

Category Theory Reading Course Meeting 1

An Ran Chen

1 Present

Yahya, Kie Seng, An Ran, Jane, James, Zach, Yao, Elke, Joan

2 Topics Covered

- definition of category
- examples of category: **Grp**, **Set**, **Ring**, discrete category
- definition of functor
- examples of functor: forgetful functor, **Grp** \rightarrow **Set** which forgets the group structure, **Ring** \rightarrow **Ab** which forgets that rings have multiplication
- definition of natural transformation
- James provided an example of natural transformation π :

$$\begin{array}{ccc} & Id & \\ \text{Grp} & \begin{array}{c} \Downarrow \pi \\ \Downarrow \end{array} & \text{Grp} \\ & Ab & \end{array}$$

where $Id : \mathbf{Grp} \rightarrow \mathbf{Grp}$ is the identity functor. and $Ab : \mathbf{Grp} \rightarrow \mathbf{Grp}$ is a functor that sends a group to its abelianization, and $\pi_G : G \rightarrow G^{ab}$ is the projection map that takes G into $G/[G, G]$

- defining vertical composition of natural transformation
- tried defining horizontal composition but ran out of time

3 Discussion

- Given two functors, is there always a natural transformation? what sorts of conditions are necessary? We tried thinking about functors from the discrete category of 5 object to the discrete category of 5 objects to try and get some ideas
- Leiston Exercise 1.2.24, is there a functor $Z : \mathbf{Grp} \rightarrow \mathbf{Grp}$ with the property that $Z(G)$ is the centre of G . Feels somewhat similar to James' example, but I think such functors do not exist, not sure about the proof
- Yahya mentioned that it is dangerous to think of objects as sets and morphisms as functions and we tried to think of some examples of this

4 Reflection

I received feedback saying that I should not be scared of silences and try to speak in full sentence if possible. It would be better if I slowed down a bit and structure my thought process before speaking.

Knowing that I have terrible boardwork, I tried to split the board into 3 sections, write down what I said in order and keep key definitions on the board. But I still ended up jumping around a bit, especially when someone asks a question. I could have also been more structured at defining notations/concept. So that I don't start defining X , realizing I need to have already defined Y but forgot to do so.