SCC120 Fundamentals of Computer Science
Unit 4: Graphs (Terminology)

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Graphs

(and also Trees) 

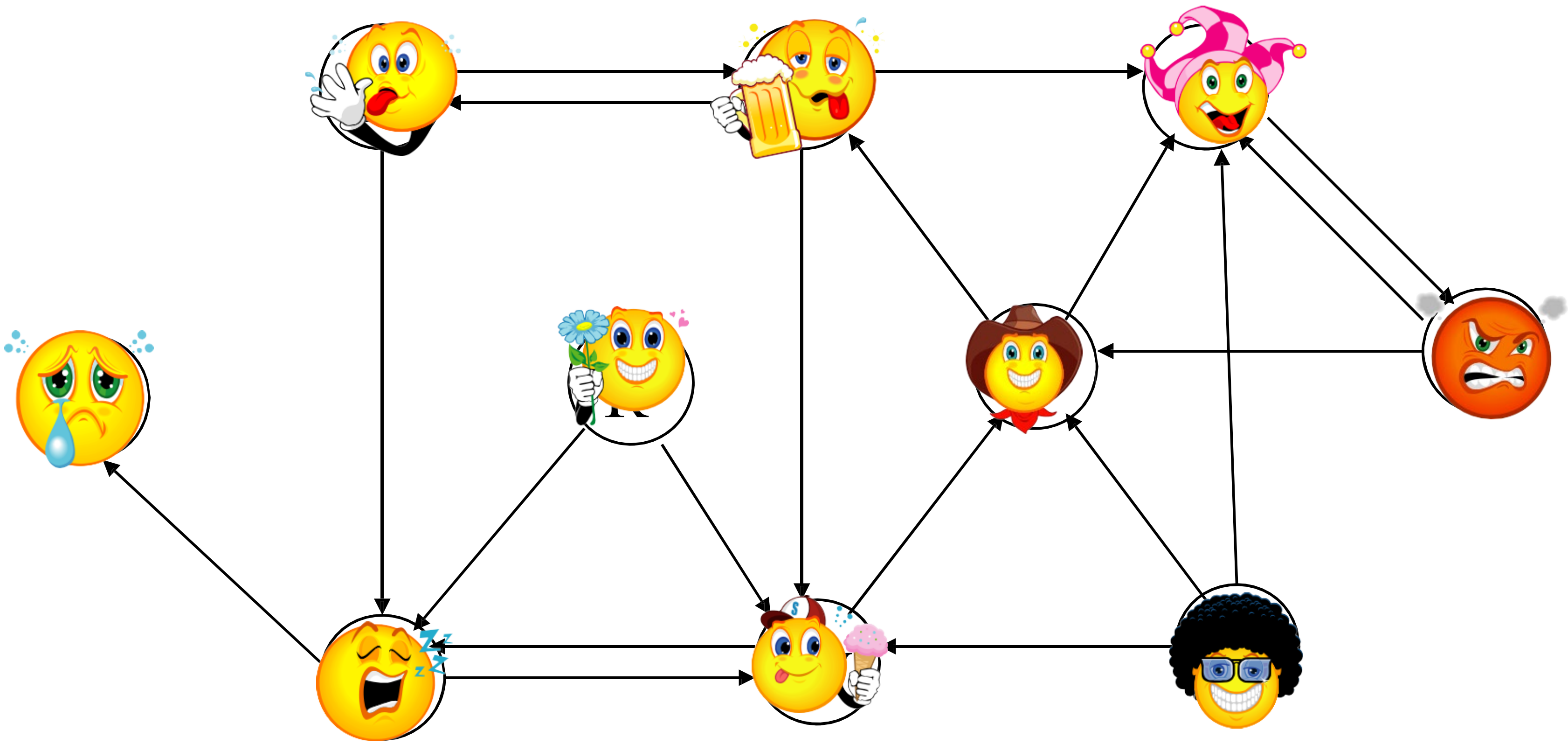
Two Types of Graph

- Graphs can be divided into two broad types
  - *directed graphs*, or *digraphs*, where each edge or arc has an associated direction (for example, from node X to node Y)
  - *non-directed graphs*, or *simple graphs*, in which every edge or arc is two-way
- We will concentrate mostly on *directed* graphs
  - but will look at simple graphs too

