

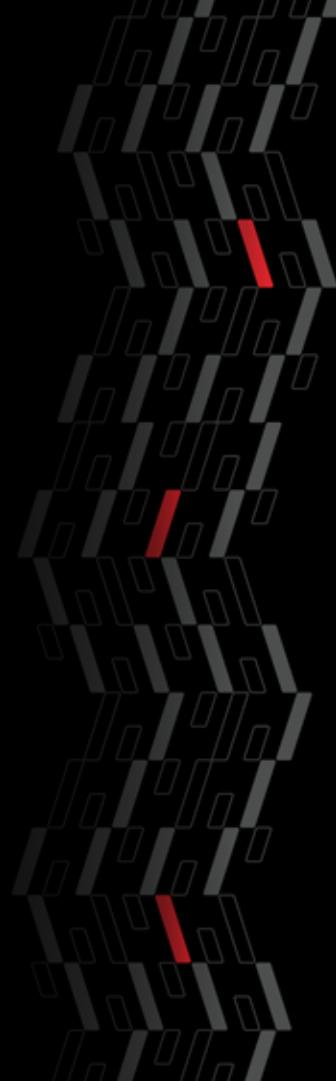
CAPM: β Time-Variation & Implications

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AGENDA

- Introduction
- Background
- Analysis
- Summary
- Conclusion



Introduction

- Capital Market Pricing Model (CAPM) can be a useful tool for modeling excess returns, and have historically been a cornerstone of finance
- Under CAPM, a critical assumption is the stability of model coefficients, specifically β 's (an assets systematic risk), and α 's are constant over time.
- As shown in class for many companies it does not appear that β is constant over time.
- This leads to the question: **Can the variation in β be explained by through estimation error? Or is this variation a flaw in the assumption? How does this affect real-world applications of CAPM?**



Background

- Under CAPM, the security characteristic line to model excess returns can be written as:

$$R_{j,t} - \mu_{f,t} = \alpha_j + \beta_j(R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

Where at time t,

β_j = Aggressiveness of an asset (quantification of market risk),

$(R_{j,t} - \mu_{f,t})$ = Excess return on the jth asset,

$(R_{M,t} - \mu_{f,t})$ = Excess return on the market,

$\epsilon_{j,t}$ = Idiosyncratic shock for jth asset

And α_j is typically set to 0 to avoid arbitration.



Background

$$R_{j,t} - \mu_{f,t} = \alpha_j + \beta_j(R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

- α and β can be estimated by a time-series regression of the excess return on the j^{th} asset on the excess return on the market.
- But, under CAPM, α and β are assumed to be constant over time, while often in practice it appears that both fluctuate over time, which can lead to implementation issues and poor empirical performance.
- Is this coefficient fluctuation in different windows estimation error surrounding a single true β ? Or is this evidence that the true β itself changes?



Analysis

Analysing beta trends of four popular stocks:

Apple (AAPL), Amazon (AMZN),
Microsoft (MSFT), and Qualcomm (QCOM)



Sample Betas

Betas from CAPM derived from 5 years of market data, all with significance levels < 0.0001.

$$Models: R_{j,t} - \mu_{f,t} = \alpha_j + \beta_j(R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

