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CSC3110
Homework 1

CSC 3110

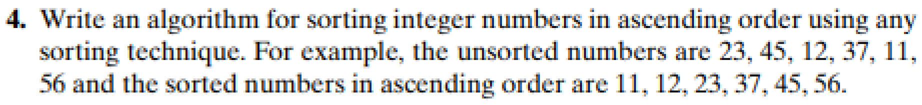
Algorithm Design and Analysis

(30 points)

Due: 01/31/2024 11:59 PM

Note: Submit answers in PDF document format. Please read the submission format for appropriate file naming conventions.

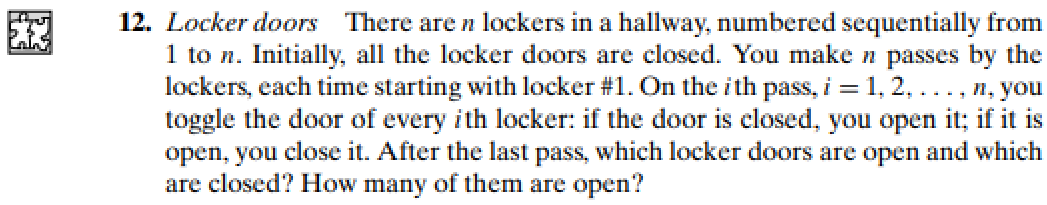
1) Exercises 1.1
   a. Problem 4 (2 points)



4. Write an algorithm for sorting integer numbers in ascending order using any sorting technique. For example, the unsorted numbers are 23, 45, 12, 37, 11, 56 and the sorted numbers in ascending order are 11, 12, 23, 37, 45, 56.

The algorithm needs to deal with an unsorted section of the array, and then put sorted sections into it.
The laziest solution is an insertion sort
1.)Define an array of length |n|
2.)Fill array with the sorted numbers (with the array starting at zero)
3.)Starting at i = 0,
4.) find the minimum from i to n using a temporary variable that compares each position against it, and swaps out the value compared against if it is less than the temporary variables value.
5.)place the minimum at position i
6.)Increase the value of i by plus one
7.)Repeat until i = (n-1)

   b. Problem 12 (2 points)



12. *Locker doors* There are n lockers in a hallway, numbered sequentially from 1 to n. Initially, all the locker doors are closed. You make n passes by the lockers, each time starting with locker #1. On the ith pass, $i = 1, 2, \ldots, n$, you toggle the door of every ith locker: if the door is closed, you open it; if it is open, you close it. After the last pass, which locker doors are open and which are closed? How many of them are open?

May Wandyez
Gq5426
CSC3110
Homework 1

Rule set:

1.) All doors initially closed

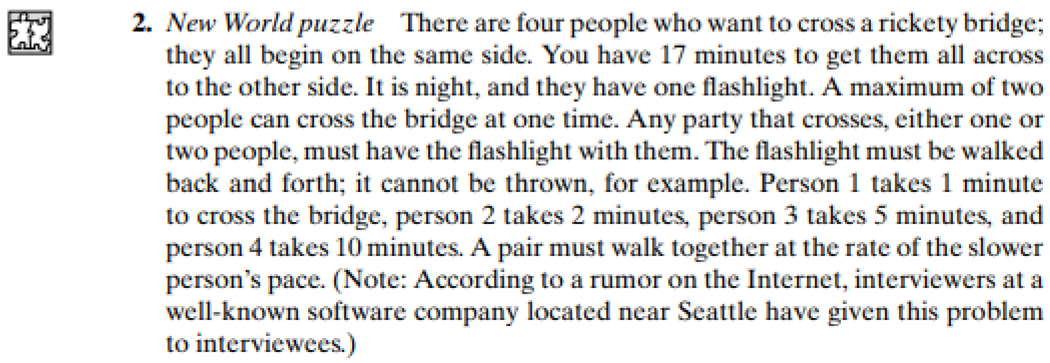
2.) Every pass begins at locker 1.

3.)The door of every ith locker is toggled on each pass. Locker/i is always toggled by pass.

In boring math terms this means that for any door on any pass, the number of times it is toggled is equal to the number of times that i can divide the value of i. In more boring math this means that unless the number dividing the ith door is a square of the number i, then there will be an even number of numbers that can divide i. This means that the doors that are in the position for squares of i are the only doors that will remain open after the last pass for ANY number of passes. This roughly works out to abs(sqrt(n)).

