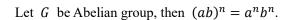
Exercise 1.3-2



 $\langle Z, + \rangle$ is a infinite cyclic group. Please show that generator of Z.

If |a| = n, then $G = \{e, a, a^2, \dots, a^{n-1}\}$ is a cyclic group of order n, |G| = |a|

< Z_{12} , $+_{12}$ > is a addition module 12 group, write the order of the group and the order of all elements of Z_{12}

Let G be a group, if $H, K \leq G$, then $H \cup K \leq G \Leftrightarrow H \subseteq K \vee K \subseteq H$

Please write all generated subgroups of group $\langle Z_{12}, +_{12} \rangle$

Let G be a group, $B \subseteq G$, then $\langle B \rangle = \bigcap \{H | H \leq G, B \subseteq H\}$ is a subgroup of G.

Let $S = \{a, b, c, d\}$, f(a) = b, f(b) = c, f(c) = d, f(d) = a, $F = \{f^0, f^1, f^2, f^3\}$, then $\langle F, \circ \rangle$ is Abelian group.

Let G is a group, H < G, then

- (1) $a, b \in G, Ha \cap Hb = \emptyset$ or Ha = Hb and $\bigcup Ha = G$
- (2) $H \approx Ha$ (equipotential)

(Remark: According to theorem 6, and the theorem about equivalence relation.)

Exercise 1.3-3

Please write the cyclic transposition of permutation.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 2 & 8 & 6 & 4 & 7 & 5 \end{pmatrix}$$

Show that $(i_1i_2\cdots i_k) = (i_1i_2)(i_1i_3)(i_1i_4)\cdots (i_1i_k)$.

Please write the product of the following cyclic transpositions.

The group
$$\langle Z_{12}, +_{12} \rangle$$
, $B = \{2,3\}$: $\langle B \rangle = ?$

The Klein 4-ary group,
$$B = \{a, b\}$$
: $\langle B \rangle = ?$

Exercise 1.4

Please show that the following algebraic systems are rings.

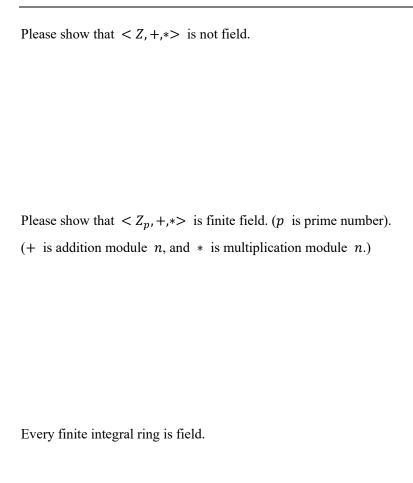
$$< Z, +, \times >, < Q, +, \times >, < Z_E, +, \times >,$$
 and $< C, +, \times >$

(The Q is the set of all rational numbers, the Z_E is the set of all even numbers, and the C is the set of all complex numbers.)

Please show that $\langle Z_n, \oplus, \otimes \rangle$ is a ring. (\oplus is addition module n, and \otimes is multiplication module n.)

Please show that the following algebraic systems are fields.

$$< Q+,*>,< R,+,*>,< C,+,*>$$



Every homomorphism image of ring is a ring.