

A Unified Method for Robust Self-calibration of 3D Field Sensor Arrays

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Abstract—Self-calibrating an array of 3D field sensors such as 3-axis magnetometers and accelerometers requires estimation of two variable sets—each sensor’s intrinsic model that maps its input field to the corresponding measurement and each sensor’s coordinates relative to a common frame of reference within the array. In this work, we propose the first unified self-calibration method for arrays of same-type 3D field sensors which is robust to anomalous sensor measurements unlike previous algorithms. The method breaks down the array calibration task into 3 steps of more easily subproblems, first estimating the intrinsic variables of each sensor independently, second computing the sensor coordinates with respect to a common reference frame, and last refining both these intrinsics and orientations jointly to minimize physically meaningful sensor estimation errors. Each stage has been carefully designed to maintain robustness to anomalies without compromising estimation quality. The performance of our method is compared against other state-of-the-art algorithms on both simulation and real data from a magnetometer array and accelerometer array, demonstrating significant improvements in accuracy and precision of the estimated array variables in versatile real-world self-calibration environments.

Index Terms—magnetometer array, accelerometer array, calibration, nonlinear optimization, robust optimization

I. INTRODUCTION

3D field sensors such as triaxial magnetometers and accelerometers are widely used devices in engineering and science. Accelerometers and magnetometers are frequently used for attitude control in navigation and mapping [1]–[4], wearable devices [5]–[7] and inertial navigation systems for aircrafts and unmanned aerial vehicles [8]. Also, both sensors form an integral part of the inertial measurement unit (IMU), which is an essential component for localization tasks.

In recent years, combining a set of same-type 3D field sensors (e.g. all magnetometers or all accelerometers) to form a sensor array have introduced a variety of additional useful applications as illustrated by Nilsson and Skog [13], [14]. To describe a few examples, Renaudin et al. [15] used an array of magnetometers to estimate orientation (with respect to the magnetic north) by averaging out heading estimates from individual sensors in a special geometric arrangement. Pang et al. [9] and Wang et al. [10] employed an array of fluxgate magnetometers for localization of ferromagnetic objects by

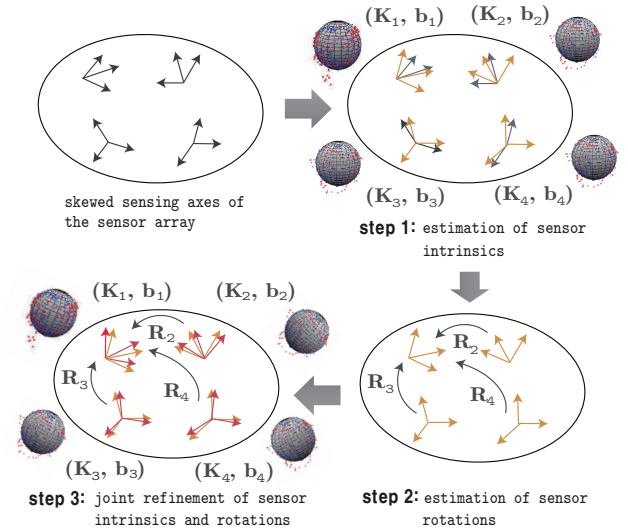


Fig. 1. Overall pipeline of our sensor array calibration method. We first estimate each sensor’s intrinsic variables (described by an ellipsoid), which transforms each measurement to the respective field direction. This yields the orthogonal axes (yellow) from the original axes (black). Then, we estimate each sensor’s absolute rotation with respect to a common frame of reference (set as the sensor 1’s coordinates for the above case). Finally, both the intrinsics and rotations of all sensors are jointly optimized for improved accuracy. The method is not limited to any particular setup of the sensors, and is robust to anomalies without compromising calibration accuracy.

applying scalar triangulation and ranging algorithm [16], [17]. Schopp et al. illustrated an accelerometer array can directly measure angular accelerations without needing to differentiate the angular velocities obtained from a gyroscope [12]. This has the benefit of avoiding the gyroscope’s saturation issue at high angular velocity [14] and reducing noise amplification arising from differentiation thereby improving the device’s noise-to-signal ratio [12]. Additionally, Brás et al. presented a way to configure fault detection and isolation systems using multiple inertial sensors [18]. Motivated by a wide range of potential applications, Skog et al. have made a small chip comprising 9 inertial measurement units (MIMU) [19]. One can also utilize MIMU to improve reliability by averaging measurements across all sensors to reduce independent stochastic errors.

In all above cases, use of a 3D field sensor array must be accompanied by accurate calibration of individual sensors, especially to account for different rotations between sensors and misalignment of the sensing axes which exists even for off-the-shelf sensors [20]. With the exception of a few earlier works calibrating sensor arrays on a rotation table, most studies [9]–[12], [15], [20] adopt a self-calibration approach that can be easily executed by end users. These methods are founded on

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TABLE I
AN OVERVIEW OF SELF-CALIBRATION ALGORITHMS FOR MAGNETOMETER ARRAYS AND ACCELEROMETER ARRAYS (SEE SEC. I-A FOR DETAILS).

Authors	Approach	Algorithmic details	Notes	Robust to outliers
Pang et al. (2013) [9]	Incremental	1. Estimate each sensor's intrinsic variables and field directions 2. Compute relative rotations between sensor 1 and others.	- Requires good initial array model and known geometry	No
Wang et al. (2017) [10]	Incremental	1. Estimate sensor 1's intrinsic variables 2. Estimate other sensors' intrinsics and rotations w.r.t. sensor 1.	- Relies on accurate calibration of sensor 1.	No
Hwangbo et al. (2013) [11]	Global (matrix factorization)	1. Rank-4 factorization of the measurement matrix 2. Upgrade solution using the q -constraint.	- Practically suboptimal due to imperfect solution upgrade	No
Schopp et al. (2016) [12]	Global (nonlinear optimization)	- Iterative minimization of sensor estimation errors and smooth calibration movement constraints over all sensor absolute rotations and 3D field directions	- Requires known intrinsics - Requires smooth movements - For accelerometer arrays only	No
Ours	Incremental → global	1. Estimate each sensor's intrinsic variables and field directions 2. Estimate absolute rotations of all sensors 3. Robust iterative minimization of sensor estimation errors over all sensor intrinsics, rotations and 3D field directions	No need for - good initial array model - Known geometry - smooth movements	Yes

the basis that i) the underlying ambient field (magnetic or gravitational) has constant magnitude and direction, and ii) the sensors are rigidly attached to an array body. Ultimately, self-calibration aims to retrieve the sensor array model which best preserves these geometric constraints.

Over the past decade, several studies pioneered in developing self-calibration algorithms for accelerometer arrays and magnetometer arrays. The methods for magnetometer arrays [9], [10], [21] are mostly based on an incremental approach, whereby a subset of the sensor array variables is first computed and used for estimation of the remaining variables. Such type of methods breaks down the calibration task into more easily solved subproblems, but becomes prone to accumulation of errors due to the nature of sequential estimation. On the other hand, the existing methods for accelerometer arrays optimize all variables jointly, but one relies on an iterative algorithm that requires a suitable initialization (i.e. prior knowledge) of the array model [12], and the other is based on a matrix factorization approach that yields an algebraic solution [11], requiring further optimization for improved accuracy (reviewed in Sec. I-A). Cherry-picking the advantages of both incremental and global approaches is a non-trivial but desirable task, providing the first motivation of this work.