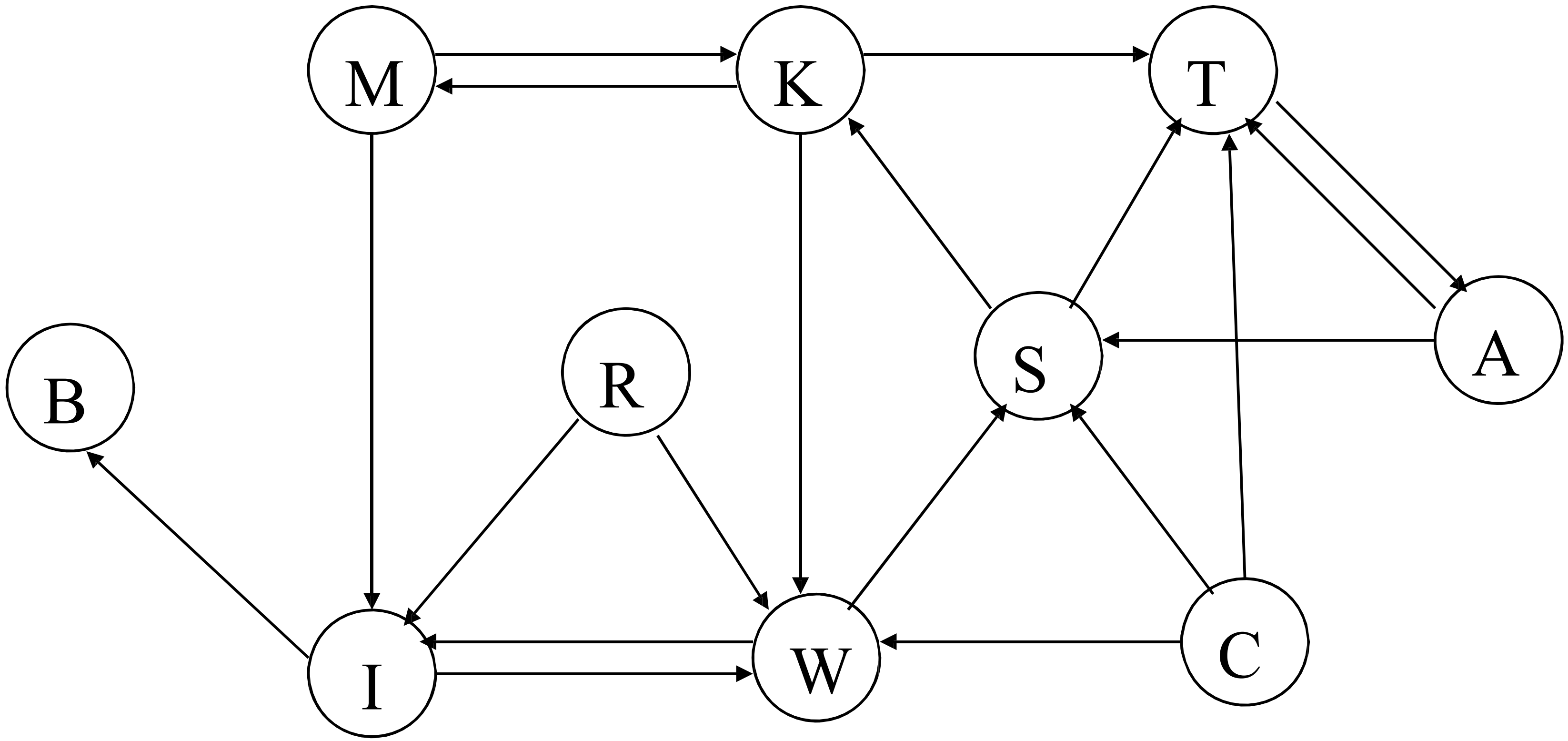
A Directed Graph

- Imagine a group of ten students on a social networking site
- Each one has up to three friends
- We can summarise this information using a diagram (or with letters for names)

A Directed Graph (Digraph)



A Directed Graph (Digraph)



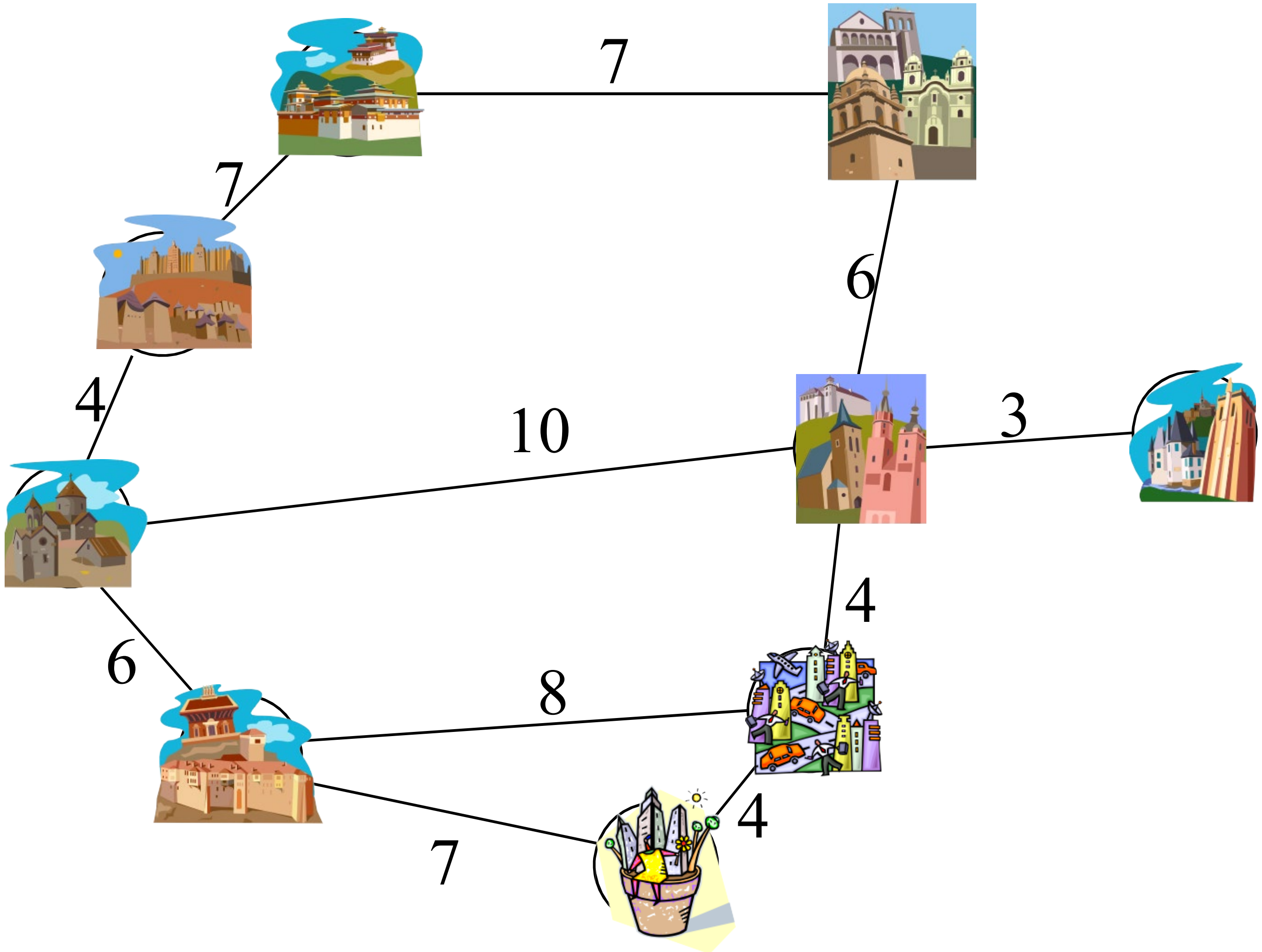
A Directed Graph (Digraph)

- That was an example of a *directed graph*
  - or *digraph* for short
- Even if there is a link from one node to another, there may or may not be a link in the reverse direction

