Discrete Maths
Workshop Unit 1 : Propositional Logic

SCC120 Fundamentals of Computer Science

Propositional Logic

Exercise 1

For the OR ($\vee$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. A $\vee$ B $\vee$ C. The table should have a heading as shown.

A	B	C	Result

Exercise 2

For the AND ($\wedge$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. A $\wedge$B $\wedge$ C. The table should have a heading as shown.

A	B	C	Result

Exercise 3

Here is a tautology. Prove that this is the case using the truth table shown below.

P $\vee$ ~(P $\wedge$ Q)

a	b	c = (a $\wedge$ b)	d = ~c	e = a $\vee$ d
P	Q	(P $\wedge$ Q)	~(P $\wedge$ Q)	P $\vee$ ~(P $\wedge$ Q)
F	F			
F	T			
T	F			
T	T			

Commutivity : $(P \vee Q) \Leftrightarrow (Q \vee P)$
DeMorgan's Law $\sim (P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$
Simple Identity $\sim Q \vee Q \Leftrightarrow True$
Domination : $(P \vee True) \Leftrightarrow True$
Implication : $(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$
Associativity : $(P \vee Q) \vee R \Leftrightarrow$
$P \vee (Q \vee R) \Leftrightarrow P \vee Q \vee R$

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1. Show that the following implications are tautologies without using truth tables. In other words, use equivalencies to manipulate the expressions until you are left with the single result, true. Give the name of each equivalence as you apply it. If the transformation is not a named equivalence, give the reasoning behind it.

The general aims of the transformations are

- Get rid of implications, if there are any.
- convert the expression to one using as few different operators as possible (ideally just one)
- end up with a single TRUE result.

To show that the proposition is a tautology, you need to convert ("reduce") the proposition to True.

a) $A \vee \sim(A \wedge B)$
b) $A \rightarrow (A \text{ v } B)$
c) $(A \wedge B) \rightarrow (A \rightarrow B)$

2. Let P(x) be the statement "x likes marmite," where the universe of discourse for x is the set of people. Express each of the following quantifications in English.

(a) $\exists x\, P(x)$ (b) $\exists x\, \neg P(x)$
(c) $\forall x\, \neg P(x)$ (d) $\neg \forall x\, \neg P(x)$

3. Working with the characters of the hilarious TV programme "The Simpsons", express the assertions given below as propositions of predicate logic using the following predicates.

Father (x,y) : x is y's father , or equivalently y is x's child.
Mother (x,y) : x is y's mother, or equivalently y is x's child.
Brother (x,y): x is y's brother.
Sister (x,y) : x is y's sister.

(a) Marge is Lisa's mother and Bart's mother
(b) There is a character in the Simpsons that is Lisa's mother and Bart's mother
(c) There is a kid whose father is Homer and whose brother is Bart
(d) There is a kid whose father is Homer and whose sister is Lisa and whose brother is Bart

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Exercises

1. Using truth tables, verify the following equivalences, which are known as the absorption laws.

(a) $p \vee (p \wedge q) \Leftrightarrow p$ (b) $p \wedge (p \vee q) \Leftrightarrow p$

2. a) Let A = "It is Wednesday" and B = "Coronation Street is on TV". Transform the English proposition "If it is Wednesday, then Coronation Street is on TV" into formal logic.

b) Let A = "You can legally drive" and B = "You have a Driver's Licence". Transform the English proposition "You can legally drive because you have a Driver's licence" into formal logic.

c) Let A = "You are a double O agent" and B = "You have a licence to kill". Transform the English proposition "You are not a double O agent so you do not have a licence to kill" into formal logic.

3. A battle between four exponents of the martial arts is under way. The following facts are known:

(e) The good Lord Alpha will not survive if the evil Count Gamma survives
(f) Either the evil Count Delta will die or the good Lord Beta will die
(g) Lord Alpha is the master of Kung Fu, while none of the other three are masters, and it is a fact that a master can only be beaten by another master
(h) Count Gamma will die only if either Lord Alpha dies or Count Delta dies

Exactly two survive. Who are they?

Hint : before transforming the third fact into a proposition, rewrite it after you have worked out what it really means.

The Process :

- List the propositions you wish to discover true and false values for.
- List the valid combinations of the propositions.
- Encode the facts ((e) to (h), as given above) in propositional logic
- Build truth tables, containing the valid combinations, for each fact.
- Produce a final table containing the valid combinations and the outcomes of the fact truth tables.
- Read off the answer from the final table.