CSC 3110 Homework 6
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4/17/2024

CSC 3110

Algorithm Design and Analysis

(30 points)

Due: 04/17/2024 11:59 PM

Note: Submit answers in PDF document format. Please read the submission format for appropriate file naming conventions.

1) Exercises 9.1:

a. Problem 1 (2 points)

1. Write pseudocode of the greedy algorithm for the change-making problem, with an amount n and coin denominations $d_1 > d_2 > \cdots > d_m$ as its input. What is the time efficiency class of your algorithm?

Given coins denominations as an array for input (organized by value, highest value starting at position 0), and an integer as the input for the number desired, start at 0, and divide the total to be returned by the largest denomination until the amount remaining is less than the value of the denomination, repeat for all denominations until value to be dispersed is zero. This should give a linear efficiency, as each value of the array needs to only be considered a few times and there should be no recursion.

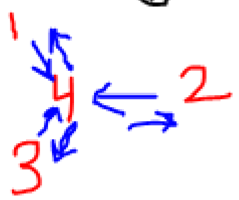
b. Problem 7 (2 points)

7. *Rumor spreading* There are n people, each in possession of a different rumor. They want to share all the rumors with each other by sending electronic messages. Assume that a sender includes all the rumors he or she knows at the time the message is sent and that a message may only have one addressee.

Design a greedy algorithm that always yields the minimum number of messages they need to send to guarantee that every one of them gets all the rumors.

This seems fairly straight forward – because everyone has a different rumor, everyone must send at least one message. If everyone sends their message to one person, and that person then messages to everyone the minimum number of messages will be 2n-2 because the person acting as the central distributor does not need to send a message to himself in the two rounds of message sending necessary.

$6 = 4 * 2 - 2$



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2) Exercises 9.2:

a. Problem 3 (2 points)

3. What changes, if any, need to be made in algorithm *Kruskal* to make it find a ***minimum spanning forest*** for an arbitrary graph? (A minimum spanning forest is a forest whose trees are minimum spanning trees of the graph's connected components.)

Kruskal's finds the minimum spanning tree for whatever vertex you start with, the solution therefore is obvious: The algorithm must be adopted to build multiple trees instead, just run it multiple times while iterating through each of the vertices and it will build the minimum spanning tree for each.

b. Problem 4 (1 point)

4. Does Kruskal's algorithm work correctly on graphs that have negative edge weights?

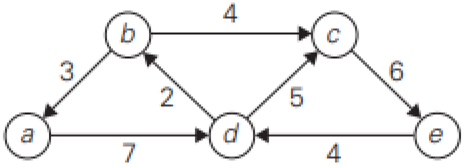
Yes.

3) Exercises 9.3:

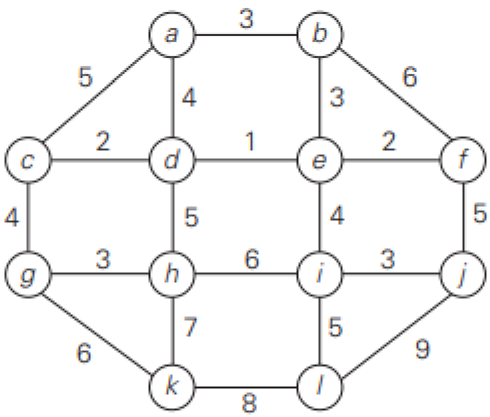
a. Problem 2 (part a & b) (2 points)

2. Solve the following instances of the single-source shortest-paths problem with vertex *a* as the source:

a.

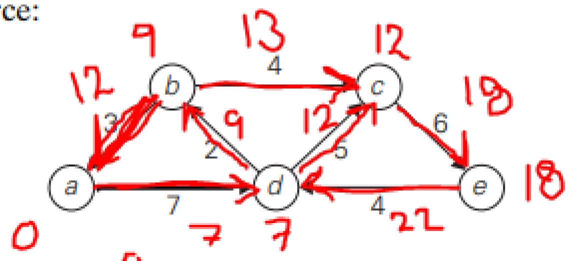


b.