AAPL



1.19



Standard Error:
0.0301

AMZN



1.43



Standard Error:
0.0391

MSFT



1.09



Standard Error:
0.0274

QCOM



1.51



Standard Error:
0.0439

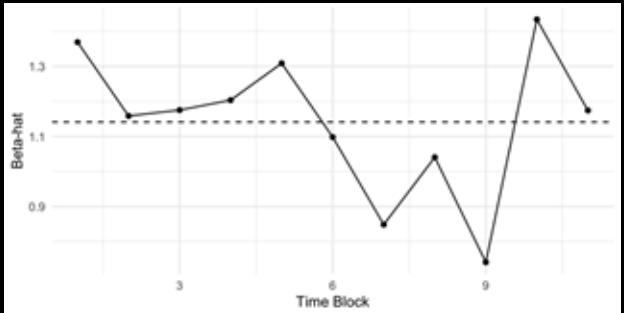


Blocked Betas

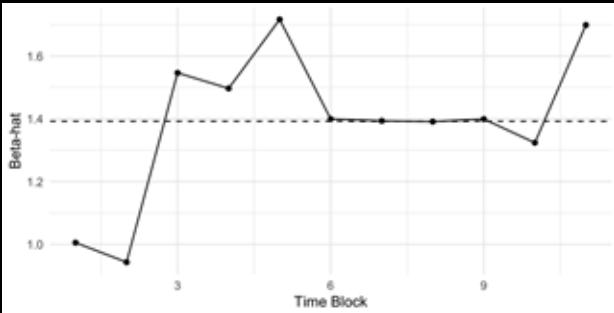
- Stock data split into five different day-increments (10, 30, 90, 120, 240)
- Linear regression performed to calculate betas for each block increment
 - Excess Asset Return \sim Excess Market Return * Block Indicator
 - Compare betas from the same block splits
 - Multiple time increments allows to see consistency in variation