2) Exercises 1.2

a. Problem 2 (2 points)



2. *New World puzzle* There are four people who want to cross a rickety bridge; they all begin on the same side. You have 17 minutes to get them all across to the other side. It is night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace. (Note: According to a rumor on the Internet, interviewers at a well-known software company located near Seattle have given this problem to interviewees.)

Ruleset:

1.)Everyone must cross the bridge

2.)The flashlight MUST be present to cross the bridge (and since it can only be walked across, this means that one person will have to constantly double back every time)

3.)The pace of the crossing party is equal to the speed of the slowest party member.

4.)The party members may cross in pairs

May Wandyez
Gq5426
CSC3110
Homework 1

5.)There is only 17 minutes to cross.
6.)The party members as represented by time it takes to cross are 1,2,5,10

My first instinct when solving this puzzle is to continuously shuttle the fastest person across the bridge as the runner for the flashlight, let's test that out to determine how long that takes.

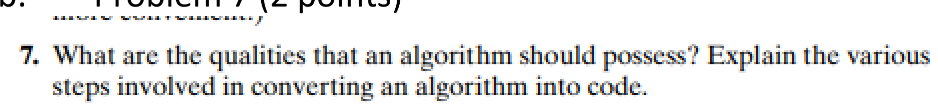
Crossing Number	1	2f	3	4f	5
Party Members	1,2	1	1,5	1	1,10
Slowest Party Member	2	1	5	1	10
Time taken to cross	2	1	5	1	10
Total time elapsed	2	3	8	9	19

Well, that didn't work, I exceeded the total allotted time by two minutes. Let's try that again but try bunching up the slowest party members together since this means that they won't delay the runner. However this also means that we need to send someone to the other side to get the runner first.

Crossing Number	1	2f	3	4f	5
Party Members	1,2	1	5,10	2	1,2
Slowest Party Member	2	1	10	2	2
Time taken to cross	2	1	10	2	2
Total time elapsed	2	3	13	15	17

This crossing order solves the puzzle.

b. Problem 7 (2 points)


7. What are the qualities that an algorithm should possess? Explain the various steps involved in converting an algorithm into code.

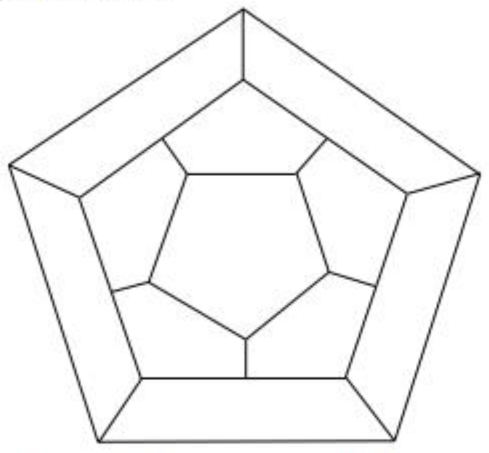
May Wandyez
Gq5426
CSC3110
Homework 1

QUALITIES AN ALGORITHM SHOULD POSSESS
1.) Efficiency: An algorithm should strive to be efficient in speed and also the amount of space that the algorithm uses.
2.) Simplicity: Programs that are easier to understand and program usually contain fewer bugs – they are also easier for future coders to work with.
3.)Generality – The more applicable an algorithm is to more circumstances, the more useful it is.

Steps to convert an algorithm to code
1.) Determine the set of rules your algorithm is bound by.
2.) Convert your algorithm to a step by step process
3.) Convert each step in the process to line of code that matches the steps iterated.
4.) Test your code
5.) Refine your code.

