

Category theory meeting 3

Yahya Fidouh

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Attendance:

Yahya, An Ran, Zak, Xiang, James, Yao, Joanne, Kie Sang

Topics covered:

- The two definitions of adjoint functors (in terms of Hom sets, and then in terms of units and counits)
- A few examples of adjoint functors:
 - The free and forgetful functors between \mathbf{Vect}_k and \mathbf{Set} .
 - The free and forgetful functors between \mathbf{Grp} and \mathbf{Set} .
 - The free and forgetful functors between \mathbf{Grp} and \mathbf{Mon} , and the forgetful and R functors.

With the last example, the forgetful functor has a right adjoint R , which corresponds to pulling out the submonoid that contains all of the invertible elements of a monoid and sending it to a group. We all had trouble understanding (or proving) why that is the case.

- Composition of adjoint functors.
- The triangle identities (lemma 2.2.2)
- Theorem 2.2.5, and corollary 2.2.6

I did not get a chance to touch on the string diagrams.

Discussion:

I didn't manage to finish the presentation before the end of the hour, however, we did get a chance to discuss some of the points during the presentation. Most of the discussion revolved around the definition of Adjoint functors, and what the natural transformations meant, Zak suggested they were equivalence relations (I think) which James later on countered by saying that from our examples, some of the categories are not equivalent, and that the natural transformations that satisfy the equivalence of categories don't satisfy the triangle identity. We also struggled to show why R is right adjoint to U in the last example. Everyone seemed to like the corollary 2.2.6. And then we ran out of time, and before everyone left, I asked for some feedback.

Feedback:

- An Ran liked the titles.
- James said I had the hardest job (which most likely means I did the worst job so far).
- An Ran said I was clear, and concise but it would have been nicer if I gave an outline of the class before starting the presentation.
- Zak suggested I reminded everyone of key concepts or definitions if I need them to explain something.

Reflection:

I felt a little bit uneasy as it was my first time presenting mathematics in front of a class, and I was not confident with the material being presented. I feel like I didn't deliver the definition of adjoint functors in a clear way, so lots of improvement can happen there. I realised that writing out definitions takes longer than I thought, so I think just writing bullet points or quick diagrams would be the best way to go.