QF634 APPLIED QUANTITATIVE RESEARCH METHODS LECTURE 8

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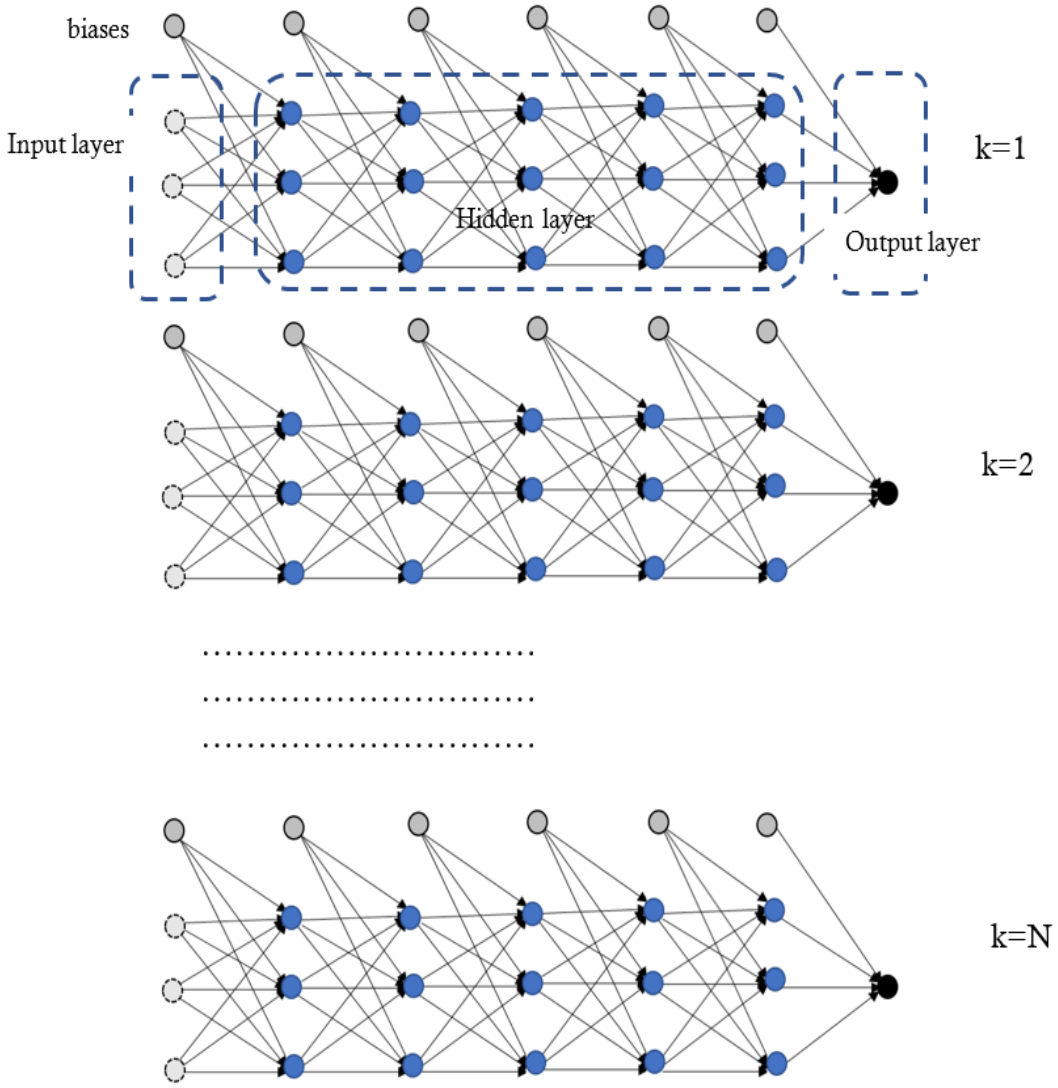
ARTIFICIAL INTELLIGENCE

Artificial Neural Network II: Deep Learning NN

In Chapter 7, we see that when cases k are independent of one another, i.e., their inputs and output do not affect other cases, then the common (across k) input weights and biases, and weights and biases in hidden layer(s) are revised via backpropagation to minimize the error/loss function L(.) that combines all cases $\sum_{k=1}^{N} \frac{1}{N} L(Y_k, \hat{Z}_k)$. $\hat{Z}_k$is the corresponding predicted output.

These computations in a single iteration can be done in parallel for each case k since the ANN is the same for each k.

In Figure 8.1, we show the parallel multiprocessing using N identical multilayer perceptrons (MLP), one to forward-propagate each case/subject or sample point k in the training data set of size N or N sample points.



Recurrent Neural Network (RNN)

Suppose now the input features for each case, instead of given altogether at the initial input layer, are given one at a time in a sequence. In a common setup to this new way of input, the number of hidden layers is now equal to the number of sequential input features so that at each additional hidden layer, there is a fresh sequential input feature – see dotted arrow in Figure 8.2. We consider the simple case where each sequential input feature is a scalar number. More general cases include an input feature which is a vector of numbers.

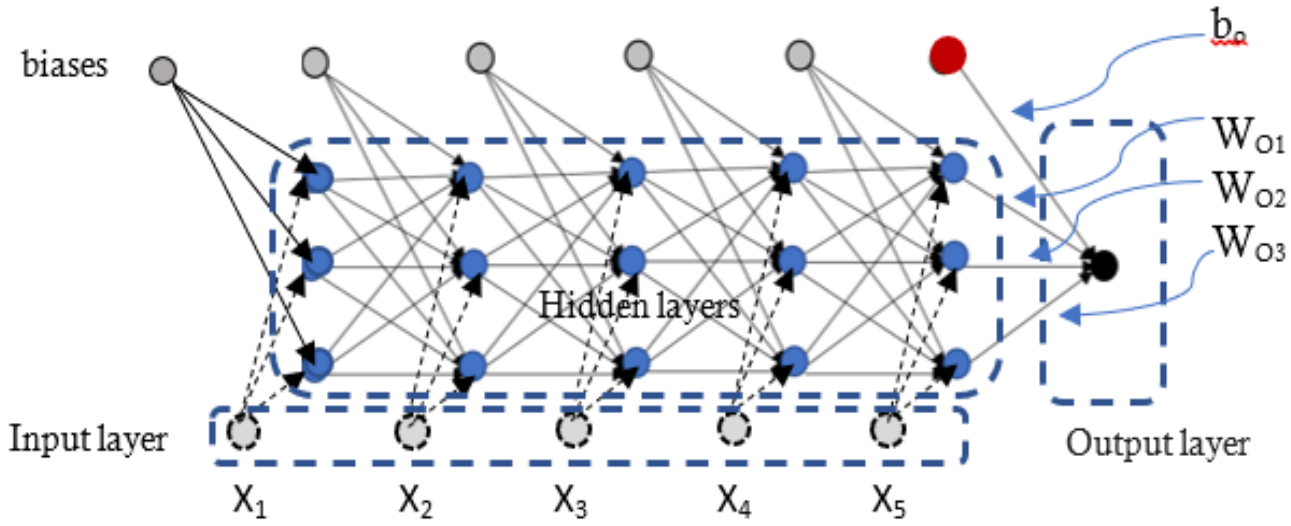


Figure 8.2

This new architecture is common in time series prediction such as predicting the next day stock price (label or target) using lagged prices in the past 5 days. These lagged prices form the features of this label. Each of these lagged prices becomes one feature in this one training case. A different window of price with lagged 5 prices forms another case, and so on.

- Figure 8.2 shows an architecture of 3 neurons per hidden layer whereby one training case is passed through the NN. The inputs for this case consist of 5 sequential features X_1, X_2, X_3, X_4, and X_5. There is one output for the case in the output layer of one neuron. As in ANN or MLP, many cases in a batch or mini batch are required to pass through this RNN before the loss function, comprising sum of loss in each case, is to be reduced via revising the parameters in the model that include all the weights and biases. The following Figure 8.3 shows the labelling convention we adopt for the RNN.
- In RNN, the weights and biases at each neuron level do not change from one hidden layer to another. This is unlike that in the ANN.
- There are 3 input weights (W_{P1}, W_{P2}, and W_{P3}), 3×3 weights W_{pq} from prior layer of neurons, and 3 biases. In addition, there are the weights W_{o1}, W_{o2}, W_{o3} and b_0 representing weights and bias from last hidden layer to the output neuron.
- In general, when the hidden layer has m neurons, then the total number of parameters to be fitted is $(m+2)(m+1) – 1$.

Labelling of Parameters on the Recurrent Neural Network (RNN)