Another major limitation of these methods is their weak robustness to anomalous measurements. For magnetometer arrays, outliers can arise due to local magnetic disturbances from nearby ferromagnetic materials that do not move with the array, inflicting inconsistent soft and hard iron effects. For accelerometer arrays, those can occur from sudden movements triggering non-negligible instantaneous accelerations. In both cases, the quality of the calibrated model can be significantly downgraded. Consequently, previous works have required a careful designated manoeuvre of the sensor array in a well-controlled environment (e.g. without ferromagnetic obstacles) to avoid anomalies, but this comes at the cost of increased time and effort required for self-calibration. Devising a robust autocalibration method to greatly simplify the data acquisition process serves as our second motivation.

In this work, we propose a unified autocalibration method for 3-axis magnetometer arrays and 3-axis accelerometer arrays that is robust to outlier measurements unlike previous

approaches. As shown in Fig. 1, our method is a 3-stage process comprising (i) estimation of each sensor's intrinsic variables (that are invariant to rotation) and 3D field directions (in its own sensor frame), (ii) computation of all sensors' absolute rotations, and (iii) joint refinement of the whole sensor variables and 3D field directions in the common frame of reference. Each stage has been equipped with an appropriate robust optimization technique to improve endurance against outliers. Our method can be viewed as an integration of incremental and global approaches, preventing the accumulation of errors whilst eliminating a need for hand-tuned initialization of the array variables for optimization.

To the best of our knowledge, this is the first work to address the issue of anomalous measurements in self-calibrating 3D field sensor arrays. Through carefully designed steps incorporating robust optimization, we will show accurate calibration can be achieved from arbitrary motions in presence of outliers.

Our main contributions are summarized below:

- a new canonical form of the triaxial sensor array model to yield a unique and meaningful solution,
- a new self-calibration method for arrays of same-type 3D field sensors which is robust to outlier observations and does not require a particular arrangement of the sensors,
- experimental comparisons against other state-of-the-art methods on synthetic and real data, and
- an ablation study of our algorithm to empirically observe the performance gains brought by different components.

The rest of this paper is structured as follows: a review of autocalibration algorithms for magnetometer arrays and accelerometer arrays is provided in Sec. I-A. We then formulate the triaxial sensor array as a mathematical model and propose its canonical form in Sec. II, yielding a unique meaningful form of the array model for optimization. Sec. III illustrates each stage of the proposed autocalibration method. In Sec. IV, our method is compared against other state-of-the-art algorithms on both synthetic and real data under various settings. Conclusions are drawn in Sec. V.

A. Related work

This section comprises review of autocalibration methods for 3-axis magnetometer arrays and accelerometer arrays. We

will use the term *intrinsic*s to define a group of sensor variables that are invariant to rotation (\mathbf{K} and \mathbf{b} in Sec. II).

Recently, Papafotis and Sotiriadis [22] showed accelerometer and magnetometer calibration share a great deal of similarity, that both sensors are assumed to output a linear transformation of the involved field (magnetic field or gravity) while the field magnitude remains constant. Despite such connection, calibration of magnetometer arrays and accelerometer arrays have been addressed separately by prior works [9]–[12], [21].

As mentioned in Sec. I, the self-calibration methods for magnetometer arrays are mostly incremental methods, sequentially estimating a subset of the sensor array variables in each stage. Pang et al. [9] devised a 2-step approach, whereby all sensor's intrinsic and their perceived 3D field directions are first estimated, after which sensor misalignments are computed by aligning each sensor's perceived fields to the common frame of reference. Since the algorithm utilizes an iterative nonlinear least squares algorithm for both stages, it requires good initialization of the array model and known array geometry without which would lead to an inaccurate solution. To circumvent the requirement for known array geometry, Pang et al. [21] and Wang et al. [10] proposed a linear solver to directly yield the product of sensor intrinsic and rotation for each sensor given a calibrated reference sensor. Nevertheless, this method still cannot avoid accumulation of errors if the reference sensor is not calibrated to high accuracy.

On the other hand, previous self-calibration algorithms for accelerometer arrays are all global joint optimization-based methods, simultaneously estimating sensor intrinsic, absolute rotations and 3D field directions. Hwangbo et al. [11] proposed a 2-step factorization approach, whereby a measurement matrix comprising all sensors' measurements at different time steps is first decomposed as two rank-4 matrices, after which are upgraded closely to a physically viable solution. The method has the advantages of being initialization-free and not limited to 3-axis sensors, but it employs an over-relaxed sensor model whereby each sensor's bias is differently scaled per measurement across different time steps, yielding an unrealistic solution. Schopp et al. [12] developed a method that can obtain each sensor's translation in addition to rotation by simultaneously minimizing the sum of sensor estimation errors (i.e. the difference between the model estimate and the observation) and regularization terms promoting smoothness of the calibration movement. However, its formulation is specific for accelerometer arrays only, and the algorithm requires known sensor intrinsic and appropriate initialization of each sensor's pose for successful convergence.

Additionally, none of the aforementioned work have explicitly addressed the issue of anomalies during self-calibration, consequently requiring carefully obtained outlier-free measurements for calibration. Recently, improving robustness to outliers has been studied for the case of single 3-axis magnetometer and accelerometer [23], but the work is not directly applicable to an array of multiple sensors.

II. CANONICAL FORM OF THE SENSOR ARRAY MODEL

We now move onto formulating the 3D field sensor array as a mathematical model and proposing its canonical form

to remove solution ambiguity. We will assume an array comprises M rigidly attached sensors (either all magnetometers or accelerometers) and that N synchronized measurements are observed across all sensors during an arbitrary movement of the array. We will also assume (as in [22]) that each 3-axis sensor i can be modelled by the equation

$$\mathbf{m}_i^{(j)} = \mathbf{y}_i^{(j)} + \epsilon_i^{(j)} = \mathbf{A}_i \hat{\mathbf{x}}_i^{(j)} + \mathbf{b}_i + \epsilon_i^{(j)} \quad (1)$$

where $\mathbf{m}_i^{(j)} \in \mathbb{R}^3$ is the sensor i 's measurement at time step j , $\mathbf{y}_i^{(j)} \in \mathbb{R}^3$ is the ideal output of sensor i at time j , $\hat{\mathbf{x}}_i^{(j)} \in S^2$ is the corresponding field direction (either magnetic field or gravity) at time step j in the sensor i 's coordinates, $\mathbf{A}_i \in \mathbb{R}^{3 \times 3}$ and $\mathbf{b}_i \in \mathbb{R}^3$ are the sensor i 's intrinsic variables (e.g. due to soft and hard iron effects for a magnetometer and axes misalignment) that are invariant to the sensor's orientation and $\epsilon_i^{(j)} \in \mathbb{R}^3$ is some observation noise.

Since all the sensors are rigidly attached to the array body, there exists a rotation matrix for each sensor i , \mathbf{R}_i , that transforms the field direction in the reference frame to the field in the sensor i 's coordinates. In terms of equation,

$$\hat{\mathbf{x}}_i^{(j)} = \mathbf{R}_i \hat{\mathbf{x}}^{(j)} \quad \forall \quad i = 1, \dots, M, \quad j = 1, \dots, N, \quad (2)$$

where $\hat{\mathbf{x}}^{(j)} \in S^2$ is the normalized field direction at time j in the reference frame and $\mathbf{R}_i \in SO(3)$ is the rotation matrix that transforms the field direction in the reference frame to that in the i -th sensor's frame. M denotes the number of sensors and N signifies the number of measurements for each sensor. (2) can be viewed as a rigidity constraint of field directions.

