Report of the Presentation in Week 7

Application of the Yoneda Lemma - Isomorphism of representables

Presenter: Xiang Li

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Attendants

Jane, Joanne, James, Kie Seng, An Ran, Xiang

Topics covered in the Presentation

- Proof that the Yoneda embedding is full and faithful.
- Explanation of the properties of full and faithful functor A -> B:
  As a consequence A is equivalent to a full subcategory of B. (This is why a full and faithful functor is an "embedding", which usually denotes a isomorphism to the image)
- Proof of Lemma 4.3.8: For a full and faithful functor A->B one can unanimously speak of isomorphism of objects in A or in the full subcategory of B
- Corollary 4.3.10 : For objects A and B of a locally small category:

$$H_A \; is \; isomorphic \; to \; H_B <=> A \; is \; isomorphic \; to \; B$$

- Examples of this Corollary (4.3.11-13)
  - In the Category of Groups: Looking at one specific group from 1, $\mathbb{Z}$ or $\mathbb{Z}/p\mathbb{Z}$ gives only partial information about this group (no information, information about the underlying set or the number of elements of order p).
  - In the Category of Sets: It is sufficient to merely look from a one-element set.
  - Adjoints are unique up to isomorphism.

Since I came very late due to my fault I could not finish the examples before some had to attend their classes. I did not facilitate a discussion, either.

Evaluation

During my presentation I forgot that the whiteboard gives structure to the presentation and is vital for knowing what the topic is and what the formulas mean. I believe that I should always have written down the statements of the propositions instead of sometimes simply talking about them and only writing down the "calculations". This could facilitate the understanding of what the calculations and formulas are for. For example I forgot to write down that I was proving the uniqueness of adjoints or that when I wrote down the functor my purpose was

recalling the Yoneda embedding. For me it is often difficult to follow spoken words in mathematical lecture. In my first mathematical presentation I learned to appreciate lecturers' ability to make board work understandable own its own. When An Ran said that the board work was a little messy, I knew that it was not only a little. Also I received the comment that everything was explained well.
Even if situations of little time I should focus more on the structure of the whiteboard rather than talking about rather trivial conclusions.