Covid19 - Forecasts ICU beds

A. Chaouch, Y. Eggli, J. Pasquier, V. Rousson and B. Trächsel
Center for Primary Care and Public Health
University of Lausanne (Switzerland)

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Definition 1. Let Y_k be the cumulative count of hospitalized patients on day k.

$$Y_k = (1 + \exp(u_k))Y_{k-1},$$

where

$$u_k \sim \mathrm{N}(\log(\lambda_k - 1), \sigma_k^2).$$

Note: $\lambda_k =$ `mlam` and $\sigma_k^2 =$ `vlam`.

A voir : $\mu_k \perp\!\!\!\perp Y_{k-1} \implies Y_k \sim$ Log-normal

Definition 2. Let p_k be the proportion of patients hospitalized on day k (incident cases) who will require intensive care. The random variable p_k follows a logit-normal distribution:

$$p_k = \frac{\exp(q_k)}{1 + \exp(q_k)},$$

where

$$q_k \sim \mathrm{N}(\log(\frac{\pi_k}{1 - \pi_k}), \tau_k^2).$$

Note: $\pi_k =$ `micp` and $\tau_k^2 =$ `vicp`.

Definition 3. Let r_k be the proportion of patients requiring intensive care on day k who will be admitted in intensive care. The random variable r_k follows a logit-normal distribution:

$$r_k = \frac{\exp(s_k)}{1 + \exp(s_k)},$$

where

$$s_k \sim \mathrm{N}(\log(\frac{..k}{1 - ..k}), ..^2_k).$$

Note: $.._k =$ `madp` and $..^2_k =$ `vadp`.

Definition 4. Let S_k be the number of days (possibly zero) between the hospitalization and the admission in the intensive care for the patients who were hospitalized on day k.

$$S_k \sim \mathrm{NegBin}(\frac{\phi_k^2 - \mu_k}{\phi_k^2}, \frac{\mu_k^2}{\phi_k^2 - \mu_k}).$$

Note: $\mu_k =$ `mlag` and $\phi_k^2 =$ `vlag`.

Definition 5. Let T_k the length of stay (possibly zero) in the intensive care for the patients who were hospitalized on day k.

$$T_k \sim \mathrm{NegBin}(\frac{\chi_k^2 - \nu_k}{\chi_k^2}, \frac{\nu_k^2}{\chi_k^2 - \nu_k}).$$

Note: $\nu_k =$ `mlos` and $\chi_k^2 =$ `vlos`.

Definition 6. Let Z_m the number of patients treated in intensive care on day m.

$$Z_m = \sum_{k=1}^{m} \sum_{j=1}^{W_k} \sum_{l=0}^{m-k} r_{k+l} I_{\{S_k = l, T_k \geq m-k-l\}}$$

where $W_k = p_k(Y_k - Y_{k-1})$ with the convention that $Y_0 = 0$ and $I_{\{\dots\}}$ is an indicator function.

Lemma 1. *Suppose that the random variables ... are independent, then*

$$E(Z_m) \approx \dots$$

Proof.

$$\begin{aligned}
E(Z_m) &= E(\sum_{k=1}^{m} W_k \sum_{l=0}^{m-k} r_{k+l} I_{\{S_k=l, T_k \geq m-k-l\}}) \\
&= \sum_{k=1}^{m} E(p_k)(E(Y_k) - E(Y_{k-1}))\dots \\
E(p_k) &\approx \frac{1}{K-1} \sum_{i=1}^{K-1} \mathrm{logit}(\Phi^{-1}_{.,.}(i/K)) \\
E(Y_k) &= (1 + \exp(log(\lambda_k - 1) + \sigma_k^2/2)E(Y_{k-1}) \\
E(I_{\{\dots\}}) &= P(S_k = l, T_k \geq m-k-l) \\
&= P(S_k = l)P(T_k \geq m-k-l)
\end{aligned}$$

$\square$