Answer for Homework 4

P272, 24:

$$a \vee (a' \wedge b) = (a \vee a') \wedge (a \vee b) = I \wedge (a \vee b) = a \vee b$$

$$a \wedge (a' \vee b) = (a \wedge a') \vee (a \wedge b) = 0 \vee (a \wedge b) = a \wedge b$$

P272, 34:

$a' = e,$

$b' = c,$

$c' = b \;\&\; d,$

$d' = c,$

$e' = a$

P278-279, 1 – 10:

1. No
2. No
3. No
4. No
5. Yes
6. No
7. Yes
8. Yes

9. Yes

10.No

P278-279, 12:

If $a \le b$, $b' = (a \vee b)' = a' \wedge b'$ and therefore, $b' \le a'$.

If $b' \le a'$, $b = (a' \wedge b')' = a \vee b$ and therefore, $a \le b$.

P278-279, 20:

$((a \vee c) \wedge (b' \vee c))' = (a \vee c)' \vee (b' \vee c)' = (a' \wedge c') \vee (b \wedge c') = (a' \vee b) \wedge c'$.

1.

(1) Since $a \le b$ and $b \le c$, $a \vee b = b = b \wedge c$.

(2) $(a \wedge b) \vee (b \wedge c) = a \vee b = b = b \wedge c = (a \vee b) \wedge (a \vee c)$.

2.

(1) Neither a nor f has any complement.

(2) No.

3.

Obviously, I is one complement of $0'$.

Assume that there exists at least one element $E \ne I$ which is also a complement of 0, and then $E<I$ because I is the greatest element. However, since 0 is the least element, it follows that $0 \vee E = E<I$, contradicting the fact that $0 \vee E = I$. Therefore, the only complement of 0 is I.