catlvm : CATegorical Latent Variable Model

Upward-Downward Algorithm

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1 Hidden Markov tree

2 Hierarchical latent class model

$$\pi_k = \Pr\left(C_1 = k\right), \quad \tau_{k|l}^{(u)} = \Pr\left(C_u = k | C_{\rho(u)} = l\right), \text{ and } \rho_{mr|k}^{(u)} = \Pr\left(Y_{m(u)} = r | C_u = k\right)$$

Upward recursion

$$\begin{aligned}
\xi_{i(u)}(k) &= \Pr\left(\mathbf{Y}_{i(u)} = \mathbf{y}_{i(u)} | C_u = k\right) = \prod_{m=1}^{M} \rho_{my_m|k}^{(u)} \\
\lambda_{i(v)}(k) &= \xi_{i(v)}(k) \\
\lambda_{i(\rho(u),u)}(l) &= \sum_k \tau_{k,l}^{(u)} \lambda_{i(u)}(k) \\
\lambda_{i(u)}(k) &= \left\{ \prod_{v \in \mathbf{c}(u)} \lambda_{i(u,v)}(k) \right\} \xi_{i(u)}(k)
\end{aligned}$$

Downward recursion

$$\alpha_{i(u)}(k) = \sum_l \frac{\tau_{k|l}^{(u)} \alpha_{i(\rho(u))}(l) \lambda_{i(\rho(u))}(l)}{\lambda_{i(\rho(u),u)}(l)}$$

Posterior probabilities

$$\begin{aligned}
\theta_{i(u)}(k) &= \Pr\left(C_u = k | \bar{\mathbf{Y}}_i = \bar{\mathbf{y}}_i\right) = \frac{\Pr\left(C_u = k, \bar{\mathbf{Y}}_i = \bar{\mathbf{y}}_i\right)}{\Pr\left(\bar{\mathbf{Y}}_i = \bar{\mathbf{y}}_i\right)} \\
&= \frac{\Pr\left(C_u = k, \bar{\mathbf{Y}}_{i(1\backslash u)} = \bar{\mathbf{y}}_{i(1\backslash u)}\right) \times \Pr\left(\bar{\mathbf{Y}}_{i(u)} = \bar{\mathbf{y}}_{i(u)} | C_u = k\right)}{\Pr\left(\bar{\mathbf{Y}}_i = \bar{\mathbf{y}}_i\right)} \\
&= \frac{\alpha_{i(u)}(k) \lambda_{i(u)}(k)}{\sum_j \alpha_{i(v)}(j) \lambda_{i(v)}(j)}
\end{aligned}$$

$$\begin{aligned}
\theta_{i(\rho(u),u)}(k,l) &= \Pr\left(C_u = k, C_{\rho(u)} = l | \bar{\mathbf{Y}}_i = \bar{\mathbf{y}}_i\right) \\
&= \frac{\lambda_{i(u)}(k) \tau_{k|l}^{(u)} \alpha_{i(\rho(u))}(j) \lambda_{i(\rho(u))}(j)}{\lambda_{i(u,\rho(u))}(j) \sum_j \alpha_{i(v)}(j) \lambda_{i(v)}(j)}
\end{aligned}$$