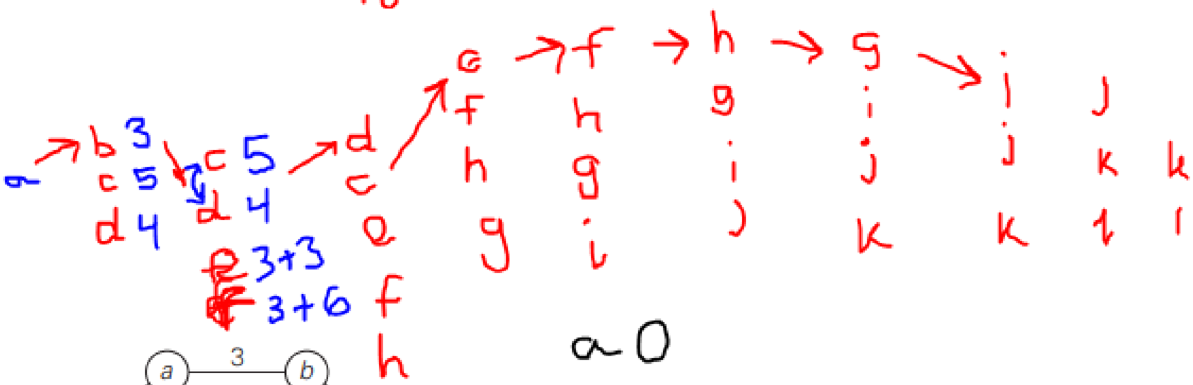


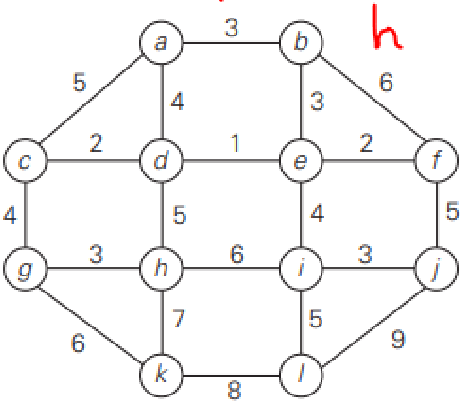
vertex a as the source:

a.



a 0
b 9
c 12
d 7
e 18





a 0
b 3
c 5
d 4
e 5
f 7
g 9
h 9
i 9
j 12
k 15
l 14

b. Problem 5 (2 points)

5. Write pseudocode for a simpler version of Dijkstra's algorithm that finds only the distances (i.e., the lengths of shortest paths but not shortest paths themselves) from a given vertex to all other vertices of a graph represented by its weight matrix.

Just write Djistra's algorithm but neglect to write down the sources or save the tree for how they got there.

4) Exercises 9.4:

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a. Problem 3 (part a & b) (2 points)

3. Indicate whether each of the following properties is true for every Huffman code.

a. The codewords of the two least frequent symbols have the same length.

b. The codeword's length of a more frequent symbol is always smaller than or equal to the codeword's length of a less frequent one.

a.) Codewords of the two least frequent symbols will always have the same length as they will always be on the same level of the tree (the bottom). This is only true if there are two though, if you start getting into odd numbers it won't work because of the binary tree nature of Huffman encoding.
b.)
Yes, the nature of Huffman encoding is that frequent symbols are placed higher in the tree. It's literally how the tree is built – it means that less probably characters are always on the same level of lower – meaning they will always have a longer or equal length.

b. Problem 4 (2 points)

4. What is the maximal length of a codeword possible in a Huffman encoding of an alphabet of n symbols?

Huffman trees can suffer from the same issues as most binary trees where all of the symbols end up on one side of the tree. However, the least frequent characters will be on the same level, meaning that the overall result is n-1.

5) Exercises 10.1

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a. Problem 3 (parts a – c) (2 points)

3. Consider the linear programming problem

$$\begin{aligned} \text{minimize} \quad & c_1x + c_2y \\ \text{subject to} \quad & x + y \geq 4 \\ & x + 3y \geq 6 \\ & x \geq 0,\ y \geq 0 \end{aligned}$$

where c_1 and c_2 are some real numbers not both equal to zero.

