Category Theory Meeting Report Week 4

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March 16, 2017

Attendance

Joanne, Kie Seng, Yao, An Ran, James, Jane, Elke, Yahya, Zac.

Topics covered

We covered exactly the first suggestion in your email:

- We defined comma and slice categories
- We showed how slice categories are special cases of comma categories.
- We defined coslice categories $A/\mathscr{A}$ and briefly discussed the equivalence $A/\mathscr{A} \cong (A \implies 1_{\mathscr{A}})$.
- We developed the comma category $(A \implies G)$ where $A \in \mathscr{A}$ is considered as a functor $1 \to \mathscr{A}$.
- We stated lemma 2.3.5 and theorem 2.3.6 (the correspondence between adjunctions and initial objects in $(A \implies G)$. However, we did not touch on their proofs. (During this time we had a quick revision of the unit-counit definition of adjunctions.)
- We discussed what it means for an object to be initial in $(A \implies G)$ and how this is similar to a universal property.
- We returned to the example about the forgetful and free functors U, F between Set and Vect being adjoints. We discussed the universal property of $F(S)$ and how this was equivalent to the initial object of $(F(S) \implies U)$.

Discussion

There wasn't much time for discussion. I presented for the whole hour. There was only some brief discussions when someone asked a question. Most people seemed to understand what was going on though, so in that sense, we didn't need discussion time.

Feedback

I got the following feedback from other people: I should explain out loud concepts after I had written them down, instead of before, as some people learn better by reading mathematics. I should try not to cram the board and instead rub things out sooner.

Reflection

I tried to give an overview of what we were doing and why we were doing it at the start and the end. I tried to motivate this new definition of adjunctions by emphasising the link between initial objects and universal properties. And then theorem 2.3.6 gives us a link between adjunctions and initial objects. This is really great, since now we have a link between adjunctions and universal properties, and we really like universal properties, since they are easy to use. But I don't think everyone understood this. (Also I'm not actually sure how correct this is!)

I thought I presented too many definitions and too few examples. I don't understand why we introduced slice categories. I assume they will come up later but so far I've yet to see their use.

I couldn't understand remark 2.3.2 (and so didn't presented it). It would have made a good discussion topic if we had time. Perhaps we could talk about this next week.