Combining (1) and (2) yields the 3-axis sensor array model

$$\mathbf{y}_i^{(j)} = \mathbf{A}_i \hat{\mathbf{x}}_i^{(j)} + \mathbf{b}_i = \mathbf{A}_i \mathbf{R}_i \hat{\mathbf{x}}^{(j)} + \mathbf{b}_i \quad \forall \quad i = 1, \dots, M. \quad (3)$$

(3) has solution ambiguity (known as gauge freedom [24]), i.e. given model variables $(\mathbf{A}_i, \mathbf{R}_i, \mathbf{b}_i)$, any solution of the form $(\mathbf{A}_i \mathbf{Q}_i, \mathbf{Q}_i^\top \mathbf{R}_i \mathbf{U}, \mathbf{b}_i)$ with arbitrary orthogonal matrices $\{\mathbf{Q}_i \in O(3)\}$ and $\mathbf{U} \in O(3)$ produce an equivalent model since

$$(\mathbf{A}_i \mathbf{Q}_i)(\mathbf{Q}_i^\top \mathbf{R}_i \mathbf{U})(\mathbf{U}^\top \hat{\mathbf{x}}^{(j)}) = \mathbf{A}_i \mathbf{R}_i \hat{\mathbf{x}}_i^{(j)} \quad \forall \quad i = 1, \dots, M. \quad (4)$$

To resolve this ambiguity, we propose a canonical form of the array model. Inspired by prior work [23], [25], for each sensor i , we constrain \mathbf{A}_i to be an upper triangular matrix, $\mathbf{K}_i \in \mathbb{R}^{3 \times 3}$, with positive diagonal entries. This effectively fixes \mathbf{Q}_i as any orthogonal matrix other than $\mathbf{Q}_i = \mathbf{I}$ will violate this constraint. Next, we set the first sensor's axes as the common frame of reference by setting $\mathbf{R}_1 = \mathbf{I}$, effectively fixing \mathbf{U} . The resulting array model equation is

$$\mathbf{y}_i^{(j)} = \mathbf{K}_i \hat{\mathbf{x}}_i^{(j)} + \mathbf{b}_i = \mathbf{K}_i \mathbf{R}_i \hat{\mathbf{x}}^{(j)} + \mathbf{b}_i \quad \forall \quad i = 1, \dots, M. \quad (5)$$

Above essentially projects the set of all possible solutions to a unique physically meaningful solution. Note this form is flexible as the outputted solution can always be transformed via simple matrix multiplication to allow a different form of \mathbf{A}_i (e.g. lower triangular or symmetric positive-definite) or set other sensor as the common frame of reference.

III. PROPOSED METHOD

In our method, we first estimate each sensor's intrinsic parameters, K_i (ambiguity-resolved A_i) and \mathbf{b}_i , along with the sensed field direction $\hat{\mathbf{x}}_i^{(j)}$ for each time step j . Second, previously obtained field directions across all sensors $\{\hat{\mathbf{x}}_i^{(j)}\}$ are used to obtain a set of relative rotations $\{\mathbf{R}_{i,i'}\}$ between all pairs of sensors. These are then used to estimate absolute sensor rotations $\{\mathbf{R}_i\}$. Last, all model variables and the field directions (in the reference frame) are jointly estimated by robustly minimizing the sum of array model estimation errors.

A. Estimation of sensor intrinsics and 3D field directions

In the first stage, we find each constituent sensor's intrinsics (K_i, \mathbf{b}_i) and the 3D field directions $\{\hat{\mathbf{x}}_i^{(j)}\}$ in its own sensor frame which best explains the corresponding measurements. For this task, we apply the 2-step robust autocalibration method developed in [23] independently for each sensor.

To briefly describe the aforementioned algorithm, it first fits an ellipsoid by minimizing a outlier-robust L_1 -norm cost function. The objective is derived from the fact that each field direction $\hat{\mathbf{x}}_i^{(j)}$ is of unit norm, which can be written as

$$\|\hat{\mathbf{x}}_i^{(j)}\|_2^2 - 1 = \|K_i^{-1}(\mathbf{y}_i^{(j)} - \mathbf{b}_i)\|_2^2 - 1 = 0. \quad (6)$$

Above can be cast as the following optimization problem

$$\arg \min_{K_i, \mathbf{b}_i, \{\hat{\mathbf{x}}_i^{(j)}\}} \sum_{j=1}^N \left\| \|K_i^{-1}(\mathbf{m}_i^{(j)} - \mathbf{b}_i)\|_2^2 - 1 \right\|_1, \quad (7)$$

where the term inside the L_1 -norm is an algebraic error directly derived from (6) with the ideal model output $\mathbf{y}_i^{(j)}$ replaced by the observation $\mathbf{m}_i^{(j)}$. (Note changing the L_1 -norm in (7) to L_2 switches to least squares ellipsoid fitting.) As stated in [23], (7) is first formulated as a convex semidefinite program proposed by Calafiore [26], which can then be efficiently solved using the SeDuMi solver [27].

The obtained solution is then refined by robustly minimizing the sum of (geometric) sensor array estimation errors each defined as the distance between the model estimate and the corresponding measurement. In terms of equation, we solve

$$\arg \min_{K_i, \mathbf{b}_i, \{\hat{\mathbf{x}}_i^{(j)} \in S^2\}} \sum_{j=1}^N \rho \left(\|K_i \hat{\mathbf{x}}_i^{(j)} + \mathbf{b}_i - \mathbf{m}_i^{(j)}\|_2^2 \right) \quad (8)$$

where $\rho: \mathbb{R} \rightarrow \mathbb{R}$ is a robust kernel implemented to lessen the effect of gross outliers. While several choices are available for $\rho(\cdot)$, we opt for the Cauchy loss $\rho(s) = \log(1 + s/\tau^2)$ like in [23], where τ is the kernel width implying the inlier radius.

Since (8) involves robust nonlinear least squares optimization, it can be solved using an iterative algorithm such as Levenberg-Marquardt [28], [29] equipped with Trigg's correction [24] as in [23], initialized by the solution of (7).

Above intrinsic estimation method is described in Algorithm 1. As shown in Fig. 2, the robust nature of the method helps to obtain more accurate intrinsic model for each sensor. Also, the fact that each sensor's intrinsics are solved independently allows us to avoid accumulation of errors.

Algorithm 1 Robust estimation of individual sensor intrinsics $\{(K_i, \mathbf{b}_i)\}$ and 3D field directions $\{\hat{\mathbf{x}}_i^{(j)}\}$

Inputs: measurements $\{\mathbf{m}_i^{(j)}\}$

1: **for** $i = 1, \dots, M$ **do**

2: Yield intrinsics (K_i, \mathbf{b}_i) and fields $\{\hat{\mathbf{x}}_i^{(j)}\}$ by solving (7).

3: Refine (K_i, \mathbf{b}_i) and $\{\hat{\mathbf{x}}_i^{(j)}\}$ by iteratively solving (8).

4: **end for**

Outputs: sensor intrinsics $\{(K_i, \mathbf{b}_i)\}$ and fields $\{\hat{\mathbf{x}}_i^{(j)}\}$

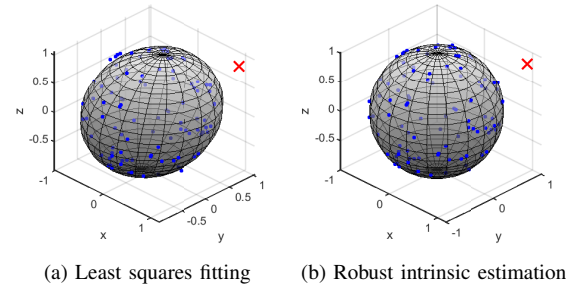


Fig. 2. Ellipsoid fitting results on 100 synthetic noisy data points with a single outlier (1%). Blue dots are inliers and the red cross is the outlier. Data points are sampled from a unit sphere with additive Gaussian noise $\mathcal{N}(0, 0.01\mathbf{I})$, and the outlier is generated by further adding uniform distribution noise $\mathcal{U}(0, 1)$ to each dimension. Robust intrinsic estimation yields a solution close to ground truth (unit sphere) while least squares fitting yields a skewed ellipsoid.