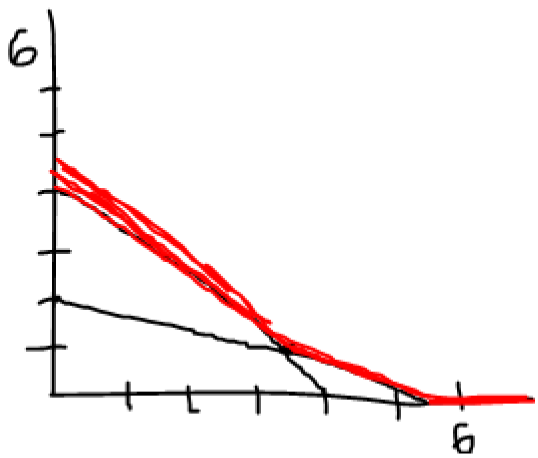
a. Give an example of the coefficient values c_1 and c_2 for which the problem has a unique optimal solution.

Iterative Improvement

b. Give an example of the coefficient values c_1 and c_2 for which the problem has infinitely many optimal solutions.

c. Give an example of the coefficient values c_1 and c_2 for which the problem does not have an optimal solution.

$$\begin{aligned} \text{minimize} \quad & c_1x + c_2y \\ \text{subject to} \quad & x + y \geq 4 \qquad Y \geq 4 - X \\ & x + 3y \geq 6 \qquad Y \geq +2 - 1/3x \\ & x \geq 0,\ y \geq 0 \end{aligned}$$



a.) 3,1 is a unique optimal solution as a minimization function will attempt to reduce the distance from 0,0 as much as possible for x and y. 3,1 is the closest point to zero in terms of distance out of any point on the line, and as such has the unique minimum. C1 = 3 and C2 =5 grant a line that runs along this line while touching no other points.

b.) There are infinitely many optimal solutions when the line described involves the regions boundary. C1 = 1 and C2 = 3 would give such a line.

c.) There are no solutions for C1 = -1, C2 = -1, as they can never be made to met the minimization criteria.

b. Problem 5 (part a & b) (2 points)

5. Trace the simplex method on

a. the problem of Exercise 2a.

b. the problem of Exercise 2b.

2

a.
$$\begin{aligned} \text{maximize} \quad & 3x + y \\ \text{subject to} \quad & -x + y \le 1 \\ & 2x + y \le 4 \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

b.
$$\begin{aligned} \text{maximize} \quad & x + 2y \\ \text{subject to} \quad & 4x \ge y \\ & y \le 3 + x \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

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$y = -3x$

$$
\begin{aligned}
\text{maximize} \quad & 3x + y \\
\text{subject to} \quad & -x + y \le 1 \qquad -x + y + u = 1 \\
& 2x + y \le 4 \qquad 2x + y + v = 4 \\
& x \ge 0,\ y \ge 0 \qquad x\ y\ u\ v \ge 0
\end{aligned}
$$

$$
\begin{array}{c|ccccc}
 & x & y & u & v & \\
\hline
u & -1 & 1 & 1 & 0 & 1 \\
v & 2 & 1 & 0 & 1 & 4 \\
 & -3 & -1 & 0 & 0 &
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & x & y & u & v & \\
\hline
u & 0 & 1.5 & 1 & 0.5 & 3 \\
x & 1 & 0.5 & 0 & 0.5 & 2 \\
 & 0 & 0.5 & 0 & 1.5 & 6
\end{array}
$$

$y = 0$

$x = 2$

$y = 0$

maximize $x + 2y$

subject to $4x \geq y$

$y \leq 3 + x$

$x \geq 0,\ y \geq 0$

$-4x + y + u = 0$

$-x + y + v = 3$

$x\ y\ u\ v \geq 0$

	x	y	u	v	
u	-4	1	1	0	0
v	-1	1	0	1	3
	-1	-2	0	0	0

y	-4	1	1	0	0
v	-3	0	-1	1	3
	-9	0	2	0	0

y	0	1	-1/3	4/3	4
x	1	0	-1/3	1/3	1
	0	0	-1	3	9

all negative = unbonded

6) Exercises 10.2

a. Problem 1 (2 points)

1. Since maximum-flow algorithms require processing edges in both directions, it is convenient to modify the adjacency matrix representation of a network as follows. If there is a directed edge from vertex i to vertex j of capacity u_{ij}, then the element in the ith row and the jth column is set to u_{ij}, and the element in the jth row and the ith column is set to $-u_{ij}$; if there is no edge between vertices i and j, both these elements are set to zero. Outline a simple algorithm for identifying a source and a sink in a network presented by such a matrix and indicate its time efficiency.