Beta trends over time

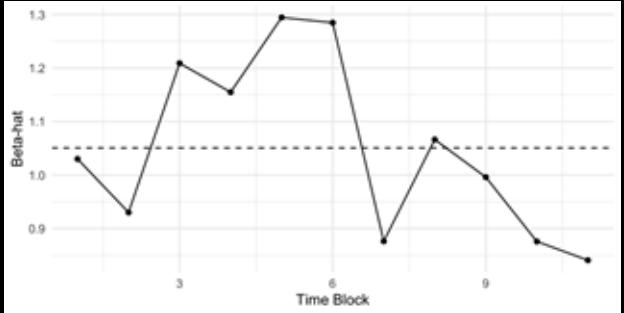
AAPL



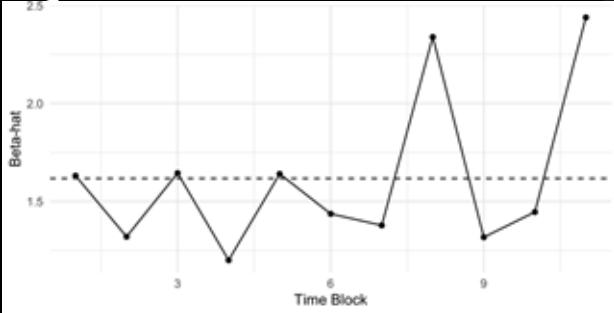
AMZN



MSFT

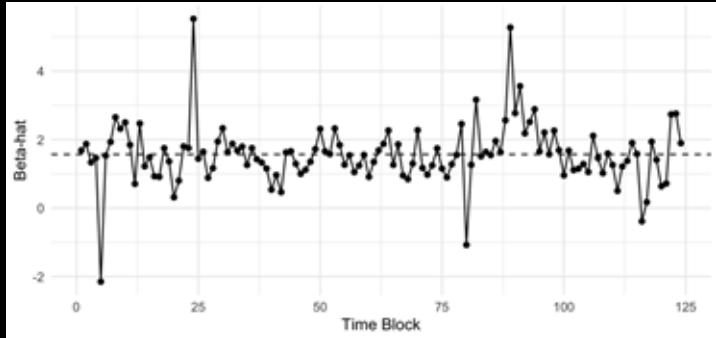


QCOM

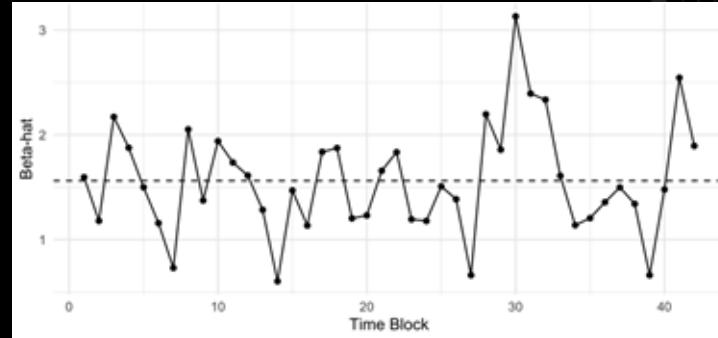


Beta trends over time by block size - QCOM

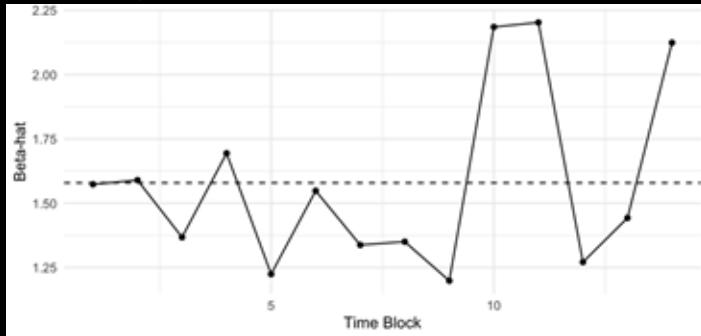
10-Day



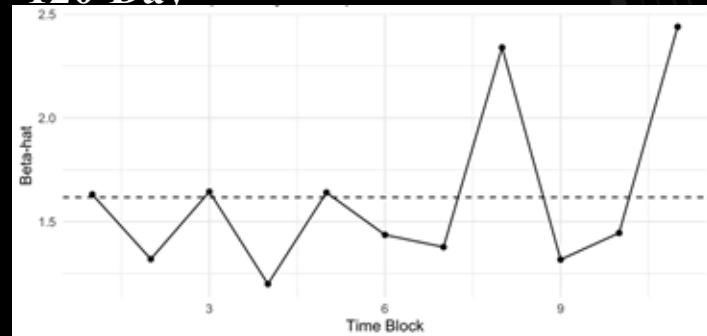
30-Day



90-Day



120-Day



Testing

- These betas do not appear to be constant over time blocks. To statistically test this, we will use a likelihood ratio test.

Model: $\hat{\beta}_j = \beta_j + e_j$

Null Hypothesis (H_0): $\beta_j = \dots = \beta_M$

Alternative Hypothesis (H_A): At least one $\beta_j \neq \beta_M$

- If at least one partitioned β_j is not equal to the entire model coefficient β_M controlling for estimation error, then we can conclude that the β for that stock is not constant over time.

Results

- ANOVA Performed on Full Model (single beta for entire sample) versus Restricted Model (one beta per time block) for each stock, by block sizes:

Ticker	240-day blocks (p)	120-day blocks (p)	90-day blocks (p)	30-day blocks (p)	10-day blocks (p)
AAPL	0.216	0.000100 **	0.000222 **	0.0121 *	0.0212 *
AMZN	0.00855 *	0.108	0.0612	0.0000635 **	0.0539
MSFT	0.000692 **	0.00625 *	0.000900 **	0.0153 *	0.0475 *
QCOM	0.0223 *	0.000171 **	0.00119 *	0.00505 *	0.103



Results

- Across most horizons and stocks, the variation observed in beta is statistically significant and therefore unlikely to be explained by estimation error alone.
- This indicates that, in practice, many stocks do not maintain a stable level of market sensitivity even over relatively short periods, underscoring that the constant-beta assumption embedded in the CAPM is often violated.
- This implies that CAPM-based risk estimates and performance evaluations can be materially misleading unless beta is treated as horizon-specific and time-varying.



Implications

- CAPM treats betas as static in order to maintain mathematical consistency and simplicity, but this assumption is often not met.
- This assumption amplifies the limitations and empirical pitfalls of the classic CAPM Model. Per MIT Sloan research, CAPM mis-valuations are 'significant' and provide inaccurate risk estimates for an asset.
- More complex models such as Fama-French 3 factor model attempt to adjust for these limitations of the classic CAPM model.



Conclusion

- Found support that the assumption that betas are non-time-variant in the CAPM model is unrealistic
- Study could be improved upon by choosing a larger sample of stocks from more diverse sectors, rather than just popular stocks today. Additionally investigating what may cause these variations in betas.
- Other models with less strict assumptions of time-variant variables may be better fit for modeling the complex behavior of financial markets



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