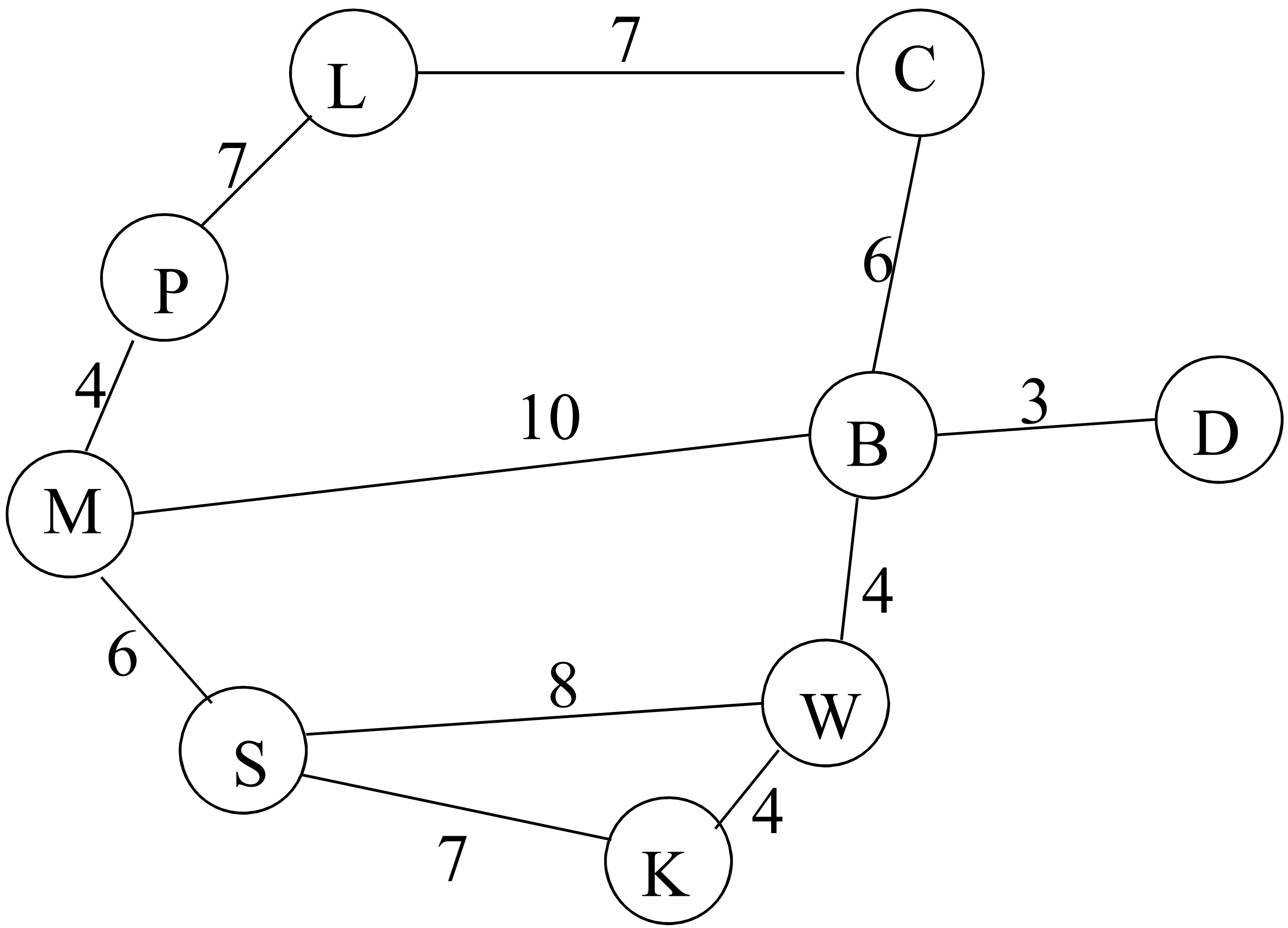
Simple Graphs

- Imagine an area of land, such as an island, with nine towns or villages
- We can represent the roads and distances between them as a graph

Simple Graphs



Simple Graphs



Simple Graphs

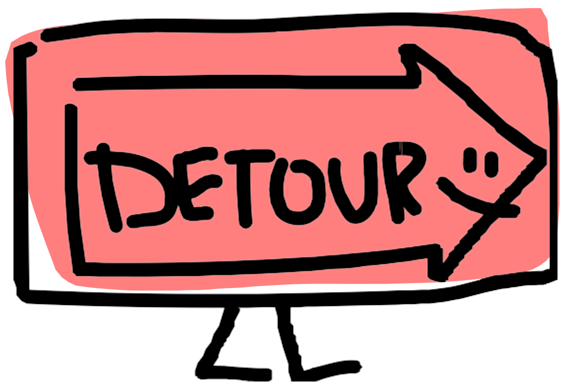
- That was an example of a *non-directed graph*, or *simple graph*
- There is no direction associated with a link
  - it simply connects node A to node B
  - and so also connects node B to node A

Numbers corresponding to edges

- The distance numbers are not necessary (although important in many cases)
  - it would still be a meaningful simple graph without this added data
  - it would show whether or not there was a (direct) road from one location to another
- Similarly, a directed graph may or may not have a number (or some other data) associated with each link

Graphs (and Trees)

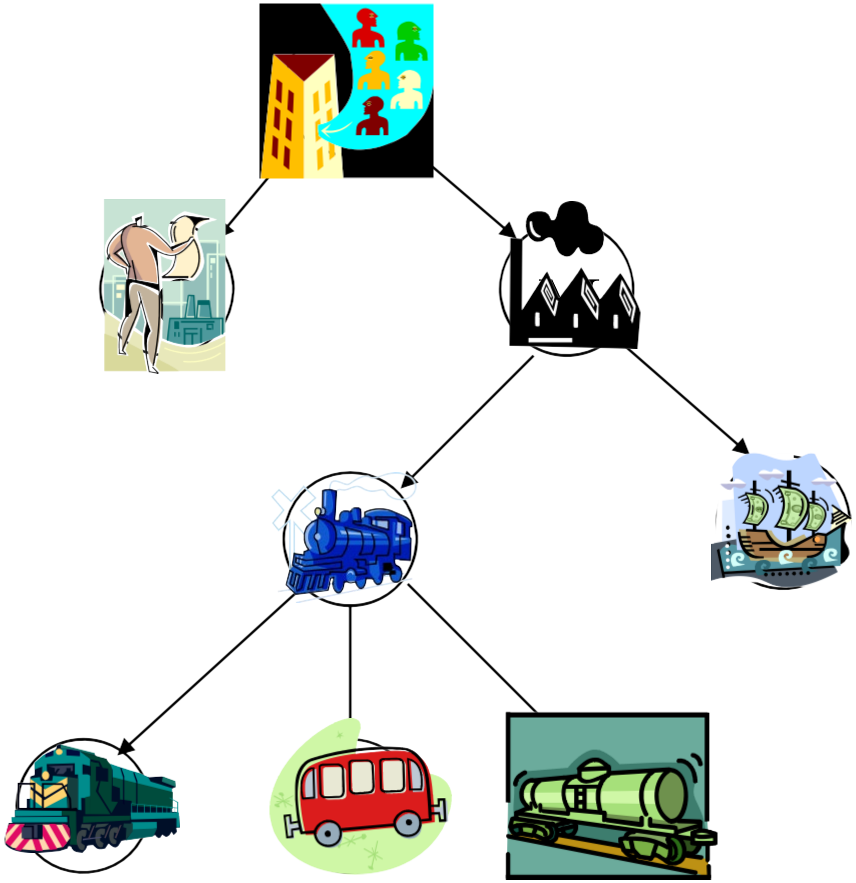
- In this unit, we are going to look at *graphs*
- Later, we will look at *trees*
- However, we need to know what a tree looks like for the discussion of graphs



