Discrete Maths : 2
Workshop Unit 1

SCC120 Fundamentals of Computer Science

Exercise 1

For the OR ($\vee$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. $A \vee B \vee C$. The table should have a heading as shown.

A	B	C	Result

Answer 1

For the OR ($\vee$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. $A \vee B \vee C$. The table should have a heading as shown.

A	B	C	Result
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Exercise 2

For the AND ($\wedge$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. $A \wedge B \wedge C$. The table should have a heading as shown.

A	B	C	Result

Answer 2

For the AND ($\wedge$) operator, create a truth table showing the outcomes of the operation when three operands are involved i.e. $A \wedge B \wedge C$. The table should have a heading as shown.

A	B	C	Result
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	T

Exercise 3

Here is a tautology. Prove that this is the case using the truth table shown below.

$P \vee \sim(P \wedge Q)$

a	b	$c = (a \wedge b)$	$d = \sim c$	$e = a \vee d$
P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$P \vee \sim(P \wedge Q)$
F	F			
F	T			
T	F			
T	T			

Answer 3

Here is a tautology. Prove that this is the case using the truth table shown below.

$P \vee \sim(P \wedge Q)$

a	b
P	Q
F	F
F	T
T	F
T	T

c=(a∧b)	d=~c	e=a∨d
(P∧Q)	~(P∧Q)	P∨~(P∧Q)
F	T	T
F	T	T
F	T	T
T	F	T

THE END

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Discrete Maths : 2 Workshop Unit 2

SCC120 Fundamentals of Computer Science

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- Prove that these are tautologies by applying the laws and equivalences of logic.
- The general aims of the transformations are
  - Get rid of implications if there are any.
  - convert the expression to one using as few different operators as possible (ideally just one)
- To show that the proposition is a tautology, you need to convert (“reduce”) the proposition to True.

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Question 1 : Fundamental Equivalences and Identities

- 1 a) $A \vee \sim(A \wedge B)$
- 1 b) $A \rightarrow (A \vee B)$
- 1 c) $(A \wedge B) \rightarrow (A \rightarrow B)$

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Step 1. Q1 a). DeMorgan’s Law

$$A \vee \sim(A \wedge B)$$

$$\sim(P \wedge Q)$$

$$\Leftrightarrow$$

$$(\sim P \vee \sim Q)$$

$$A \vee (\sim A \vee \sim B)$$

Mappings

A	P
B	Q

$$\sim(P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$$

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Step 2. Q1 a). By Associativity

Mappings

A	P
~A	Q
~B	R

$$A \vee (\sim A \vee \sim B)$$

$$P \vee (Q \vee R)$$

$$\Leftrightarrow$$

$$(P \vee Q) \vee R$$

$$(A \vee \sim A) \vee \sim B$$

Mappings

P	A
Q	~A
R	~B

$$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$$

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Step 3. Q1 a). By “simple” identity

Mappings

A	Q

$$(A \vee \sim A) \vee \sim B$$

$$Q \vee \sim Q$$

$$\Leftrightarrow$$

$$\text{True}$$

$$\text{True} \vee \sim B$$

$$Q \vee \sim Q \Leftrightarrow \text{True}$$

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Step 4. Q1 a). Commutivity

- Swap the order using commutivity.

Mappings

True	P
~B	Q

True ∨ ~B

P ∨ Q

⇔

Q ∨ P

~B ∨ True

Mappings

P	True
Q	~B

P ∨ Q ⇔ Q ∨ P

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Step 5. Q1 a). Domination law

Mappings

~B	P

~B ∨ True

P ∨ True

⇔

True

True

P ∨ True ⇔ True

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Q1 a) Summary

- Step 1. *A ∨* ~(A ∧ B). Apply DeMorgan's Law on ~(A ∧ B).
- Step 2. A ∨ (~A ∨ ~B). Apply Associativity.
- Step 3. (A ∨ ~A) *∨ ~B.* Apply Identity to (A ∨ ~A)
- Step 4. True ∨ ~B. Apply commutivity.
- Step 5. ~B ∨ True. Apply Dominion law.
- Finish. True.

