Linear Algebra (MATH 3333) Spring 2009 Section 2
Homework 12
Due: Wed. May. 6, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may *not* use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Reading

Section 8.1

Written Assignment

Total: 100 points

Problem A: (15 points) Let A be a 3×3 matrix with real entries and $p(\lambda)$ its characteristic polynomial. As we did for the 2×2 case in class, make a list of all possible combinations of the number of distinct real eigenvalues and complex eigenvalues with multiplicity. For each case, say whether A (i) is diagonalizable, (ii) may or may not be diagonalizable, or (iii) is not diagonalizable.

Problem B: (20 points) Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(i) Find the eigenvalues and eigenspaces of A.
(ii) Diagonalize A (i.e., write $A = PDP^{-1}$ where D is diagonal)
(iii) Use (ii) to show $A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$.

Problem C: (15 points) Let

$$A = \frac{1}{2}\begin{pmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}.$$

(i) Find the eigenvalues and eigenspaces for A.
(ii) Diagonalize A.
(ii) Use (ii) to compute A^{10}.

Problem D: (15 points) Let $A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ where $x \neq 0$. Show

$$P^{-1}AP = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

for some invertible P. (*Hint:* If you recall what we did in class, you should only need to change the second basis vector to get A into the desired form.)

Problem E: (15 points) Let $A = \begin{pmatrix} 3 & 1 \\ -4 & -4 \end{pmatrix}$. Compute A^{100}.

Problem F: (20 points) Let $c(t)$ and $r(t)$ denote the number of coyotes and roadrunners in your backyard t years from now. Suppose they satisfy the (oversimplified and bad) predator-prey relations $c(t+1) = 0.5c(t) + 0.25r(t)$ and $r(t+1) = -0.5c(t) + 1.25r(t)$.

(i) Find formulas for $c(t)$ and $r(t)$ in terms of the initial populations $c(0)$ and $r(0)$.

(ii) In particular, what will the population of coyotes and roadrunners be in 5 years if $c(0) = 10$ and $r(0) = 100$? What is the long term tendency of this system, i.e., what are the limits of $c(t)$ and $r(t)$ as $t \to \infty$?