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Discrete Maths : 1
Workshop Unit 1 : Sets
Sample Solutions

SCC120 Fundamentals of Computer Science

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Exercise/Answer 1

- a) {1, 3, 5} and {5,3,1} are equal. Remember, sets are unordered. Both sets contain the same number of elements, and the same elements.
- b) {1,3,5} and {5,1,6} are not equal. Both sets do have the same number of elements, but not the same elements.

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Exercise 2

Let A = {1, 2, 3, 4, 5} and B= {0, 3, 6}. Find

a) $A \cap B$

b) $A \cup B$

c) $A – B$

d) $B - A$

U = {	0,	1,	2,	3,	4,	5,	6	}
A = {		1,	2,	3,	4,	5		}
B = {	0,			3,			6	}

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Answer 2 (a)

- a) $A \cap B$ = {3}

U = {	0,	1,	2,	3,	4,	5,	6	}
A = {		1,	2,	3,	4,	5		}
B = {	0,			3,			6	}
$A \cap B$ = {				3				}

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Answer 2(b)

- b) $A \cup B$ = {0, 1, 2, 3, 4, 5, 6}

U = {	0,	1,	2,	3,	4,	5,	6	}
A = {		1,	2,	3,	4,	5		}
B = {	0,			3,			6	}
$A \cup B$ = {	0,	1,	2,	3,	4,	5,	6	}

6

Answer 2 (c)

- c) $A – B$ = {1, 2, 4, 5}

U = {	0,	1,	2,	3,	4,	5,	6	}
A = {		1,	2,	3,	4,	5		}
B = {	0,			3,			6	}
$A – B$ = {		1,	2,		4,	5		}

Answer 2 (d)

- d) B – A = {0, 6}

U = {	0,	1,	2,	3,	4,	5,	6	}
A = {		1,	2,	3,	4,	5		}
B = {	0,			3,			6	}
B – A = {	0,						6	}

Exercise 3

Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6} and C = {4, 5, 6, 7, 8, 9, 10}. Find

a)A ∩ B ∩ C

b)A ∪ B ∪ C

c)(A ∪ B) ∩ C

d)(A ∩ B) ∪ C

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}
A = {	0,		2,		4,		6,		8,		10	}
B = {	0,	1,	2,	3,	4,	5,	6,					}
C = {					4,	5,	6,	7,	8,	9,	10	}

Answer 3 (a)

- a) A ∩ B ∩ C = {4, 6}

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}
A = {	0,		2,		4,		6,		8,		10	}
B = {	0,	1,	2,	3,	4,	5,	6,					}
C = {					4,	5,	6,	7,	8,	9,	10	}
{					4,		6					}

Answer 3 (b)

- b) A ∪ B ∪ C = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}
A = {	0,		2,		4,		6,		8,		10	}
B = {	0,	1,	2,	3,	4,	5,	6,					}
C = {					4,	5,	6,	7,	8,	9,	10	}
{	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}

Answer 3 (c)

- c) (A ∪ B) ∩ C = {4, 5, 6, 8, 10}

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}
A = {	0,		2,		4,		6,		8,		10	}
B = {	0,	1,	2,	3,	4,	5,	6,					}
(A ∪ B) = {	0,	1,	2,	3,	4,	5,	6,		8,		10	}
C = {					4,	5,	6,	7,	8,	9,	10	}
(A ∪ B) ∩ C = {					4,	5,	6,		8,		10	}

Answer 3 (d)

- d) (A ∩ B) ∪ C ={0, 2, 4, 5, 6, 7, 8, 9, 10 }

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}
A = {	0,		2,		4,		6,		8,		10	}
B = {	0,	1,	2,	3,	4,	5,	6,					}
(A ∩ B) = {	0,		2,		4,		6					}
C = {					4,	5,	6,	7,	8,	9,	10	}
(A ∩ B) ∪ C = {	0,		2,		4,	5,	6,	7,	8,	9,	10	}

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Exercise 4

if A = {a, b, c, y}, B = {a, b, c , d, e}, C = {x, y} evaluate:

a) $A \cup (B \cap C)$
b) $(A \cup B) \cap C$
c) $C - A$
d) $(A - B) - C$
e) $A - (B - C)$
f) $(A \cap C) \cup B$
g) $A \cap (C \cup B)$

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Answer 4(a)

- (a) $A \cup (B \cap C) = A$
- $(B \cap C)$ is the empty set.