- **DeMorgan's Law** ~ (P ∧ Q) ⇔ (~P ∨ ~Q)
- **Associativity** : (P ∨ Q) ∨ R ⇔ P ∨ (Q ∨ R) ⇔ P ∨ Q ∨ R
- **Simple Identity** ~Q ∨ Q ⇔ True
- **Commutivity** : (P ∨ Q) ⇔ (Q ∨ P)
- **Dominion** : (P ∨ True) ⇔ True

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Q1 a) Summary

- Step 1. *A ∨* ~(A ∧ B). Apply DeMorgan's Law on ~(A ∧ B).
- Step 2. A ∨ (~A ∨ ~B). Apply Associativity.
- Step 3. (A ∨ ~A) *∨ ~B.*
  Apply Identity to (A ∨ ~A)
- Step 4. True ∨ ~B. Apply commutivity.
- Step 5. ~B ∨ True. Apply Dominion law.
- Finish. True.

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Q1 b)

- b) A → (A ∨ B)
- To solve this you will need the following identities.
- Implication : (P → Q) ⇔ (~ P ∨ Q)
- Associativity : (P ∨ Q) ∨ R ⇔ P ∨ (Q ∨ R)
- Commutivity : (P ∨ Q) ⇔ (Q ∨ P)
- Domination: (P ∨ True) ⇔ True
- And a further "simple" identity :
  ~Q ∨ Q ⇔ True

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Step 1. Q1 b). By Implication

Mappings

A	P
(A ∨ B)	Q

A → (A ∨ B)

(P → Q)

⇔

(~P ∨ Q)

~A ∨ (A ∨ B)

Mappings

P	A
Q	(A ∨ B)

(P → Q) ⇔ (~ P ∨ Q)

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Step 2. Q1 b). By Associativity

Mappings

~A	P
A	Q
B	R

$\sim A \lor (A \lor B)$

$P \lor (Q \lor R)$

$\Leftrightarrow$

$(P \lor Q) \lor R$

$(\sim A \lor A) \lor B$

Mappings

P	~A
Q	A
R	B

$(P \lor Q) \lor R \Leftrightarrow P \lor (Q \lor R)$

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Step 3. Q1 b). By “simple” identity

Mappings

A	Q

$(\sim A \lor A) \lor B$

$\sim Q \lor Q$

$\Leftrightarrow$

True

True $\lor$ B

$\sim Q \lor Q \Leftrightarrow$ True

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Step 4. Q1 b). Commutivity

- Swap the order using commutivity.

Mappings

True	P
B	Q

True $\lor$ B

$P \lor Q$

$\Leftrightarrow$

$Q \lor P$

B $\lor$ True

Mappings

P	True
Q	B

$P \lor Q \Leftrightarrow Q \lor P$

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Step 5. Q1 b). Domination law

Mappings

B	P

B $\lor$ True

P $\lor$ True

$\Leftrightarrow$

True

True

P $\lor$ True $\Leftrightarrow$ True

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Q1 a) Summary

- Step 1. $A \to (A \lor B)$. Apply Implication law.
- Step 2. $\sim A \lor (A \lor B)$. Apply Associativity.
- Step 3. $(\sim A \lor A) \lor B$. Apply Identity law on $(\sim A \lor A)$.
- Step 4. True $\lor$ B. Apply Commutivity.
- Step 5. B $\lor$ True. Apply Domination law.
- Finish. True.

- Implication : $(P \to Q) \Leftrightarrow (\sim P \lor Q)$
- Associativity : $(P \lor Q) \lor R \Leftrightarrow P \lor (Q \lor R)$
- Commutivity : $(P \lor Q) \Leftrightarrow (Q \lor P)$
- Domination: $(P \lor \text{True}) \Leftrightarrow$ True
- And a further “simple” identity : $\sim Q \lor Q \Leftrightarrow$ True

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Q1 a) Summary

- Step 1. $A \to (A \lor B)$. Apply Implication law.
- Step 2. $\sim A \lor (A \lor B)$. Apply Associativity.
- Step 3. $(\sim A \lor A) \lor B$.
  Apply Identity law on $(\sim A \lor A)$.
- Step 4. True $\lor$ B. Apply Commutivity.
- Step 5. B $\lor$ True. Apply Domination law.
- Finish. True.

Step 1. Q1 c). By Implication

The bit in *blue italics* shows that not all of the proposition has to be altered in each step.

