M202204W Discrete Mathematics

Term 2 Coursework (II)

Deadline for submission: **14th June 2023, at 14:00**

Please print the document **single-sided on A3 paper**, **hand write** it and submit on time. This coursework is worth **20%** of the final mark.

Please answer all questions. You must do this coursework completely by yourself. In this instance, it means you are not even allowed to discuss the coursework questions with other students. Your submitted files will automatically be checked for plagiarism.

You are allowed to ask the instructor about these questions. But I can only clarify the questions or give you suggestions if you get stuck, and I will try not to tell you whether you have the correct answer (to be fair to all students).

Name: _____

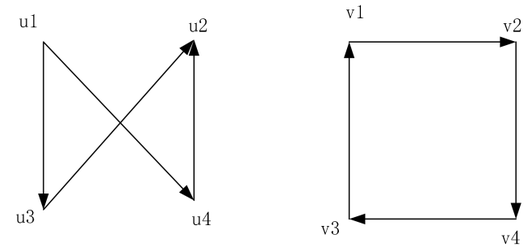
ID: _____

Total score: _____

1. Answer the following questions. (2 marks for total, 1mark for a) to d), 1mark for e) to f))
   a) How many edges does the completed graph K_n have _____
   b) Draw the Q_4

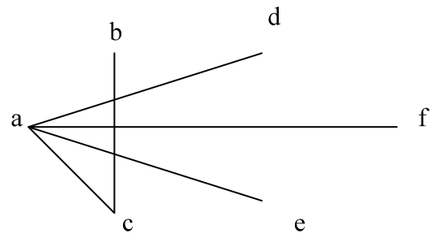
   c) Is a complete matching always a maximum matching? (Y/N)_____
   d) What is the chromatic number of W_8 _____
   e) If the undirected graph $G = (V, E)$ contains 7 vertices, the minimum number of edges needed to ensure that the graph G is connected in all cases is _____
   f) It is known that an undirected graph G contains 16 edges, of which the number of vertices of degree 4 is 3, the number of vertices of degree 3 is 4, and the degree of all other vertices is less than 3. The number of vertices contained in the graph G is at least _____
2. Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq \frac{v^2}{4}$. (1 mark)

3. Answer the following questions. (2 marks for total)
   a) Determine whether the given pair of directed graphs in Figure 1 are isomorphic. (Y/N) _____ (0.5 mark)

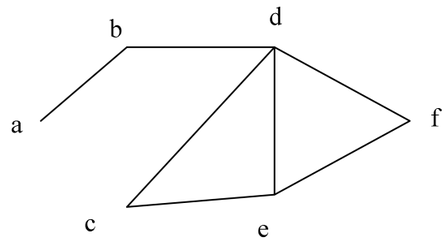


(Figure1)

b) Draw the union graph of Figure 2 and Figure3. Is it the union graph a bipartite graph? (1 mark)
c) Use adjacency list and adjacency matrix to represent your union group. (0.5 mark)



(Figure 2)

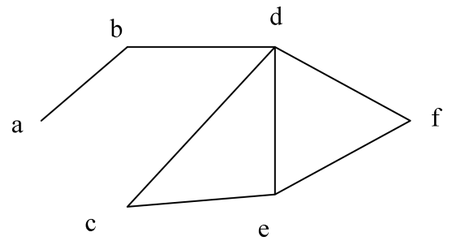


(Figure 3)

4. Show that how many subgraphs with at least one vertex does K_5 have? (1 mark)

5. Show that if a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$. (1 mark)

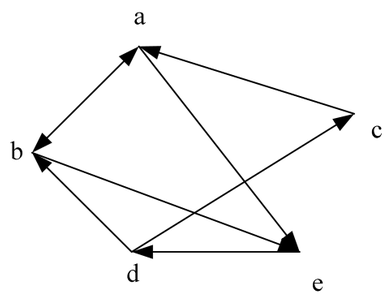
6. How many paths of length five are there from **a** to **f** in Figure 4? (1 mark)



(Figure 4)

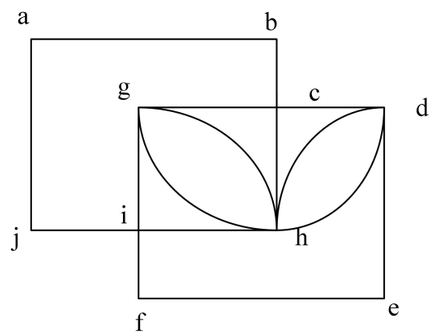
7. Answer the following questions. (2 marks for total, 1 mark for two questions)

a) Determine whether the Figure 5 is strongly connected and whether it is weakly connected.



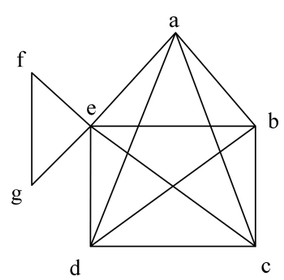
(Figure 5)

b) Find the Euler circuit of the Figure 6
Draw the circuit and its direction in Figure 6 using different color ink (don't use red and black ink)



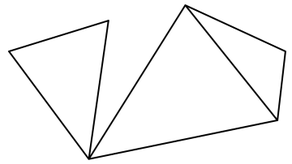
(Figure 6)

c) Is there a Euler circuit in the Figure 7? If so, draw the circuit and its direction in the diagram using different color ink (don't use red and black ink). If not, explain why.