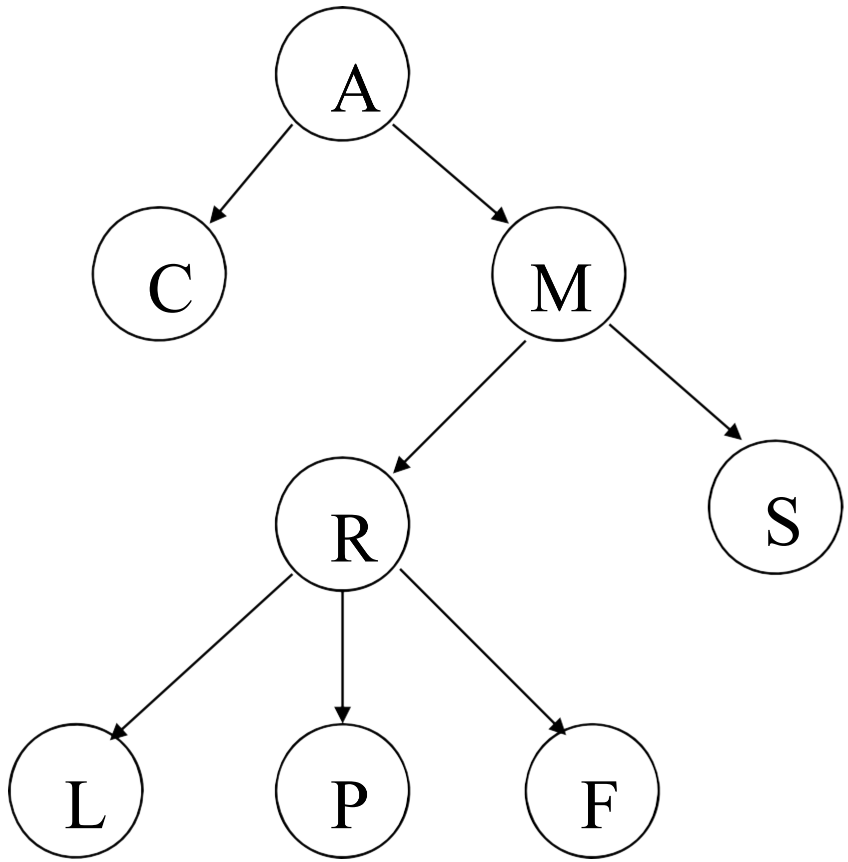
A Tree

- A certain engineering company A is divided into a consultancy division C and a manufacturing division M
- The manufacturing division M is divided into a railway section R and a marine engine section S
- Section R is divided into three departments, building locomotives (L), passenger coaches (P), and freight wagons (F)

A Tree

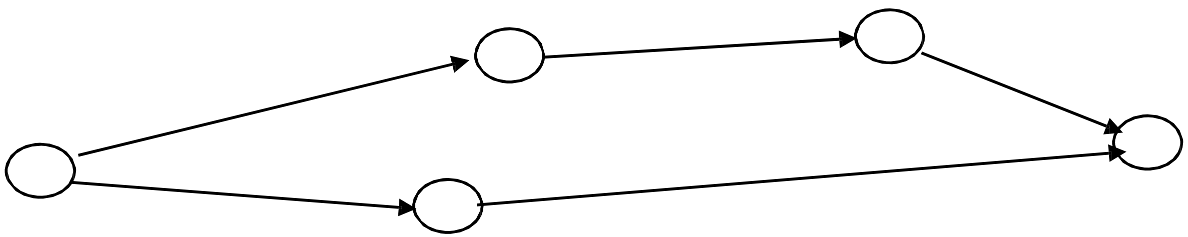


A Tree



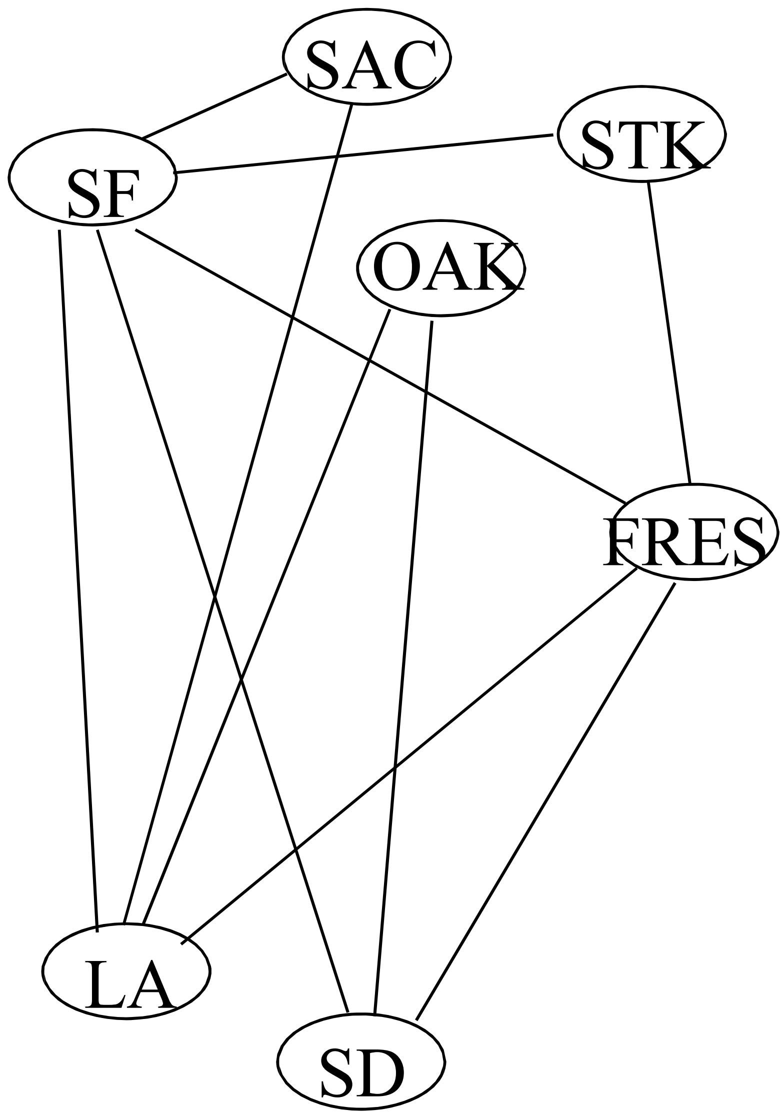
A Tree

- That was an example of a *tree*
- The node at the top is called the *root*
- Trees are used for holding hierarchical information
- A tree is “a graph with no loops”
  – we must also have no paths “separating and then rejoining”



Back to “Graphs”