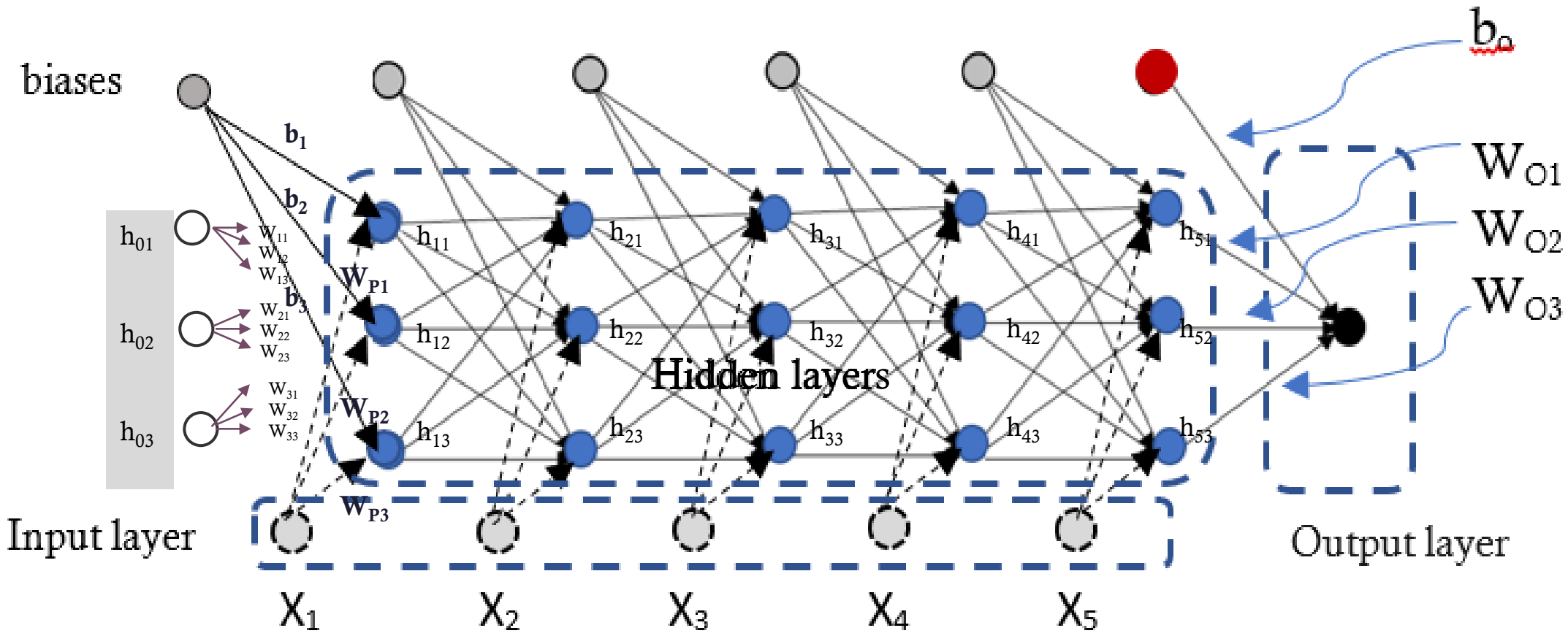
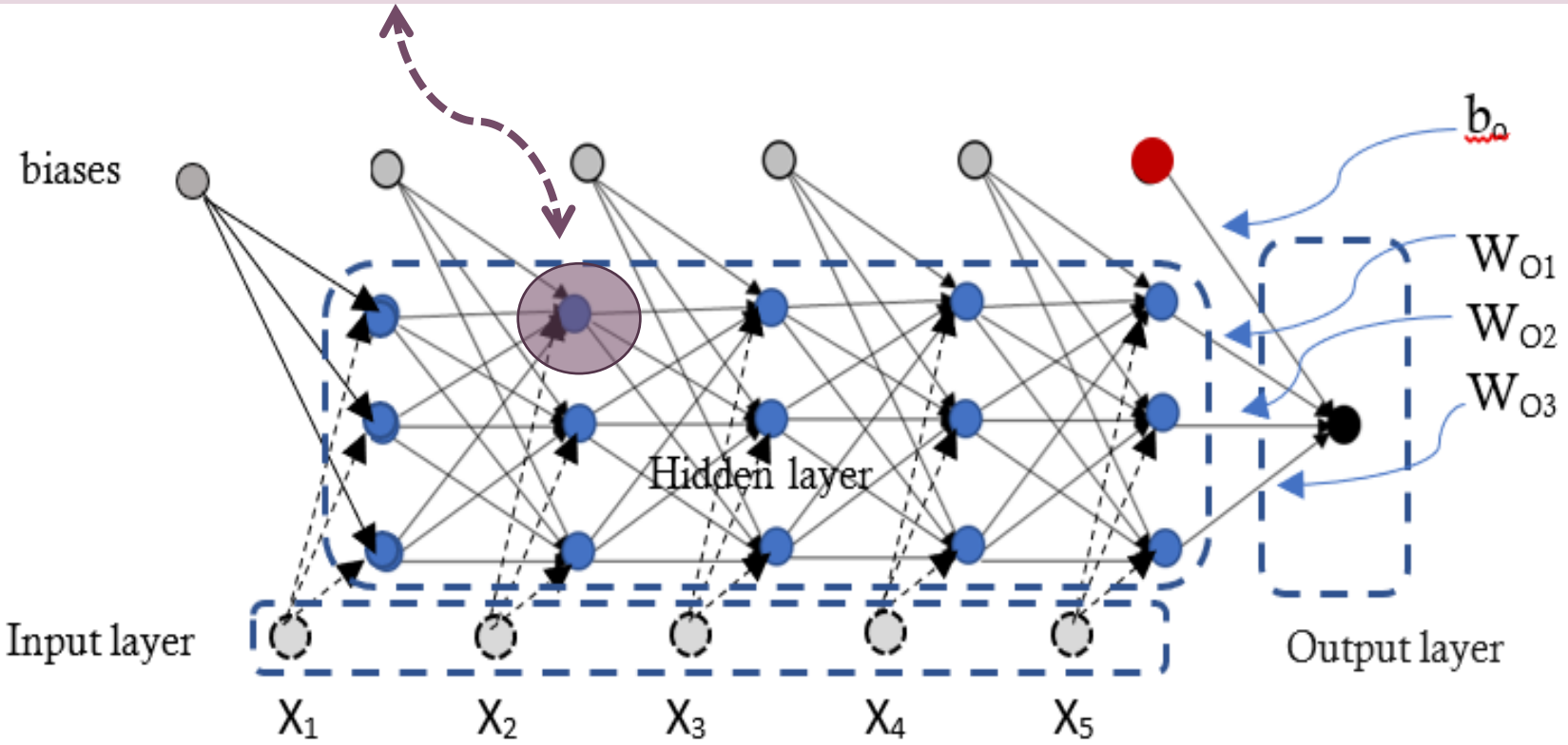


Figure 8.3

For example, the parameters to the first (uppermost) neuron in the 2nd layer are (1) W_{P1}, the weight on exogenous input X_2 from the input layer I, (2) the bias b_1, (3) W_{11}, the weight on forward pass from previous hidden layer first neuron, (4) W_{21}, the weight on forward pass from previous hidden layer second neuron, (5) W_{31}, the weight on forward pass from previous hidden layer third neuron.



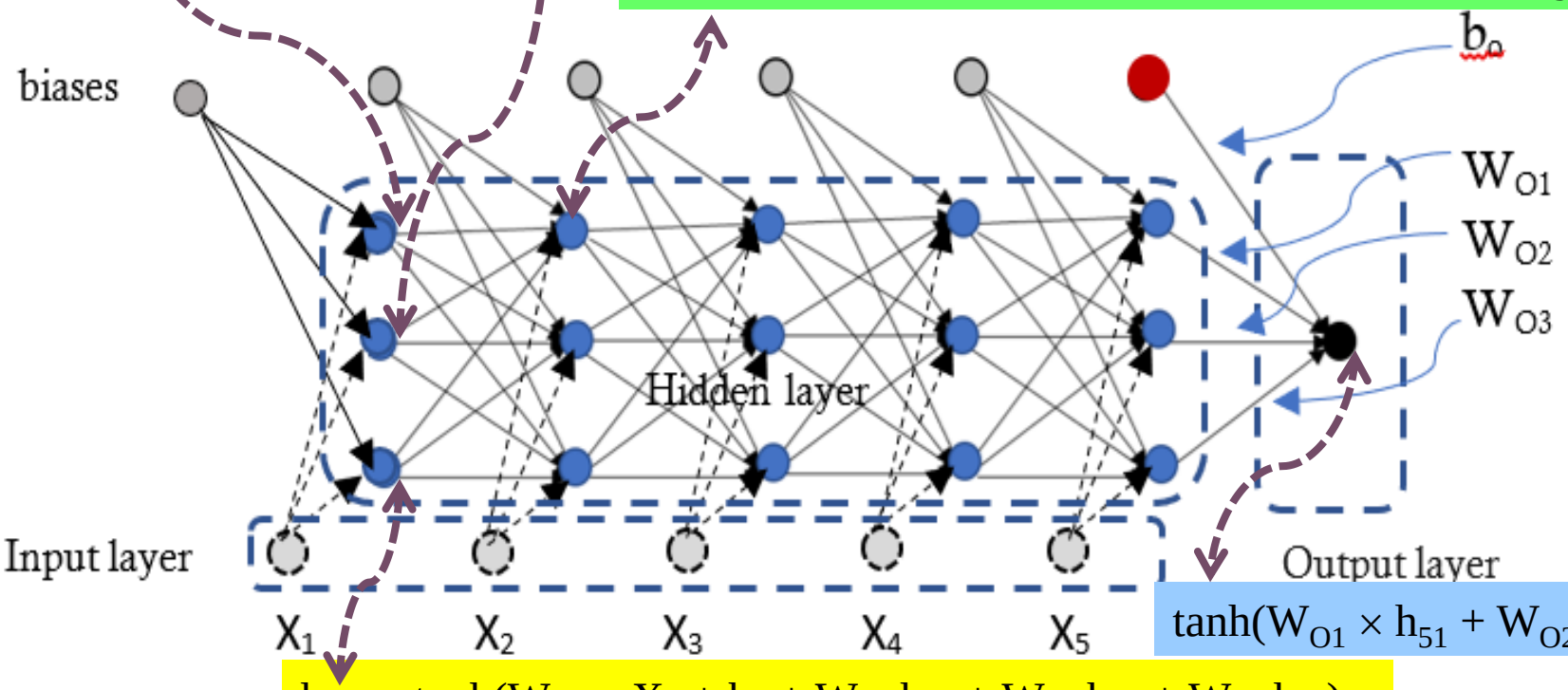
Output from the first neuron in the first hidden layer is

$h_{11} \equiv \tanh(W_{P1} \times X_1 + b_1 + W_{11}\, h_{01} + W_{21}\, h_{02} + W_{31}\, h_{03})$

Outputs (hidden state) h_{tj} at sequence step t and j^{th} position neuron

$h_{12} \equiv \tanh(W_{P2} \times X_1 + b_2 + W_{12}\, h_{01} + W_{22}\, h_{02} + W_{32}\, h_{03})$

$h_{21} \equiv \tanh(W_{P1} \times X_2 + b_1 + W_{11}\, h_{11} + W_{21}\, h_{12} + W_{31}\, h_{13})$