***U* = {**	***a*,**	***b*,**	***c*,**	***d*,**	***e*,**	***x*,**	***y***	**}**
A = {	a,	b,	c,				y	}
B = {	a,	b,	c,	d,	e			}
C = {						x,	y	}
$(B \cap C)$ = {								}
$A \cup (B \cap C)$ = {	a,	b,	c,				y	}

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Answer 4(b)

- b) $(A \cup B) \cap C = \{y\}$

***U* = {**	***a*,**	***b*,**	***c*,**	***d*,**	***e*,**	***x*,**	***y***	**}**
A = {	a,	b,	c,				y	}
B = {	a,	b,	c,	d,	e			}
$(A \cup B)$ = {	a,	b,	c,	d,	e,		y	}
$(A \cup B)$ = {	a,	b,	c,	d,	e,		y	}
C = {						x,	y	}
$(A \cup B) \cap C$ = {							y	}

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Answer 4 (c)

- c) $C - A = \{x\}$

***U* = {**	***a*,**	***b*,**	***c*,**	***d*,**	***e*,**	***x*,**	***y***	**}**
C = {						x,	y	}
A = {	a,	b,	c,				y	}
C – A = {						x		}

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Answer 4 (d)

- d) $(A - B) - C = \varnothing$

***U* = {**	***a*,**	***b*,**	***c*,**	***d*,**	***e*,**	***x*,**	***y***	**}**
A = {	a,	b,	c,				y	}
B = {	a,	b,	c,	d,	e			}
(A – B) = {							y	}
(A – B) = {							y	}
C = {						x,	y	}
(A – B) – C = {								}

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Answer 4 (e)

- e) $A - (B - C) = \{y\}$
- Note that $(B - C) = B$.

***U* = {**	***a*,**	***b*,**	***c*,**	***d*,**	***e*,**	***x*,**	***y***	**}**
B = {	a,	b,	c,	d,	e			}
C = {						x,	y	}
(B – C) = {	a,	b,	c,	d,	e			}
A = {	a,	b,	c,				y	}
(B – C) = {	a,	b,	c,	d,	e			}
A – (B – C) = {							y	}

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Answer 4(f)

- f) $(A \cap C) \cup B = \{a, b, c, d, e, y\}$

U = {	***a,***	***b,***	***c,***	***d,***	***e,***	***x,***	***y***	***}***
A = {	a,	b,	c,				y	}
C = {						x,	y	}
(A ∩ C) = {							y	}
(A ∩ C) = {							y	}
B = {	a,	b,	c,	d,	e			}
(A ∩ C) ∪ B = {	a,	b,	c,	d,	e,		y	}

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Answer 4 (g)

- g) $A \cap (C \cup B) = A$

U = {	***a,***	***b,***	***c,***	***d,***	***e,***	***x,***	***y***	***}***
B = {	a,	b,	c,	d,	e			}
C = {						x,	y	}
(C ∪ B) = {	a,	b,	c,	d,	e,	x,	y	}
A = {	a,	b,	c,				y	}
(C ∪ B) = {	a,	b,	c,	d,	e,	x,	y	}
A ∩ (C ∪ B) = {	a,	b,	c,				y	}

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Exercise 5

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets, in English.

a) $A \cap B$

b) $A \cup B$

c) $A – B$

d) $B - A$

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Answer 5 (a)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

a)Answer: $A \cap B$ = ? (intersection)

The set of students who live within one mile of school and who walk to classes.