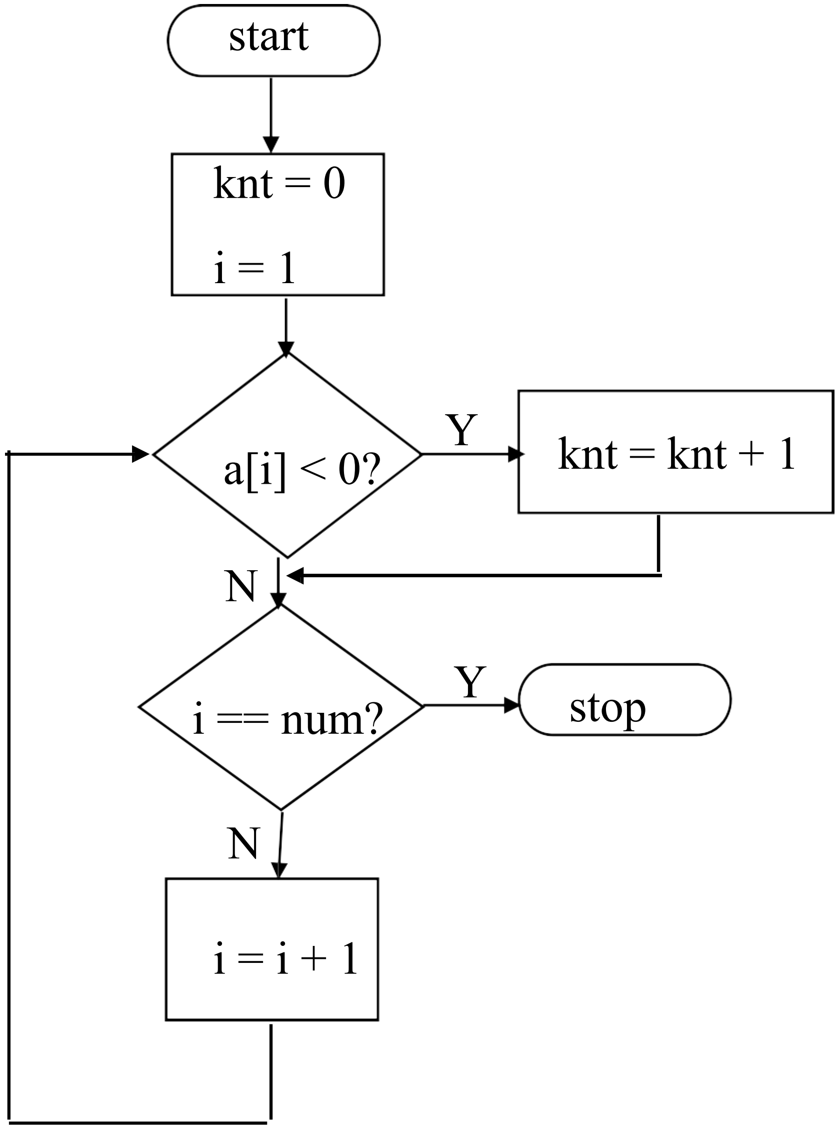
Example 1: Airline Routes



Example 1: Airline Routes

- A line connects two cities if and only if there is a non-stop flight between them in both directions
- Some possible questions:
  - Is there a nonstop flight between SD and SAC?
  - What is the cheapest way to fly from STK to SD?
  - Which route involves the least flying time?
  - If one city’s airport is closed by bad weather, can you still fly between any other pair of cities?
- The 2nd and 3rd questions require additional information on each route

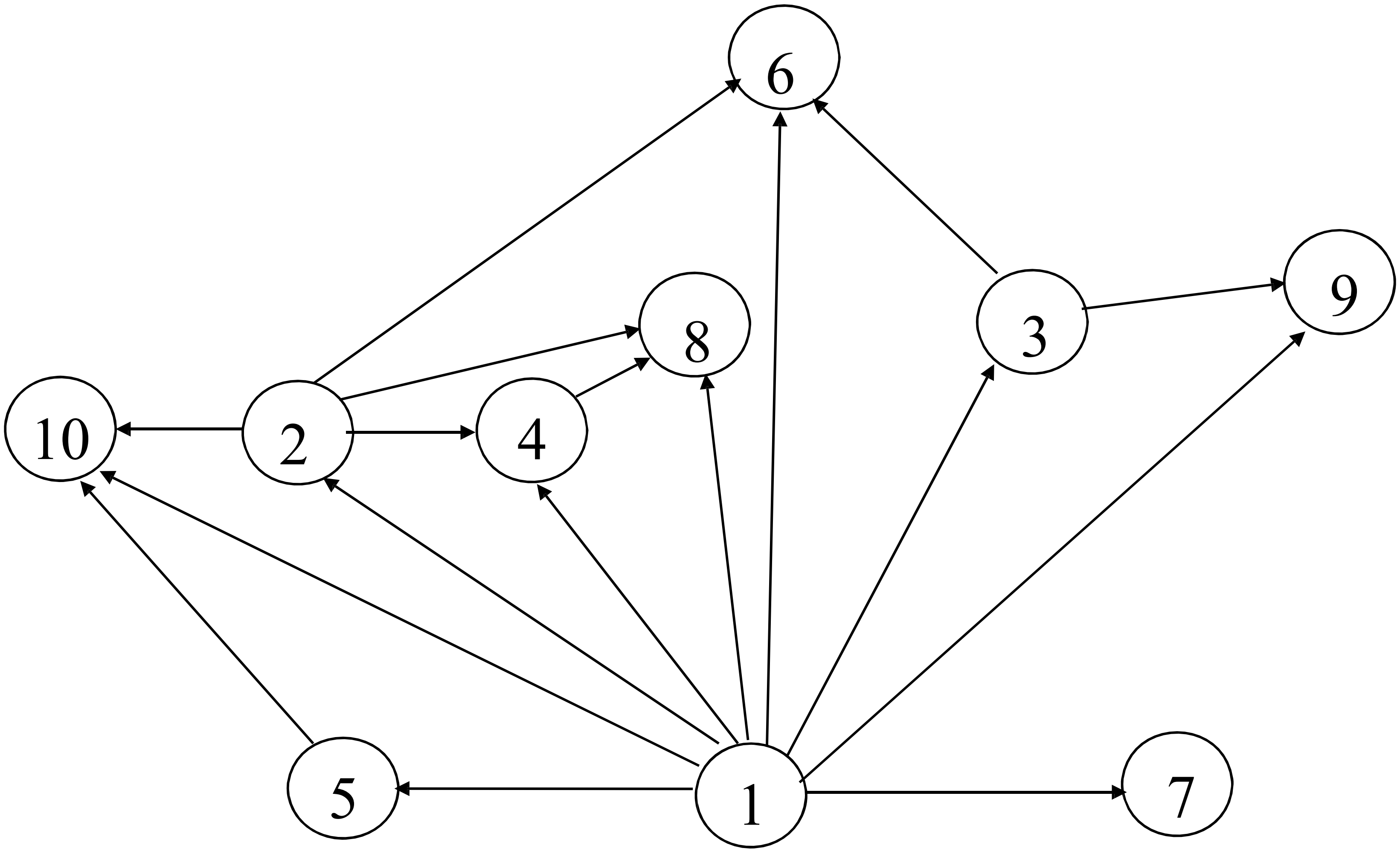
Example 2: Program Flowchart



Example 2: Program Flowchart

- The points or nodes are the flowchart boxes; the lines or edges are the flowchart arrows
- Some possible questions are:
  - Does this flowchart contain a loop?
  - How many separate paths are there through the flowchart?