B. Estimation of absolute sensor rotations

After each sensor's intrinsics (K_i, \mathbf{b}_i) and field directions $\{\hat{\mathbf{x}}_i^{(j)}\}$ (in its own frame) are computed, the next task is to estimate the rotation of each sensor (\mathbf{R}_i) with respect to the common reference frame. We accomplish this by computing the relative rotation between each pair of sensors, from which we estimate most plausible absolute sensor rotations.

1) Estimation of relative rotations between sensor pairs:

Relative rotation between a pair of sensors i and i' can be estimated by finding a rotation matrix $\mathbf{R}_{i,i'}$ that transforms the field directions in the sensor i 's frame ($\{\hat{\mathbf{x}}_i^{(j)}\}$) to the fields in the sensor i' 's frame ($\{\hat{\mathbf{x}}_{i'}^{(j)}\}$), since ideally $\hat{\mathbf{x}}_{i'}^{(j)} = \mathbf{R}_{i,i'} \hat{\mathbf{x}}_i^{(j)}$ (following the convention from [30]). This can be formulated as Wahba's problem [31], in which one finds the most feasible rotation between two sets of 3D points.

One thing to note here is that miscomputed fields during the intrinsic estimation phase can yield outlier pairs of field directions. To avoid these outlier correspondences, we incorporate Fischler and Bolles' random sample consensus (RANSAC) algorithm [32], which is a popular choice for robust model estimation. The basic idea is simple—rather than utilizing all field pairs at once, the algorithm iteratively samples a few field pairs at random to estimate multiple potential relative rotations, after which it outputs the best relative rotation matrix across all iterations. Through consecutive random sampling, the algorithm is encouraged to once draw a set of all inlier samples to fit a correct relative rotation matrix.

Implementing RANSAC requires two components, a fast model fitting algorithm known as *minimal solver*, and the objective function used to quantify the model quality. For the

Algorithm 2 Robust estimation of relative rotations

Inputs: field directions of all sensors $\{\hat{\mathbf{x}}_i^{(j)}\}$

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1: for  $i = 1, \dots, (M-1)$  do
2:   for  $i' = (i+1), \dots, M$  do
3:     repeat
4:       Sample 2 field indices  $(\Omega)$  between 1 and  $N$ .
5:        $[U, S, V] \leftarrow \text{svd}(\sum_{j \in \Omega} \hat{\mathbf{x}}_{i'}^{(j)} \hat{\mathbf{x}}_i^{(j)\top})$ 
6:        $R_{i,i'} \leftarrow U \text{diag}([1, 1, \det(UV^\top)]) V^\top$ 
7:       Compute  $f_{i,i'} := f(R_{i,i'})$  from (11).
8:       if  $f_{i,i'} < f_{i,i'}^{\text{best}}$  or iteration == 1 then
9:          $f_{i,i'}^{\text{best}} \leftarrow f_{i,i'}$ 
10:         $R_{i,i'}^{\text{best}} \leftarrow R_{i,i'}$ 
11:       end if
12:     until max. # iterations reached
13:   end for
14: end for
Outputs: relative rotations  $\{R_{i,i'}^{\text{best}}\}$  and costs  $\{f_{i,i'}^{\text{best}}\}$ 

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minimal solver, we choose the singular value decomposition (SVD) based method illustrated in the work of Markley and Mortari [4], which solves the following Wahba's problem

$$\arg \min_{R_{i,i'} \in SO(3)} \sum_{j \in \Omega} \left\| \hat{\mathbf{x}}_{i'}^{(j)} - R_{i,i'} \hat{\mathbf{x}}_i^{(j)} \right\|_2^2, \quad (9)$$

where Ω denotes a set of sampled field-pair indices. The solution of (9) is derived via SVD as follows:

$$\sum_{j \in \Omega} \hat{\mathbf{x}}_{i'}^{(j)} \hat{\mathbf{x}}_i^{(j)\top} = USV^\top$$

$$R_{i,i'} = U \text{diag}([1, 1, \det(UV^\top)]) V^\top. \quad (10)$$

It is known in the literature [33] above computation only requires 2 unique field vector pairs (i.e. $|\Omega| = 2$). In addition to (10) being a closed form solution, this is a big advantage for a RANSAC-based algorithm since (10) is more likely to draw all-inlier samples (which is required to fit a correct model) than other solvers requiring more than 2 data points.

For the objective function $f_{i,i'}$, we combine the estimation errors of sensors i and i' to check how well the relative rotation $R_{i,i'}$ suits the measurements of both sensors, i.e. we compute

$$f(R_{i,i'}) := \frac{1}{2} \sum_{j=1}^N \left[\rho \left(\left\| R_{i,i'}^\top \hat{\mathbf{x}}_{i'}^{(j)} + \mathbf{b}_i - \mathbf{m}_i^{(j)} \right\|_2^2 \right) + \rho \left(\left\| R_{i,i'} \hat{\mathbf{x}}_i^{(j)} + \mathbf{b}_{i'} - \mathbf{m}_{i'}^{(j)} \right\|_2^2 \right) \right]. \quad (11)$$

The rotation $R_{i,i'}$ with the lowest objective value is selected.

The overall procedure is illustrated in Algorithm 2.

2) *Estimation of absolute rotations:* From the above relative rotations, we can form a graph (V, E) as shown in Fig. 3, whereby each node V represents a sensor and each edge E signifies the relative rotation between the adjacent sensors. Each edge has a cost value assigned from computing (11).

Without loss of generality, when computing absolute sensor rotations, one sensor coordinates can be set as the common frame of reference from which other sensor coordinates are defined. According to our canonical array formulation from Sec. II, the first sensor is set as the reference frame, but we are free to choose any other sensor as the reference if necessary.

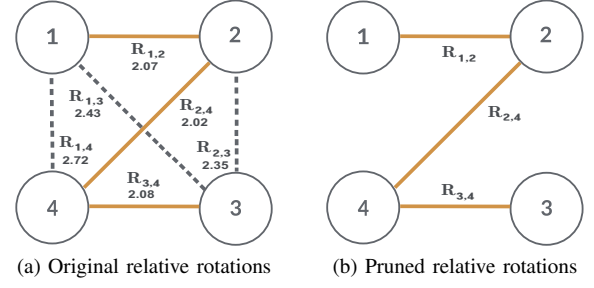


Fig. 3. Graphs of relative rotations between pairs of sensors (a) before pruning and (b) after pruning. We employ Prim's algorithm [34] to extract the best set of edges (relative rotations) with minimum overall cost (sum of pairwise sensor estimation errors). The pruned edges are used to uniquely determine initial absolute sensor rotations, which are then passed as inputs to the robust rotation averaging stage in Algorithm 3.

Other sensor rotations can then be computed by transforming the reference frame by a single or multiple relative rotations. For example, sensor 3's rotation (R_3) can be obtained by computing $R_{1,3}R_1$, $R_{4,3}R_{1,4}R_1$ or $R_{2,3}R_{4,2}R_{1,4}R_1$, etc. In an ideal noiseless environment, these would all yield the same rotation for sensor 3. However, in presence of noise, each of the above is likely to yield a different estimate for R_3 , with some more accurate than the others. Hence, there should be a way of finding the set of (relative rotation) edges leading to the most accurate absolute rotations of all sensors.

To achieve above, we first employ Prim's algorithm [34] to prune the set of (relative rotation) edges and form a tree of best edges spanning all sensor nodes with minimum sum of edge costs (widely known as the *minimum spanning tree*). This can be interpreted as extracting the set of relative rotations linking all sensors with lowest sum of sensor estimation errors (see Fig. 3 for an example). These pruned relative rotations are then used to yield initial absolute rotations of the array sensors.