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Answer 5 (b)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

b) Answer: $A \cup B$ = ? (union)

The set of students who live within one mile of school or who walk to classes (or who do both).

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Answer 5(c)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

c) $A - B$ = ? (difference : who appears in set A and not in set B)

The set of students who live within one mile of school but do not walk to classes.

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Answer 5 (d)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

d) B - A = ?

The set of students who walk to classes but live more than one mile away from school.

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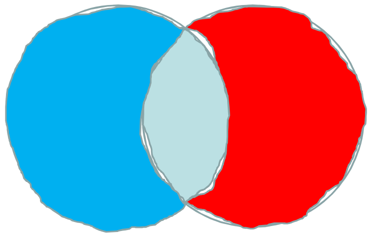
Sets A and B



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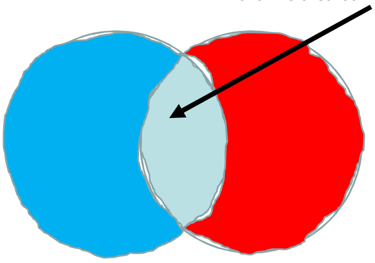
Sets A and B : $A \cap B$



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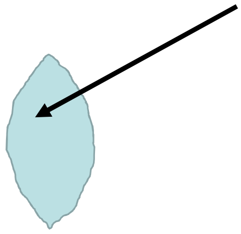
Sets A and B : $A \cap B$



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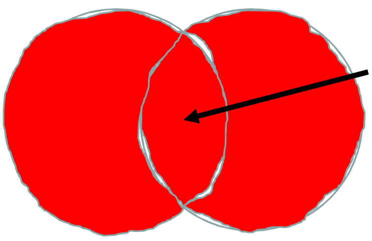
Sets A and B : $A \cup B$



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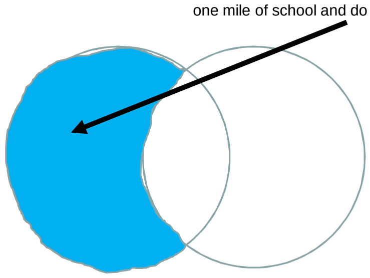
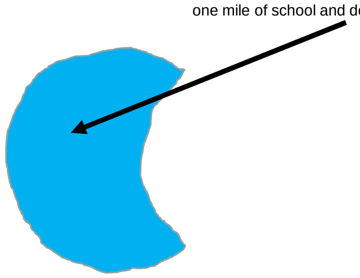
Sets A and B : A - B



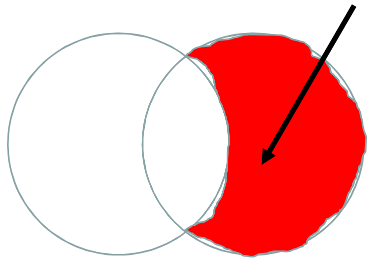
Sets A and B : A - B

A - B : Students who live within one mile of school and do not walk to classes



Sets A and B : B - A

B - A : Students who do not live within one mile of school and walk to classes

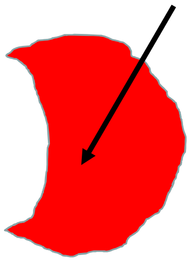


A : Students who live within one mile of school

B : Students who walk to classes

Sets A and B : B - A

B - A : Students who do not live within one mile of school and walk to classes



Exercise 6

Let A = {a, b, c} and B = {y, z}. Find

a) A x B

b) B x A

Answer 6(a)

Let A = {a, b, c} and B = {y, z}. Find

a) A x B

Answer: A x B = { <a, y>, <a, z>, <b, y>, <b, z>, <c, y>, <c, z>}

	y	z
a	<a, y>	<a, z>
b	<b, y>	<b, z>
c	<c, y>	<c, z>

Answer 6(b)

Let A = {a, b, c} and B = {y, z}. Find

b) B x A

Answer: B x A = { <y, a>, <y, b>, <y, c>, <z, a>, <z, b>, <z, c>}

	a	b	c
y	<y, a>	<y, b>	<y, c>
z	<z, a>	<z, b>	<z, c>

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Exercise/Answer 7

How many different elements does A x B have if A has m elements and B has n elements?

