QF600 Assignment 2 : Capital Asset Pricing Model (CAPM)

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1 Market Model

Estimate the intercept coefficient (α) and slope coefficient (β) for each of the ten industry portfolio using the market model.

- Create a table showing the intercept and slope coefficients for the ten industry portfolios.

Table 1 : Portfolio names and corresponding values for Y-intercept and slope (β)

Portfolio name	Y-intercept	Slope
NoDur	0.36944	0.65265
Durbl	-0.41560	1.64854
Manuf	0.15977	1.16985
Enrgy	0.50172	0.96985
HiTec	-0.06402	1.13297
Telcm	0.19469	0.90073
Shops	0.27549	0.82649
Hlth	0.23784	0.67304
Utils	0.44458	0.53809
Other	-0.38713	1.20731
Market	0.00000	1.00000

- Briefly explain the economic significance of the intercept and slope coefficients.

  The market risk model is a one-factor linear regression model which describes the excess return of an asset is a function of β and excess market return.

$$\tilde{R_i} - R_f = \alpha_i + \beta_i(\tilde{R_m} - R_f) + \tilde{\epsilon}_i$$

  The intercept coefficient can be considered as a measure of how different is an asset priced relative to the price calculated when using CAPM (i.e. pricing error). In the formula above, this is represented as α. If CAPM is true for an asset, then it should have been equal to 0 . If the intercept is > 0, then it is underpriced relative to the pricing by CAPM, it is overpriced if intercept is < 0.

  The slope coefficient can be used as an estimator for β of an asset. β itself is a measure of exposure of systematic "market" risk (i.e. risk that will affect every type of asset). The greater the value of β, the more exposed to market risk the asset is. The β of the market portfolio equals to 1 since the market portfolio is used as a reference when calculating β.

2 Security Market Line

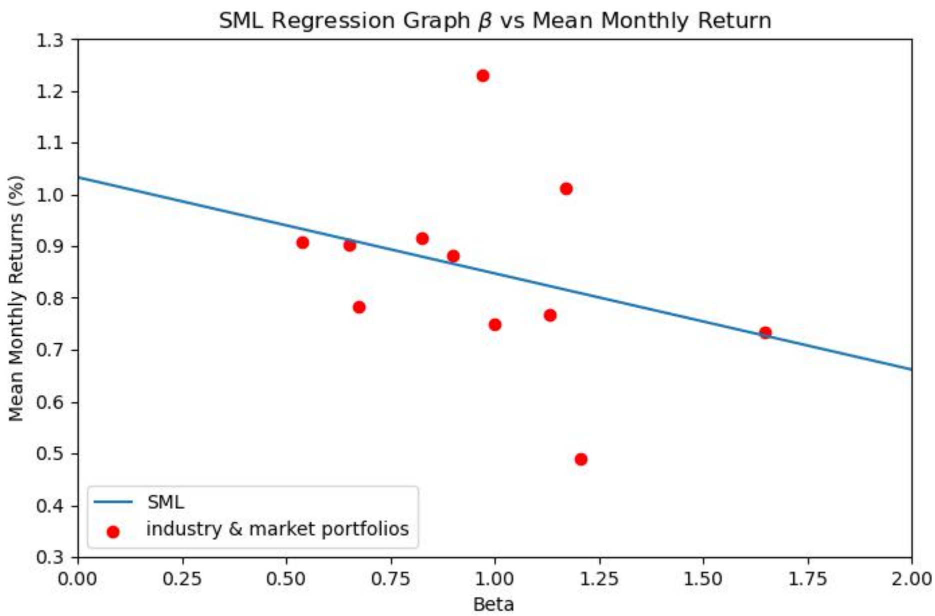
Regress the mean monthly returns of the ten industry portfolios and the market portfolio on the corresponding β's.

- Use the estimated intercept and slope coefficients for the SML to plot the SML in the range of β from zero to two on the horizontal axis.

Table 2 : Portfolios with corresponding estimated β and mean monthly return

Portfolio name	Estimated β	Portfolio mean
NoDur	0.65265	0.90283
Durbl	1.64854	0.73333
Manuf	1.16985	1.01283
Enrgy	0.96985	1.23117
HiTec	1.13297	0.76625
Telcm	0.90073	0.88142
Shops	0.82649	0.91633
Hlth	0.67304	0.78383
Utils	0.53809	0.90717
Other	1.20731	0.48908
Market	1.00000	0.74808

- Also plot the positions of the ten industry portfolios and the market portfolio. (You are NOT required to label the individual portfolios.)