Second, since the obtained absolute rotations are inevitably biased towards pruned relative rotations, we perform *rotation averaging* [30] to find optimal absolute sensor rotations over the entire set of relative rotations. For algorithmic illustration, suppose the initial rotations of sensor i and i' are R_i and $R_{i'}$ respectively, and the relative rotation between sensors i and i' is computed as $R_{i,i'}$. Ideally, $R_i' = R_{i,i'}R_i$ (following the convention from [30]), but due to noise, $R_{i,i'} = R_i'R_i^\top$ may not hold. We wish to make updates ΔR_i and $\Delta R_{i'}$ such that the updated rotations $R_{i'}\Delta R_{i'}$ and $R_i\Delta R_i$ satisfy the relative rotation constraint $R_{i,i'} = R_{i'}\Delta R_{i'}\Delta R_i^\top R_i^\top$.

By using Rodrigues' formula $\Delta R =: \text{expm}(\Delta \omega)$, where expm denotes an exponential map from the lie algebra $so(3)$ to $SO(3)$, we can rewrite this as

$$R_{i,i'} = R_{i'} \text{expm}(\Delta \omega_{i'}) \text{expm}(-\Delta \omega_i) R_i^\top$$

$$= R_{i'} BCH(\Delta \omega_{i'}, -\Delta \omega_i) R_i^\top,$$

where BCH stands for the Baker-Campbell-Hausdorff formula [35]. Chatterjee and Govindu [30] first order approximated $BCH(\Delta \omega_{i'}, -\Delta \omega_i) \approx \text{expm}(\Delta \omega_{i'} - \Delta \omega_i)$, yielding

$$BCH(\Delta \omega_{i'}, -\Delta \omega_i) = R_{i'}^\top R_{i,i'} R_i \approx \text{expm}(\Delta \omega_{i'} - \Delta \omega_i)$$

$$\Rightarrow \Delta \omega_{i'} - \Delta \omega_i \approx \logm(R_{i'}^\top R_{i,i'} R_i), \quad (12)$$

Algorithm 3 L_1 -norm averaging of absolute sensor rotations

Inputs: relative rotations between pairs of sensors $\{R_{i,i'}\}$ and absolute sensor rotations $\{R_i\}$

- 1: Form D from (13).
- 2: **repeat**
- 3: Initialize e as an empty vector.
- 4: **for** $i = 1, \dots, (M-1)$ **do**
- 5: **for** $i' = (i+1), \dots, M$ **do**
- 6: $e \leftarrow [e; R_{i,i'}^\top R_{i,i'} R_i]$
- 7: **end for**
- 8: **end for**
- 9: $\Delta\omega \leftarrow \arg \min_{\Delta\omega} \|D\Delta\omega - e\|_1$
- 10: **for** $i = 1, \dots, M$ **do**
- 11: Retrieve $\Delta\omega_i$ from $\Delta\omega$
- 12: $R_i \leftarrow R_i \expm(\Delta\omega_i)$
- 13: **end for**
- 14: **until** max. # iterations reached or $\|\Delta\omega\|_2 < \epsilon$

Output: absolute sensor rotations $\{R_i\}$

where \logm is a logarithmic map from $SO(3)$ to $so(3)$. Now, stacking (12) for all relative rotation edges yields

$$\begin{bmatrix} -I & I & \cdots & 0 \\ -I & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \\ \vdots \\ \Delta\omega_M \end{bmatrix} \approx \begin{bmatrix} \logm(R_2^\top R_{1,2} R_1) \\ \logm(R_3^\top R_{1,3} R_1) \\ \vdots \\ \logm(R_M^\top R_{M-1,M} R_{M-1}) \end{bmatrix} \Rightarrow D\Delta\omega \approx e, \quad (13)$$

where $\Delta\omega \in \mathbb{R}^{3M}$ is a stacked vector of updates in all absolute sensor rotations. So long as $\|\Delta\omega\|_2$ is small, the above approximation holds.

As written in Algorithm 3, we iteratively minimize $\|D\Delta\omega - e\|_1$ over $\Delta\omega$ with e being updated per iteration. Since this is a convex objective, we can use SDPT3, a default solver included in the CVX library [36].

C. Joint refinement of sensor variables and field vectors

Since the intrinsic and rotation estimations have been carried out in a sequential manner, the errors from intrinsic estimation are likely to have lowered the accuracy of absolute rotations. To mitigate this issue, we propose to refine all sensor array variables by minimizing the sum of robustified distances between the observations and the corresponding model estimates over all sensor intrinsics $\{K_i, b_i\}$, absolute rotations $\{R_i\}$ and field directions $\{\hat{x}^{(j)}\}$, i.e. solve

$$\arg \min_{\{K_i, R_i, b_i\}, \{\hat{x}^{(j)}\}} \sum_{i=1}^M \sum_{j=1}^N \rho \left(\|K_i R_i \hat{x}^{(j)} + b_i - m_i^{(j)}\|_2^2 \right) \quad (14)$$

where $\rho: \mathbb{R} \rightarrow \mathbb{R}$ is a robust kernel implemented to reduce the effect of gross outliers. Like in Sec. III-A, we choose the Cauchy kernel [23] $\rho(s) = \log(1 + s/\tau^2)$, where τ is the inlier radius, and each field vector is parameterized to lie on a unit 3D sphere. Additionally, each rotation matrix is parameterized as an axis-angle vector [37] $\omega \in \mathbb{R}^3$ such that $R_i := \expm(\omega_i)$, and we set the first sensor's rotation as the common frame of reference by fixing $R_1 = I$.

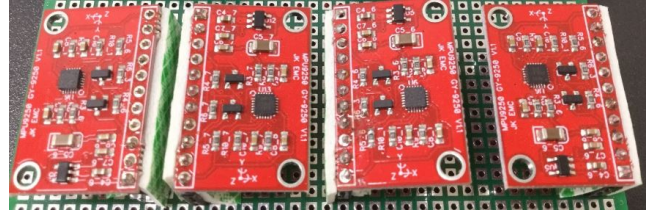


Fig. 4. Four InvenSense™ MPU9250 IMUs used for collecting real data

Since (14) is a robust nonlinear problem, we solve this by using the Levenberg-Marquardt algorithm [28] equipped with Trigg's correction [24], similar to Sec. III-A. This requires deriving the first order derivative (Jacobian) of the residual $K_i R_i \hat{x}^{(j)} + b_i - m_i^{(j)}$ with respect to all the optimized variables. While these Jacobians are analytically tractable, one can apply a publicly available auto-differentiation library such as Ceres Jet [37] to minimize (14) without deriving analytic derivatives.

Although solving (14) is non-trivial without good initialization due to its nonlinear nature, the obtained sensor array model variables from steps 1 and 2 yield a near-optimal solution as will be demonstrated in Sec. IV, serving as appropriate initialization for robust nonlinear optimization.

IV. EXPERIMENTAL RESULTS

We have carried out experiments to observe two characteristics, namely our method's performance compared with other state-of-the-art array self-calibration algorithms, and the accuracy improvement brought by different components of our method also known as ablation study.

For the first comparison, we chose Wang et al.'s method [10] and Hwangbo et al.'s method [11], both of which are the only algorithms from Table I not requiring prior knowledge of sensor intrinsics or array geometry (like ours).

For ablation study, we compared 3 settings, i) our proposed method, ii) ours without refinement (steps 1 and 2 only) and iii) ours without robustness (the L_2 -norm version). Specifically, our L_2 -norm version replaces the Cauchy kernel by a trivial function $\rho(s) = s$ and Algorithm 2 by solving (9) over all data points (not just 2). We selected these to demonstrate the quality of our solution before and after refinement and the impact of robustness in improving the calibration accuracy.