Answer: m x n

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Exercise 8

List the members of the following sets

a) {x | x is a positive integer less than 12}

b) {x | x is the square of an integer and x < 100}

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Answer 8 (a)

List the members of the following sets

a) {x | x is a positive integer less than 12}

Answer: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

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Answer 8 (b)

List the members of the following sets

b) {x | x is the square of an integer and x < 100}

The set consist of all squares less than 100

$0^2 = 0$ $1^2= 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

$5^2 = 25$ $6^2 = 36$ $7^2= 49$ $8^2 = 64$ $9^2 =81$

Answer: {0, 1, 4, 9, 16, 25, 36, 49, 64, 81}

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Exercise 9

For each of the following sets, determine if 2 is an element of that set

a){x $\in$ R | x is an integer greater than 1}

b){x $\in$ R | x is the square of an integer}

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Answer 9(a)

For each of the following sets, determine if 2 is an element of that set

a){x $\in$ R | x is an integer greater than 1}

Answer: This set contains the element 2

(2 is an integer and 2 > 1)

Answer 9(b)

For each of the following sets, determine if 2 is an element of that set

b) $\{x \in R \mid x$ is the square of an integer$\}$

Answer: This set does not contains the element 2 as 2 is not a square

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Discrete Maths : 1
Workshop Unit 2 : Relations and Functions

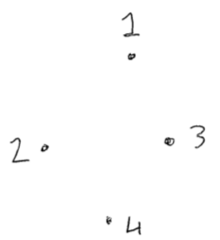
SCC120 Fundamentals of Computer Science

2

PART 1. RELATIONS

3

Exercise 1



- Draw diagraphs of the 3 relations, using this as your starting point.

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Exercise 1 : R1

R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}

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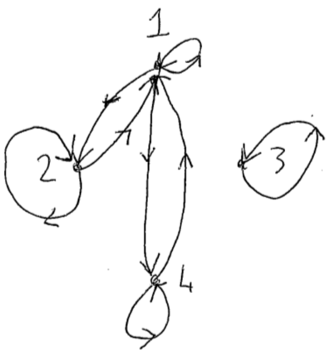
Exercise 1 : R3



R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}

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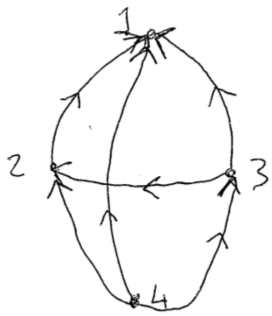
Exercise 1 : R4



R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

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Rules

- Reflexive
- <a, a> ∈ R
- Symmetric
- If <a,b> ∈ R,
  then <b,a> must be ∈ R
- Transitive
- If <a,b> and <b,c> ∈ R
  then <a,c> must be ∈ R

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Exercise 2

- Consider the following relations on { 1, 2, 3, 4 }:
- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Which of these relations are reflexive?

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Answer 2

- R3 since it contains all pairs of the form <a, a>, namely <1, 1>, <2, 2>, <3, 3 >, <4, 4>
- Consider the following relations on {1, 2, 3, 4}:
- R3 = { <1, 1> , <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3> , <4,1 >, <4, 4> }
- Which of these relations are reflexive?

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Answer 2

- R1 does not contain <3, 3>
- R4 does not contain <1,1>, <2,2>, <3,3> or <4,4>
- Consider the following relations on {1, 2, 3, 4}:
- R1 = { <1, 1> , <1, 2>, <2, 1>, <2, 2> , <3, 4>, <4,1>, <4,4> }
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Why are the other relations R1 and R4 not reflexive?

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Exercise 3

- Consider the following relations on { 1, 2, 3, 4}:
- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Which of these relations are symmetric?

