1 Assumptions

1. Population is well mixed i.e. there is an equal probability of any two individuals interacting
2. No reinfection i.e. once removed, individuals cannot become susceptible

2 SEIIR Model

$$S \xrightarrow{\alpha_P I_P + \alpha_S I_S} E \xrightarrow{\gamma_P} I_P \xrightarrow{\gamma_S} I_S \xrightarrow{\rho} R$$

2.1 Equations

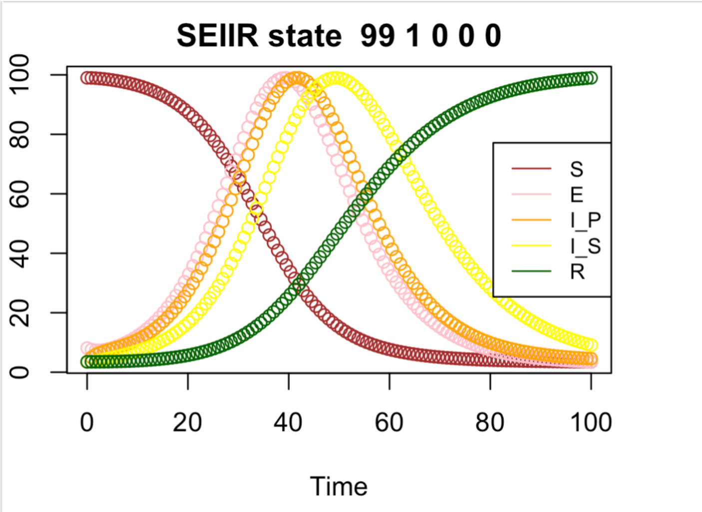
$$\begin{aligned}
\dot{S} &= -(\alpha_P I_P + \alpha_S I_S)\frac{S}{N} \\
\dot{E} &= (\alpha_P I_P + \alpha_S I_S)\frac{S}{N} - \gamma_E E \\
\dot{I_P} &= \gamma_E E - \gamma_P I_P \\
\dot{I_S} &= \gamma_P I_P - \rho I_S \\
\dot{R} &= \rho I_S \\
N &= S + E + I_S + I_P + R
\end{aligned}$$

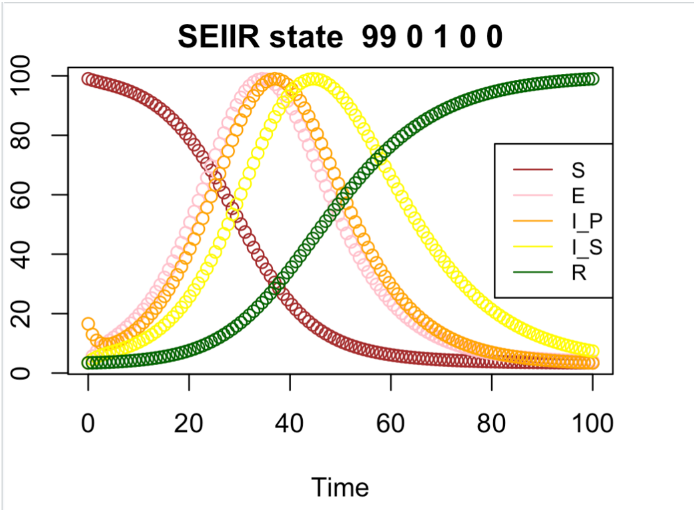
2.2 Parameter Estimates

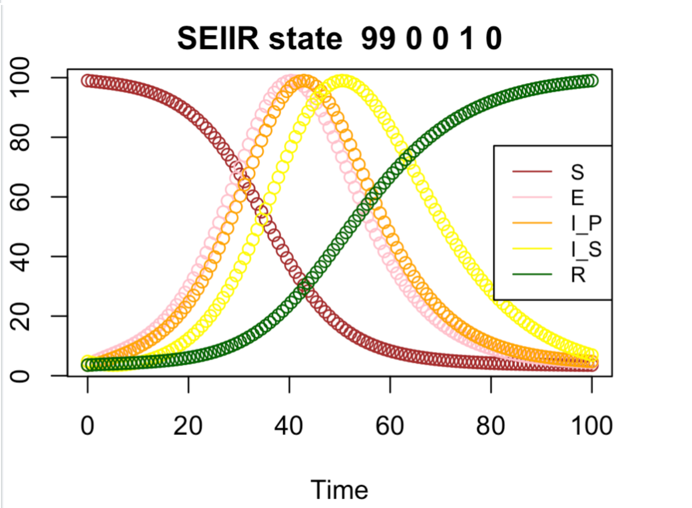
$\alpha_P = 0.6$	presymptomatic rate
$\alpha_S = 0.2$	symptomatic rate
$\gamma_E = 0.16$	inverse incubation period
$\gamma_P = 0.4$	inverse time presymptomatic to symptomatic
$\rho = 0.1$	inverse time until removal
$R_0 = \frac{S_0}{N}(\frac{\alpha_P}{\gamma_P} + \frac{\alpha_S}{\rho})$	rate of transmission per individual