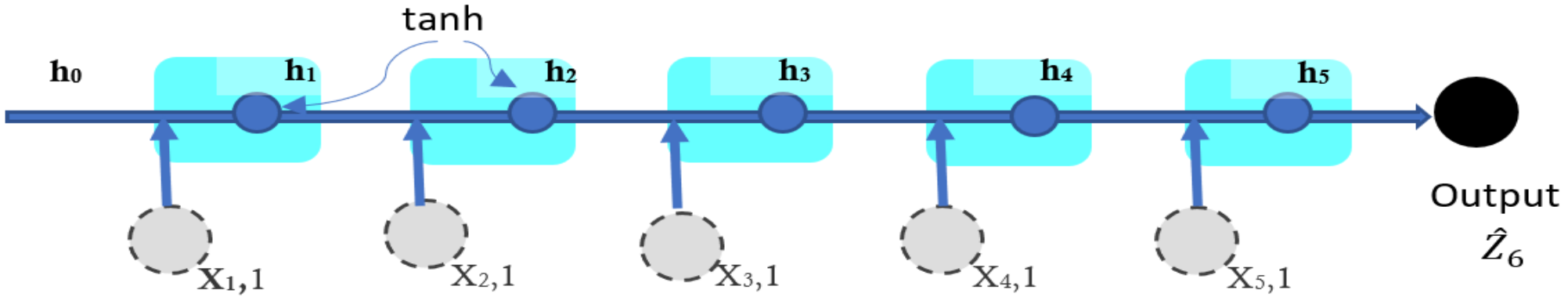
$\tanh(W_{O1} \times h_{51} + W_{O2}\, h_{52} + W_{O3}\, h_{53} + b_0)$

$h_{13} \equiv \tanh(W_{P3} \times X_1 + b_3 + W_{13}\, h_{01} + W_{23}\, h_{02} + W_{33}\, h_{03})$

A concise representation of Recurrent Neural Network

Each box represents a hidden layer of neurons

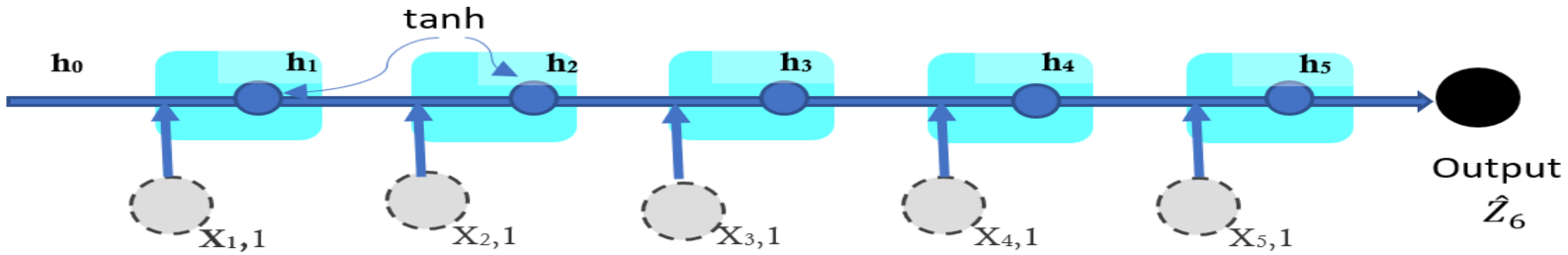
It is noted that the hidden states (at each level of neuron) change with each step in the sequence. Each hidden state at sequence step t, h_{tj}, contains information of lagged exogenous input features X_{t-1}, X_{t-2}, X_{t-3}, and so on. This type of architecture is important if indeed current input features are not independent from lagged input features that have useful information in predicting the output. This type of neural network that allows for recurrence of influences of past features is thus called Recurrent Neural Network (RNN).



The additional "1" in the input box reflects the "input" to multiply with the bias coefficient.

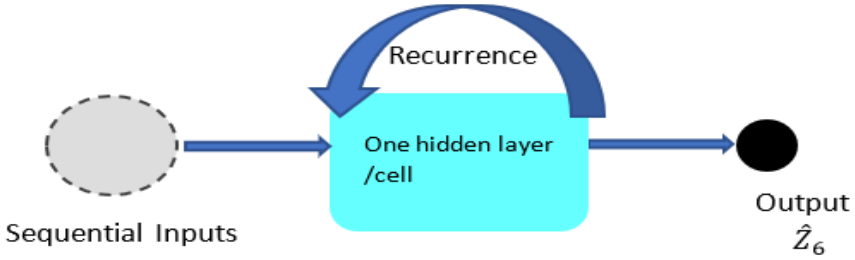
A concise representation of Recurrent Neural Network

Each box represents a hidden layer of neurons



There is concatenation of the inputs (X_t , 1) and $\mathbf{h_{t-1}} \equiv (h_{t-1,1}, h_{t-1,2}, h_{t-1,3})$ at their junction, and this is passed through the hidden layer where the dark circle represents the activation function on the dot product of the concatenated inputs and parameters. The output of that hidden layer is then $\mathbf{h_t}$ which enters into the computations at the next hidden layer. Each of the hidden layer may be viewed as a recurrent cell since the computations are recurrent on the updated hidden states as the sequence progresses until the output.

Above is sometimes called the unfolded NN of the diagram on the right where recurrence is explicitly indicated.



Recurrent Neural Network

Revision/Updating of Parameters

- In general, a training data set could have a time series or a sequence of T data points. The T sample points are divided into cases or packs each with R, e.g., 5, number of recurrent cells/hidden layers. The number of cases or packs is T/R. A certain number of cases/packs are grouped into batches with batch size S. So, there are T/(RS) number of mini batches (each size S).
- Each sample point, a time series element, in each of the T/(RS) number of mini batches is one case and yields one set of partial derivatives of the loss function with respect to the (m+2) (m+1) – 1 number of parameters. S number of elements in each mini batch yield S sets of partial derivatives of the loss function. The aggregate loss function is the sum of losses of the S number of cases.
- Thus, over each mini batch, the forward propagation and then backward propagation in updating the parameters based on the sum of S partial derivatives is considered one iteration. The training sample points or elements can be run in parallel for each iteration. Over E number of epochs and T/(RS) number of mini batches, there will be E × T/(RS) number of iterations to update/revise the parameters.

Backward Propagation in RNN

Consider the loss function of one case

$$\frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial W_{P1}} = \frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial \hat{Y}_{t+6}} \times \frac{\partial \hat{Y}_{t+6}}{\partial W_{P1}} = \frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial \hat{Y}_{t+6}} \times \left(1 - \hat{Y}_{t+6}{}^2\right) \times \frac{\partial (W_{O1} \times \boldsymbol{h_{51}} + W_{O2} \times h_{52} + W_{O3} \times h_{53} + b_o)}{\partial \boldsymbol{W_{P1}}}$$

where

$$\frac{\partial \boldsymbol{h_{51}}}{\partial \boldsymbol{W_{P1}}} = \left(1 - h_{51}{}^2\right) \frac{\partial (W_{P1}\, X_5 \,+\, W_{11}\, h_{41} \,+\, W_{21}\, h_{42} \,+\, W_{31}\, h_{43} \,+\, b_1)}{\partial W_{P1}} = \left(1 - h_{51}{}^2\right) \left(X_5 + W_{11} \frac{\partial h_{41}}{\partial W_{P1}} + W_{21} \frac{\partial h_{42}}{\partial W_{P1}} + W_{31} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{52}}{\partial W_{P1}} = \left(1 - h_{52}{}^2\right) \frac{\partial (W_{P2}\, X_5 \,+\, W_{12}\, h_{41} \,+\, W_{22}\, h_{42} \,+\, W_{32}\, h_{43} \,+\, b_2)}{\partial W_{P1}} = \left(1 - h_{52}{}^2\right) \left(0 + W_{12} \frac{\partial h_{41}}{\partial W_{P1}} + W_{22} \frac{\partial h_{42}}{\partial W_{P1}} + W_{32} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{53}}{\partial W_{P1}} = \left(1 - h_{53}{}^2\right) \frac{\partial (W_{P3}\, X_5 \,+\, W_{13}\, h_{41} \,+\, W_{23}\, h_{42} \,+\, W_{33}\, h_{43} \,+\, b_3)}{\partial W_{P1}} = \left(1 - h_{53}{}^2\right) \left(0 + W_{13} \frac{\partial h_{41}}{\partial W_{P1}} + W_{23} \frac{\partial h_{42}}{\partial W_{P1}} + W_{33} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{41}}{\partial W_{P1}} = \ldots\ldots$$

and so on. This clearly involves summation of partial derivative terms across each time-step or each sequential recurrent unit or hidden layer, e.g., $\frac{\partial \boldsymbol{h_{5j}}}{\partial \boldsymbol{W_{P1}}}$, $\frac{\partial h_{4j}}{\partial W_{P1}}$, $\frac{\partial h_{3j}}{\partial W_{P1}}$, $\frac{\partial h_{2j}}{\partial W_{P1}}$, $\frac{\partial h_{1j}}{\partial W_{P1}}$ for every j, involving terms of inputs X_1, …., X_5 and previous parameter values $W_{ij}^{[t]}$, and so on. This is unlike the feedforward NN case where a partial derivative would involve summation only across paths originating from the parameter edge.

Backward Propagation in RNN

- The above partial derivatives are for one case. They are summed across all cases in one mini batch size S.
- If loss across all cases in the mini batch of size S is $L = 1/S\ \Sigma^{S}\ L\left(Y_{t+R}, \hat{Y}_{t+R}\right)$, then $\partial L/\partial W_{ij}^{[t]} = 1/S\ \Sigma^{S}\ \partial L\left(Y_{t+R}, \hat{Y}_{t+R}\right) / \partial W_{ij}^{[t]}$ at the t^{th} iteration.
- After the updated partial derivatives are found, the usual adjustments of the parameter weights are done, viz.

$$W_{ij}^{[t+1]} = W_{ij}^{[t]} - \alpha(t+1)\frac{\partial L}{\partial W_{ij}^{[t]}}$$

Advantages and Disadvantages of RNN

- While the advantage of RNN over MLP is the ability to recognize the input of lagged (historical) information in affecting current output, and the ability to take in long sequence of inputs (since the number of parameters is not blown up as the parameters are constant at each recurrent unit), the computational time can be longer as the back propagation involves more computations of the complicated partial derivatives.
- RNN also has two drawbacks: (1) the partial derivatives in the RNN backpropagation may in some situations progressively collapse to zero (as the tanh activation on arguments close to zero in turn produces output close to zero) or else they may explode exponentially (when large aggregation of partial derivatives > 1 produces increasing gradients); (2) the design does not allow it to consider future input for training since the time-sequence uses information or input from the past till the current and is meant to predict the future data. In general, (2) is a limitation for financial time series where known future state cannot be used to predict an earlier state – this introduces serious issues of spuriously high accuracies.