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Answer 3

- R3, because in each case <b, a> belongs to the relation whenever <a, b> does.
- Consider the following relations on {1, 2, 3, 4}:
- R3 = { <1, 1> , <1, 2> , <1, 4> , <2, 1> , <2, 2> , <3, 3> , <4,1 > , <4, 4> }
- Which of these relations are symmetric?

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R3 : symmetric

- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}

	<a,b>	<b,a>	
a	<1,1>	<1,1>	(a)
b	<1,2>	<2,1>	(d)
c	<1,4>	<4,1>	(g)
d	<2,1>	<1,2>	(b)
e	<2,2>	<2,2>	(e)
f	<3,3>	<3,3>	(f)
g	<4,1>	<1,4>	(c)
h	<4,4>	<4,4>	(h)

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Answer 3

- The rest of relations are not symmetric: find a pair <a, b> so that it is in the relation but <b, a> is not.
- Consider the following relations on {1, 2, 3, 4}:
- R1 = {<1, 1>, <1, 2> , <2, 1> , <2, 2>, <3, 4> , <4,1> , <4,4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 > }
- Which of these relations are symmetric?

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Exercise 4

- Consider the following relations on { 1, 2, 3, 4}:
- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Which of these relations are transitive?

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Answer 4

- R4 since if <a, b> and <b, c> is in relation, then <a, c> is.
- R4 : <4,2> and <2, 1> then <4,1>
- Consider the following relations on {1, 2, 3, 4}:
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Which of these relations are transitive?

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R4 : transitive

R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

		Match?		
a	<2,1>	no		
b	<3,1>	no		
c	<3,2>	<2,1>	∴	<3,1> (b)
d	<4,1>	no		
e	<4,2>	<2,1>	∴	<4,1> (d)
f	<4,3>	<3,1>	∴	<4,1> (d)
		<3,2>	∴	<4,2> (e)

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Answer 4

- R1: <3, 4> and <4, 1> belong to it, while <3, 1> does not
- R3: <4, 1> and <1, 2> belong to it, while <4, 2> does not
- Consider the following relations on {1, 2, 3, 4}:
- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- Why the other relations R1 and R3 are not transitive?

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R1: non - transitive

a	<1,1>	<1,2>	∴	<1,2> (b)
b	<1,2>	<2,1>	∴	<1,1> (a)
		<2,2>	∴	<1,2> (b)
c	<2,1>	<1,1>	∴	<2,1> (c)
		<1,2>	∴	<2,2> (d)
d	<2,2>	<2,1>	∴	<2,1> (c)
e	<3,4>	<4,1>	∴	<3,1> NO!
f	<4,1>	<1,1>	∴	<4,1> (f)
		<1,2>	∴	<4,2> NO!
g	<4,4>	<4,1>	∴	<4,1> (f)

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PART 2. FUNCTIONS

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Exercise 1

- Let A = {a, b, c, d, e} and B = {1, 2, 3, 4} with
- f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 1 and f(e)= 1.
- (a) What is the domain of this function?
- (b) What is the co-domain?
- (c) What is the range of this function?

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Answer 1 a)

- Answer: set A = {a, b, c, d, e}
- Let A = {a, b, c, d, e} and B = {1, 2, 3, 4} with
- f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 1 and f(e)= 1.
- What is the domain of this function?

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Answer 1 b)

- Answer: set B = {1, 2, 3, 4}
- Let A = {a, b, c, d, e} and B = {1, 2, 3, 4} with
- f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 1 and f(e)= 1.
- What is the co-domain?

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Answer 1 c)

- Answer: set {1, 2, 4}
- Let A = {a, b, c, d, e} and B = {1, 2, 3, 4} with
- f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 1 and f(e)= 1.
- What is the range of this function?

Exercise 2

- Let f_1 and f_2 be two functions from A to B such that
  $f_1(x) = x^2$ and $f_2(x) = x – x^2$.
- What is the function $f_1 + f_2$?
- What is the function $f_1 x\ f_2$?

