Category Theory Reading Course Week 2

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2.1 Attendance

Students who were present: Jane, An Ran, James, Yahya, Nge Kie Seng, Joanna, Likun Yao, Yao Fan, Jianing (Elke) Zhao, and Zac.

2.2 Material

- Faithful and Full Functors - Definition and the distinction between injective and faithful functors with a simple example to illustrate this point.

- Subcategories and Full Subcategories - Included into talk due to questions that arose from discussion. Definition and discussion over if the image of a functor is a subcategory (it is not).

- Essentially Surjective on Objects - Definition and explanation.

- Equivalence of categories - Mostly redefined so that everyone had it fresh in mind, explained its use over isomorphism of categories in analogy to using isomorphism over equality on objects.

- Statement of Proposition - A functor is an equivalence if and only if it is full, faithful, and essentially surjective on objects.

- Initial and Terminal Objects - Defined, an example on pointed sets.

2.3 Discussion

- The first point of discussion was whether a faithful functor need be injective, there was discussion on this and on the counterexample as to why it was true. We collectively constructed one from a category with two objects and one arrow that was sent to a category with only one object.

- Following from this it was asked if the image of a functor is itself a category, this lead to an impromptu addition of talking about subcategories and full subcategories and an attempt to find a functor that did not have a subcategory for its image.

- During the discussion phase of the essentially surjective on objects we attempted to find an example of an essentially surjective map, Yahya provided an example of a category with one object being set to a category with two isomorphic objects.

- The main point of confusion that resulted from the discussion was if a category that had two objects and one map between them was equivalent to a category that had just one object. We had determined that the functor from the first category to the second category was faithful and essentially surjective but accidentally determined that it was full. So there was some confusion on how to construct the equivalence between the two categories. In the problem solving session the following day James pointed out that we had misidentified the functor as being full and showed how it was not full.

2.4 Reflection

This was my first time presenting mathematics in front of a group so I was very uncertain if what I was saying actually made sense or if what I was saying was too straightforward and boring. So throughout my presentation of the material after I would deliver a definition/theorem I would ask if that made sense, or if there were any questions or if anyone had an example. This accidentally resulted in the talk and discussion alternating, so there wasn't a distinct period of talking and period of discussion.

I planned several examples but I didn't write the details of the examples down on my notes, so a couple of the times I would present the example and state "this is an example of this property" and then promptly forget why it was true. We ended up working through these examples collectively to see why they were in fact true.

For some of the content I moved a bit fast, for example when we were discussing the lemma on functors being equivalent I had to slow down and double back because I realised I hadn't sufficiently explained what it meant for categories to be equivalent.