Statapult

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Designs

There were two possible designs for the data of this form: a 2_V^{5-1} design, and a 2_{III}^{5-2} design.

The Resolution III Design

The 2_{III}^{5-2} design is a quarter-fraction design which uses five factors over 8 runs. This design uses the constraints $D \equiv AB$ and $E \equiv AC$. Its defining relation is $I \equiv ABD \equiv ACE \equiv BCDE$. We will refer to this resolution III design as design 1, notated as d_1.

The Resolution V Design

The 2_{V}^{5-1} design is a half-fraction design which uses five factors over 16 runs. This design uses the constraint $E \equiv ABCD$. Its defining relation is $I \equiv ABCDE$. We will refer to this resolution V design as design 2, notated as d_2.

Minimum Aberration Model Robustness

Minimum aberration is a model robustness metric used to select a design shape for a dataset. It uses the wordlength vectors from two or more designs to determine the model with the minimum aberration. Less aberration is desirable.

The wordlength for Design 1 is $W = (0, 1, 0)$, whereas the wordlength for Design 2 is $W = (2, 1, 0)$, based on the defining relation for each of the designs.

Given the d_1 design of 2^{5-2} (quarter-fraction) and the d_2 design of 2^{5-1} (half-fraction), let r be the smallest integer such that $A_r(d_1) \neq A_r(d_2)$, where $A_r(\cdot)$ is one element of the design's wordlength pattern.

Design d_1 has less aberration than design d_2, since $A_r(d_1) = 0 < 2 = A_r(d_2)$. Therefore, d_1 is desired over d_2 due to its lower aberration.

Clear and Strong Clear Effects for Model Robustness

Determining model clear and strong clear factors can also be used to determine model robustness. It uses the number of clear effects and strong clear effects from two or more designs to determine the model with the most strong clear, then the most clear effects. More strong clear effects are desirable. If all models have the same number of strong clear effects or no models have any strong clear effects, then more clear effects are desirable.

We define a main effect or two-factor interaction as clear if none of its aliases are main effects or two-factor interactions, and we define it as strong clear if none of its aliases are main effects, two-factor interactions, or three-factor interactions.

To determine strong clear and clear effects, we consider the alias structure. Design d_1 has the defining relation $I \equiv ABD \equiv ACE \equiv BCDE$. We get the following aliases with two-factor interactions.

$A \equiv BD \equiv CE (\equiv BCDE)$

$B \equiv AD (\equiv ACE \equiv CDE)$

$C \equiv AE (\equiv ABCD \equiv BDE)$

$D \equiv AB (\equiv ACDE \equiv BCE)$

$E \equiv AC (\equiv ABDE \equiv BCD)$

Observe how all main effects have an alias with a two-factor interaction. Therefore, none of the effects are strong clear.

Design d_2 has the defining relation $I \equiv ABCDE$. For each main effect, there are no aliases with two- or three-factor interactions. Therefore, each main effect is considered strong clear.

Therefore, design d_2 is more desirable due to its high number of strong clear factors.

Fries and Hunter (1980) established that not all 2^{k-p} designs of maximum resolution are equally desirable, and they introduced the minimum aberration criterion for further discriminating designs of the same resolution.

Data

We began by collecting our data. The data were collected on April 7, 2022 using a "statapult." Out of six possible factors, we chose to analyze five of them, leaving the last factor (POST) at the low level for all runs. The factors are as follows.

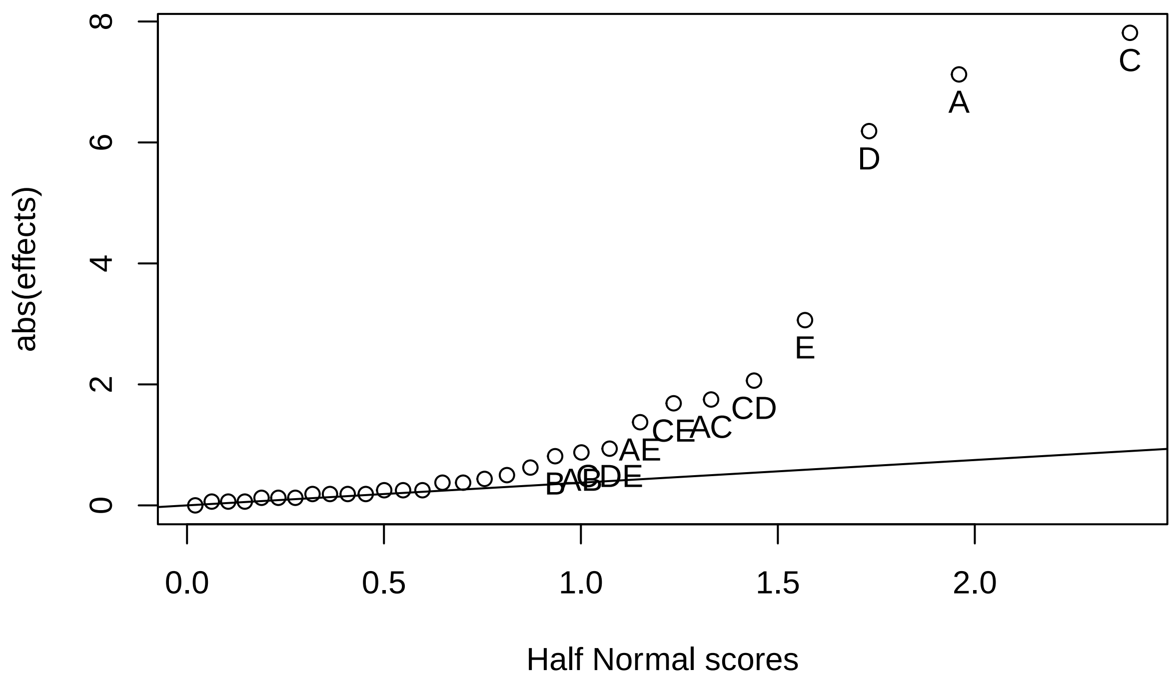
- `arm` - rubber band attachment point on the arm: 1, 3
- `ball` - ball type: ping-pong ball (low), wiffle ball (high)
- `position` - ball position: 1, 3
- `height` - arm draw back height: 1, 3
- `tilt` - catapult tilt: 1, 3

The last factor (`post`) was the rubber band attachment point on the post. It was left at the high level (1) for all runs.

Analysis

We started by analyzing the full 2^5 fractional design. To determine significant effects, we used a half-normal plot on the full dataset with no replicates.

```
myout = lm(distance ~ A*B*C*D*E, data = statapult)
effects = myout$coefficients[2:31]
halfnorm(effects, labs=names(effects), alpha = 0.10, refline = "TRUE")
```




```
## zscore= 0.02089009 0.06270678 0.1046335 0.146745 0.1891184 0.2318344
   0.2749777 0.3186394 0.3629173 0.4079187 0.4537622 0.5005801 0.5485223
   0.5977601 0.6484922 0.7009514 0.755415 0.8122178 0.871771 0.9345893
   1.001331 1.072861 1.150349 1.23544 1.330562 1.439531 1.56892 1.731664
   1.959964 2.39398effp= 1.251781e-15 0.0625 0.0625 0.0625 0.125 0.125
   0.125 0.1875 0.1875 0.1875 0.1875 0.25 0.25 0.25 0.375 0.375 0.4375 0.5
    0.625 0.8125 0.875 0.9375 1.375 1.6875 1.75 2.0625 3.0625 6.1875 7.125
    7.8125
```

Discussion

Conclusion