Answer 2 a)

- Answer : x
- $f_1(x) = x^2$ and $f_2(x) = x – x^2$.
- What is the function $f_1 + f_2$?
- $(f_1 + f_2)(x)$
  $= f_1(x) + f_2(x)$
  $= x^2 + x – x^2 = x^2 – x^2 + x$
  $= x$

Answer 2b)

- Answer = $x^3 - x^4$
- $f_1(x) = x^2$ and $f_2(x) = x – x^2$.
- b) What is the function $f_1 x\ f_2$?
- $(f_1\ f_2)(x)$
  $= f_1(x)\ f_2(x)$
  $= x^2 (x – x^2) = (x^2 * x) – (x^2 * x^2)$
  $= x^3 - x^4$

Exercise 3

- Let f and g be two functions from the set of integers to the set of integers defined by
- $f(x) = 2x + 3$ and $g(x)=3x + 2$
- a) What is the composition of f and g?
- b) What is the composition of g and f?

Answer 3a)

- Answer : 6x + 7
- $f(x) = 2x + 3$ and $g(x)=3x + 2$
- a) What is the composition of f and g?
- $(f \circ g)(x)$
  $= f(g(x))$
  $= f(3x + 2)$
  $= 2(3x+ 2) + 3$
  $= 6x + 4 + 3$
  $= 6x + 7$

Answer 3b)

- Answer 6x + 11
- $f(x) = 2x + 3$ and $g(x)=3x + 2$
- b) What is the composition of g and f?
- $(g \circ f)(x)$
  $= g(f(x))$
  $= g(2x + 3)$
  $= 3(2x+ 3) + 2$
  $= 6x + 9 + 2$
  $= 6x + 11$

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Exercise 4

- Let f(x)= x2 + 1 and g(x)= x + 2 be two functions from A to B.
- Find the following functions
- a) *f + g*
- b) *fg (or f x g)*
- c) *f* ° *g*
- d) *g* ° *f*

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Answer 4 a) *f + g*

- Answer $x^2 + x + 3$
- $f(x) = x^2 + 1$ and $g(x)= x + 2$
- $(f + g)(x)$
  $= f(x) + g(x)$
  $= x^2 + 1 + x + 2$
  $= x^2 + x + 1 + 2$
  $= x^2 + x + 3$

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Answer 4 b) *fg*

- Answer $x^3 + 2x^2 + x + 2$
- $f(x) = x^2 + 1$ and $g(x) = x + 2$
- $(f g)(x)$
  $= f(x)\, g(x)$
  $= (x^2 + 1)(x + 2)$
  $= x^2 (x + 2) + 1 (x + 2)$
  $= x^3 + 2x^2 + x + 2$

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Answer 4 c) *f* ° *g*

- Answer $x^2 + 4x + 5$
- $f(x) = x^2 + 1$ and $g(x) = x + 2$
- $(f \circ g)(x)$
  $= f(g(x))$
  $= f(x+2)$
  $= (x + 2)^2 +1$
  $= (x + 2)(x + 2) +1$
  $= x(x + 2)+2(x + 2) +1$
  $= x^2 + 2x + 2x + 4 +1$
  $= x^2 + 4x + 5$

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Answer 4 d) *g* ° *f*

- Answer $x^2 + 3$
- $f(x) = x^2 + 1$ and $g(x) = x + 2$
- $(g \circ f)(x)$
  $= g(f(x))$
  $= g(x^2 +1)$
  $= (x^2 + 1) + 2$
  $= x^2 + 3$

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Answer 5

- Answer $f^{-1}(x) = (x-5)/a$
- Let the function $f(x) = ax + 5$.
- Find its inverse
- $f(x) = y$ means $ax + 5 = y$
- $ax + 5 – 5 = y – 5$
- $ax = y – 5$
- $ax / a = (y – 5) / a$
- So $x = (y – 5)/a$
- So $f^{-1}(y) = (y – 5)/a$
- So the inverse function is $f^{-1}(x) = (x-5)/a$

