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Stereoscopic Operators and Their Application

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Direct and inverse mathematical operators of stereo transformation (stereo operators) are studied in this paper. The stereo operators install a one-to-one correspondence between three dimensional coordinates of any point in space and the stereo coordinates which can be displayed on the screen under the given conditions, i.e. stereo vision base and the position of viewer. The stereo operators can be applied to the analyses of stereoscopic image distortions when the stereo vision base and the position of viewer are changed.

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I. INTRODUCTION

In recent years, the interest in stereoscopic images for the purposes of simulations, virtual reality, and entertainment has increased rapidly. The stereoscopic images are displayed to our eyes as a pair of photographs, which can be fused into a scene with its own depth. The pair of photographs should correspond to the pair of images which can be seen on our left and right eyes, respectively. This photographic pair triggers binocular parallax to our eyes.

The 3-D imaging methods based on the stereoscopic image can be easily realized due to their simplicity and compatibility with current 2-D imaging systems. However, the perceived 3-D image is distorted when the stereo camera position is different from the viewer's position or the distance of the two exit pupils is different from that of the two cameras. For the delivery of a high quality stereoscopic image, the distortion should be minimized. Distortions of stereoscopic images have both psychological and geometrical reasons. Puppet theater and cardboard effects are appropriate examples of distortions psychologically generated by the human perception mechanisms [1]. And the keystone and nonlinearity are geometrically generated distortions caused by the photographing and projection mechanisms [2]. Geometrical distortions are analyzed quantitatively by several authors [2,3]. However, in those references there are no analyses of the distortions in the stereo image perceived by a viewer due to the changes in the viewer's position relative to the screen.

II. STEREOSCOPIC OPERATOR

As shown in Fig. 1, the point $\mathbf{I}(X_i, Y_i, Z_i)$ is an arbitrary point on the object Q. The point $\mathbf{M}(x_0, y_0, z_0)$ is the middle point of the left camera position $\mathbf{C}(x_0 - a, y_0, z_0)$ and right camera position $\mathbf{D}(x_0 + a, y_0, z_0)$. And the point $\mathbf{M}'(x_0', y_0', z_0')$ is the middle point of the left viewer position $\mathbf{C}'(x_0' - b, y_0', z_0')$ and right viewer position $\mathbf{D}'(x_0' + b, y_0', z_0')$. The camera position can be generalized to be another arbitrary viewer position as shown in Fig. 1. Thus we can consider the camera and viewer positions to be "viewer 1" and "viewer 2" positions as are shown in Fig. 1, respectively. The line connecting the two points $\mathbf{C}$ (Left Eye of viewer 1) and $\mathbf{D}$ (Right Eye of viewer 1) is parallel to the line which connects the two points $\mathbf{C}'$ (Left Eye of viewer 2) and $\mathbf{D}'$ (Right Eye of viewer 2). And these two lines are also parallel to the screen, which is the x-y plane as shown in Fig. 1. The distances of each viewer's eyes are given to be $2a$ and $2b$ as in Fig. 1. The *stereo operator* ($S_a\{\}$) [4] is defined as the transformation matrix, which transforms the point $\mathbf{I}(X_i, Y_i, Z_i)$ on the object Q to the column vector $\mathbf{V}_i(x_{iL}, x_r, y_i)$ as shown in Eq. (1). The subscript a in $S_a\{\}$ indicates that the distance of the two eyes, a is contained in the stereo operator formula. The Direct stereo operator ($S_a\{\}$) transforms the point $\mathbf{I}(X_i, Y_i, Z_i)$ on the object Q to the column vector $\mathbf{V}_i(x_{iL}, x_{ir}, y_i)$. The inverse stereo operator ($S_a^{-1}\{\}$) transforms the column vector $\mathbf{V}_i(x_{iL}, x_{ir}, y_i)$ to the point $\mathbf{I}(X_i, Y_i, Z_i)$ on the object Q. The two coordinates $\mathbf{C_s}(x_{iL}, y_i, 0)$ and $\mathbf{D_s}(x_{iR}, y_i, 0)$ are the stereo pair for the given

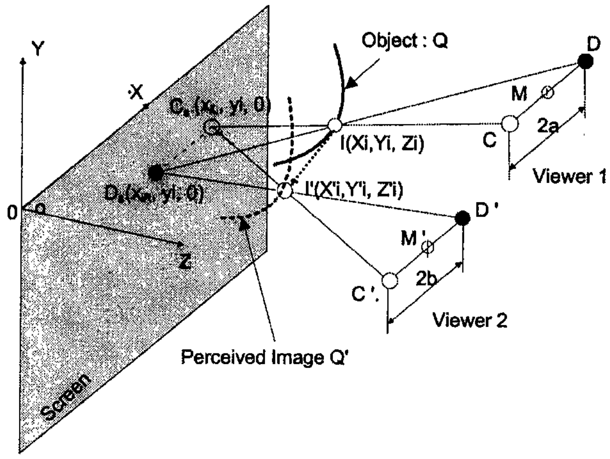

FIG. 1. The coordinate system for the stereo image.

