Covid19 - Forecasts ICU beds

A. Chaouch, J. Pasquier, V. Rousson and B. Trächsel
Center for Primary Care and Public Health
University of Lausanne (Switzerland)

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Definition 1. Let Y_k be the cumulative count of hospitalized patients on day k.

$$Y_k = (1 + \exp(u_k))Y_{k-1},$$

where

$$u_k \sim \mathrm{N}(\log(\lambda_k - 1), \sigma_k^2).$$

Note: $\lambda_k =$ `mlam` and $\sigma_k^2 =$ `vlam`.

A voir : $\mu_k \perp\!\!\!\perp Y_{k-1} \implies Y_k \sim$ Log-normal

Definition 2. Let p_k be the proportion of patients hospitalized on day k (incident cases) who will then be admitted to intensive care. The random variable p_k follows a logit-normal distribution:

$$p_k = \frac{\exp(q_k)}{1 + \exp(q_k)},$$

where

$$q_k \sim \mathrm{N}(\log(\frac{\pi_k}{1 - \pi_k}), \tau_k^2).$$

Note: $\pi_k =$ `mpic` and $\tau_k^2 =$ `vpic`.

Definition 3. Let S_k be the number of days (possibly zero) between the hospitalization and the admission in the intensive care for the patients who were hospitalized on day k.

$$S_k \sim \mathrm{NegBin}(\frac{\phi_k^2 - \mu_k}{\phi_k^2}, \frac{\mu_k^2}{\phi_k^2 - \mu_k}).$$

Note: $\mu_k =$ `mlag` and $\phi_k^2 =$ `vlag`.

Definition 4. Let T_k the length of stay (possibly zero) in the intensive care for the patients who were hospitalized on day k.

$$T_k \sim \mathrm{NegBin}(\frac{\chi_k^2 - \nu_k}{\chi_k^2}, \frac{\nu_k^2}{\chi_k^2 - \nu_k}).$$

Note: $\nu_k =$ `mlos` and $\chi_k^2 =$ `vlos`.

Definition 5. Let define the following random variable:

$$I_{j,k,l} = \begin{cases} 1 & \text{if the } j\text{th patient hospitalized on day } k \text{ is being treated in intensive care } l \text{ day later,} \\ 0 & \text{if not.} \end{cases}$$

Lemma 1. $P(I_{j,k,l} = 1) = \sum_{i=0}^{l} P(S_k = i, T_k \geq l - i)$

Proof.

$$\begin{aligned} P(I_{j,k,l} = 1) &= P(S_k \leq l \leq S_k + T_k) \\ &= \sum_{i=0}^{l} P(S_k = i, T_k \geq l - i) \end{aligned}$$

□

Definition 6. Let Z_m the number of patients treated in intensive care on day m.

Lemma 2.

$$Z_m = \sum_{k=1}^{m} \sum_{j=1}^{W_k} I_{j,k,m-k}$$

where $W_k = p_k(Y_k - Y_{k-1})$ with the convention that $Y_0 = 0$.

Lemma 3. *Suppose that the random variables ... are independent, then*

$$E(Z_m) \approx \ldots$$

Proof.

$$\begin{aligned} E(Z_m) &= E(\sum_{k=1}^{m} W_k I_{j,k,m-k}) \\ &= \sum_{k=1}^{m} E(p_k)(E(Y_k) - E(Y_{k-1}))E(I_{j,k,m-k}) \\ E(p_k) &\approx \frac{1}{K-1} \sum_{i=1}^{K-1} \mathrm{logit}(\Phi_{.,.}^{-1}(i/K)) \\ E(Y_k) &= (1 + \exp(log(\lambda_k - 1) + \sigma_k^2/2)E(Y_{k-1}) \\ E(I_{j,k,m-k}) &= \sum_{i=0}^{l} P(S_k = i, T_k \geq l - i) \\ &= \sum_{i=0}^{l} P(S_k = i)P(T_k \geq l - i) \end{aligned}$$

□