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Answer 6

- Answer $f^{-1}(x) = {}^3\sqrt{(x-1)}$
- Find the inverse function of $f(x)= x^3 + 1$, where $x \in N$.
- $f(x)= x^3 + 1$ means $y = x^3 + 1$,
  so $x^3 = y - 1$
- So $x = {}^3\sqrt{(y-1)}$
- So $f^{-1}(x) = {}^3\sqrt{(x-1)}$

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Exercise 7

- For each of the following relations defined on the positive integers:

  >, <, =, ≥, ≤
- justify whether the relation is:
  - reflexive
  - symmetric
  - transitive

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Hint

- Build the 5 sets required where $R \subseteq A \times A$ and $A = \{1, 2, 3, 4, 5\}$.
- E for equal, L for less than, G for greater than, LE for less than or equal, GE for greater than or equal.
- Then test each set for the 3 qualities.

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- A = { 1, 2, 3, 4, 5}
- A x A =

	1	2	3	4	5
1	<1, 1>	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	<2, 1>	<2, 2>	<2, 3>	<2, 4>	<2, 5>
3	<3, 1>	<3, 2>	<3, 3>	<3, 4>	<3, 5>
4	<4, 1>	<4, 2>	<4, 3>	<4, 4>	<4, 5>
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	<5, 5>

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=

- E = { <1,1> , <2,2> , <3,3> , <4,4>, <5,5> }

E	1	2	3	4	5
1	**<1, 1>**	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	<2, 1>	**<2, 2>**	<2, 3>	<2, 4>	<2, 5>
3	<3, 1>	<3, 2>	**<3, 3>**	<3, 4>	<3, 5>
4	<4, 1>	<4, 2>	<4, 3>	**<4, 4>**	<4, 5>
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	**<5, 5>**

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<

- L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5> }

L	1	2	3	4	5
1	<1, 1>	**<1, 2>**	**<1, 3>**	**<1, 4>**	**<1, 5>**
2	<2, 1>	<2, 2>	**<2, 3>**	**<2, 4>**	**<2, 5>**
3	<3, 1>	<3, 2>	<3, 3>	**<3, 4>**	**<3, 5>**
4	<4, 1>	<4, 2>	<4, 3>	<4, 4>	**<4, 5>**
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	<5, 5>

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>

- G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>}

G	1	2	3	4	5
1	<1, 1>	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	**<2, 1>**	<2, 2>	<2, 3>	<2, 4>	<2, 5>
3	**<3, 1>**	**<3, 2>**	<3, 3>	<3, 4>	<3, 5>
4	**<4, 1>**	**<4, 2>**	**<4, 3>**	<4, 4>	<4, 5>
5	**<5, 1>**	**<5, 2>**	**<5, 3>**	**<5, 4>**	<5, 5>

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- LE = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5>, <1,1>, <2,2>, <3,3>, <4,4>, <5,5> }
- GE = {<2,1>, <3,1>, <3,2>, <4,1>, <4, 2>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>, <1,1>, <2,2>, <3,3>, <4,4>, <5,5>}

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Reflexive

R $\subseteq$ ***A x A*** is **reflexive** if and only if
< a, a > $\in$ ***R*** for every element ***a*** of ***A***
- every element of A is in relation with itself

- So for A = { 1, 2, 3, 4, 5} R *must* contain <1,1>, <2,2>, <3,3>, <4,4> and <5, 5>.

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Reflexive?

E = { **<1,1> , <2,2> , <3,3> , <4,4>, <5,5>** }	yes
L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5> }	no
G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>}	no
LE = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5>, **<1,1>, <2,2>, <3,3>, <4,4>, <5,5>** }	yes
GE = {<2,1>, <3,1>, <3,2>, <4,1>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>, **<1,1>, <2,2>, <3,3>, <4,4>, <5,5>**}	yes

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Symmetric

R $\subseteq$ ***A x A is*** **symmetric** if and only if
for any ***a***, and ***b*** in ***A***,
whenever **<*a, b*>** $\in$ ***R*** then **<*b, a*>** $\in$ ***R*.**

Symmetric?