Example 3: A Binary Relation

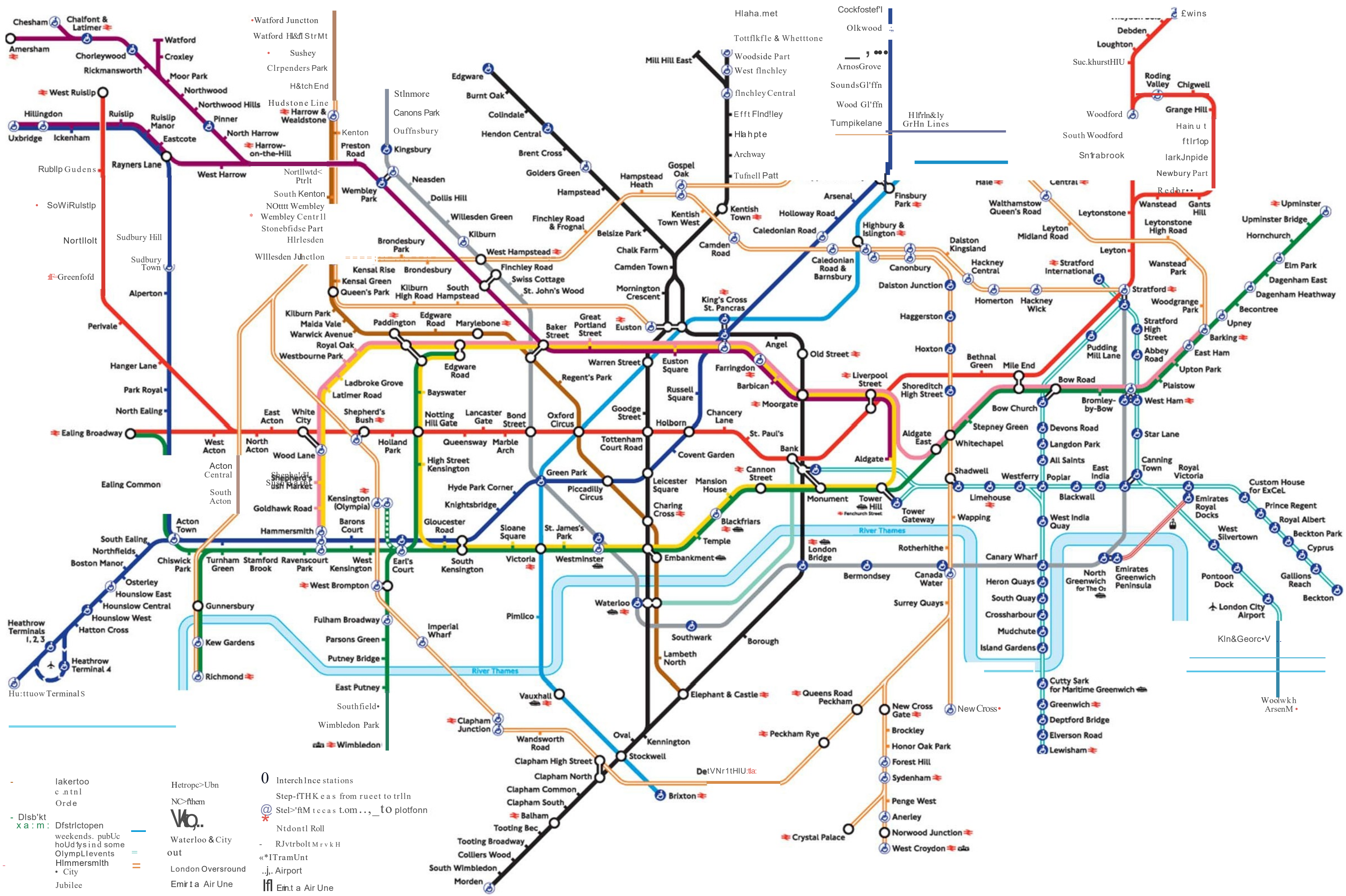


Example 3: A Binary Relation

- Let S be the set { 1, 2, 3, …, 9, 10 }
- Let R be the relation on S defined by xRy if and only if $x \neq y$ and x divides y (that is, with no remainder)
- So 3R6 and 3R9, but not 3R3 or 3R7
- A possible question is:
  - Is the relation R *transitive* (that is, aRb and bRc together imply aRC)?

Other Examples

- A road map or railway network
  - What is the shortest, or cheapest, or fastest, route from A to B?