All the experiments were run on MATLAB R2019b with a PC comprising AMD Ryzen 7 2700X CPU and 32GB RAM.

A. Implementation details

Below are details for implementation of the tested methods.

1) *Data normalization:* For each tested algorithm, we applied the data normalization technique in [23] to the measurements $\{m_i^{(j)}\}$ across all array sensors, subtracting by the mean of all measurements then dividing by the respective standard deviation. This ensures a level playground for comparison by improving numerical stability of all methods.

2) *Optimization hyperparameters:* For all iterative nonlinear optimization processes (steps 1 and 3 in our method and step 1 in Wang et al.'s method), we employed the Levenberg-Marquardt algorithm with the maximum number of iterations

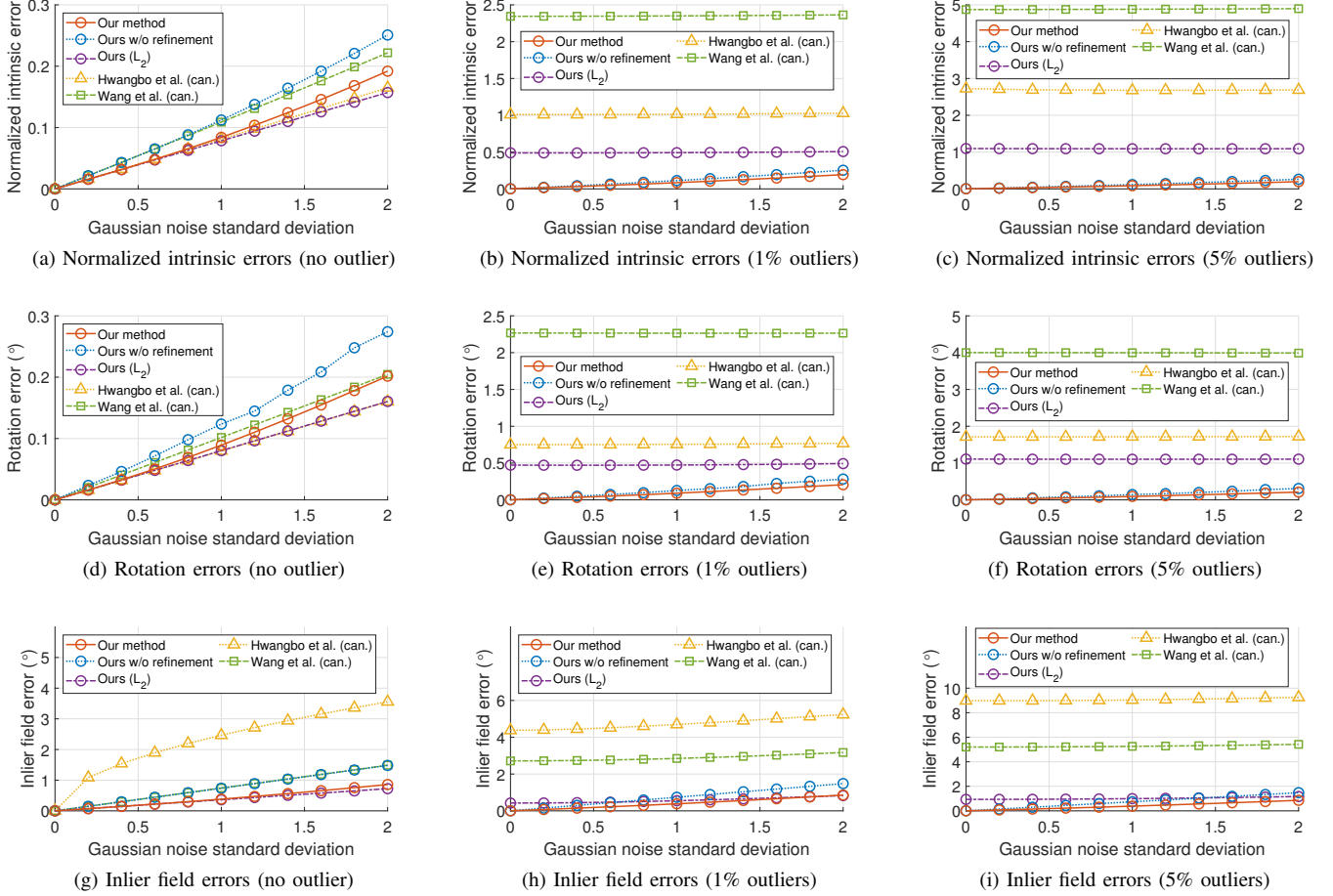


Fig. 5. Results on synthetic data as a function of additive Gaussian noise standard deviation (σ) with different proportions of gross outliers. More details about the used metrics for comparison can be found in Sec. IV-B.

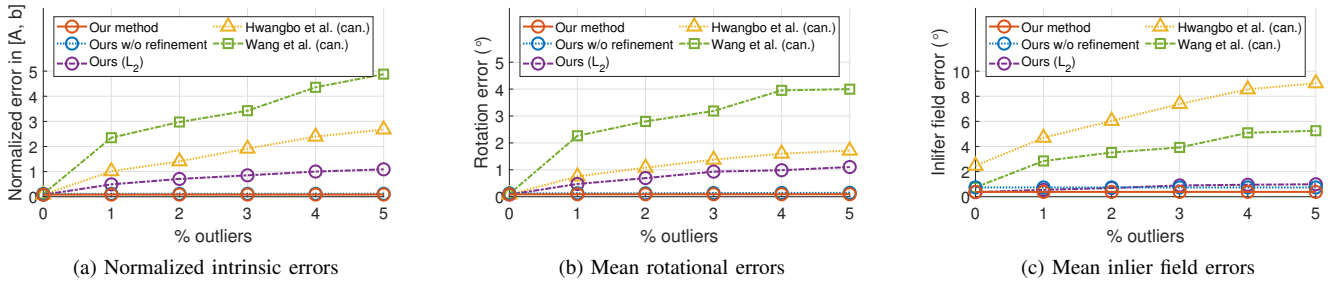


Fig. 6. Results on synthetic data as a function of proportion of gross outliers (η). The standard deviation of added Gaussian noise (σ) is fixed at 1.0, similar to that obtained from real magnetometer readings (0.69). Note ours w/o refinement is just slightly above our method in both plots.

set to 300 and the function tolerance set to 10^{-9} . For robust nonlinear optimization in our method, we set the inlier radius τ for the Cauchy kernel to $\sqrt{3}$, implying error of 1.0 per dimension for a data point to be considered as an inlier. For estimating relative rotations, the number of RANSAC iterations was set to 1000. ϵ in Algorithm 2 was set to 10^{-3} .

3) *Solution canonicalization*: Neither Wang et al. or Hwangbo et al. have explicitly stated how to resolve the solution ambiguity mentioned in Sec. II, making it difficult to compare between the solutions from different algorithms. For a fair comparison, we applied our canonical form of the

sensor array model from Sec. II to the output of each method (denoted as *can.*) in Figs. 5 and 6 and Table II). In addition, Hwangbo et al.'s method outputs an unrealistic algebraic array model with an additional scale factor $t^{(j)}$ per field vector, i.e. $\mathbf{y}_i^{(j)} = \mathbf{A}_i \hat{\mathbf{x}}^{(j)} + t^{(j)} \mathbf{b}_i$, so we added a final step of setting each $t^{(j)}$ to 1 to yield a physically meaningful array model.

B. Simulation study

We compared the accuracy of different algorithms for a range of additive Gaussian noise levels (σ) and proportion of gross outlier measurements (η).