3) Exercises 1.3
   a. Problem 5 (2 points)

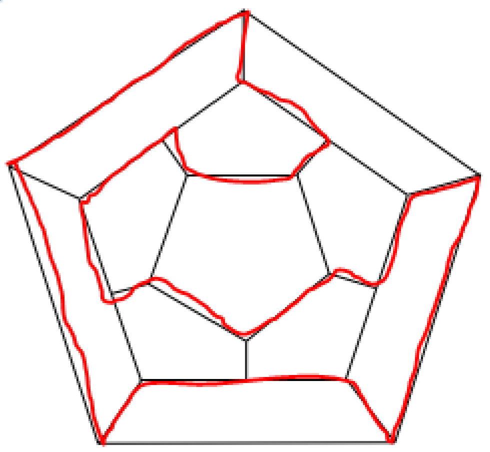
5. *Icosian Game* A century after Euler's discovery (see Problem 4), another famous puzzle—this one invented by the renowned Irish mathematician Sir William Hamilton (1805–1865)—was presented to the world under the name of the Icosian Game. The game's board was a circular wooden board on which the following graph was carved:



Find a ***Hamiltonian circuit***—a path that visits all the graph's vertices exactly once before returning to the starting vertex—for this graph.

The shape of the graph is an outer shape with an inner shape, if one were to ONLY complete the outer shape or inner shape, they would be forced to double back, therefore the path MUST enter the inner shape, complete a circuit within, then exit back out to the outer shape.

ollowing graph was carved:

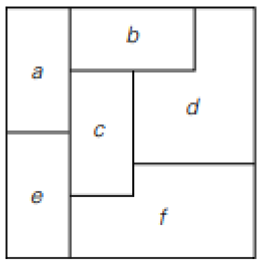


[illegible]

Look at this squiggly solution, it sure is.

b. Problem 8 (parts a & b) (2 points)

8. Consider the following map:

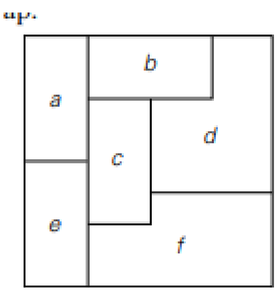


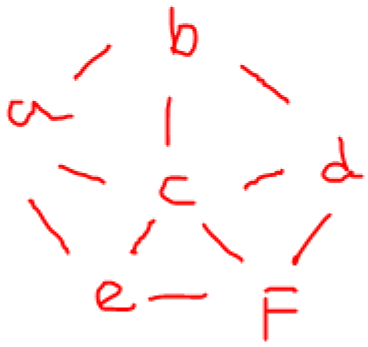
a. Explain how we can use the graph-coloring problem to color the map so that no two neighboring regions are colored the same.

b. Use your answer to part (a) to color the map with the smallest number of colors.

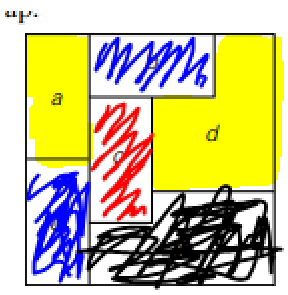
a.) We can use the graph coloring problem to color the map so that no two neighboring regions are colored the same by creating a graph where there are vertices for each of the regions, with edges between the vertices where the regions share a border.

May Wandyez
Gq5426
CSC3110
Homework 1



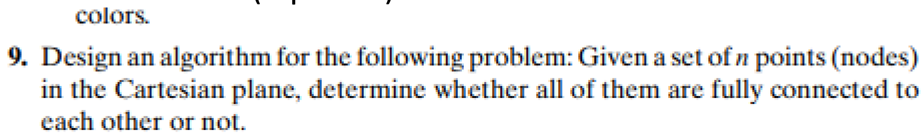


b.) Since no two colors of the same can border each other, we can think of the solution like a pinwheel, with a central color in the middle, and alternating colors on each side. This should minimize the number of colors needed.



Looks like it took four colors.

c. Problem 9 (2 points)



colors.

9. Design an algorithm for the following problem: Given a set of *n* points (nodes) in the Cartesian plane, determine whether all of them are fully connected to each other or not.

Fully connected means the node has an edge connecting it to every other node. Therefore we need to check to see if each node has a direct connection to every other node.

1.) Start at node I = 0
2.) Check to see if i connects to nodes i+1 to n, return false if there is no connection, return true after checking to connection n and no false has been returned.
3.) Increase i by one

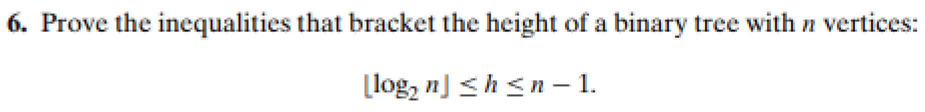
May Wandyez
Gq5426
CSC3110
Homework 1

4.) Repeat 3 and 4 until I = n

This algorithm should work, it also takes into account connections that have already been checked previously in the algorithm due to the increasing value of i.