point $\mathbf{I}(X_i, Y_i, Z_i)$ on the object Q where the subscript s in $\mathbf{C_s}$ and $\mathbf{D_s}$ indicates that $\mathbf{C_s}$ and $\mathbf{D_s}$ are the stereo coordinates on the screen. The concept of this stereo operator can be summarized as follows. $S_a\{\mathbf{I}\} = \mathbf{V}_i$, $S_a^{-1}\{\mathbf{V}_i\} = \mathbf{I}$ for two points $\mathbf{C}(x_0 - a, y_0, z_0)$ and $\mathbf{D}(x_0 + a, y_0, z_0)$ as Fig. 1.

$$S_a\{\bullet\} = \frac{A_a}{(z_0 - Z_i)} \cdot \bullet \tag{1}$$

$$S_a^{-1}\{\bullet\} = (z_0 - Z_i) \cdot A_a^{-1} \cdot \bullet \tag{2}$$

where $\bullet$ must be a 3×1 column vector

$$A_a = \begin{bmatrix} z_0 & 0 & a - x_0 \\ z_0 & 0 & -a - x_0 \\ 0 & z_0 & -y_0 \end{bmatrix}.$$

The 3×3 square matrix A_a is a nonsingular matrix. Thus, we can get the stereo coordinates on the screen in the component forms

$$\begin{aligned} x_{iL} &= \frac{X_i z_0 + Z_i(a - x_0)}{z_0 - Z_i} \\ x_{iR} &= \frac{X_i z_0 - Z_i(a + x_0)}{z_0 - Z_i} \\ y_i &= \frac{z_0 Y_i - y_0 Z_i}{z_0 - Z_i} \end{aligned} \tag{3}$$

III. THEIR APPLICATION TO THE DISTORTION ANALYSIS OF STEREOSCOPIC IMAGES

The position of the perceived image can be calculated by adopting the stereo operator. And this concept can be applied to the distortion analysis in the stereoscopic images.

For two points $\mathbf{C}(x_0 - a, y_0, z_0)$ and $\mathbf{D}(x_0 + a, y_0, z_0)$ as in Fig. 1, the point $\mathbf{I}(X_i, Y_i, Z_i)$ on the object is written as

$$\mathbf{I} = (z_0 - Z_i) \cdot A_a^{-1} \cdot \mathbf{V_i} \tag{4}$$

where

$$A_a = \begin{bmatrix} z_0 & 0 & a - x_0 \\ z_0 & 0 & -a - x_0 \\ 0 & z_0 & -y_0 \end{bmatrix}$$

By using the geometrical similarity, the point $\mathbf{I}'(X_i', Y_i', Z_i')$ on the object Q' for two points $\mathbf{C}'(x_0' - b, y_0', z_0')$ and $\mathbf{D}'(x_0' + b, y_0', z_0')$ is written as

$$\mathbf{I}' = (z_0' - Z_i') \cdot A_a^{-1} \cdot \mathbf{V_i} \tag{5}$$

where

$$A_b = \begin{bmatrix} z_0 & 0 & b - x_0 \\ z_0 & 0 & -b - x_0 \\ 0 & z_0 & -y_0 \end{bmatrix}.$$

Therefore we can combine the Eqs. (1), (4), and (5) together to get the compact form in which the stereo coordinate $\mathbf{V_i}$ on the screen is eliminated

$$\mathbf{I}' = \frac{(z_0' - Z_i')}{z_0 - Z} \cdot A_b^{-1} A_a \cdot \mathbf{V_i} \tag{6}$$

where

$$A_b^{-1} \cdot A_a = \begin{bmatrix} \frac{z_0}{z_0'} & 0 & \frac{bx_0' - bx_0}{az_0'} \\ 0 & \frac{z_0}{z_0'} & \frac{by_0' - by_0}{az_0'} \\ 0 & 0 & \frac{b}{a} \end{bmatrix}.$$

The Eq. (6) can be applied to the distortion analysis for the stereoscopic image. The amount of distortion is defined as the differences between the perceived image and the original object position.

$$\begin{aligned} \Delta X &= X_i' - X_i \\ &= \frac{(a - b)X_i + (bx_0' - ax_0)}{az_0 + Z_i(b - a)} \\ \Delta Y &= Y_i' - Y_i \\ &= \frac{(a - b)Y_i + (by_0' - ay_0)}{az_0 + Z_i(b - a)} \\ \Delta Z &= Z_i' - Z_i \\ &= \frac{(a - b)Z_i + (bz_0' - az_0)}{az_0 + Z_i(b - a)} \end{aligned} \tag{7}$$

In this case the position of viewer 1 in Fig. 1 can be regarded as the camera position. From Eq. (7), we can find that the distortions will disappear when the camera position is the same as the viewer's osition and the distance of the two cameras is the same as that of the viewer's eyes. And when the distance of the two cameras is equal to that of the two eyes, the distortions have linear properties which are not affected by the object position but are proportional to the differences between the object and the stereo image. From the Eq. (7), the stereo image has the minimum distortion if the viewer's eyes are located on each line that connects the object point with the corresponding position of the two cameras. Therefore the viewer's left eye should be on the line which connects the object point with the position of the left camera, and the viewer's right eye should be positioned in the same manner to minimize the distortion of the stereoscopic images.

IV. CONCLUSION

The stereo operator can be applied to stereoscopic image analysis as a useful mathematical tool. The concept of the stereo operator stems from the need for geometrical simplification in stereo image analysis. In this paper, the stereo operator has been applied to the perceived stereo image formulations and the properties of the distortion in the stereo image were studied.

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