Worked Example I -- Data

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

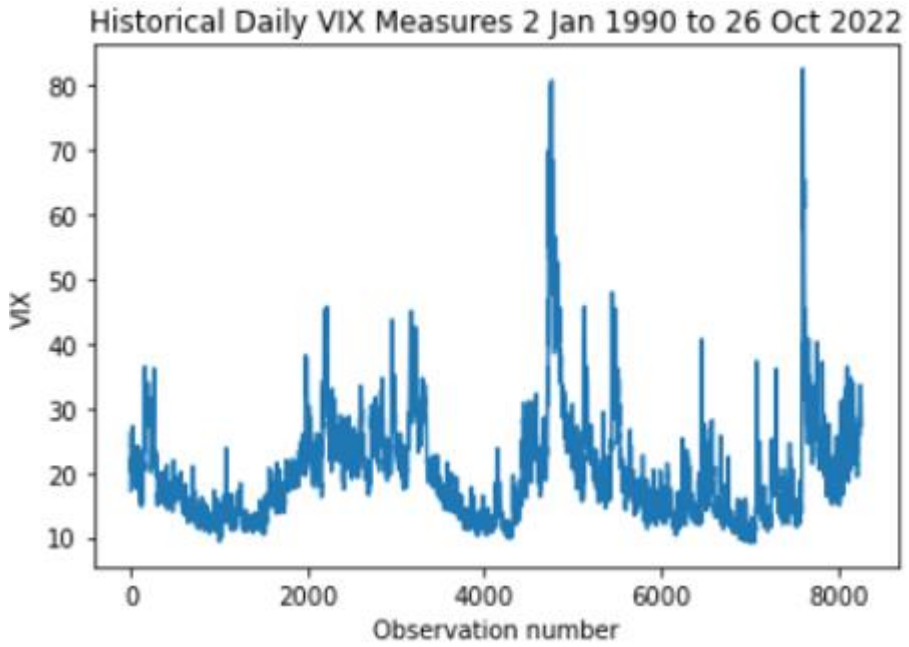
- The CBOE Volatility Index (VIX) is a real-time index derived from the prices of SPX (S&P 500) index options with near-term (approximately 1 month) expiration dates. It is market expectation of the SPX risk-neutral volatility over the short term. VIX has been called a 'fear gauge' and is often seen as a measure of negative market sentiment. Data is downloaded from Yahoo Finance.
- The following exercise uses daily VIX data from 2 Jan 1990 to 26 Oct 2022 (8270 time-sequenced sample points) to perform a RNN prediction of next day VIX based on the observed VIX of the past 10 days (approximately 2 trading weeks), the inputs. The total data set is split into 80% training data (6616 observations) and 20% test data (1654 observations) – see code line [5], [6], [7].

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

Snapshot of the data is shown as follows.

	Date	Open	High	Low	Close	Adj Close	Volume
0	2/1/1990	17.240000	17.240000	17.240000	17.240000	17.240000	0
1	3/1/1990	18.190001	18.190001	18.190001	18.190001	18.190001	0
2	4/1/1990	19.219999	19.219999	19.219999	19.219999	19.219999	0
3	5/1/1990	20.110001	20.110001	20.110001	20.110001	20.110001	0
4	8/1/1990	20.260000	20.260000	20.260000	20.260000	20.260000	0
...	...	...	...	...	...	...	...
8265	20/10/2022	31.299999	31.320000	29.760000	29.980000	29.980000	0
8266	21/10/2022	30.209999	30.440001	29.240000	29.690001	29.690001	0
8267	24/10/2022	30.650000	30.950001	29.780001	29.850000	29.850000	0
8268	25/10/2022	29.799999	30.000000	28.219999	28.459999	28.459999	0
8269	26/10/2022	28.440001	28.520000	27.270000	27.280001	27.280001	0

[8270 rows x 7 columns]

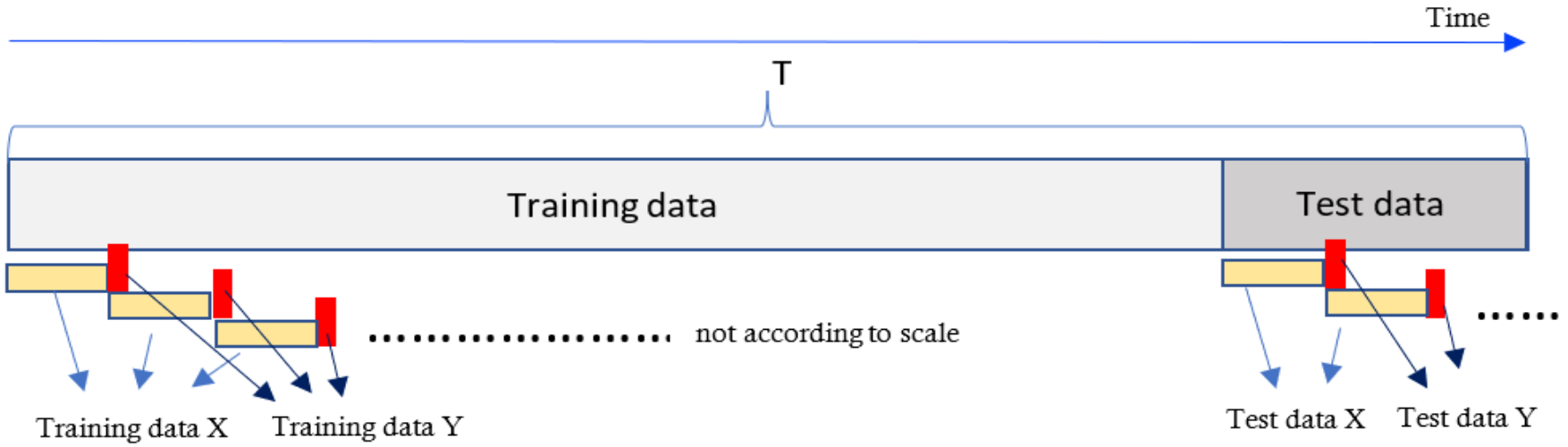


Transforming the Data

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

```
In [9]: #Performing Feature Scaling
        #from sklearn.preprocessing import StandardScaler
        from sklearn.preprocessing import MinMaxScaler
        scaler = MinMaxScaler(feature_range=(0, 1))  ### putting all positive is suitable as vola are all pos nos.
        train_data = scaler.fit_transform(train_data.values.reshape(-1, 1)) ### (-1,1) reshapes it to a 2D array;
        ### without .values - it may not work; .values .values convert to np array, the axes labels will be removed.
        test_data = scaler.fit_transform(test_data.values.reshape(-1, 1)) ### (-1,1) reshapes it to a 2D array
```

Arranging Data for Training and Testing



Arranging Data for Training and Testing

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

In [13]:
```
### Preparing the input X and target Y
def get_XY(dat, time_steps):
    ### Indices of target array
    C_ind = np.arange(time_steps, len(dat), time_steps)
        ### example np.arange(start=1, stop=10, step=3) gives array([1, 4, 7]), ends up to/before stop
    C = dat[C_ind]
        ### example: ray=np.arange(2, stop=10, step=3); print(ray) --- gives [2 5 8]
        ### c=np.array([1,3,6,8,9,10,12,15,18,20]); c[ray] --- gives array([ 6, 10, 18])
        ### with elements from the 2nd, 5th, 8th positions of c. c's 1st position starts at '0'

    ### Prepare X
    rows_x = len(C)                              ### here len(C) is 661
    X = dat[range(time_steps*rows_x)]            ### range(L) is 0,1,2,..., L-1. L is 10 x 661 = 6610. X is array 1,2,...,6610
    X = np.reshape(X, (rows_x, time_steps, 1))   ### X rshaped as (661,10,1)
    return X, C

time_steps = 10  ### hence C_ind = array ([10, 20, 30, 40, ....,661]), 661 number of 10 steps
                 ### C = array(10th position, 20th position of dat, etc.)
                 ### -- approx two weeks (10 trading days) interval for one prediction point of VIX
trainX, trainY = get_XY(train_data, time_steps)
testX, testY = get_XY(test_data, time_steps)
```

We use a sequence of 10 steps (approximately two weeks of 10 trading days) to form one pack of 10 recurrent units. These 10 daily VIX prices of training data X (inputs) are then followed by the following 11^{th} day of VIX price as the training data for output Y. The training data set is divided into T/R number of the packs. The test data are similarly arranged into packs of 10 inputs followed by 1 output. The cases or packs are non-overlapping.

RNN Structure

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

In [18]:
```
def create_RNN(hidden_units, dense_units, input_shape, activation):
    model = Sequential()
    model.add(SimpleRNN(hidden_units, input_shape=input_shape, activation=activation[0]))
       ### See SimpleRNN apps in https://www.tensorflow.org/api_docs/python/tf/keras/layers/SimpleRNN
    model.add(Dense(units=dense_units, activation=activation[1]))
       ### Using model = Sequential() allows defining -- no. of inputs,#neurons in hidden,#neurons in output layer
    model.compile(loss='mean_squared_error', optimizer='adam')
       ### .compile in Sequential carries loss and optimizer options
    return model
```

In [19]:
```
# Create model and train
model1 = create_RNN(hidden_units=8, dense_units=1, input_shape=(time_steps,1), activation=['tanh', 'tanh'])
   ### calls function create_RNN, fills in the arguments that were sub-defined in last codeline via .add that defines operations
   ###   at the input layer and at the hidden layer and at dense/output layer
### hidden_units = 8 means that there are 8 neurons in each hidden layer
### input_shapes = (time-steps,1) with time_steps=10 means that each time-step in a pack of 10 is an input
###   (the output for that time-step is ignored) -- only the end of pack time variable is used in trainY
```

Fitting and Prediction

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

Model1.fit is then called (via the Sequential app) to execute the training fit (minimizing loss based on number of iterations specified in number of epochs and batch size: 30×661 (T/R)) to find the optimal $(m+2)(m+1) - 1 = 10 \times 9 - 1 = 89$ parameters.

```
model1.fit(trainX, trainY, epochs=30, batch_size=1, verbose=2)
   ### time series of trainX is reshaped as (661,10,1), trainY is (661,)
   ### time series of testX is (165, 10, 1)), testY is (165,)
```

