Date.

2.

(a)

$$\mu(r_t) = E[r_t] = E\left[\frac{0.01}{1-0.2} + \sum_{j=0}^{\infty} 0.2^j a_{t-j}\right]$$
$$= E\left[\frac{0.01}{1-0.2}\right] + E\left[\sum_{j=0}^{\infty} 0.2^j a_{t-j}\right]$$
$$= \frac{0.01}{1-0.2} = 0.0125$$

$$Var(r_t) = \frac{\sigma^2}{1-0.2^2} = \frac{\sigma^2}{1-0.04} \simeq 1.04\sigma^2$$

(b)

$$\rho(1) = Corr(r_t, r_{t-1}) = \frac{\gamma(1)}{\gamma(0)} = \phi_1^1 = 0.2$$
$$\rho(2) = Corr(r_t, r_{t-2}) = \frac{\gamma(2)}{\gamma(0)} = \phi_1^2 = 0.04$$

(c)

$$\hat{r}_{101} = \phi_1 r_{100} = 0.2 \times -0.01 = -0.002$$
$$\hat{r}_{102} = \phi_1 \hat{r}_{101} = 0.2 \times -0.002 = -0.0004$$
$$e_{100}(1) = r_{101} - \hat{r}_{101} = a_{101} \sim WN(0, \sigma^2)$$
$$e_{100}(2) = r_{102} - \hat{r}_{102} = \phi_1 a_{101} + a_{102} \sim WN(0, (1+\phi_1^2)\sigma^2)$$
$$\Rightarrow e_{100}(k) = r_{100+k} - \hat{r}_{100+k} = \phi_1^{k-1} a_{100} + \cdots + a_{100+k} \sim WN(0, (1+\phi_1^2+\cdots+\phi_1^{2(k-1)})\sigma^2)$$

3.

The characteristic equation of r_t is

$$1 - 1.1z - 0.4z^2 = 0.$$

The roots is $z_1 = \frac{1.1+\sqrt{2.81}}{0.8} \simeq 3.47$, $z_2 = \frac{1.1-\sqrt{2.81}}{0.8} \simeq -0.72.$

$|z_2| \leq 1$, so the process is not stationary.

4.

Since $\sum_{j=0}^{\infty} 0.2^j$ is absolutely summable $\left(\sum_{j=0}^{\infty} 0.2^j = 1.25\right)$, the process is stationary.

5.

$$\rho(1) = Corr(r_t, r_{t-1}) = \frac{\gamma(1)}{\gamma(0)} = \frac{\theta_1 + \theta_1\theta_2}{1+\theta_1^2+\theta_2^2}.$$

$$\rho(2) = Corr(r_t, r_{t-2}) = \frac{\gamma(2)}{\gamma(0)} = \frac{\theta_2}{1+\theta_1^2+\theta_2^2}$$

$$\rho(3) = Corr(r_t, r_{t-3}) = \frac{\gamma(3)}{\gamma(0)} = \frac{0}{1+\theta_1^2+\theta_2^2}$$

$\vdots$

$$\rho(k) = Corr(r_t, r_{t-k}) = 0 \quad (k \geq 3)$$

ACF for MA(2) process