$(A \wedge B) \rightarrow (A \rightarrow B)$

$(P \rightarrow Q)$

$\Leftrightarrow$

$(\sim P \vee Q)$

$(A \wedge B) \rightarrow (\sim A \vee B)$

Mappings

A	P
B	Q

$(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

Step 2. Q1 c). Again, Implication

$(A \wedge B) \rightarrow (\sim A \vee B)$

$(P \rightarrow Q)$

$\Leftrightarrow$

$(\sim P \vee Q)$

$\sim(A \wedge B) \vee (\sim A \vee B)$

Mappings

$A \wedge B$	P
$\sim A \vee B$	Q

$(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

Step 3. Q1 c). DeMorgan's Law

$\sim(A \wedge B) \vee (\sim A \vee B)$

$\sim(P \wedge Q)$

$\Leftrightarrow$

$(\sim P \vee \sim Q)$

$(\sim A \vee \sim B) \vee (\sim A \vee B)$

Mappings

A	P
B	Q

$\sim(P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$

Step 4. Q1 c). By Associativity

- $(\sim A \vee \sim B) \vee (\sim A \vee B)$
- Remove the brackets
- $\sim A \vee \sim B \vee \sim A \vee B$

$(P \vee Q) \vee (R \vee S) \Leftrightarrow P \vee Q \vee R \vee S$

Step 5. Q1 c) By Commutivity

- re-order
- $\sim A \vee \sim B \vee \sim A \vee B$
- Commutivity : $(P \vee Q) \Leftrightarrow (Q \vee P)$
  $(P \vee Q \vee R \vee S) \Leftrightarrow (S \vee Q \vee R \vee P)$
- $\sim B \vee B \vee \sim A \vee \sim A$

Step 6. Q1 c). "Simple" identity

$\sim B \vee B \vee \sim A \vee \sim A$

$\sim Q \vee Q$

$\Leftrightarrow$

True

True $\vee \sim A \vee \sim A$

Mappings

~B	~Q
B	Q

$\sim Q \vee Q \Leftrightarrow$ **True**

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Step 7. Q1 c) By Commutivity

- True ∨ ~A ∨ ~A
- Reorder (commutivity)
- ~A ∨ ~A ∨ True

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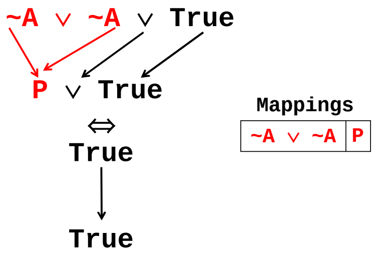
Step 8. Q1 c) Domination law



P ∨ True ⇔ True

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Q1 c) Summary

- Step 1. (A ∧ B) → (A → B). Apply Implication to (A → B).
- Step 2. (A ∧ B) → (~A ∨ B). Apply Implication.
- Step 3. ~(A ∧ B) ∨ (~A ∨ B). Apply DeMorgan's Law to ~(A ∧ B).
- Step 4. (~A ∨ ~B) ∨ (~A ∨ B). By Associativity, remove all brackets.
- Step 5. ~A ∨ ~B ∨ ~A ∨ B. By Commutivity, reorder operands.
- Step 6. ~B ∨ B ∨ ~A ∨ ~A. Apply Simple Identity law to ~B ∨ B.
- Step 7. True ∨ ~A ∨ ~A. Reorder using Commutivity.
- Step 8. ~A ∨ ~A ∨ True . Apply Domination law to ~A ∨ ~A.
- Finish. True.

- Implication : (P → Q) ⇔ (~ P ∨ Q)
- DeMorgan's Law ~ (P ∧ Q) ⇔ (~P ∨ ~Q)
- Associativity : (P ∨ Q) ∨ R ⇔ P ∨ (Q ∨ R) ⇔ P ∨ Q ∨ R
- Commutivity : (P ∨ Q) ⇔ (Q ∨ P)
- Simple Identity ~Q ∨ Q ⇔ True
- Domination : (P ∨ True) ⇔ True

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Q1 c) Summary

- Step 1. (A ∧ B) → (A → B).
  Apply Implication to (A → B).
- Step 2. (A ∧ B) → (~A ∨ B). Apply Implication.
- Step 3. ~(A ∧ B) ∨ (~A ∨ B).
  Apply DeMorgan's Law to ~(A ∧ B).
- Step 4. (~A ∨ ~B) ∨ (~A ∨ B). By Associativity, remove all brackets.
- Step 5. ~A ∨ ~B ∨ ~A ∨ B.
  By Commutivity, reorder operands.
- Step 6. ~B ∨ B ∨ ~A ∨ ~A.
  Apply Simple Identity law to ~B ∨ B.
- Step 7. True ∨ ~A ∨ ~A. Reorder using Commutivity.
- Step 8. ~A ∨ ~A ∨ True .
  Apply Domination law to ~A ∨ ~A.
- Finish. True.

