Assignment 3

Due Friday, February 22 at 11:59 pm.

Please submit all work on Canvas as a PDF or Word file. Make sure you clearly label each solution, and include the answers to the in-class quiz at the beginning of the assignment. Scan or take a clear picture of any decisions trees to include it in your submission.

Part 1: Sensitivity Analysis

1) Split or Steal Recall the "Split or Steal" game:

> Two players each have the option of choosing either **Split** or **Steal**.
>
> - If both players choose **Split**, then they split the jackpot and get $50,000 each.
> - If one chooses **Split** and the other chooses **Steal**, then the player who chose **Steal** takes all $100,000, and the player who chose **Split** gets nothing.
> - If both players choose **Steal**, then both players get nothing.

Suppose you play a round of this game.

(a) **You think there is a 50-50 chance of your opponent choosing either "Split" or "Steal". Draw the decision tree for this problem and report the decision you should make based on the expected value criteria. (2 points)**

(b) **Suppose you think there is an 20% chance of your opponent choosing "Steal" instead. What decision should you make based on the expected value criteria? Make sure you show your calculations of all expected values. (2 points)**

(c) **Create a strategy table showing which decision you should make based as the probability of your opponent choosing "Split" goes from 0 to 1 in increments of 0.1. (3 points)**

2) Lottery Suppose that you are deciding whether to buy a $1 lottery ticket. The jackpot is $2 million and there is a 1/1,000,000 chance of winning. In addition, you may choose to receive the jackpot as a lump sum immediately or in yearly payments over 10 years. If you choose the lump sum, you get 60% of the money all at once. If you choose the yearly payments, you will receive 75% of the money over 10 years of equal payments with the first payment made in one year. The interest rate is 3% per year.

(a) **Draw the decision tree for this problem. (2 points)**

(b) **Make a strategy table of what you decision you should make as you change the jackpot from $1.5 million to $2.5 million in increments of $100,000, and as you change the interest rate from 0 to 5% in increments of 1%. Show the work for at least one cell in the table, then show the table with all decisions. (5 points)**

3) Investments You are considering 4 investments, A, B, C, D. The payoff from each investment is a function of the economic climate over the next 2 years. The economy can be weak or strong. You think that the probability of a declining economy is 50% and an expanding economy is 50%. Since you are not an expert on market tendencies, you also have the option of hiring a consultant. Suppose if the consultant's research suggests optimism about the economy, then the probability of a declining economy is 10% and an expanding economy is 90%, whereas if the consultant's research gives reasons to be pessimistic, the probability of a declining economy is 60% and an expanding economy is 40%. The payoff matrix below shows the outcomes for each scenario, in thousands of dollars.

Investment	Weak Economy	Strong Economy
A	-30	120
B	20	60
C	30	35
D	15	30

(a) **Assuming that you don't hire the consultant, draw the decision tree for this problem. What decision should you make? (2 points)**

(b) **If the consultant's research suggests optimism about the economy, what decision should you make? What if the research suggests pessimism? Show all expected value calculations that you used to reach your answer. (4 points)**

(c) **Suppose you think there's a 20% chance that the consultant comes back with an optimistic report, and an 80% chance they come back with a pessimistic report. What is the Expected Value with Sample Information? (2 points)**

(d) **If the consultant charges $5,000, would it be worth hiring them to determine whether the research suggests optimism or pessimism? (2 points)**

Part 2: Bayes' Theorem

4) Monty Hall Problem Consider the following scenario:

> As part of a game show, you are presented with three identical closed doors: A, B, and C. Two of the doors lead to a goat, while the third contains a car. You must choose one of the doors. Then, the host reveals one of the other two remaining doors containing a goat. You then have the choice of switching doors, or sticking with the door you chose. For example, suppose you chose door A. Then, the host might reveal door B to contain a goat, then offer you the choice of switching to door C or sticking with door A.

For the purposes of this problem, assume that there is no difference among the doors (that is, each door is equally likely to be chosen as the one to contain the car by the game show hosts). Since the door label doesn't matter, without loss of generality, assume that you choose door A and the host reveals door B.

(a) **What is the probability that any given door contains the car? What about the goat? Express the probabilities as P(A=car), P(A=goat), etc. (Don't overthink this one — it's given in the description.) (2 points)**

(b) **What is the probability that the host reveals door B if the car is behind door A? If the car is behind door B? Door C? Assume that the host chooses randomly if there are two possible doors to reveal. Show the expression of your answers as conditional probabilities. (3 points)**

(c) **What is the probability that the host reveals a goat behind door B? (This is the sum of the probabilities of revealing a goat behind door B for all possibilities of where the car is.) (2 points)**

(d) **What is the probability that the car is behind door A if the host reveals door B? (2 points)**

(e) **Should you switch? Use Bayes' Theorem and the calculations you found in this problem to justify your answer. (2 points)**