This is followed by making predictions on trainY based on trainX (using the optimized .fit parameters), and then making predictions on testY based on testX (using the optimized .fit parameters).

```
### make predictions
train_predict = model1.predict(trainX) ### Using the fitted model with trainX, trainY
test_predict = model1.predict(testX)   ### Using the same fitted model with trainX, trainY

### above, more appropriately test_predict is run after train_predict error is minimized or loss is minimized after the
### training set trainX, trainY are used in .fit -- and then the hyperparameters are tuned, before applying in a separate
### model1.fit to the testX, testY data. Above assumes the model1.fit on trainX, trainY is already the optimal one
```

Minimized Loss

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

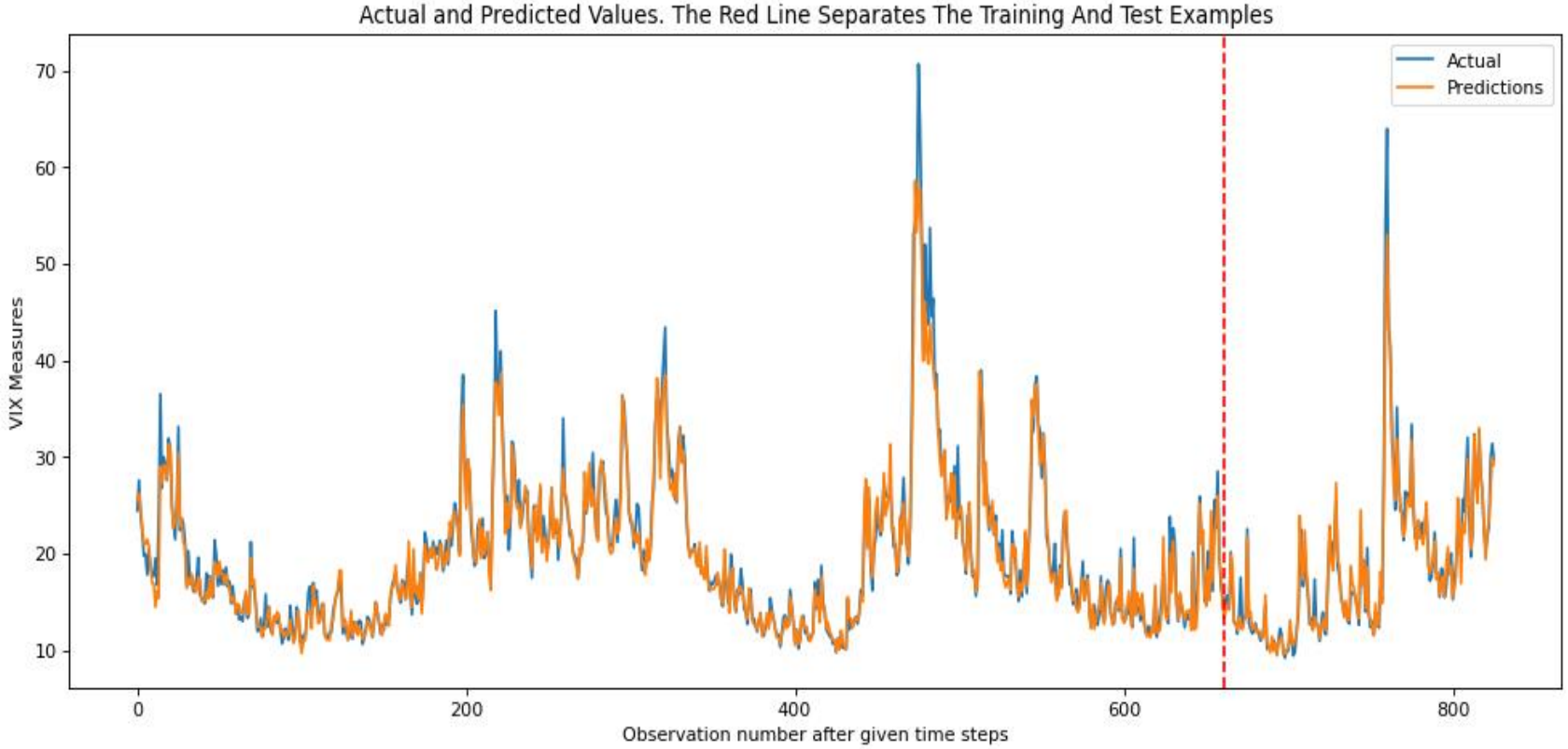
After iterating over 661 cases (packs of 10) each epoch, and then repeatedly over 30 epochs, the optimized .fit parameters produce the following root-mean-square-errors (RMSE) in the prediction of training data Y (trainY) and test data Y (testY) respectively:

```
In [21]: def print_error(trainY, testY, train_predict, test_predict):
             ### Error of predictions
             train_rmse = math.sqrt(mean_squared_error(trainY, train_predict))
             test_rmse = math.sqrt(mean_squared_error(testY, test_predict))
             ### Print RMSE
             print('Train RMSE: %.3f RMSE' % (train_rmse))
             print('Test RMSE: %.3f RMSE' % (test_rmse))

         print_error(trainY, testY, train_predict, test_predict)

         Train RMSE: 0.023 RMSE
         Test RMSE: 0.024 RMSE
```

Prediction versus Actual

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

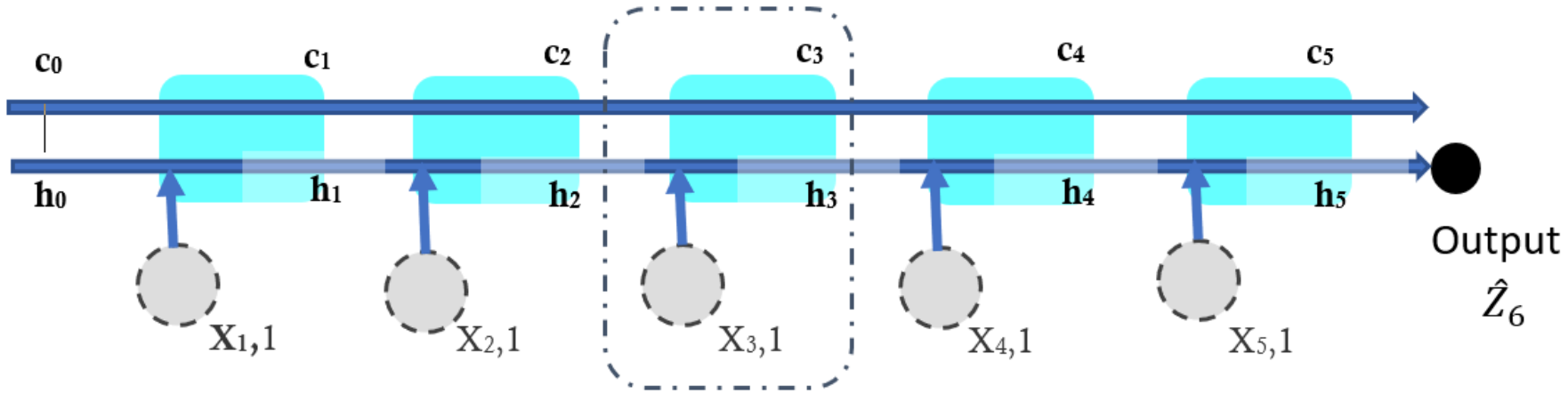
The RMSEs are about 2.3% for the training and 2.4% for the testing. The plot of the rescaled (inverse of scaling in [9] actual versus predicted outputs in both the training set and the test set are shown below.



Variants of RNN

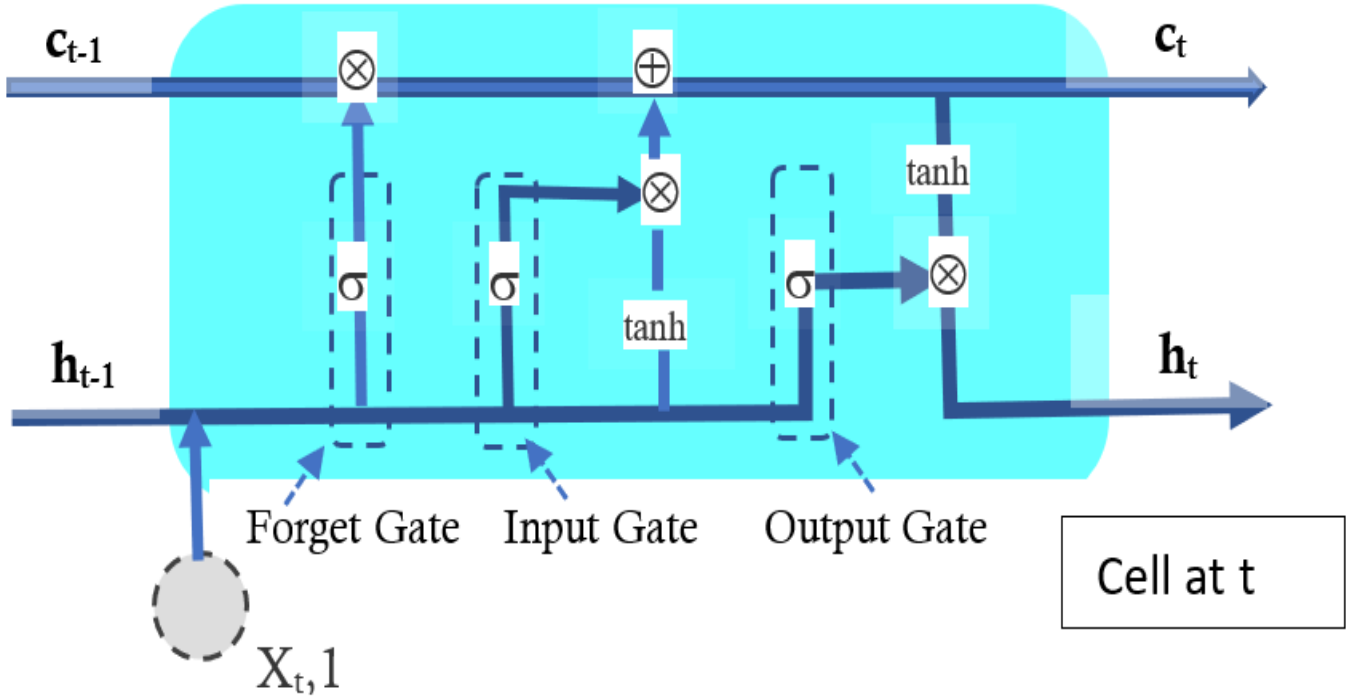
Mitigating lack of long term memory with disappearing gradients

In the LSTM architecture, the recurrent cell in Figure 8.4 is redesigned as follows in Figure 8.7 There is an additional state (also not explicitly observed) called the 'cell state'. Like the hidden states $\mathbf{h}_t$ in a traditional RNN as in Figure 8.3 that stores 'short-term' memory of effects of past inputs, the cell state $\mathbf{c}_t$ stores 'long-term' memory of effects of past inputs. As in Figure 8.3 each "box" with each fresh input represents one hidden layer in the RNN.