4) Exercises 1.4:

a. Problem 6 (2 points)



6. Prove the inequalities that bracket the height of a binary tree with n vertices:

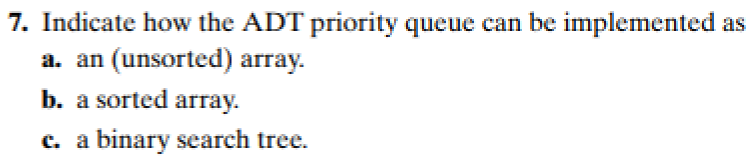
$$\lfloor \log_2 n \rfloor \le h \le n - 1.$$

The inequality presented says that the height of a binary tree lies between the log2(n) of its vertices, and the value of its total number of vertices minus one.

The height of a tree is the largest possible number of edges from the root to the leaf. Since it is not possible for there to be more edges than the total number of edges minus one, the right side of the inequality is true.

Assuming a binary tree is perfectly full, this means there are two edges for any single edge at any point in the tree 2^n, this means that the shortest a tree can be is represented by taking the logarithm of this. This means the left side of the inequality is true.

b. Problem 7 (parts a-c) (2 points)



7. Indicate how the ADT priority queue can be implemented as

a. an (unsorted) array.

b. a sorted array.

c. a binary search tree.

ADT priority queue means 'abstract data type' priority queue, so….just a priority queue. Which requires the following operations:

1.) Finding its largest element

2.)Deleting its largest element

3.)Adding a new element.

a.) Finding the largest element in an unsorted array can be accomplished by scanning through the array to find the largest element (use a temporary variable assigned value array[0], and compare it against all other elements, replacing it if the element is larger), once the largest element is found it can either be returned as a value, or deleted. Because the array is unsorted the new values to the queue can be added anywhere, usually this is done with a .append type function which either directly adds items to the array, or creates a new array out of all the previous values of the old array, and adds the new value in there too.

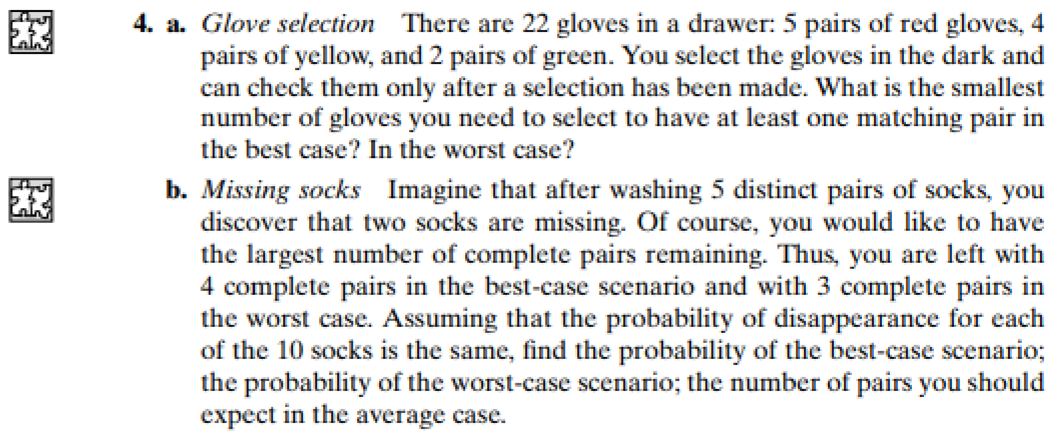
b.)Assuming the array is already sorted in ascending order, this means that the largest element of the array can be found at array[n], where it can also be deleted. Adding a new element requires scanning the array up to array [n-1] starting at position array[i+1], and comparing the values of array[i] and array[i+1], if the inserted value is between the values, it should be placed at that position. (Or create a new array of length n+1, copy the original array into it, and then perform swaps of the new element until it is no longer greater than the item being compared against)

c.)The largest element in a binary search tree can be found on the right most edge of the binary search tree. Deleting it involves deleting the edge indicating that the element should point there. Adding a new element to the binary search tree involves starting at the root of the binary search tree, and comparing the new item against the value of the item stored in the node, if the new element is greater it is compared against the value of the node to the right, if it is less is compared against the value of the node to the left. This process repeats until the bottom of the tree is reached, and the new value is added in via an edge.

