

# Homework 2

---

- Textbook p421: 6, 8, 22, 30, 34
- 1. Let  $\langle G, * \rangle$  be a monoid with identity  $e$ . Show that if  $a * a = e$  for all  $a$  in  $G$ , then  $\langle G, * \rangle$  is Abelian.
- 2. Let  $\langle G, * \rangle$  be a group. Prove that if  $g \in G$  has the property  $g * g = g$ , then  $g$  is the identity element of  $G$ .
- 3. Let  $\langle G, * \rangle$  be an Abelian group with identity  $e$ , and let  $H = \{x \mid x * x = e\}$ . Show that  $\langle H, * \rangle$  is a subgroup of  $\langle G, * \rangle$ .

# Homework 2

- 4. Let  $G=\{a, b, c, d\}$ . Prove  $\langle G, * \rangle$  is a cyclic group and find out the generator.

$*$	$a$	$b$	$c$	$d$
$a$	$a$	$b$	$c$	$d$
$b$	$b$	$a$	$d$	$c$
$c$	$c$	$d$	$b$	$a$
$d$	$d$	$c$	$a$	$b$

- 5. Let  $S=\{a, b, c, d\}$ ,  $P=\{p_1, p_2, p_3, p_4\}$ . Show that  $\langle P, \circ \rangle$  is a permutation group on  $S$ .

$$p_1 = \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}$$
$$p_2 = \begin{pmatrix} a & b & c & d \\ b & a & c & d \end{pmatrix}$$
$$p_3 = \begin{pmatrix} a & b & c & d \\ a & b & d & c \end{pmatrix}$$
$$p_4 = \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix}$$