Variants of RNN

Long Short Term Memory NN



The concatenated inputs (X_t,1) and the previous hidden states (outputs of last hidden layer) h_{t-1} are passed through (1) the Forget Gate before they flow through to update the last cell state c_{t-1} , (2) the Input Gate before they flow through to make the final update on the cell state that is preliminarily updated by (1), (3) the Output Gate before they update the hidden state $\mathbf{h_{t-1}}$ to $\mathbf{h_t}$.

Variants of RNN

Long Short Term Memory NN

- Suppose we create 50 neurons in the hidden layer or the cell. In the Forget Gate (1), concatenated inputs (X_t,1, $\mathbf{h_{t-1}}$) after weighting is transformed by activation σ -function, viz.

$$\sigma\,(W_{FX}\,X_t + W_{Fh}\,\mathbf{h_{t-1}} + b_F\,)$$

- In (2), input gate is multiplied by cell update candidate

$$\sigma\,(W_{IX}\,X_t + W_{Ih}\,\mathbf{h_{t-1}} + b_I\,) \otimes \tanh\,(W_{CX}\,X_t + W_{Ch}\,\mathbf{h_{t-1}} + b_C\,)$$

This Hadamard product is used to update cell state

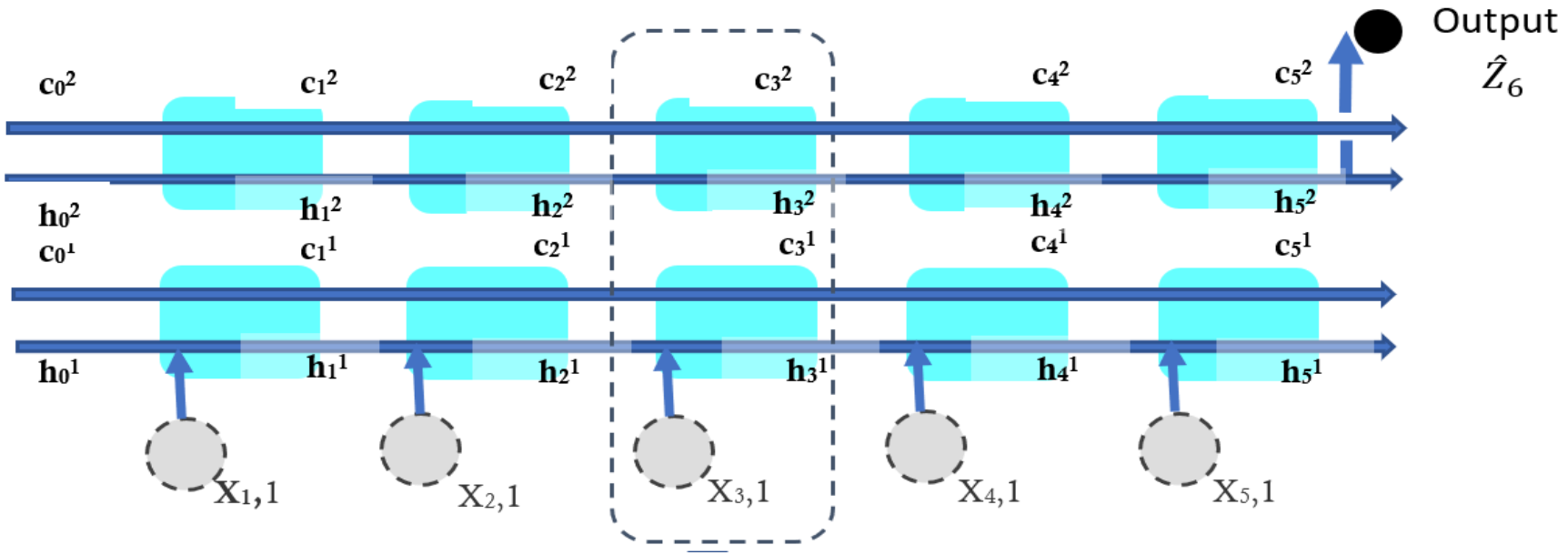
$$\mathbf{c_t} = \sigma\,(W_{FX}\,X_t + W_{Fh}\,\mathbf{h_{t-1}} + b_F\,) \otimes \mathbf{c_{t-1}}$$
$$+ \sigma\,(W_{IX}\,X_t + W_{Ih}\,\mathbf{h_{t-1}} + b_I\,) \otimes \tanh\,(W_{CX}\,X_t + W_{Ch}\,\mathbf{h_{t-1}} + b_C\,)$$

- In Output Gate (3), $\mathbf{h_t} = \sigma\,(W_{OX}\,X_t + W_{Oh}\,\mathbf{h_{t-1}} + b_O\,) \otimes \tanh\,(\mathbf{c_t})$

Variants of RNN

Stacked Long Short Term Memory NN

However, single hidden layer (even with a large number of neurons per layer or width) and many epochs may not be able to perform good training results. A deeper version is the stacked LSTM model with more hidden layers (that are stacked up at an input time-sequence).



Variants of RNN

Stacked Long Short Term Memory NN

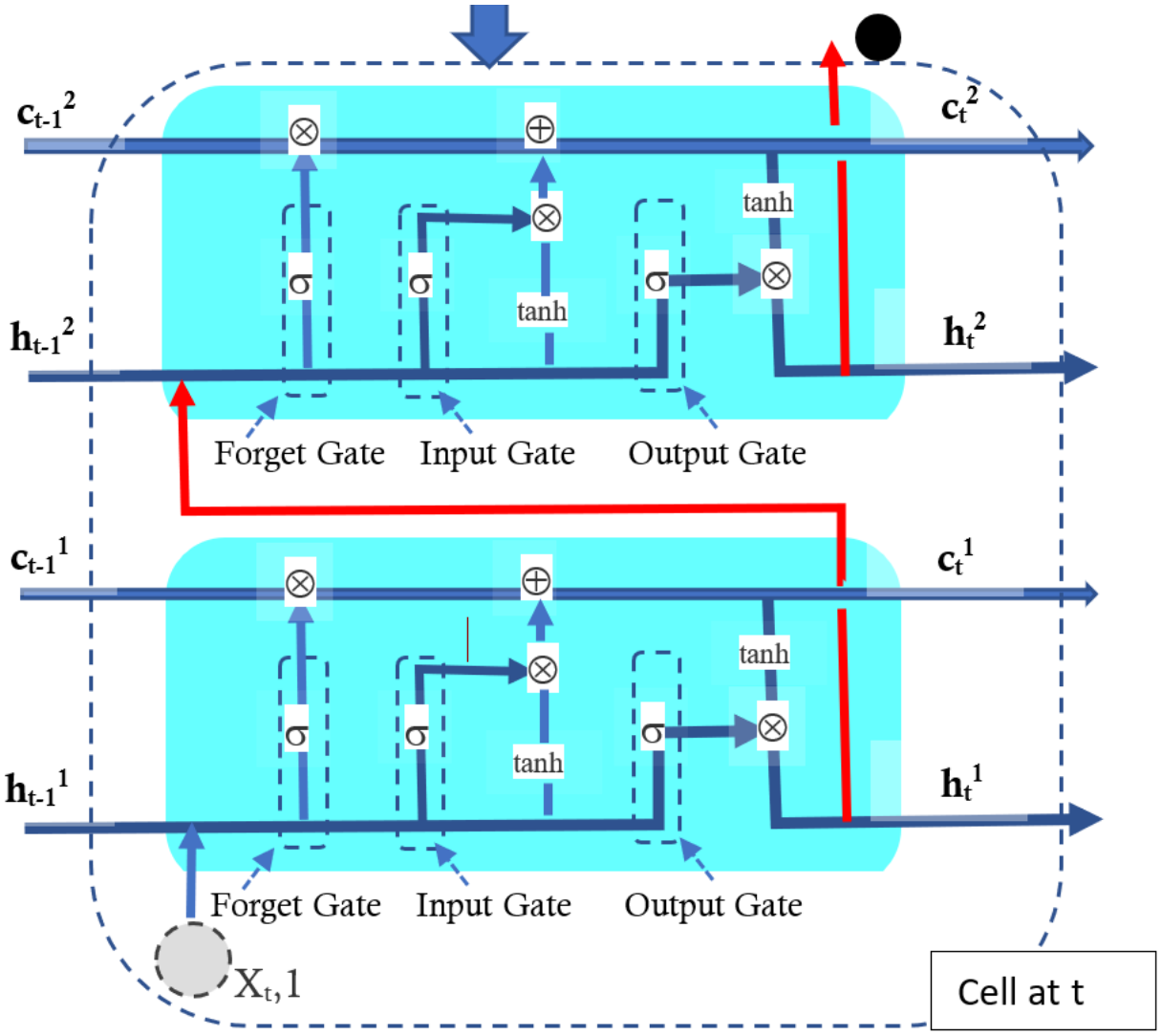
The Output Gate from first level cell provides 50 hidden states at level 1, viz.

$$\mathbf{h}_t^1 = \sigma\,(W_{OX}^1 X_t + W_{Oh}^1 \mathbf{h}_{t-1} + b_O^1) \otimes \tanh(\mathbf{c}_t^1).$$

Forget Gate output fraction vector
$\sigma(W_{FX}^2 \mathbf{h}_t^1 + W_{Fh}^2 \mathbf{h}_{t-1}^2 + b_F^2)$

The same operations as in level 1 occur at the Input Gate, the candidate for cell update, and the Output Gate. Hence in total there are $4 \times [(N + N) \times N + N]$ parameters at the second level LSTM cell/stacked layer.

$$\mathbf{h}_t^2 = \sigma\,(W_{OX}^2 \mathbf{h}_t^1 + W_{Oh}^2 \mathbf{h}_{t-1}^2 + b_O^2) \otimes \tanh(\mathbf{c}_t^2)$$



Worked Example II -- Data

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

In this second worked example, we show how a LSTM RNN can be used to predict Google stock prices. Historical data are collected from Yahoo Finance consisting of daily (distribution and split) adjusted closing Google stock prices from 3 Jan 2012 till 29 Dec 2016, with a total of 1257 sample points.

```
In [2]: ### import training set
        dataset=pd.read_csv('GOOG.csv')
        dataset.head()
```

Out[2]:

	Date	Open	High	Low	Close	Adj Close	Volume
0	3/1/2012	16.262545	16.641375	16.248346	16.573130	16.573130	147611217
1	4/1/2012	16.563665	16.693678	16.453827	16.644611	16.644611	114989399
2	5/1/2012	16.491436	16.537264	16.344486	16.413727	16.413727	131808205
3	6/1/2012	16.417213	16.438385	16.184088	16.189817	16.189817	108119746
4	9/1/2012	16.102144	16.114599	15.472754	15.503389	15.503389	233776981

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

The data structure is created to feed the inputs to the NN. T = 1100 for the training data set. The first 60 data points are used for training data X or X_train while the next data point (point 61) is used as training data Y or Y_train. The concatenated X_train is reshaped into 1040 rows (overlapping cases) each with 60 columns (second dimension size) of timed inputs each.

