CSI 2110
Assignment 1

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Question 1

2^{n+a} is $O(n^2)$ for $a>0$

$\log\log(n) < \log(n) < n^a < 2^n$ or $b^n < n! < b^{b^n}$

$\rightarrow$ Asymptotically $\rightarrow a > 0$ (int)
$\rightarrow b > 0$ (int)

$\boxed{\text{False}}$

2^{2n+a} is $O(n^2)$ for $a>0$

Exp >> polyn. but $2^{2n+a} >> n^2$

$\rightarrow$ Asymptotically

$\boxed{\text{False}}$

$\sum_{k=1}^{n} k^3$ is $O(n^4)$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2}{4}(n+1)^2$$

$$= \frac{1}{4}(n^4 + 2n^3 + n^2) \cdot O(n^4)$$ $\boxed{\text{True}}$

$\sum_{k=1}^{n} k^3$ is $\Theta(n^3)$

If the upper bound of $\sum_{k=1}^{n}$ is $O(n^4)$ it can never be $\Theta(n^3)$

Θ is only true if the upper and lower bound is the same

$(2n+8)\log(n^{10})$ is $O(n\log(n))$

$= (2n+8)\log[\log n_1 + \log n_2 + \log n_3 + \log n_4 + \dots + \log n_{10}]$

$= (2n+8) \times (10 \log(n))$

$= 20n\log(n) + 80\log(n)$

$< 21n\log(n)$ which is $O(n\log(n))$

for $c = 21$, it is $\boxed{\text{True}}$

$(2n+8)\log(n^{10})$ is $\Theta(n\log(n))$

Lower bound is the same as the upper bound so see a)

$\boxed{\text{True}}$

Question 3

There will be $\log_2 n$ recursive calls in the worst case and a maximum of 3 comparison in each call.

$3\log_2 n + 1 \rightarrow$ the 1 is for the final 1 comparison

a) # of comparisons $= 3\log_2(8)+1$
$= 10$

b) # of comp. $= 3\log_2(16)+1$
$= 13$

c) $T(n) = T(n/2) + 3$, $T(1) = 1$
so $O(\log_2(n))$

Question 3

a) Finds the max length of the subarray where all elements are arranged in ascending order

b) Best : O(n)
Worst : O(n²)

c) Best: array is already sorted in ascending order
Worst: array is sorted in descending order

Question 4

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duplicates (array):
for i to length
    for j = i+1 to length
        if (array[i] == array[j])
            return false
return true
```

$\Theta(n^2)$