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Q2

- 2. Let P(x) be the statement "x likes marmite," where the universe of discourse for x is the set of people. Express each of the following quantifications in English.
- (a) ∃x P(x)
- (b) ∃x ¬P(x)
- (c) ∀x ¬P(x)
- (d) ¬∀x ¬P(x)

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Q2 a)

- P(x) : "x likes marmite,"
- (a) ∃x P(x)
- (a) there is at least one person who likes marmite

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Q2 b)

- P(x) : "x likes marmite,"
- (b) $\exists x \neg P(x)$

- (b) there is at least one person who doesn't like marmite

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Q2 c)

- P(x) : "x likes marmite,"
- (c) $\forall x \neg P(x)$

- (c) everyone doesn't like marmite

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Q2 d)

- P(x) : "x likes marmite,"
- (d) $\neg\forall x \neg P(x)$

- (d) "Not everyone doesn't like marmite", or an equivalent simpler form "there is someone that likes marmite".

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Q2 d)

- If we take $\neg\forall x \neg P(x)$ to be $\neg(\forall x \neg P(x))$ then according to logic unit 4, $\sim(\forall x\, Q(x))$ is equivalent to $\exists x \sim Q(x)$
- So we can rewrite (d) above as
- $\exists$ x P(x)
- which is the same Q3(a) : there is at least one person who likes Marmite

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Q2 d)

- If we take $\neg\forall x \neg P(x)$ to be $\neg(\forall x \neg P(x))$ then according to logic unit 4, $\sim(\forall x\, Q(x))$ is equivalent to $\exists x \sim Q(x)$
- So we can rewrite (d) above as
- $\exists$ x P(x)
- which is the same Q3(a) : there is at least one person who likes Marmite

$$\neg(\forall x\, Q(x)) \Leftrightarrow \exists x\, \neg Q(x)$$
$$Q(x) \Rightarrow \neg P(x) \qquad \neg Q(x) \Rightarrow P(x)$$
$$\neg(\forall x\, \neg P(x)) \Leftrightarrow \exists x\, P(x)$$

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Q3

- 3. Working with the characters of the hilarious TV programme "The Simpsons", express the assertions given below as propositions of predicate logic using the following predicates.

- Father (x,y) : x is y's father, or equivalently y is x's child
- Mother (x,y) : x is y's mother, or equivalently y is x's child
- Brother (x,y): x is y's brother
- Sister (x,y) : x is y's sister

- (a) Marge is Lisa's mother and Bart's mother
- (b) There is a character in the Simpsons that is Lisa's mother and Bart's mother
- (c) There is a kid whose father is Homer and whose brother is Bart
- (d) There is a kid whose father is Homer and whose sister is Lisa and whose brother is Bart

Q3 a)

- Mother (x,y) : x is y's mother, or equivalently y is x's child
- (a) Marge is Lisa's mother and Bart's mother
- *(a) Mother (Marge,Bart) $\wedge$ Mother (Marge, Lisa)*

Q3 b)

- Mother (x,y) : x is y's mother, or equivalently y is x's child
- (b) There is a character in the Simpsons that is Lisa's mother and Bart's mother
- *(b) $\exists x$ (Mother(x, Lisa) $\wedge$ Mother (x, Bart))*

Q3 c)

- Father (x,y) : x is y's father, or equivalently y is x's child
- Brother (x,y): x is y's brother
- (c) There is a kid whose father is Homer and whose brother is Bart
- *(c) $\exists x$ (Father(Homer, x) $\wedge$Brother (Bart, x))*

Q3 d)

- Father (x,y) : x is y's father , or equivalently y is x's child
- Brother (x,y): x is y's brother
- Sister (x,y) : x is y's sister
- (d) There is a kid whose father is Homer and whose sister is Lisa and whose brother is Bart
- *(d) $\exists x$ (Father(Homer, x) $\wedge$ Brother (Bart, x) $\wedge$ Sister(Lisa, x))*

THE END

Q1 using different notation

APPENDIX ONE

1

Discrete Maths : 2
Workshop Unit 3

SCC120 Fundamentals of Computer Science

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Q1 a)

- 1. Using truth tables, verify the following equivalences, which are known as the absorption laws.
- a) $p \vee (p \wedge q) \Leftrightarrow p$

a	b	c = a ∧ b	d = a ∨ c
P	Q	(P ∧ Q)	P ∨ (P ∧ Q)
F	F		
F	T		
T	F		
T	T		

3

Q1 a)