In [7]:
```
### creating data structure with 60 time-steps and 1 output
X_train=[]
y_train=[]
for i in range(60,1100):
    X_train.append(training_set_scaled[i-60:i, 0])
    y_train.append(training_set_scaled[i, 0])
X_train, y_train = np.array(X_train), np.array(y_train)
print(X_train.shape, y_train.shape)
X_train=np.reshape(X_train, (X_train.shape[0], X_train.shape[1],1))
### this step converts X_train to 3D from (1040,60) to (1040,60,1) for input to the keras app
```

```
(1040, 60) (1040,)
```

Stacked LSTM NN is used with four stacked LSTM cells at each timed input for each case of 60 inputs followed by prediction of the output at the end of 60 time-sequenced inputs. There are 1040 cases as we use overlapping cases here. Each stacked LSTM cell contains 50 neurons. The output layer specified one neuron, so there is only one predicted scalar output.

Stacked LSTM

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

In [9]:
```
### Initializing RNN
model = Sequential()
```

In [10]:
```
### Add first LSTM layer and add Dropout Reegularization
model.add(LSTM(units=50,return_sequences=True,input_shape=(X_train.shape[1],1))) ### Sequential reads input as 3D
model.add(Dropout(0.2))
```

In [11]:
```
### Add second LSTM layer and Dropout
model.add(LSTM(units=50,return_sequences=True))
model.add(Dropout(0.2))
```

In [12]:
```
### Add third LSTM layer and Dropout
model.add(LSTM(units=50,return_sequences=True))
model.add(Dropout(0.2))
```

In [13]:
```
### Add fourth LSTM layer and Dropout
model.add(LSTM(units=50)) ### note: last LSTM layer does not carry argument 'return_sequences=True'
model.add(Dropout(0.2))
```

In [14]:
```
### Add output layer
model.add(Dense(units=1))  ### not capital "U"nit
```

In [15]:
```
### Compiling the RNN
model.compile(optimizer='adam',loss='mean_squared_error')
```

Fitting and Prediction

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

In [16]:
```
### Run the training set with the LSTM (specialized RNN here)
model.fit(X_train,y_train,epochs=100,batch_size=10)
```

In [18]:
```
predict_train=model.predict(X_train)
print(predict_train.shape)
### this output is (1040,60,1), we want only the first no. in each row of 1040
### this 3D structure makes it more difficult to interpret comparison of prediction in training set vs output in trg set
```

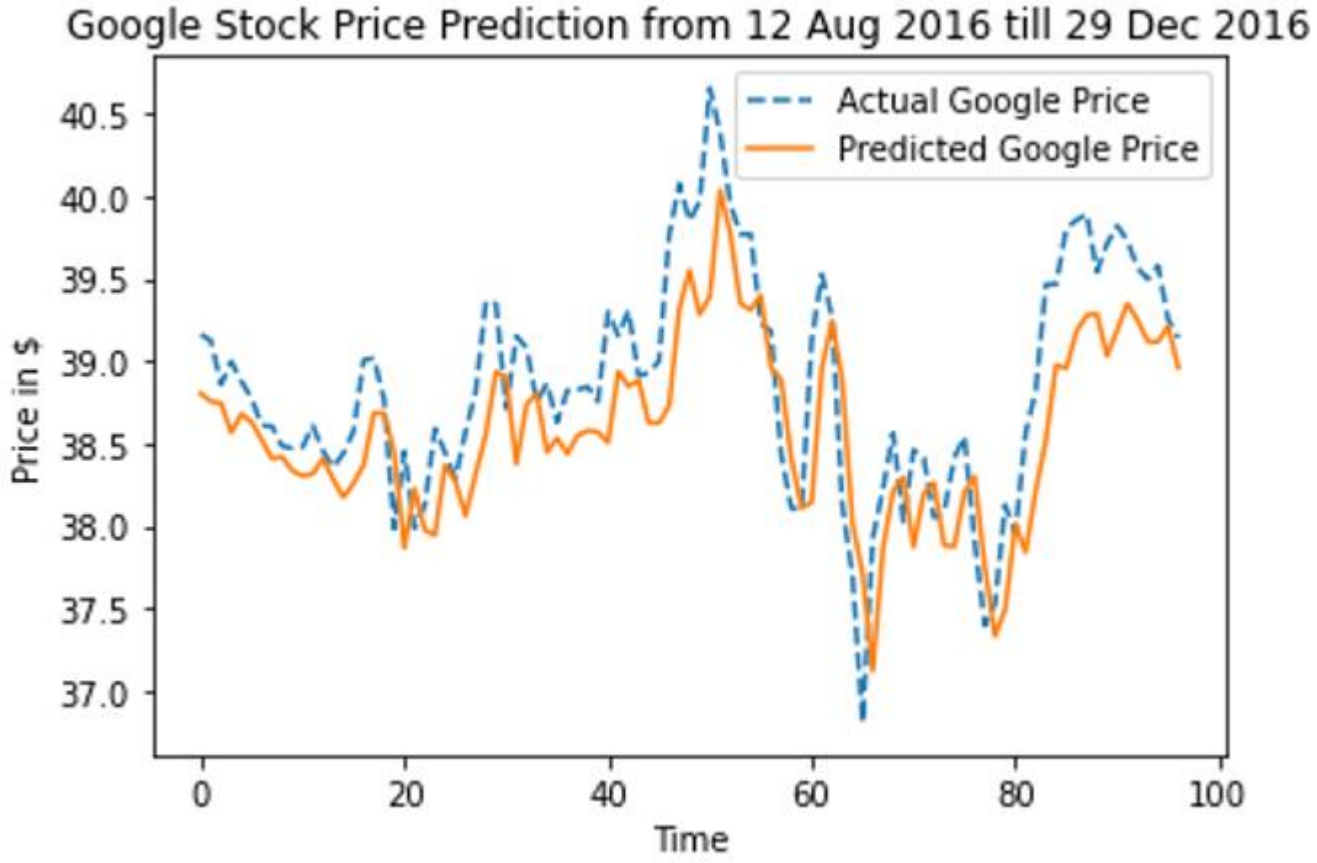
In [25]:
```
### Prediction
predicted_stock_price=model.predict(X_test)
predicted_stock_price=predicted_stock_price[:,0]
#predicted_stock_price1=sc.inverse_transform(predicted_stock_price)
#predicted_stock_price.shape
```

In [27]:
```
from sklearn.metrics import mean_squared_error
import math
def print_error(trainY, testY, train_predict, test_predict):
    ### Error of predictions
    train_rmse = math.sqrt(mean_squared_error(trainY, train_predict))
    test_rmse = math.sqrt(mean_squared_error(testY, test_predict))
    ### Print RMSE
    print('Train RMSE: %.3f RMSE' % (train_rmse))
    print('Test RMSE: %.3f RMSE' % (test_rmse))

print_error(y_train, y_test, predict_train, predicted_stock_price)
```

```
Train RMSE: 0.020 RMSE
Test RMSE: 0.019 RMSE
```

