Financial Econometrics | In Choi
Sogang Universtiy | Spring 2022

Homework 4

Due: November 10, 2022

1. (6 points) Consider the model

$$
\begin{aligned}
y_t &= y_t^* + \varepsilon_t, \\
y_t^* &= \mu + \phi_1 y_{t-1}^* + \zeta_t + \theta \zeta_{t-1}
\end{aligned}
$$

   for $t = 1, \ldots, n$, where y_t is the observed time series and μ is an unknown constant. The disturbances $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\zeta_t \sim N(0, \sigma_\zeta^2)$ are mutually and serially independent at all times and lags. Assume $\{y_t\}$ is stationary.

   (a) Represent this model in the state space form.

   (b) State the recursive relations for the Kalman filter.

   (c) State the recursive relations for the Kalman smoother.

2. (4 points) Assume the local linear trend model for Boston's monthly temperature data uploaded in the class website. Plot the Kalman filter and smoother for the data (Use either Python or EViews; see https://www.statsmodels.org/stable/examples/notebo for the Python code).