- 1. Using truth tables, verify the following equivalences, which are known as the absorption laws.
- a) $p \vee (p \wedge q) \Leftrightarrow p$

a	b	c = a ∧ b	d = a ∨ c
P	Q	(P ∧ Q)	P ∨ (P ∧ Q)
F	F	F ∧ F = F	F ∨ F = F
F	T	F ∧ T = F	F ∨ F = F
T	F	T ∧ F = F	T ∨ F = T
T	T	T ∧ T = T	T ∨ T = T

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Q1 b)

- (b) $p \wedge (p \vee q) \Leftrightarrow p$

a	b	c = a ∨ b	d = a ∧ c
P	Q	(P ∨ Q)	P ∧ (P ∨ Q)
F	F		
F	T		
T	F		
T	T		

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Q1 b)

- (b) $p \wedge (p \vee q) \Leftrightarrow p$

a	b	c = a ∨ b	d = a ∧ c
P	Q	(P ∨ Q)	P ∧ (P ∨ Q)
F	F	F ∨ F = F	F ∧ F = F
F	T	F ∨ T = T	F ∧ T = F
T	F	T ∨ F = T	T ∧ T = T
T	T	T ∨ T = T	T ∧ T = T

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Q2

1. a) $A \rightarrow B$ *Let A = "It is Wednesday" and B = "Coronation Street is on TV". Transform the English proposition "If it is Wednesday, then Coronation Street is on TV" into formal logic.*

b) $B \rightarrow A$ *Let A = "You can legally drive" and B = "You have a Driver's Licence". Transform the English proposition "You can legally drive because you have a Driver's licence" into formal logic.*
(If you have a driver's licence you can legally drive)

Q2

1 c) ~A → ~B

Let A = "You are a double O agent" and B = "You have a licence to kill". Transform the English proposition "You are not a double O agent so you do not have a licence to kill" into formal logic.

(If you are not a double O agent then you do not have a licence to kill)

Q3 : Truth Table Puzzle

- A battle between four exponents of the martial arts is under way. The following facts are known:
- (e) The good Lord Alpha will not survive if the evil Count Gamma survives
- (f) Either the evil Count Delta will die or the good Lord Beta will die
- (g) Lord Alpha is the master of Kung Fu, while none of the other three are masters, and it is a fact that a master can only be beaten by another master
- (h) Count Gamma will die only if either Lord Alpha dies or Count Delta dies.
- Exactly two survive. Who are they?

Propositions

- What do we want to find out?
- We want to know who survives ...

A = "Lord Alpha Survives"
B = "Lord Beta Survives"
C = "Count Gamma Survives"
D = "Count Delta Survives"

We know from the problem statement that 2 of these propositions should be true and 2 should be false

The Process

- List the valid combinations of the propositions.
- Encode the facts in propositional logic
- Build truth tables, containing the valid combinations, for each fact.
- Produce a final table containing the valid combinations and the outcomes of the fact truth tables.
- Read off the answer from the final table.

Combinations

Row	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

- There are 2^4 possible outcomes, ranging from all 4 dying to all 4 surviving.
- We can list this as a truth table.
- What are the numbers of the rows we need to consider?

Combinations

Row	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

- We need to consider the rows which have 2 ones in them.
- So rows 3, 5, 6, 9, 10 and 12.

Row	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

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Convert the 4 facts into propositions

- E. The good Lord Alpha will not survive if the evil Count Gamma survives
- F. Either the evil Count Delta will die or the good Lord Beta will die
- G. Lord Alpha is the master of Kung Fu, while none of the other three are masters, and it is a fact that a master can only be beaten by another master
- H. Count Gamma will die only if either Lord Alpha dies or Count Delta dies.
- We take the four facts, lettered (E) to (H) above, and convert them from English into propositional logic.

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Baby Steps Example: Fact F

- "Either the evil Count Delta will die or the good Lord Beta will die."
- We need to transform this into a logic proposition, that involves one or more of the four propositions we are seeking the truth value of.

A = "Lord Alpha Survives"
B = "Lord Beta Survives"
C = "Count Gamma Survives"
D = "Count Delta Survives"

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Baby Steps Example: Fact F

- "Either the evil Count Delta will die or the good Lord Beta will die."
- We know propositions B and D; they regarding surviving, not dying.
- But if we negate them ..
  ~B = "Lord Beta does not survive" or "Lord Beta dies"
- ~D = "Count Delta does not survive" or "Count Delta dies"

A = "Lord Alpha Survives"
B = "Lord Beta Survives"
C = "Count Gamma Survives"
D = "Count Delta Survives"

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Baby Steps Example: Fact F

- "Either the evil Count Delta will die or the good Lord Beta will die."
- So we substitute the propositions ~B and ~D in the English statement.
- "Either ~D or ~B"
- So it looks like an OR operator is likely.