E = { **<1,1> , <2,2> , <3,3> , <4,4>, <5,5>** } yes

L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5> } i.e <5,4> absent no

G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>} i.e. <4,5> absent no

LE = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5>, <1,1>, <2,2>, <3,3>, <4,4>, <5,5> } i.e. <5,4> absent no

GE = {<2,1>, <3,1>, <3,2>, <4,1>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>, <1,1>, <2,2>, <3,3>, <4,4>, <5,5>} i.e. <4,5> absent no

Transitive

- R $\subseteq$ A x A is transitive if and only if
  for any a, b, and c $\in$ A,
  if <a, b> $\in$ R , and <b, c> $\in$ R
  then <a, c> $\in$ R
- '=' is transitive, for if a = b and b = c then a = c. (1 = 1 and 1 = 1 then 1 = 1)
- '>' is transitive, for if a > b and b > c then a > c. (5 > 4 and 4 > 3 then 5 > 3)
- '<' is transitive, for if a < b and b < c then a < c. (3 < 4 and 4 < 5 then 3 < 5)
- '>=' and '<=' are also transitive.

THE END

Discrete Maths : 1
Workshop Unit 3 :
Recursion

SCC120 Fundamentals of Computer Science

Exercise 1

Suppose that f is defined recursively by

$f(0) = 3$

$f(n) = 2 f(n-1) + 3$

Find f(1), f(2), f(3), and f(4).

Answer 1

$f(0) = 3$

$f(n) = 2 f(n-1) + 3$

$f(1) = 2\ f(0) + 3 = 2 * 3 + 3 = 6 + 3 = 9$
$f(2) = 2\ f(1) + 3 = 2 * 9 + 3 = 18 + 3 = 21$
$f(3) = 2\ f(2) + 3 = 2 * 21 + 3 = 42 + 3 = 45$
$f(4) = 2\ f(3) + 3 = 2 * 45 + 3 = 90 + 3 = 93$

Exercise 2

define the following function recursively, using the formula f(n) = f(n-1) + B : f(n) = 3n – 4.

Solve using algebra rather than numerically. Also, work out the value of the base clause, f(0).

Answer 2
Base Clause

Firstly, work out the value of the Base Clause

$f(0) = 3 * 0 – 4 = 0 – 4 = -4$

Answer 2
Recursive Clause

- $f(n) = 3n - 4$
- To work out what f(n-1) is, we “plug in” n-1 in the above formula.
- We replace “n” by “n-1” throughout.
- $f(n-1) = 3(n-1) - 4$
  $= 3n – 3 - 4$
  $= 3n – 7$

Answer 2
Recursive Clause

- We wish to express f(n) in terms of f(n-1).
- f(n) = f(n-1) + B.
- "B" is some unknown quantity that we need to find.
- On the previous slide, we defined f(n-1) as
- f(n-1) = 3n – 7
- So we substitute "3n -7" for "f(n-1)" in the first equation above, giving
  f(n) = 3n – 7 + B

Answer 2
Recursive Clause

- f(n) = 3n – 7 + B
- We know from the original specification that f(n) = 3n - 4
- So we substitute "3n - 4" for "f(n)" in the first equation on this slide, giving
- 3n - 4 = 3n – 7 + B
- Which is the same as
- 3n – 7 + B = 3n - 4

Answer 2
Recursive Clause

- 3n – 7 + B = 3n - 4
- We are trying to find out what B is, so we want an equation with B on its own on the LHS.
- 3n -7 + B - 3n = 3n - 4 - 3n
  -7 + B = -4
  -7 + B + 7 = -4 + 7
  B = 3
- f(n) = f(n-1) + B
- f(n) = f(n-1) + 3
- This is our recursive definition for f(n)

THE END