Selected Problems in CLRS

Section 7

Quicksort is similar to mergesort but has many different interesting features.

Notifications

Problem Difficulty (count with star)

1. you can solve w/o the brain
2. you can solve if you think a bit
3. you can solve if you think carefully
4. you might solve if you push yourself
5. you can solve if you use other's brain

Exercise

7.1-1 $\star\star$

Using Figure 7.1 as a model, illustrate the operation of PARTITION on the arry $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 \rangle$.

7.1-3 $\star\star$

Give a brief argument that the running time of PARTITION on a subarray of size n is $\Theta(n)$

7.2-2 $\star\star$

What is the running time of QUICKSORT when all elements of array A have the same value?

7.2-3 $\star\star$

What is the running time of QUICKSORT when all elements of array A have the same value?

7.2-6 $\star\star\star$

Argue that for any constant $0 < \alpha \leq 1/2$, the probability is approximately $1 - 2\alpha$ that on a random input array, PARTITION produces a split more balanced than $1 - \alpha$ to α

7.4-1 $\star\star\star$

Show that in the recurrence

$$T(n) = \max_{0 \leq q \leq n-1} (T(q) + T(n - q - 1)) + \Theta(n)$$

7.4-4 $\star\star\star$

Show that the RANDOMIZED-QUICKSORT's expected running time is $\Omega(n \lg n)$

7.4-6 $\star\star\star\star$

Consider modifying the PARTITION procedure by randomly picking three elements form array A and partitioning about their median (the middle value of the three elements). Approximate the probabilitu of getting at worst an α-to-$(1 - \alpha)$ split, as a function of α in the rans $0 < \alpha < 1$

Problems

7.1 Hoare partition correctness $\star\star\star$

Note : sample implementation code of quick sort using this partition method

```
HOARE-PARTITION(A, p, r)
x = A[p]
i = p - 1
j = r + 1
while TRUE
  repeat
    j = j - 1
  until A[j] <= x
  repeat
    i = i + 1
  until A[i] >= x
  if i < j
    exchange A[i] with A[j]
  else return j
```

a. Demonstrate the operation of HOARE-PARTITION on the arry $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21 \rangle$, showing the values of the array and auxiliary valuees after each iteration of the **while** loop in lines 5-14

b. Prove it, the indices i and j are such that we never access an element of A outside the subarrya $A[p..r]$

c. Prove it, when HOARE-PARTITION terminates, it returns a value j such aht $p \leq j < r$

d. Prove it, every element of $A[p..j]$ is less than or equal to every element of $A[j+1..r]$ when HOARE-PARTITION terminates.

Note : there are very good problems in this section, I recommand that to solve every problems in this section. It will be helpful.