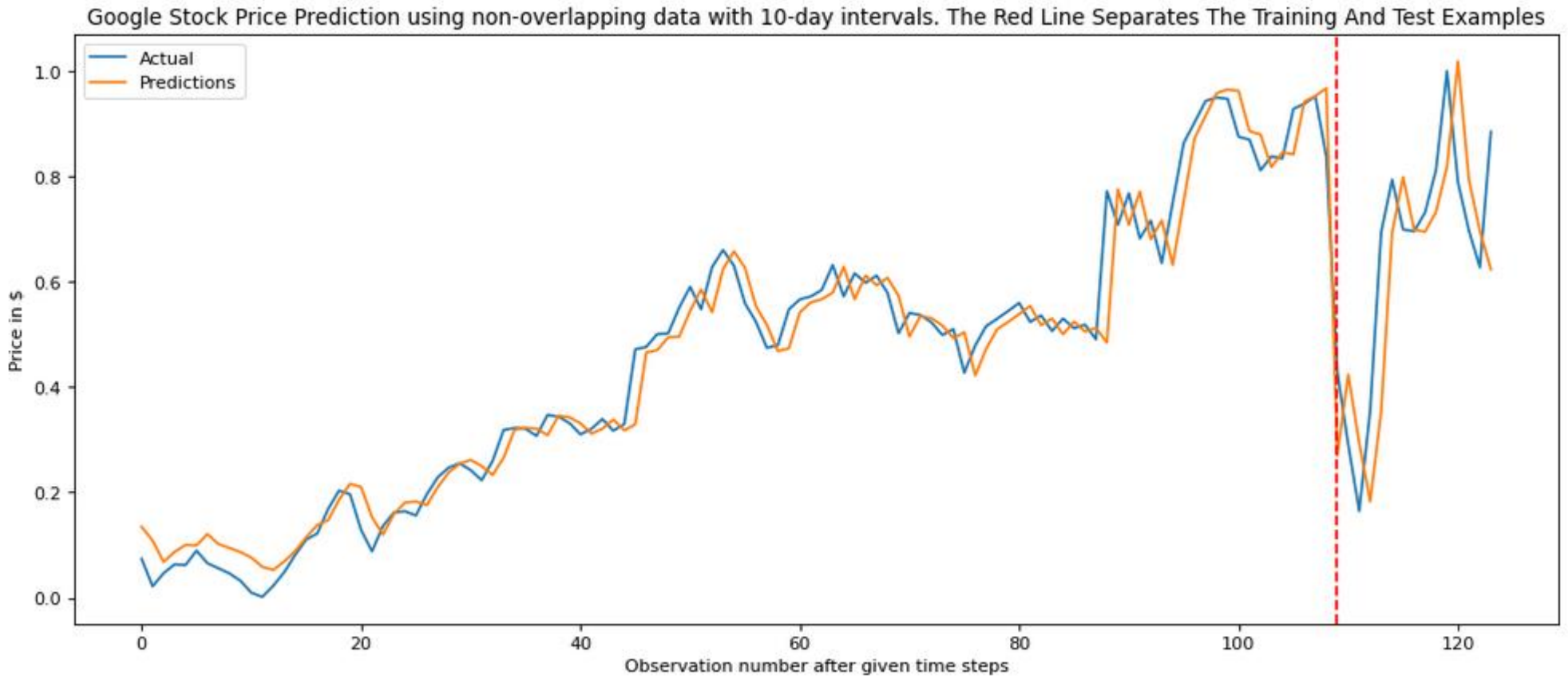
Overlapping Cases

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook



Non-overlapping Cases

Please upload Chapter8-3.ipynb and follow the computing steps in Jupyter Notebook



CAVEATS

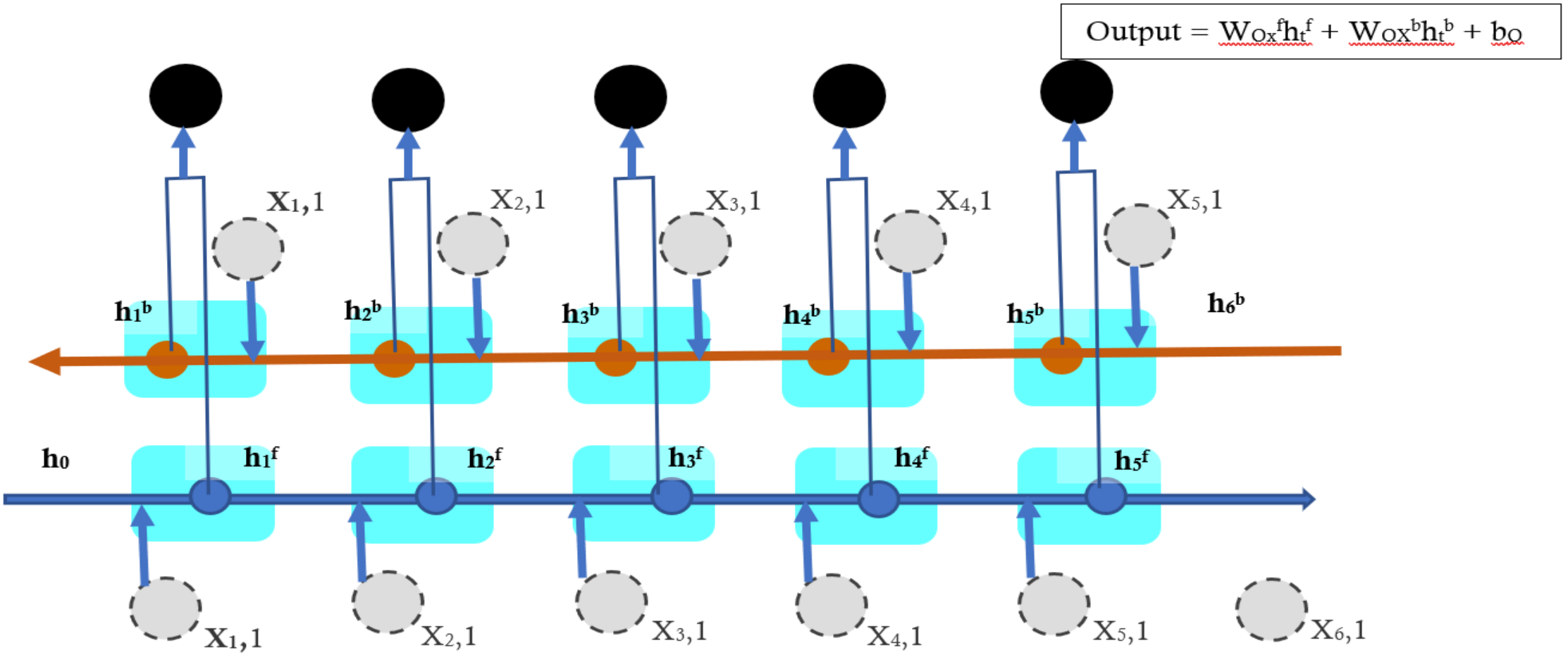
- There are good predictions (1) and there are good predictions (2). Price chasing (1) using just a time series trained on past prices could be a good machine predictor but may be a bad idea for trading. In the latter, an alternative is to enable better trading if based on momentum trading – AR process – faster computing time than a complex NN, the few microseconds could mean gain or loss.
- Big caveats when one proceeds to transport a predictive machine such as the Google prediction to something economic/downright market finance such as trading – can you make a steady/consistent profit?
  - transaction fees/costs
  - impact costs/widening bid-ask spread if more want to buy (ask goes up), if more want to sell (bid goes down)
  - Liquidity risk – when it is not possible to trade as frequently as in the training/testing using past data
  - slippages in market order – getting higher purchase price or lower selling price for your order (effect same as in execution risk when latency - time delay between order and actual trade - is slower than several microseconds, 1/1,000,000 second)

CAVEATS

- A trend following prediction (that happens often in minimizing MSE) not considering above costs may do well in long runs of up or long runs of down – but will lose heavily in sharp market turnaround

- More meaningful ML models for trading (2) could include training based not just on past prices, but using market microstructure features in high frequency trading (HFT) such as queue order level 2 quotes (market depth) or even level 3 (who are the buyers/sellers in the queue), the momentum – e.g. average price movements in the last few intervals, bid-ask spread, volume, a market-sentiment measure, firm size, P/E, P/B ratios, profit news, competitor prices, industry price, market movements, interest rates, etc.

Bidirectional RNN/LSTM



Gated Recurrent Unit (GRU)

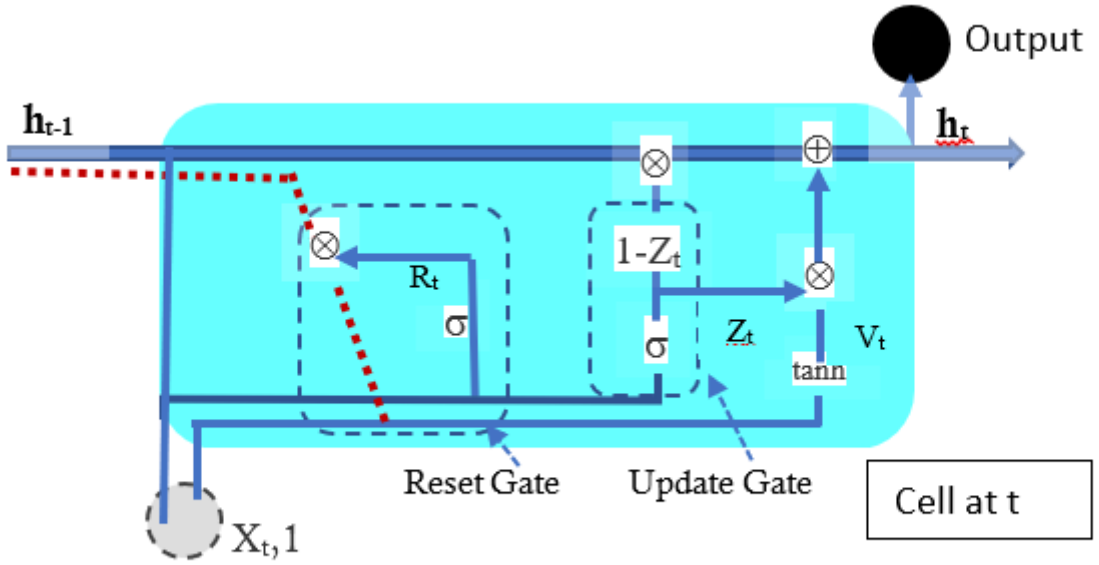


Figure 8.10

In the Reset Gate, the output R_t is given by $R_t = \sigma (W_{RX} X_t + W_R \mathbf{h_{t-1}} + b_R)$. This R_t is multiplied with $\mathbf{h_{t-1}}$ in a Hadamard product (see dotted line) and then activated using tanh function to create a candidate activation vector, $\mathbf{V_t}$. R_t is between 0 and 1 and could be close to zero, implying small V_t , where $V_t = \tanh (W_{VX} X_t + W_V (R_t \otimes \mathbf{h_{t-1}}) + b_V)$.

The Reset Gate decides how much of the past information in $\mathbf{h_{t-1}}$ is to be neglected or if the hidden state is important or not.

In the Update Gate, the output Z_t is given by $Z_t = \sigma (W_{UX} X_t + W_Z \mathbf{h_{t-1}} + b_Z)$. The output hidden state $\mathbf{h_t} = Z_t \otimes V_t + (1 - Z_t) \otimes \mathbf{h_{t-1}}$. The Update Gate determines a weighted average of V_t and past $\mathbf{h_{t-1}}$.

In-Class Practice Exercise (not graded):

Chapter8-4.ipynb

Use the VIX price data for prediction – VIX.csv

Construct a stacked LSTM NN with 50 neurons in each stacked layer, and 4 layers, plus an output layer. Use batch size = 10 and # epochs = 100. Here time-steps = 60. This is an input in the input_shape=(X_train.shape[1],1)) of Keras LSTM App.

Use Adam optimizer, and mean-squared-error loss.

Use Overlapping training cases.

Find the training and the test prediction RMSEs.

End of Class