TABLE II

RESULTS ON REAL DATA ACHIEVED BY TESTED ARRAY AUTOCALIBRATION ALGORITHMS. M1 AND A1 ARE OUTLIER-FREE AND USED AS TEST SETS AND OTHERS ARE FOR MODEL ESTIMATION (TRAINING). ON EACH TRAINING DATA, EACH METHOD LEARNS A MODEL $\{(\mathbf{A}_i, \mathbf{b}_i)\} = \{(\mathbf{K}_i \mathbf{R}_i, \mathbf{b}_i)\}$, WHICH IS COMPARED AGAINST THE TEST SET-DERIVED MODEL (PRECISION) AND TESTED ON M1/A1 MEASUREMENTS (ACCURACY).

Dataset info.	Array type	MAGNETOMETER ARRAY							ACCELEROMETER ARRAY			
	Sequence ID	M1	M2	MP1	MP2	MS1	MS2	MS3	A1	A2	AO1	AO2
	# sensors # measurements (% outliers)	4 326 (0.0%)	4 426 (0.0%)	4 454 (3.3%)	4 422 (6.9%)	4 454 (2.6%)	4 476 (5.9%)	4 421 (4.3%)	4 362 (0.0%)	4 589 (1.2%)	4 492 (13.2%)	4 477 (12.6%)
Model deviation	Our method	-	0.54	0.61	0.37	0.78	0.41	0.67	-	0.66	0.83	0.72
	Ours w/o refinement	-	0.52	0.66	0.79	1.09	0.45	0.69	-	1.13	1.09	1.04
	Ours (L_2)	-	0.52	0.86	1.25	1.61	2.19	1.79	-	0.69	4.80	2.51
	Hwangbo et al. [11]	-	0.53	0.93	1.33	3.85	3.13	2.66	-	0.69	231.36	5.13
	Wang et al. [10]	-	0.39	17091.16	9359.68	15318.34	3.62	1.63	-	0.74	72968.19	5.00
Test set fitting error	Our method	-	0.58	0.67	0.63	0.67	0.60	0.62	-	1.30	1.34	1.32
	Ours w/o refinement	-	0.59	0.62	0.77	0.99	0.62	0.60	-	1.32	1.36	1.32
	Ours (L_2)	-	0.58	0.89	0.80	0.79	1.17	1.31	-	1.29	2.13	1.44
	Hwangbo et al. [11]	-	0.58	0.94	0.83	1.54	2.14	1.48	-	1.30	95.87	3.59
	Wang et al. [10]	-	0.58	13.52	12.79	17.49	1.55	1.28	-	1.29	48.10	2.53

1) *Generating ground truth data:* We sampled 1000 arbitrary 3D field directions commonly observed by 4 sensors. Each sensor's absolute rotation \mathbf{R}_i was randomly sampled from the $SO(3)$ manifold [38] apart from the first sensor's rotation \mathbf{R}_1 , which was set to \mathbf{I} . Similar to [23], each sensor's upper triangular \mathbf{K}_i was produced as a diagonal-dominant matrix $100(\mathbf{I} + 0.1\mathbf{E}_i)$, where \mathbf{I} is the identity matrix and \mathbf{E}_i is a random upper triangular matrix with each element sampled from $\mathcal{N}(0, 1)$. The elements of each sensor's bias \mathbf{b}_i were each drawn from $\mathcal{N}(0, 1)$ and scaled by factor of 10. Each ideal model output $\mathbf{y}_i^{(j)}$ was calculated using (5).

To simulate the sensor measurements $\{\mathbf{m}_i^{(j)}\}$, we perturbed each model output $\mathbf{y}_i^{(j)}$ by adding Gaussian noise $\mathcal{N}(0, \sigma)$. Then, a predefined fraction (η) of measurements were further corrupted by adding random gross outlier noise, where each element was sampled from $\mathcal{U}(0, 1)$, multiplied by 100 and randomly flipped sign with 50% probability.

2) *Experimental conditions:* The tested ranges of Gaussian noise level (σ) and proportion of gross outliers (η) are between 0 to 2.0 (twice the standard deviation obtained from outlier-free real magnetometer readings) and 0 to 5% respectively. For each algorithm, we made 25 repeats with different ground truth data on each experimental condition (σ, η).

3) *Accuracy metrics:* Each algorithm's accuracy is reported in terms of three metrics, namely the normalized sensor intrinsic error defined as $\sqrt{\frac{1}{12M} \sum_{i=1}^M \|\mathbf{K}_i, \mathbf{b}_i\| - \|\mathbf{K}_i^*, \mathbf{b}_i^*\|_2^2}$, the mean absolute rotation error (in degrees) defined as $\sqrt{\frac{1}{M} \sum_{i=1}^M \|\log(\mathbf{R}_i^T \mathbf{R}_i^*)\|_2^2}$, and the mean inlier field error (in degrees) defined as $\frac{1}{|C|} \sum_{j \in C} \cos^{-1}(\hat{\mathbf{x}}^{(j)T} \hat{\mathbf{x}}^{*(j)})$, where the quantities with asterisks are ground truth values and C denotes the set of inlier measurements (considering fields from anomalous measurements can incur misleading results).

4) *Results:* Figs. 5 (per σ) and 6 (per η) demonstrate our proposed (robust) method outperforms other methods by some margin in all cases except for the ideal situation with zero outliers, which is narrowly won by our L_2 -norm variant (see Figs. 5a and 5d). This shows incorporating robustness is crucial for stable, accurate and precise sensor array calibration. Additionally, the figures show joint refinement (step 3) in our

method improves the accuracy and that the solution from step 2 is still quite accurate. Also, our L_2 -norm variant achieves the best accuracy across all tested non-robust algorithms, demonstrating the appropriateness of our method's architecture.

Additionally, it is interesting to observe Hwangbo et al. wins over Wang et al. in terms of model accuracies $\{\mathbf{K}_i, \mathbf{b}_i, \mathbf{R}_i\}$ but loses in estimating field directions $\{\hat{\mathbf{x}}_j\}$ for inlier measurements. We envisage this is related to the fact that Hwangbo et al.'s method yields an unrealistic solution in presence of noise as discussed in Sec. I-A and Sec. IV-A. This results in each field $\hat{\mathbf{x}}_j$ experiencing a different degree of the sensor bias which, besides not being physically accurate, potentially deteriorates the accuracy of estimated field directions.

C. Results on real data

We have also compared the calibration precision and accuracy of different methods on real sensor array data.

1) *Data acquisition:* We obtained measurements from 4 IMUs (InvenSenseTM MPU 9250), each comprising a 3-axis accelerometer and magnetometer, all rigidly attached to an array board. These were collected by an arduino microprocessor via I²C protocol. To avoid potential address conflicts, we used an I²C multiplexer (Texas Instruments TCA9548A). Magnetometer readings and accelerometer readings were obtained in separate occasions to mimic different types of array sensors.

The obtained measurement sequences are listed in Table II. Each was acquired without following a particular conventional calibration procedure such as the one proposed in [22]. Dataset IDs with M are from a magnetometer array and the ones with A are from an accelerometer array. Since the accelerometer readings were about a factor of 100 greater than those of magnetometers, we divided the accelerometer readings by 100 to keep the same effective width of the robust kernel. M{1-2}, A{1-2} are mostly, if not all, outlier-free tracks whilst MP{1,2} (magnetometers near a phone), MS{1-3} (magnetometers near a stapler) and AO{1-2} (accelerometers with sudden accelerations) contain a significant proportion of outliers (1.2–12.6%) arising from real-world disturbances.