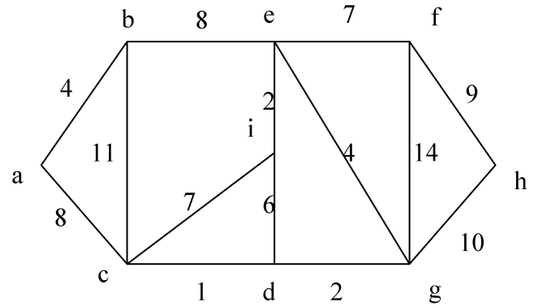
(Figure 7)

d) Determine whether Figure 8 has a Hamiltonian circuit? Whether there is a Hamiltonian path?



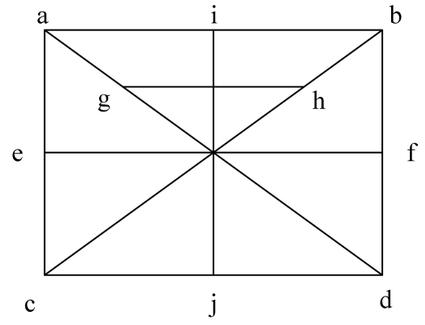
(Figure 8)

8. Use Dijkstra's algorithm to find the length of a shortest path between the vertices **a** and **h** in the weighted graph displayed in Figure 9. (1 mark)



(Figure 9)

9. Show whether the graph G shown in Figure 10 is planar. (1 mark)



(Figure 10)

10. Show that $\kappa(G) \leq \lambda(G) \leq min_{v \in V} deg(v)$. (1 mark)

11. Answer the following questions. (2 marks for total)
    a) Known character set $\{a, b, c, d, e, f, g, h\}$ if the Huffman code of each character is $0100, 10, 0000, 0101, 001, 011, 11, 0001$, then the decoding result of the code sequence 01000110010010011110101 is _____
    b) (Choose all the correct answers) Converting a forest to a corresponding binary tree, if in a binary tree, node u is the parent of node v's parent node, then in the original forest, the relationship that u and v may have is/are _____
    I. Parent-child relationship II. Brotherhood III. The parent node of u is sibling to the parent node of v
    c) (Multiple-Choice Questions) If the preorder traversal sequence and postorder traversal sequence of a binary tree are $1,2,3,4$ and $4,3,2,1$, respectively, then the inorder traversal sequence of the binary tree will not be _____$(A.\,1,2,3,4 \quad B.\,2,3,4,1 \quad C.\,3,2,4,1 \quad D.\,4,3,2,1)$
    d) A tree with 101 nodes is known to have 66 leaf nodes, and the number of nodes without right children in the corresponding binary tree is _____
12. Use Huffman coding to encode the following symbols with the frequencies listed: A: 0.06, B: 0.12, C: 0.24, D: 0.26, E: 0.32. (Find the code for each letter and illustrate each step). What is the average number of bits used to encode a character? (1 mark for total)

13. Complete the in-order traversal code. (1 mark for all places)

```
public static void inOrder(TreeNode root) {
    if (root != null) {
        //complete the code----------------

        //---------------------------------
        System.out.print(root.val + " ");
        //complete the code ----------------

        //---------------------------------
    }
}
public class TreeNode {
    int val;
    TreeNode left;
    TreeNode right;
    TreeNode(int val) {
        this.val = val;
    }
    TreeNode(int val, TreeNode left, TreeNode right) {
        this.val = val;
        this.left = left;
        this.right = right;
    }
}
```

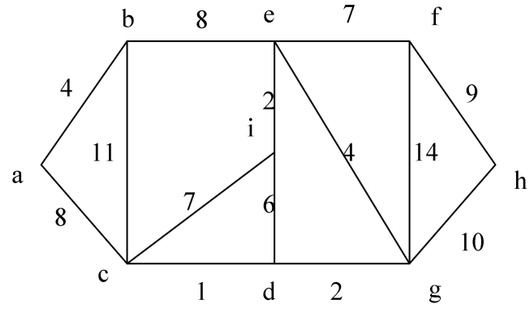
14. a) Represent the expression $((x-1) \div 3) \times ((2+y) \div z)$ using a binary tree. Write this expression in

b) prefix notation.

c) postfix notation.

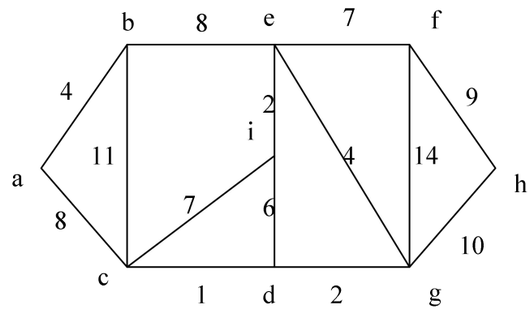
d)infix notation. (1 mark for a) to d))

15. Use depth-first and breadth-first search to produce a spanning tree for Figure 9. Choose **a** as the root of this spanning tree and assume that the vertices are ordered alphabetically. (1 mark for total)



(Figure 9)

16. Use Prim's and Kruskal's algorithm to design a minimum spanning tree in Figure 9. (1 mark for total)



(Figure 9)