A = "Lord Alpha Survives" ~B = "Lord Beta Dies"
B = "Lord Beta Survives" ~D = "Count Delta Dies"
C = "Count Gamma Survives"
D = "Count Delta Survives"

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Baby Steps Example: Fact F

- F = "Either ~D or ~B"
- So it looks like an OR operator is likely.
- Because it states that one or the other will die, but not both, we should use an exclusive OR.
- F = ~B $\underline{\vee}$ ~D

A = "Lord Alpha Survives" ~B = "Lord Beta Dies"
B = "Lord Beta Survives" ~D = "Count Delta Dies"
C = "Count Gamma Survives"
D = "Count Delta Survives"

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F = ~B $\underline{\vee}$ ~D

F	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

~B	~D	F = ~B $\underline{\vee}$ ~D

F = ~B $\veebar$ ~D

F	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

~B	~D	F = ~B $\veebar$ ~D
1	0	T
0	0	F
0	1	T
1	0	T
1	1	F
0	1	T

Fact E

- "The good Lord Alpha will not survive if the evil Count Gamma survives"

If Gamma survives, Alpha does not.

E = C → ~A

Fact F

- "Either the evil Count Delta will die or the good Lord Beta will die."
- It appears that only one of them will die.

F = ~B $\veebar$ ~D

Need to use exclusive OR.

Fact G

- "Lord Alpha is the master of Kung Fu, while none of the other three are masters, and it is a fact that a master can only be beaten by another master"
- Rewrite this as a much simpler fact

This is a very convoluted way of saying that Alpha will be one of the survivors.

G = A

Fact H

- "Count Gamma will die only if either Lord Alpha dies or Count Delta dies."

Deceptively complex.
Rewrite as "if Alpha dies or Delta dies, Gamma will die".

Also, it says nothing about what happens if Alpha and Delta both die, so it is an exclusive OR.

H = ~C → ~A $\veebar$ ~D

Build the Truth Tables

- Produce the truth tables for each of the 4 propositions. Present the rows in numerical order.

$E = C \rightarrow \sim A$

E	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

C	~A	E = C → ~A

$E = C \rightarrow \sim A$

E	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

C	~A	E = C → ~A
1	1	T
0	1	T
1	1	T
0	0	T
1	0	F
0	0	T

$F = \sim B \veebar \sim D$

F	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

~B	~D	F = ~B $\veebar$ ~D

$F = \sim B \veebar \sim D$

F	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

~B	~D	F = ~B $\veebar$ ~D
1	0	T
0	0	F
0	1	T
1	0	T
1	1	F
0	1	T

$G = A$

G	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

G = A

$G = A$

G	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

G = A
F
F
F
T
T
T

$H = \sim C \rightarrow \sim A \veebar \sim D$

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- R = ~A $\veebar$ ~D
- H = ~C→R

G	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

		R		H	
~A	~D	~A $\veebar$~D	~C	H = ~C→R	H

$H = \sim A \veebar \sim D \leftrightarrow \sim C$

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- H = ~A $\veebar$ ~D ↔ ~C
- R = ~A $\veebar$ ~D
- H = R ↔ ~C
- F ↔ F = T, T ↔ T = T otherwise false

G	A	B	C	D
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

		R		H	
~A	~D	~A $\veebar$~D	~C	H = ~C→R	H
1	0	T	F	F → T = T	T
1	0	T	T	T →T = T	T
1	1	F	F	F→ F = T	T
0	0	F	T	T→ F = F	F
0	1	T	F	F → T = T	T
0	1	T	T	T →T = T	T

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Fill in the final table.

A	B	C	D
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	**1**	**0**	**0**

E	F	G	H
T	T	F	T
T	F	F	T
T	T	F	T
T	T	T	F
F	F	T	T
T	**T**	**T**	**T**

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- From the final table, find out the names of the two survivors.
- All of E, F, G and H D must be true; this only occurs in the last row, so we look at the corresponding values of A, B, C and D in that row, and find that A is true, and B is true.
- From this we can see that the propositions A : Alpha lives, and B: Beta lives, are true and therefore the names of the two survivors are Alpha and Beta.

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THE END