5) Exercises 2.1:

May Wandyez
Gq5426
CSC3110
Homework 1

a. Problem 4 (parts a & b) (2 points)



4. a. *Glove selection* There are 22 gloves in a drawer: 5 pairs of red gloves, 4 pairs of yellow, and 2 pairs of green. You select the gloves in the dark and can check them only after a selection has been made. What is the smallest number of gloves you need to select to have at least one matching pair in the best case? In the worst case?

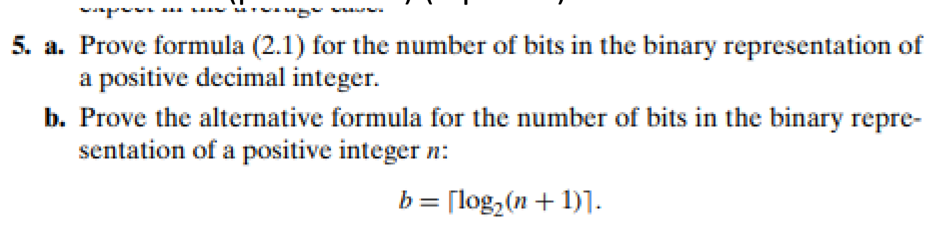
b. *Missing socks* Imagine that after washing 5 distinct pairs of socks, you discover that two socks are missing. Of course, you would like to have the largest number of complete pairs remaining. Thus, you are left with 4 complete pairs in the best-case scenario and with 3 complete pairs in the worst case. Assuming that the probability of disappearance for each of the 10 socks is the same, find the probability of the best-case scenario; the probability of the worst-case scenario; the number of pairs you should expect in the average case.

a.) Remember, gloves come in right and left handed pairs The best case scenario is that a matching pair of gloves is pulled on the first try, so that only two gloves need to be pulled.

The worst case scenario is that you pull the left handed variants of all colors of gloves before drawing the right handed variant of the glove, meaning that worst case is n+1, or 5+4+2+1 = 12 tries

b.)The best case scenario is that two of the same pair are missing, and the worst case scenario two of different pairs are missing. There are a total of 10 socks, the total number of different outcomes to choose 2 socks out of the 10 is 45. The number of best case scenarios is 5/45 (the 5 times where it's a pair that is missing), the number of worst case scenarios is 40/45 (all other scenarios). The average scenario is roughly 3 + 5/45 socks are missing.

b. Problem 5 (parts a & b) (2 points)



5. a. Prove formula (2.1) for the number of bits in the binary representation of a positive decimal integer.

b. Prove the alternative formula for the number of bits in the binary representation of a positive integer n:

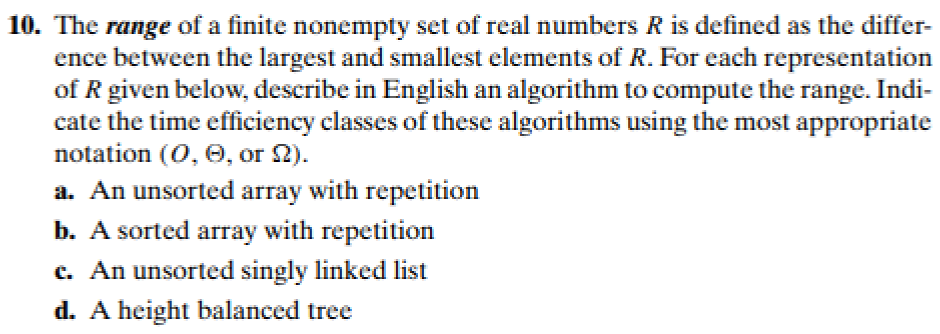
$$b = \lceil \log_2(n+1) \rceil.$$

a.) Positive decimal integer means whole number above the value of zero. Two digits places are needed for every value of 1 for a base 10 number system, this requires taking the logarithm of that number log2(n) and adding +1 (there is always one digit required for the decimal place in the event that the number produces a 0 from the log function). The resulting function is therefore always log2(n) +1.

b.) Remember, n>0, which means that there will always be at least one decimal place occupied by the resulting binary number. See previous answer in a why log2(n) digit places(bits) are needed. The resulting answer is therefore log2(n+1). To be honest I am deeply uncertain in this answer, it does not feel right – I fail to see the purpose of this in algorithm design.

