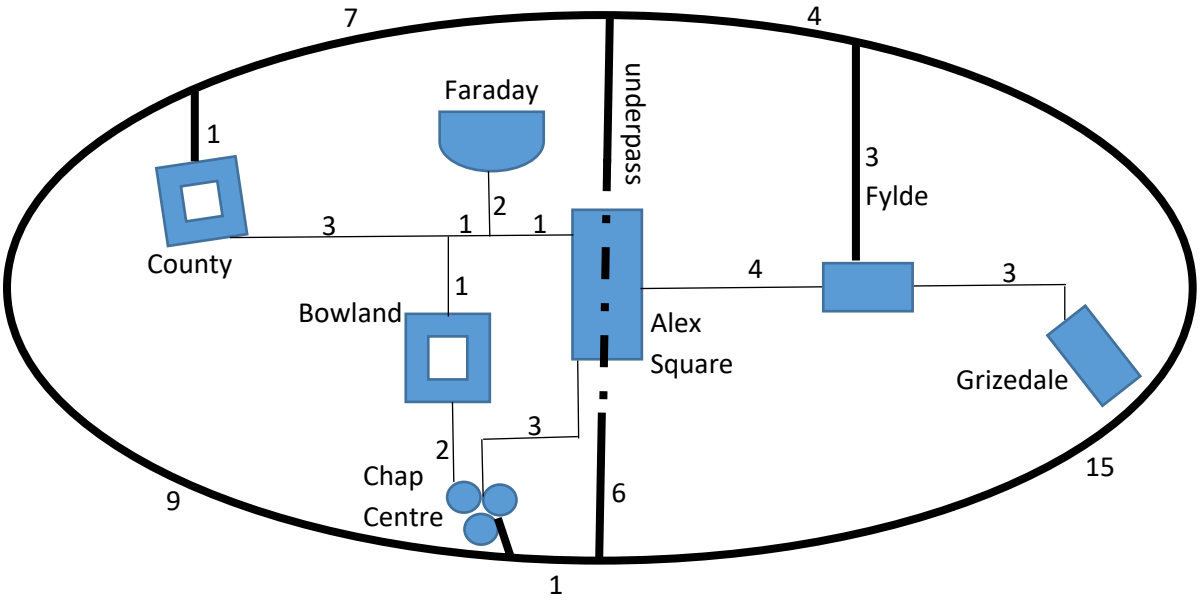
[3 marks]

b) How would removing an element differ if the queue was replaced by a stack? Describe the process of removing an element from a stack. (3 marks)
What is this process typically called? (1 mark)

[4 marks]

Question 4 continues on next page...

Question 4 continued…



c)

i. Given the map above, draw a weighted graph G representing routes around campus (Note – the underpass does not connect to Alex Square).

[6 marks]

ii. Works are added between Alex Square and County, closing the whole of the path between them. Draw a partial graph H of G (from 4.c part i), to show the connections with this path closed (2 marks). What is the change of density? (this can be given as a fraction or a decimal)

[3 marks]

iii. An inspector wants to see how badly the university's path network is affected by the works at the north end of campus. A subgraph N of H (from 4.c part ii) is defined to represent the north end of campus, containing only the nodes for Faraday, County, Bowland, Alex Square and the Chaplaincy Centre. Draw the complementary graph J of N. Will graph J give the inspector the information he's looking for?

[2 marks]

d) Your parents are visiting and have parked at county college. You have provided them with a robotic guide to find you in Grizedale. Your robot must calculate its path first.

Using graph G, start at County and search for Grizedale using Dijkstra's Algorithm. Show your working.

[7 marks]

[Total 25 Marks]

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