Based on the definition, a source can have no negative elements in it same row otherwise there would be material flowing TO it, similarly a sink can have no material flowing FROM it. A quick check would be to follow the row, and if all the numbers are positive it is a source, if all the numbers are negative, it is a sink. However, the entire adjacency matrix would have to be scanned, meaning that it would take n^2 time to run.

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b. Problem 3 (part a & b) (2 points)

3. a. Does the maximum-flow problem always have a unique solution? Would your answer be different for networks with different capacities on all their edges?

b. Answer the same questions for the minimum-cut problem of finding a cut of the smallest capacity in a given network.

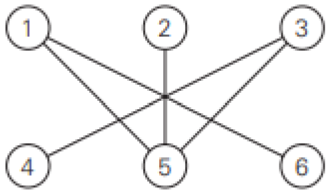
a.) There can be more than one optimal solutions for a maximum flow problem even in a network with different capacities on all edges as what matters is that the maximum capacity of a path is the same.

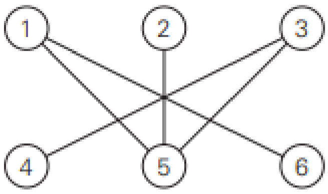
b.)There can be more than one optimal solution to the minimum cut problem for the same reason, because even if the edges are distinct what matters is the values along the entire path.

7) Exercises 10.3

a. Problem 2 (1 point)

2. Apply the maximum-matching algorithm to the following bipartite graph:

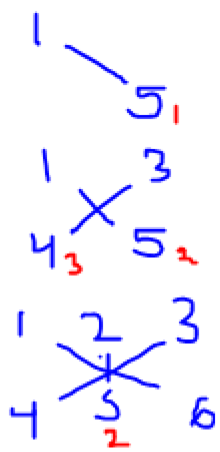




Q 1 2 3

2 3

2



b. Problem 3 (part a & b) (2 points)

3. a. What is the largest and what is the smallest possible cardinality of a matching in a bipartite graph $G = \langle V, U, E \rangle$ with n vertices in each vertex set V and U and at least n edges?

b. What is the largest and what is the smallest number of distinct solutions the maximum-cardinality-matching problem can have for a bipartite graph $G = \langle V, U, E \rangle$ with n vertices in each vertex set V and U and at least n edges?

a.)Cardinality means how many connections between each set, The smallest is 1 (because there must be at least one match between each set), and the maximum is when all vertices of a set match with all other vertices of the other set. In which case the maximum would be equal to the number of edges.

b.) n! is the largest distinct solution as that represents every vertex having a connection to another vertex in the opposite set without having overlap (factorial is used for combinations). The smallest number of distinct matchings however is 1, where only one in each set matches to only one other in each set.

8) Exercises 10.4

a. Problem 4 (2 points)

4. Find a stable marriage matching for the instance defined by the following ranking matrix:

	A	B	C	D
α	1, 3	2, 3	3, 2	4, 3
β	1, 4	4, 1	3, 4	2, 2
γ	2, 2	1, 4	3, 3	4, 1
δ	4, 1	2, 2	3, 1	1, 4

Stability checks are when a man and woman prefer each other over their current partners.

Round 1

	A	B	C	D
α	1, 3	2, 3	3, 2	4, 3
β	1, 4	4, 1	3, 4	2, 2
γ	2, 2	1, 4	3, 3	4, 1
δ	4, 1	2, 2	3, 1	1, 4

Round 2

	A	B	C	D
α	1, 3	2, 3	3, 2	4, 3
β	1, 4	4, 1	3, 4	2, 2
γ	2, 2	1, 4	3, 3	4, 1
δ	4, 1	2, 2	3, 1	1, 4

No parties prefer each other over their current partner in this match up.

b. Problem 6 (2 points)

6. Prove that a man-optimal stable marriage set is always unique. Is it also true for a woman-optimal stable marriage matching?

The stable marriage problem does not allow ties, this means that a solution cannot have a parallel 'also correct' solution, someone must be better and someone must be worse in preference sorting. As a result every solution is unique. This is also true for woman-optimal problems as it's the same problem just starting with woman instead.