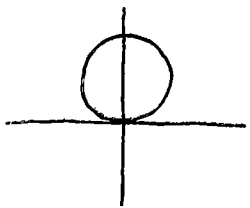


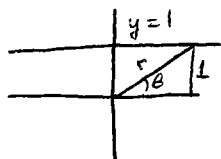
1.



parametric :  $x = \cos t, y = 1 + \sin t$  or  $x = \sin t, y = 1 + \cos t$

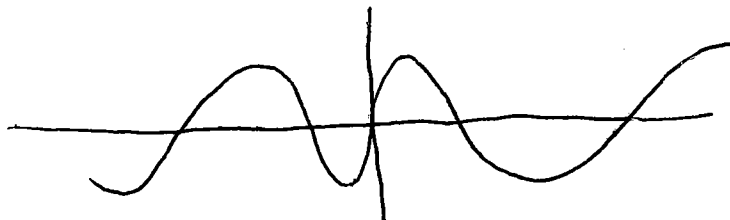
polar :  $r = 2 \sin \theta$

2.



$$\sin \theta = \frac{1}{r} \Leftrightarrow r = \frac{1}{\sin \theta} = \csc \theta, \quad 0 < \theta < \pi$$

3.



$$\begin{aligned} x' &= 3t^2 \\ y' &= \cos t \end{aligned}$$

$$t = \pi/4 \Rightarrow \frac{dy}{dx} = \frac{1/\sqrt{2}}{3(\pi/4)^2} = \frac{16}{3\sqrt{2}\pi^2}$$

$$\text{tang. line @ } t = \pi/4 : y - \frac{1}{\sqrt{2}} = \frac{16}{3\sqrt{2}\pi^2} \left( x - \frac{\pi^3}{4^3} \right)$$

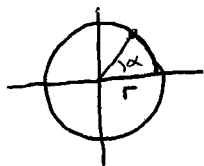
$$x' = 0 \Leftrightarrow t = 0$$

$$y' = 0 \Leftrightarrow t = \pi/2 + n \quad (n = \dots, -3, -2, -1, 0, 1, 2, 3, \dots)$$

vert. tang @  $t = 0$ , i.e.  $(x, y) = (0, 0)$ .

horiz. tang @  $t = \pi/2 + n$ , i.e.  $(x, y) = \left( 3\left(\frac{\pi}{2} + n\right)^3, \cos\left(\frac{\pi}{2} + n\right) \right)$

4.



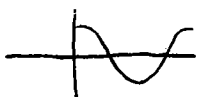
$$\text{parametric : } \begin{aligned} x &= r \cos t \\ y &= r \sin t \end{aligned} \Rightarrow \text{arclen} = \int_0^\alpha \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_0^\alpha \sqrt{r^2 \sin^2 t + r^2 \cos^2 t} dt$$

$$= \int_0^\alpha r dt = r\alpha$$

$$\text{polar : } r = r_0 \Rightarrow \text{arclen} = \int_0^\alpha \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \int_0^\alpha \sqrt{r_0^2 + 0} d\theta = \int_0^\alpha r_0 d\theta = \alpha r_0$$

5.

 $\cos \theta$ 


$$r = 1 + \cos \theta$$



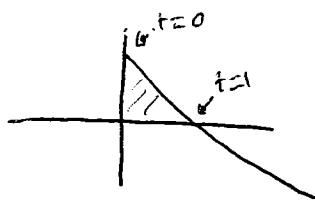
(cardioid)

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\text{area} = \int_0^{2\pi} \frac{1}{2} r^2 d\theta = \frac{1}{2} \int_0^{2\pi} (1 + \cos \theta)^2 d\theta = \frac{1}{2} \int_0^{2\pi} [1 + 2\cos \theta + \cos^2 \theta] d\theta$$

$$= \frac{1}{2} \cdot 2\pi + \int_0^{2\pi} \cos \theta + \frac{1}{4} \int_0^{2\pi} (1 + \cos 2\theta) d\theta = \pi + \frac{1}{4} \int_0^{2\pi} d\theta = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

6.



$$\begin{aligned} \text{area} &= \int_{t=0}^{t=1} y dx = \int_0^1 (1-t^2)(1+2t) dt = \int_0^1 (1+2t-t^2-2t^3) dt \\ &= \left[ t + t^2 - \frac{t^3}{3} - \frac{t^4}{2} \right]_0^1 = 1 + 1 - \frac{1}{3} - \frac{1}{2} = \frac{7}{6} \end{aligned}$$