6) Exercises 2.2:

a. Problem 10 (2 points)



10. The ***range*** of a finite nonempty set of real numbers R is defined as the difference between the largest and smallest elements of R. For each representation of R given below, describe in English an algorithm to compute the range. Indicate the time efficiency classes of these algorithms using the most appropriate notation (O, Θ, or Ω).

a. An unsorted array with repetition

b. A sorted array with repetition

c. An unsorted singly linked list

d. A height balanced tree

To determine range we have to find the:

1.) Maximum

2.) Minimum

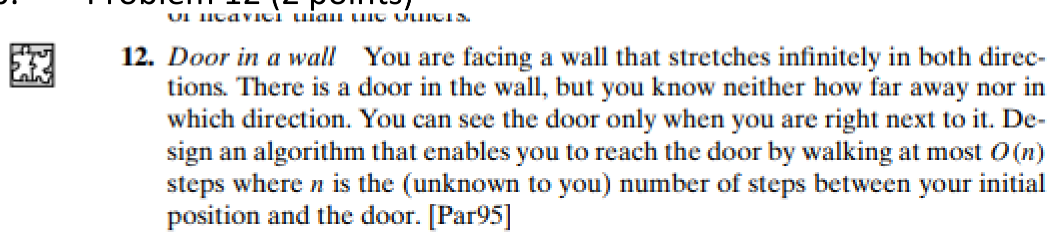
a.)The array is unsorted, which means every element must be considered because it is unknown where each element resides. which means one pass has to be made through the array while comparing temporary variables for maximum and minimum, therefore the efficiency of moving through the unsorted array. (I don't know how to insert symbols) Is Theta(R).

b.)The array is sorted, but contains repetition, this however represents no issue, because the array is sorted we known that the largest and smallest elements reside at the ends of the arrays, therefore the time necessary to find the maximum and minimum is Theta(1) A[n-1]-A[0].

May Wandyez
Gq5426
CSC3110
Homework 1

c.)The linked list is unsorted, which means it is necessary to go through every item in the list. This requires Theta(R) efficiency (linear) because a pass must be made through the entire list.
d.)A height balanced tree has the largest element in the rightmost node, and the smallest element in the left most node, because the tree is balanced the difference between its rightmost and leftmost nodes is at maximum 1. This means that the height of the tree is at minimum log(n+1) -1, which must be traversed twice, so it would be a Theta(log) type efficiency.

b. Problem 12 (2 points)


or heavier than the others.

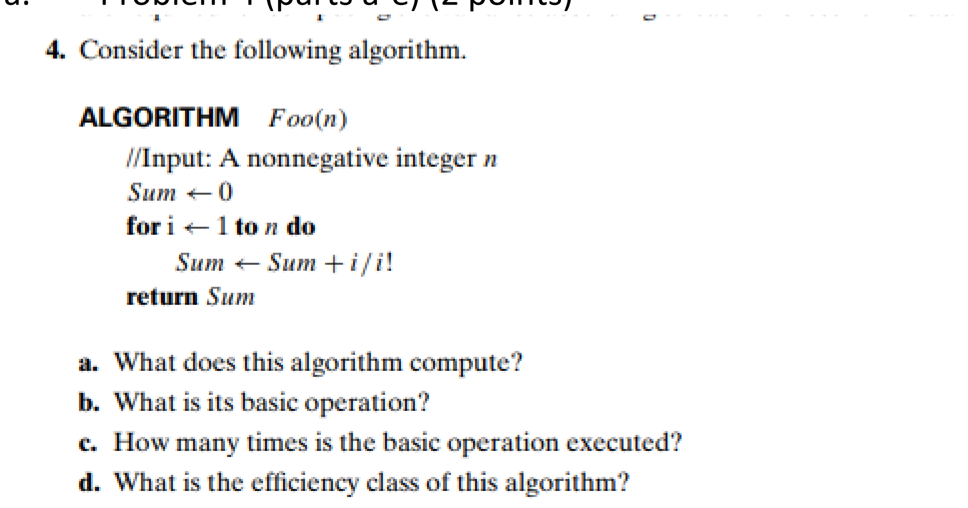
12. *Door in a wall* You are facing a wall that stretches infinitely in both directions. There is a door in the wall, but you know neither how far away nor in which direction. You can see the door only when you are right next to it. Design an algorithm that enables you to reach the door by walking at most $O(n)$ steps where n is the (unknown to you) number of steps between your initial position and the door. [Par95]

