

### Problem 1

A → 4 days unsuccessful

B → 3 days unsuccessful

C → 1 day successful

$t_i$  → trial  $i$

D → Number of days

Solving for  $E[D]$

$$\begin{aligned} E[D] = & E[D | t_1 = A, t_2 = B] P(t_1 = A, t_2 = B) + \\ & E[D | t_1 = A, t_2 = C] P(t_1 = A, t_2 = C) + \\ & E[D | t_1 = B, t_2 = A] P(t_1 = B, t_2 = A) + \\ & E[D | t_1 = B, t_2 = C] P(t_1 = B, t_2 = C) + \\ & E[D | t_1 = C] \end{aligned}$$

$$E[D] = 8 \cdot \left(\frac{1}{3} \cdot \frac{1}{2}\right) + 5 \cdot \left(\frac{1}{3} \cdot \frac{1}{2}\right) + 8 \cdot \left(\frac{1}{3} \cdot \frac{1}{2}\right) + 4 \cdot \left(\frac{1}{3} \cdot \frac{1}{2}\right) + 1 \cdot \left(\frac{1}{3}\right)$$

$$E[D] = \frac{8}{6} + \frac{5}{6} + \frac{8}{6} + \frac{4}{6} + \frac{2}{6} = \boxed{\frac{27}{6}}$$

$$\text{Var}[D] = E[D^2] - (E[D])^2$$

$$\text{Var}[D] = 8^2 \left(\frac{1}{6}\right) + 5^2 \left(\frac{1}{6}\right) + 8^2 \left(\frac{1}{6}\right) + 4^2 \left(\frac{1}{6}\right) + 1^2 \left(\frac{1}{3}\right) - \left[\frac{27}{6}\right]^2$$

$$\text{Var}[D] = \frac{64}{6} + \frac{25}{6} + \frac{64}{6} + \frac{16}{6} + \frac{2}{6} - \frac{729}{36}$$

$$\text{Var}[D] = \frac{171}{6} - \frac{729}{36} = \frac{1026}{36} - \frac{729}{36} = \frac{297}{36} = \frac{99}{12} = \boxed{\frac{33}{4}}$$