Chesham
Chalfont & Latimer
Amersham
Chorleywood
Rickmansworth
Watford
Croxley
Moor Park
Northwood
Northwood Hills
Pinner
North Harrow
Harrow-on-the-Hill
West Harrow
West Ruislip
Hillingdon
Ruislip
Ruislip Manor
Eastcote
Uxbridge
Ickenham
Rayners Lane
Rubllp Gudens
SoWiRulstlp
Nortllolt
Greenfofd
Sudbury Hill
Sudbury Town
Alperton
Perivale
Hanger Lane
Park Royal
North Ealing
Ealing Broadway
Ealing Common
West Acton
North Acton
East Acton
White City
Acton Central
South Acton
Acton Town
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow Central
Hounslow West
Hatton Cross
Heathrow Terminals 1, 2, 3
Heathrow Terminal 4
Hu:ttuow Terminal S
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
Watford Junctton
Watford Hl&fl StrMt
Sushey
Clrpenders Park
H&tch End
Hudstone Line
Harrow & Wealdstone
Kenton
Preston Road
Nortllwtd< Ptrlt
South Kenton
NOtttt Wembley
Wembley Centrl l
Stonebfidse Part
Hlrlesden
Wlllesden Juhctlon
Stlnmore
Canons Park
Ouffnsbury
Kingsbury
Wembley Park
Neasden
Dollis Hill
Willesden Green
Kilburn
West Hampstead
Brondesbury Park
Kensal Rise
Brondesbury
Kensal Green
Queen's Park
Kilburn High Road
South Hampstead
Kilburn Park
Maida Vale
Warwick Avenue
Royal Oak
Westbourne Park
Paddington
Edgware Road
Marylebone
Ladbroke Grove
Latimer Road
Shepherd's Bush
Shepherd's Bush Market
Goldhawk Road
Hammersmith
Kensington (Olympia)
Barons Court
West Kensington
Holland Park
Notting Hill Gate
Queensway
Lancaster Gate
Marble Arch
Bond Street
Bayswater
High Street Kensington
Gloucester Road
Earl's Court
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfield
Wimbledon Park
Wimbledon
Imperial Wharf
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Finchley Road & Frognal
Finchley Road
Swiss Cottage
St. John's Wood
Belsize Park
Chalk Farm
Camden Town
Mornington Crescent
Hampstead Heath
Gospel Oak
Kentish Town West
Kentish Town
Camden Road
Mill Hill East
Hlaha.met
Tottflkfle & Whetttone
Woodside Part
West flnchley
flnchley Central
Efft Flnd!ley
Hlahpte
Archway
Tufnell Patt
Cockfostef'l
Olkwood
ArnosGrove
SoundsGl'ffn
Wood Gl'ffn
Tumpikelane
Hlfrln&ly GrHn Lines
Arsenal
Holloway Road
Caledonian Road
Caledonian Road & Barnsbury
Finsbury Park
Highbury & Islington
Canonbury
Dalston Junction
Dalston Kingsland
Haggerston
Hoxton
Hackney Central
Homerton
Hackney Wick
King's Cross St. Pancras
Euston
Euston Square
Warren Street
Great Portland Street
Baker Street
Regent's Park
Oxford Circus
Goodge Street
Tottenham Court Road
Russell Square
Holborn
Chancery Lane
Farringdon
Barbican
Angel
Old Street
Moorgate
Liverpool Street
St. Paul's
Bank
Covent Garden
Leicester Square
Piccadilly Circus
Green Park
Hyde Park Corner
Knightsbridge
South Kensington
Sloane Square
Victoria
St. James's Park
Westminster
Charing Cross
Embankment
Temple
Blackfriars
Mansion House
Cannon Street
Monument
Tower Hill
Fenchurch Street
Aldgate
Aldgate East
Tower Gateway
Shoreditch High Street
Whitechapel
Bethnal Green
Mile End
Stepney Green
Bow Road
Bow Church
Shadwell
Limehouse
Wapping
London Bridge
Bermondsey
Rotherhithe
Canada Water
Surrey Quays
River Thames
Waterloo
Southwark
Borough
Lambeth North
Elephant & Castle
Pimlico
Vauxhall
Oval
Kennington
Stockwell
Brixton
Wandsworth Road
Clapham Junction
Clapham High Street
Clapham North
Clapham Common
Clapham South
Balham
Tooting Bec
Tooting Broadway
Colliers Wood
South Wimbledon
Morden
Queens Road Peckham
Peckham Rye
DetVNr1tHIU:tla:
New Cross Gate
New Cross
Brockley
Honor Oak Park
Forest Hill
Sydenham
Crystal Palace
Penge West
Anerley
Norwood Junction
West Croydon
Walthamstow Queen's Road
Leyton Midland Road
Leytonstone
Leyton
Stratford International
Stratford
Pudding Mill Lane
Devons Road
Bromley-by-Bow
West Ham
Langdon Park
All Saints
Westferry
Poplar
East India
Blackwall
West India Quay
Canary Wharf
Heron Quays
South Quay
Crossharbour
Mudchute
Island Gardens
Cutty Sark for Maritime Greenwich
Greenwich
Deptford Bridge
Elverson Road
Lewisham
North Greenwich for The O2
Emirates Greenwich Peninsula
Emirates Royal Docks
Canning Town
Star Lane
Royal Victoria
Custom House for ExCeL
Prince Regent
Royal Albert
Beckton Park
Cyprus
Gallions Reach
Beckton
West Silvertown
Pontoon Dock
London City Airport
Kln&Georc•V
Woolwkh ArsenM
Stratford High Street
Abbey Road
Plaistow
Upton Park
East Ham
Barking
Upney
Becontree
Dagenham Heathway
Dagenham East
Elm Park
Hornchurch
Upminster Bridge
Upminster
Wanstead Park
Woodgrange Park
Leytonstone High Road
Wanstead
Gants Hill
Redbr••
Newbury Part
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Hainu t
Grange Hill
Chigwell
Roding Valley
Woodford
South Woodford
Sn'rabrook
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Other Examples

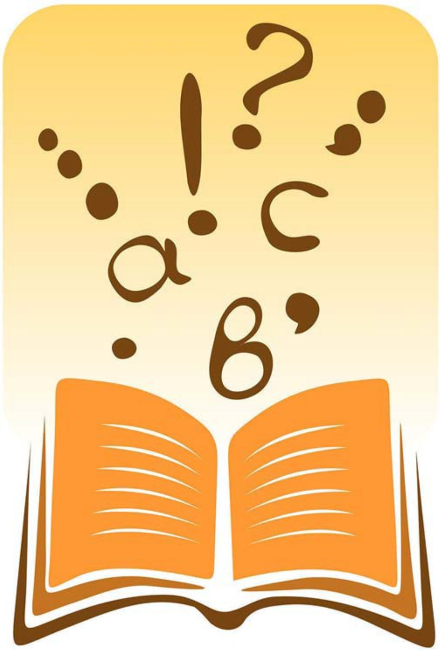
- A computer network
  - If one computer in a network goes down, can messages still be sent between any other pair of computers in the network?

Other Examples

- An electrical circuit
  - How can we interconnect various pieces of electrical equipment so that we use the minimum amount of wire?
  - Is the graph *planar*?
    - That is, can we draw the circuit on a flat surface without crossings?
    - If not, what is the minimum number of crossings to draw it on a flat surface?

Terminology

- There is a lot of terminology associated with graphs
- Some concepts have more than one name



Representing a (Directed) Graph

- We can represent a graph as an ordered pair
  - a list of *nodes* (or *vertices*)
  - and a list of *arcs* (or *edges*)
- G = (N, A)
  - N = { a, b, c }
  - A = { (a, a), (a, c), (b, a), (c, a), (c, b) }
- Or
  - G = ({ a, b, c }, { (a, a), (a, c), (b, a), (c, a), (c, b) })
- This is a graph of 3 nodes and 5 arcs

Terminology

- The number of nodes is sometimes called its *weight*
- Node b is said to be *adjacent* to another node a if there is an arc (a, b)
- If we have an arc (a, b), a is the *out-node* and b is the *in-node*
- Similarly we can define the *in-arcs* and *out-arcs* of a node
- An arc whose out-node and in-node is the same is a *self-loop;* for example, G has a self-loop (a, a)

Numbers of Arcs

- An arc cannot be duplicated
  - there cannot be two arcs (a, c)
  - though (in a *directed* graph) there may or may not be an arc (c, a) as well
- So if the weight (number of nodes) of a graph is w, the maximum possible number of arcs is w^2
- A directed graph which contains the maximum possible number of arcs is called a *full* graph

Numbers of Arcs

- The *density* of a graph is the proportion of the w^2 possible arcs which are actually present
  - so if there are a arcs and w nodes, the density is a/w^2
  - so for the graph G, density = 5/9 = 0.56
  - for the full graph, density = 9/9 = 1.0