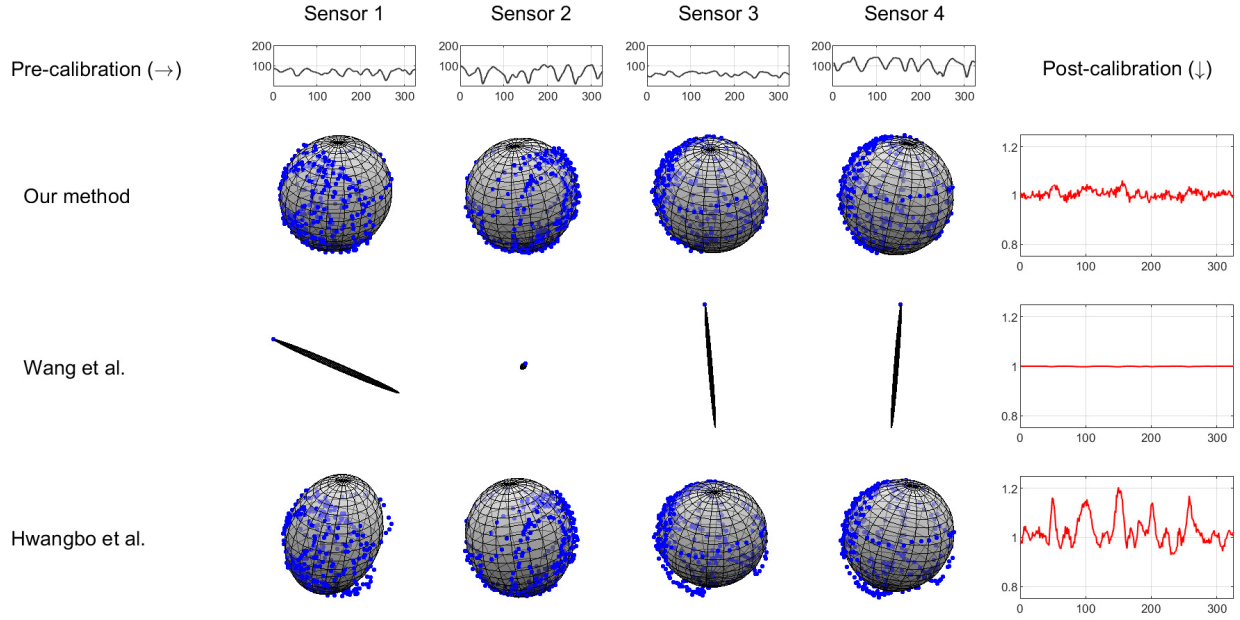


Fig. 7. Visualization of the array sensor model and respective post-calibration plot obtained by each algorithm on real data. Each algorithm has estimated the array model of 4 3-axis magnetometers from the outlier-present MS1 dataset in Table II. The quality of each array model is visualized via ellipsoid fitting results and post-calibration plots (field magnitude over time) on the inlier-only M1 sequence (used as test set). Blue dots are individual measurements in M1.

2) *Experimental procedure*: Since no ground truth data is available for real-world autocalibration, we have used outlier-free M1 (326 measurements) and A1 (362 measurements) as the test sets and the other sequences as training data. For each algorithm on each training sequence, we estimated (trained) an array model. Each of these obtained models was tested on either M1 or A1 depending on the type of sensor array.

3) *Accuracy and precision metrics*: To quantify the model precision, we compared the output model $\{(A_i, b_i)\}$ against the de facto ground truth $\{(A_i^*, b_i^*)\}$ estimated from the test set (M1 or A1) by computing the normalized array model deviation error $\sqrt{\frac{1}{12M} \sum_{i=1}^M \| [A_i, b_i] - [A_i^*, b_i^*] \|_2^2}$. ($A_i = K_i R_i$.) For quantifying the model accuracy, we computed the model's fitting error on the test set (M1 or A1) by calculating

$$\min_{\{\hat{\mathbf{x}}^{(j)} \in S^2\}} \frac{1}{12M} \sum_{i=1}^M \sum_{j=1}^N \| A_i \hat{\mathbf{x}}^{(j)} + \mathbf{b}_i - \mathbf{m}_i^{(j)} \|_2^2. \quad (15)$$

This involves estimating optimal 3D field directions for the test track given the sensor variables $\{(A_i, b_i)\}$ which can be solved in closed-form followed by vector normalization as no robust kernel is present in (15) (since M1/A1 are outlier-free).

4) *Accuracy and precision results*: All the results on real data are included Table II. These confirm the trend shown in the simulation study, with our method winning most of the entries especially on the tracks with outliers. Additionally, Wang et al.'s method fails for many of the tested sequences.

5) *Visual verification of results*: Additionally, we have applied verification techniques such as ellipsoid fitting and post-calibration plots to visualize and verify above numerical results between our method and prior studies. For this purpose, each algorithm first fits a magnetometer array model on the outlier-existing MS1 dataset, and then this array model is verified on the inlier-only M1 sequence by checking two

criteria, i) the quality of the ellipsoid fitting results on 4 sensors and ii) the respective post-calibration plot as shown in Fig. 7. Each post-calibration plot shows the field magnitude ($\{\|\mathbf{x}^{(j)}\|_2\}$) over time (in the M1 sequence) jointly estimated across the 4 sensors by the respective algorithm, where each field $\mathbf{x}^{(j)}$ is computed by solving

$$\min_{\mathbf{x}^{(j)} \in \mathbb{R}^3} \sum_{i=1}^M \| A_i \mathbf{x}^{(j)} + \mathbf{b}_i - \mathbf{m}_i^{(j)} \|_2^2 \quad (16)$$

through least squares. Note, for the purpose of plotting post-calibration plots, the field vectors remain un-normalized (i.e. not enforced to be of unit norm) to show discrepancies in the absolute field magnitude at test time [22], [39], [40].

To summarize the findings from Fig. 7, our method finds feasible intrinsics of the array model from outlier-present MS1, overall fitting ellipsoids well to the outlier-free M1 measurements. On the other hand, Hwangbo et al.'s method shows some disparity between the ellipsoids and the M1 observations especially for sensors 1, 3, and 4, and the method of Wang et al. ends up with significantly distorted ellipsoids across all sensors. These results are in line with the algorithm ranking produced by our numerical comparisons in Table II.

Regarding the post-calibration plots, our method shows less variation in field magnitude than Hwangbo et al.'s, demonstrating better calibration performance. Surprisingly though, Wang et al.'s method seems to yield an almost flat line, which may seem optimal at first sight. Nevertheless, a previous study [23] has intuitively explained this can be triggered by badly scaled ellipsoids, which can be generated by self-calibration algorithms not being robust to outlier measurements. For this reason, the study has also emphasized a need to always consult the ellipsoid fitting results jointly. We believe this is the case

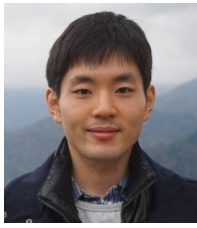
for Wang et al.'s algorithm since it reports significant errors at test time in Table II, implying its weakness against anomalies.

V. CONCLUSIONS

In this work, we have addressed the problem of self-calibrating an array of same-type 3D field sensors, such as all 3-axis magnetometers or all accelerometers, from an arbitrary movement which can trigger anomalous measurements. For this purpose, we proposed a canonical form of the sensor array model for optimization and developed a multistage robust autocalibration method, yielding a solution that minimizes the array model estimation errors. Our method has consistently achieved better accuracy and precision than other state-of-the-art algorithms on synthetic and real data with outliers. Additionally, we have shown, through ablation study, each added component of the method improves the average solution accuracy. Future work should focus on making the pipeline more efficient to allow faster autocalibration of the sensor arrays while maintaining robustness to gross anomalies.

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