Starting at n=0
1.) Walk to the right n steps and check for the door.
2.) Walk n steps to the left to return to the middle.
3.)Walk to the left n steps and check for the door.
4.)Increase the value of n by one
5.)Repeat steps 1-4 until the door is located.

7) Exercises 2.3:

May Wandyez
Gq5426
CSC3110
Homework 1

a. Problem 4 (parts a-e) (2 points)



4. Consider the following algorithm.

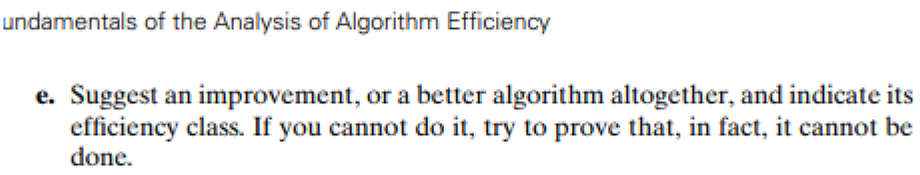
```
ALGORITHM  Foo(n)
    //Input: A nonnegative integer n
    Sum ← 0
    for i ← 1 to n do
        Sum ← Sum + i / i!
    return Sum
```

a. What does this algorithm compute?

b. What is its basic operation?

c. How many times is the basic operation executed?

d. What is the efficiency class of this algorithm?



undamentals of the Analysis of Algorithm Efficiency

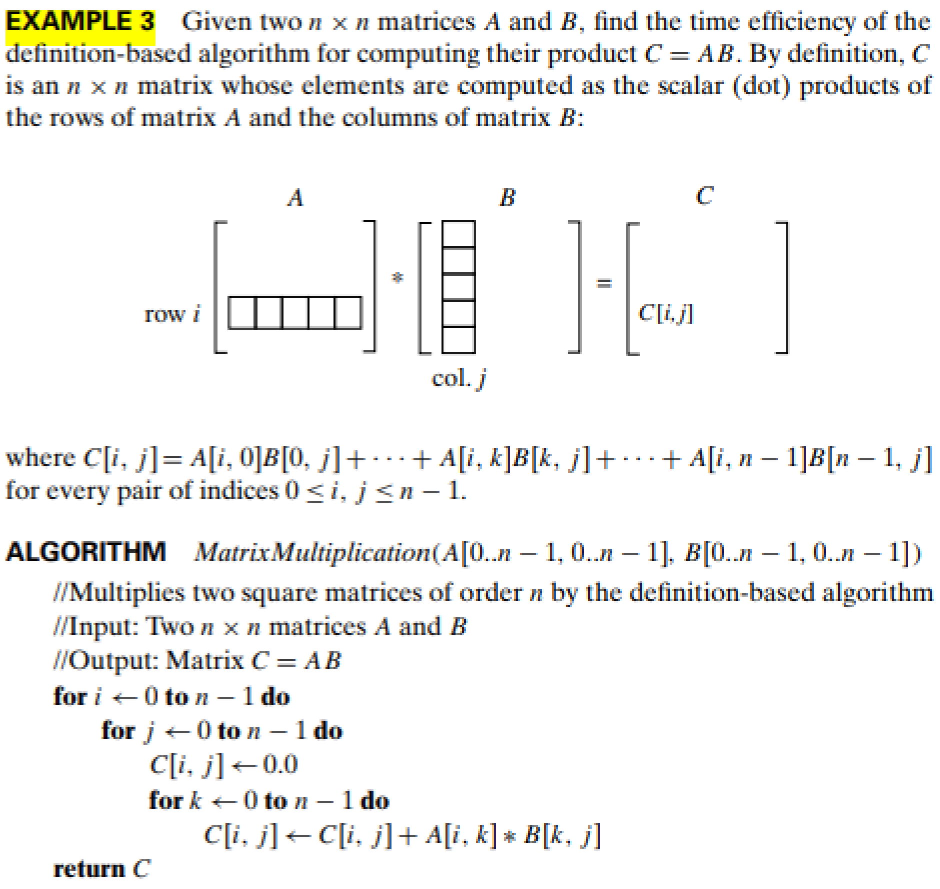
e. Suggest an improvement, or a better algorithm altogether, and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.