Numbers of Arcs

- A graph with a density of zero (that is, no arcs at all) is said to be *empty*
  - or *totally disconnected*
- A graph with very few arcs compared to the number of nodes (that is, whose density is low) is *sparse*
  - this is not a precise term

The Complementary Graph

- The *complementary graph* of G contains
  - all the nodes of G
  - all the possible arcs not present in G
- The complementary graph of G is denoted G*
  - density(G*) = 1 – density(G)
  - (G*)* = G

A Partial (Reduced) Graph

- A *partial* (or *reduced*) graph of G is one with
  - all the nodes of G
  - one or more arc deleted

A Subgraph

- A *subgraph* of G is a graph derived from G with
  - one or more node deleted
  - all the arcs deleted which are associated with one or more of the deleted nodes
- We can use the term subgraph to refer to some portion of G which we are focussing on at present, as though it were separate from the rest of the graph

Paths and Distances

- A *path* of a graph G is an ordered set of nodes such that each node is adjacent to its predecessor
  - for example P = (b, a, c, a, a)
  - we could also write it as a sequence of arcs P = ((b, a), (a, c), (c, a), (a, a))
- The *length* of a path is the number of arcs it contains
  - so the length of P is 4

Paths and Distances

- A *simple* (or *elementary*) path is one which does not visit any node more than once
  - for example, (c, b, a) is a simple path of G
- A *loop* (or *cycle*) is a path whose initial and final nodes are the same
  - for example, (c, b, a, a, c) is a loop in G
- A *simple loop* is a loop which becomes a simple path if its final node is deleted
  - for example, (c, b, a, c) is a simple loop in G

Paths and Distances

- The *distance* between two nodes is the length of the shortest path between the nodes
  - a shortest path is always simple (why?)
- The *diameter* of a graph is the largest distance which can be found in it
  - so in G, there are shortest paths of length 2
  - ((b, a), (a, c)) and ((a, c), (c, b))
  - and none of length 3
  - so the diameter of G is 2

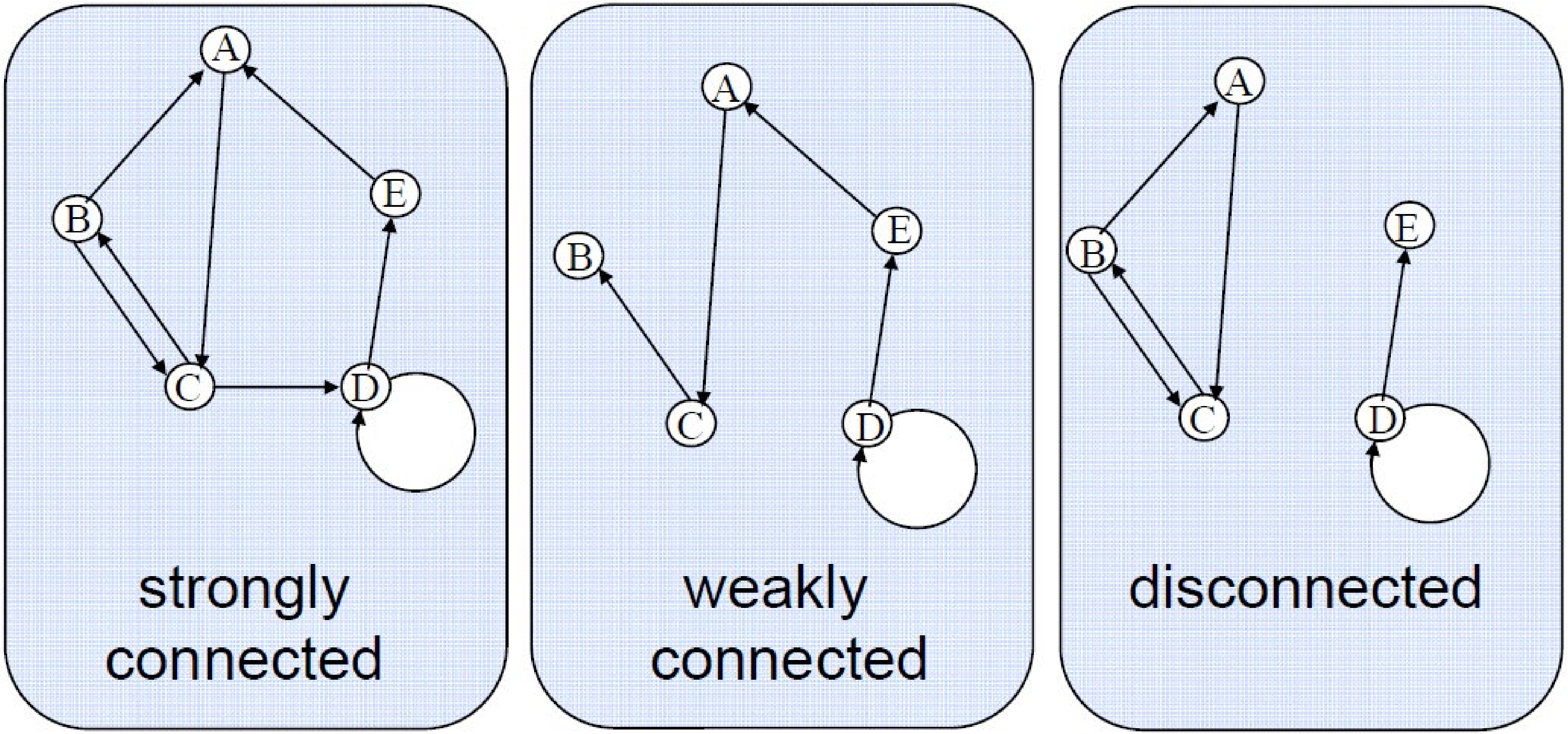
Paths and Distances

- There are algorithms to calculate the distance (shortest path) between two nodes in a graph
- We will look at some of these later

In- and Out-Degree

- A node has *degree* (p, q) if p arcs point to it, and q arcs leave it
  - so in graph G node “a” is of degree (3, 2)
  - p is called the *in-degree* of the node
  - q is called the *out-degree* of the node
- In a full graph, every node has degree (w, w)

Connectivity



Connectivity

- For some applications, it is very important that a digraph be strongly connected
- For example, suppose we represent a one-way traffic network as a digraph, where each stretch of road is an arc, and each cross-roads is a node
  - What are the consequences if the network is not strongly connected?
- In a simple (non-directed) graph, every arc can be traversed in either direction, so it is either connected or disconnected
  - There is no strong/weak connectivity concept

SCC120 ADT (weeks 5-10)

- Week 5 Abstractions; Set
  Stack
- Week 6 Queues
  Priority Queues
- Week 7 Graphs (Terminology)
- Week 8
- Week 9
- Week 10