## Homework 1

Due: September 27, 2022

Note: For Problem #1, write your answers into a Jupyter Notebook file and submit the file.

Total points: 8

- 1. (2) Using Python, perform the following for the KOSPI index returns during the period 2021:01:02-2021:12:30 (on the basis of daily, closing prices; data available at https://ecos.bok.or.kr/flex/EasySearch.jsp).
  - (a) Plot the sample autocorrelation function of the simple returns of the KOSPI index (log-differences of the index). Do they indicate serial correlation?
  - (b) Test the null of no serial correlation using the Ljung-Box test at the 5% level. Set the lag length at 10.
- 2. (3) <sup>1</sup>Suppose that the daily log return of a security follows the model

$$r_t = 0.01 + 0.2r_{t-1} + a_t$$

where  $\{a_t\}$  is a Gaussian white noise series with mean zero and variance 0.02.

- (a) What are the mean and variance of the return series  $r_t$ ?
- (b) Compute the lag-1 and lag-2 autocorrelations of  $r_t$ .
- (c) Assume that  $r_{100} = -0.01$ , and  $r_{99} = 0.02$ . Compute the 1- and 2-step ahead forecasts of the return series at the forecast origin T = 100. What are the associated standard deviations of the forecast errors?
- 3. (1) Suppose that  $r_t$  is represented by the AR(2) process

$$r_t = 1.1r_{t-1} - 0.4r_{t-2} + a_t, \ a_t \sim WN(0, \sigma^2).$$

Is this process stationary?

4. (1) Consider the process

$$r_t = 1 + \sum_{j=0}^{\infty} 0.2^j a_{t-j}, \ a_t \sim WN(0, \sigma^2).$$

Is this process stationary?

<sup>&</sup>lt;sup>1</sup>This is from Chapter 2 of Tsay's book.

5. (1) Find the ACF function of the  $\mathrm{MA}(2)$  process

$$r_t = a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}, \ a_t \sim WN(0, \sigma^2).$$