a.) The above algorithm adds the summation of the initial sum of zero plus the value of the iterator/factorial of the iterator. So the summation of i/i! from i to n. This looks a lot like the formula used to calculate e. (I've seen it before).
b.)The basic operation is the sum being added to the the iterator divided by the factorial of the iterator.
c.)The basic operation is executed n-1 times (this assumes that in the way it is written that 'to' only means 'up to' as it means in most programming languages where the right end of the range is NOT included)
d.) The efficiency class of this algorithm is factorial, because a factorial is used in its calculation, meaning it will grow more and more inefficient as the value of n expands.
e.) The summation of this formula as n approaches infinite is e, so at a certain point it
i/i! can be simplified to 1/(i-1)!, which removes one basic component of

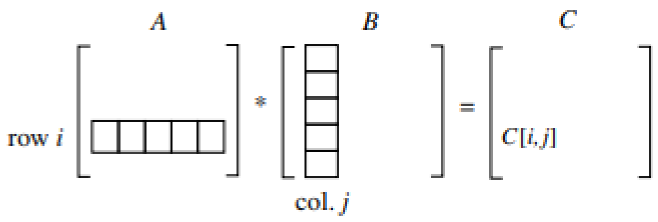
May Wandyez
Gq5426
CSC3110
Homework 1

multiplication. As I approaches infinite, the value of -1 becomes less and less relevant, this means the formula simplifies down to 1/n!, which is the formula for e. Simplifying this function down to ~e would make the efficiency class O(1), which is as good as it gets, so let's ignore the approximation bit and just skip to making it e.

b. Problem 7 (2 points)



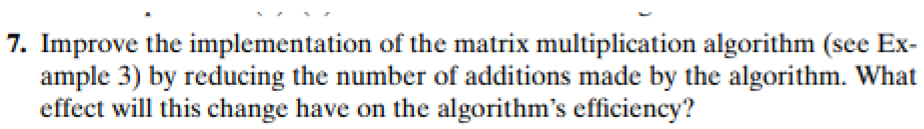
EXAMPLE 3 Given two $n \times n$ matrices A and B, find the time efficiency of the definition-based algorithm for computing their product $C = AB$. By definition, C is an $n \times n$ matrix whose elements are computed as the scalar (dot) products of the rows of matrix A and the columns of matrix B:



where $C[i, j] = A[i, 0]B[0, j] + \cdots + A[i, k]B[k, j] + \cdots + A[i, n-1]B[n-1, j]$ for every pair of indices $0 \le i, j \le n-1$.

ALGORITHM *MatrixMultiplication*($A[0..n-1, 0..n-1], B[0..n-1, 0..n-1]$)

```
//Multiplies two square matrices of order n by the definition-based algorithm
//Input: Two n × n matrices A and B
//Output: Matrix C = AB
for i ← 0 to n − 1 do
    for j ← 0 to n − 1 do
        C[i, j] ← 0.0
        for k ← 0 to n − 1 do
            C[i, j] ← C[i, j] + A[i, k] ∗ B[k, j]
return C
```



7. Improve the implementation of the matrix multiplication algorithm (see Example 3) by reducing the number of additions made by the algorithm. What effect will this change have on the algorithm's efficiency?

The current algorithm as implemented above has three iterative loops, whereupon each row is multiplied by the corresponding column, where a third iterative loop actually handles the calculation for the length of the column n. The current total is added to the total for a specific multiplication. I guess you could reduce it by moving from of the initial calculations for A[I,0] and B[0,j] which will save one pass or so through the loop, which is entirely negligible for n as it approaches large sizes, the fact there are three loops means that saving a pass in one of the loops is meaningless, and the efficiency of the algorithm will continue to be cubic.