SML intercept = 1.03277

SML slope = -0.18547

- Briefly explain the economic significance of the SML.

  The SML is the graphical representative of the CAPM formula. It shows what should be the appropriate expected return of an asset (asset here can be any type, any combination) against a varying level of exposure to systematic risk represented by β. Assuming CAPM is true, the investor can expect the market portfolio return for β equals to 1 and risk-free rate (R_f) return for β equal to 0 (i.e. SML will intersect the Y-axis at R_f). The SML is a static description of the equilibrium state, meaning assets with a given value of β that are overpriced or underpriced will settle to their appropriate levels when the supply for an asset equals the demand. For example, in a conventional- upward sloping SML, assets above the line are considered underpriced for their amount of exposure to systematic risk. Investors will try to purchase these assets which will eventually cause its price to increase, bringing the return of the asset down to SML. The opposite occurs for assets below the SML. The slope of the SML is also known as the Treynor Ratio. The Treynor Ratio represents the the ratio of risk premium to β. All risky assets in equilibrium will eventually have the same Treynor ratio, indicating the return will be equally attractive across all possible combinations on the SML.

  In our case, it seems that the SML line is downward sloping, meaning that the expected return decreases when increasing one's exposure to the market risk. Additionally, the SML does not intersect the Y-axis at the risk-free rate return $R_f = 0.13$. This could indicate a weakness in the CAPM (hence SML) where there are certain risk factors beyond just market risk that should be considered when evaluating risk and reward.

```
import numpy as np
import scipy as sp
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker
pd.set_option("display.precision", 8)

#we assume risk free rate remains stable at 0.13
R_f = 0.13

#load data
industry_portfolio_df = pd.read_excel('Industry_Portfolios.xlsx')
industry_portfolio_df = industry_portfolio_df.drop(['Date'],axis =1)
market_portfolio_df = pd.read_excel('Market_Portfolio.xlsx')
market_portfolio_df = market_portfolio_df.drop(['Date'],axis =1)
concated_df = pd.concat([industry_portfolio_df,market_portfolio_df],axis=1)
concated_df_mean = concated_df.mean().to_numpy()
concated_excess_df = concated_df - R_f

# regression for market model
market_model_result =[]
for n,portfolio in enumerate(concated_excess_df.columns):
    y = concated_excess_df[portfolio].to_numpy()
    x = concated_excess_df['Market'].to_numpy()
    market_model_result.append(np.polyfit(x,y,1)[::-1])

# regression result output
regression_result = np.column_stack((concated_excess_df.columns, market_model_result))
regression_result_df = pd.DataFrame(regression_result)\
    .rename(columns={0:'portfolio name',1:'y intercept',2:'slope'})
regression_result_df
print(regression_result_df.to_latex(index=False,
                  float_format="{:.5f}".format,
))
#prepare data for security market line
sml_data = np.column_stack((concated_excess_df.columns,\
                            regression_result_df['slope'].to_numpy(),\
                            concated_df.mean().to_numpy() ))
sml_data_df = pd.DataFrame(sml_data).rename(columns={0:'portfolio name',1:'estimated
beta from regression',2:'portfolio mean'})
sml_data_df
print(sml_data_df.to_latex(index=False,
                  float_format="{:.5f}".format,
))

# regression for SML
sml_x = list(sml_data_df['estimated beta from regression'].to_numpy())
sml_y = list(sml_data_df['portfolio mean'].to_numpy())
sml_result = np.polyfit(sml_x,sml_y,1)
sml_slope = sml_result[0]
sml_intercept = sml_result[1]
print(sml_x)
print(sml_x[0:-1])
```

```
#create y points for SML graph plotting
possible_beta  = np.arange(0,2.01,0.01)
sml_return = []
for x in possible_beta:
    y = sml_intercept + sml_slope * x
    sml_return.append(y)
f1,ax1 = plt.subplots(1,figsize=(8, 5))
ax1.plot(possible_beta,sml_return,label='SML')
ax1.xaxis.set_major_locator(plt.MultipleLocator(0.25))
ax1.yaxis.set_major_locator(plt.MultipleLocator(0.10))
ax1.set(xlim=(0,2),ylim=(0.3,1.3))
ax1.set_xlabel('Beta')
ax1.set_ylabel('Mean Monthly Returns (%)')
ax1.set_title(r'SML Regression Graph ${\beta}$ vs Mean Monthly Return')
ax1.scatter(sml_x, sml_y,label='industry & market portfolios',color='red')
ax1.legend(loc='lower left')
plt.show(f1)
f1.savefig('sml.jpg')
```