2.3 Modeling in R

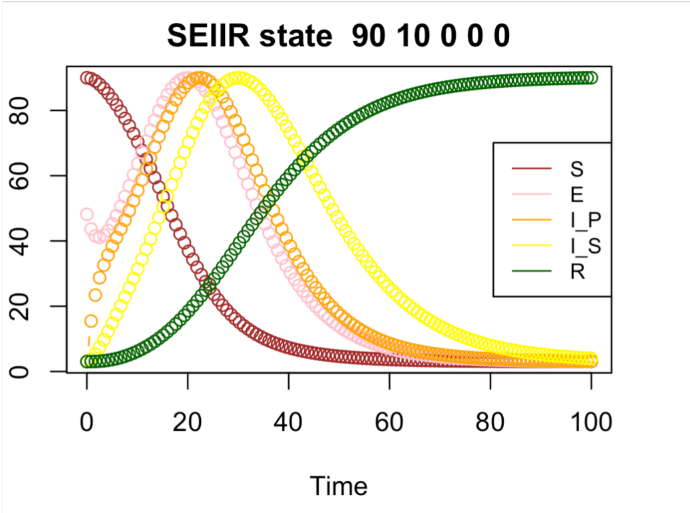
2.3.1 Changes in State

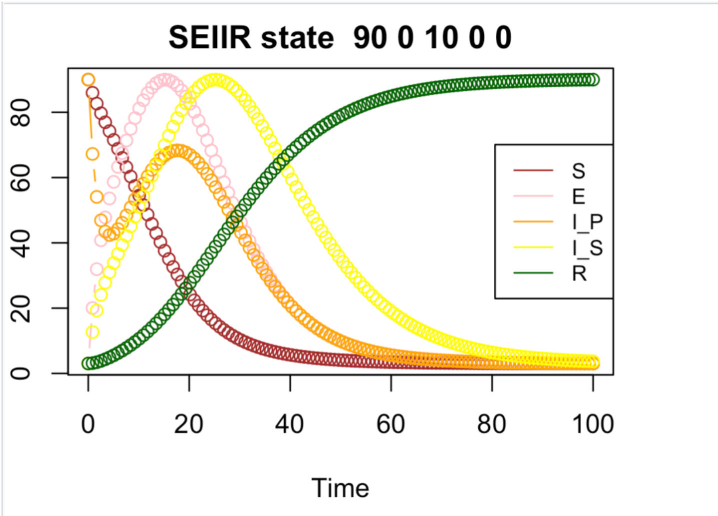


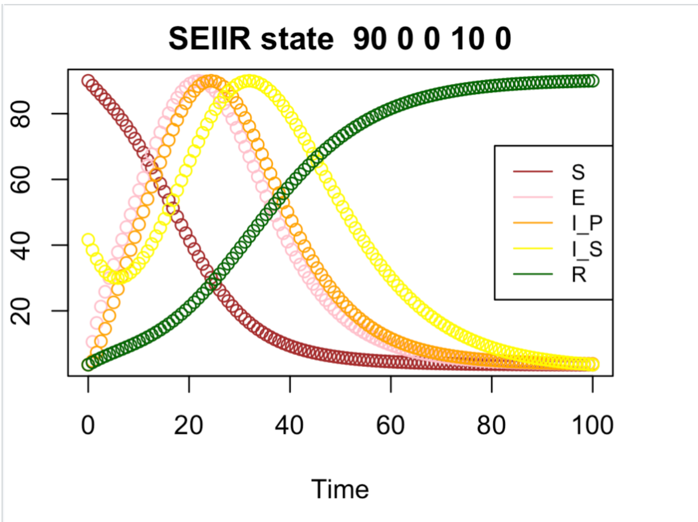




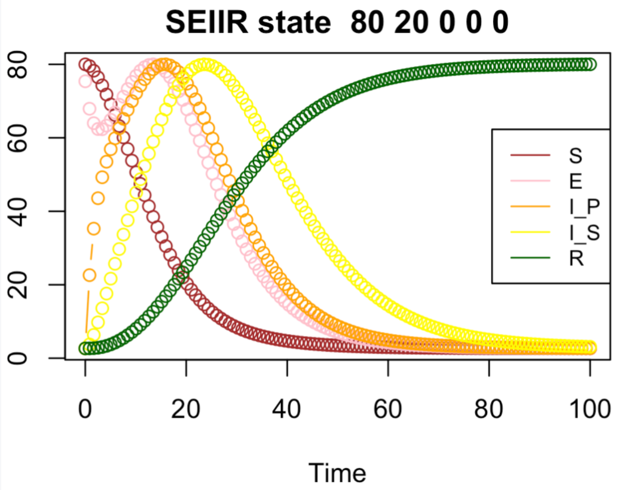
When there is a single individual in either E, I_P, I_S, there is a clear shift in when S and R cross (call this the tipping point). In this simulation, the I_P individual shifts the tipping point to the left.

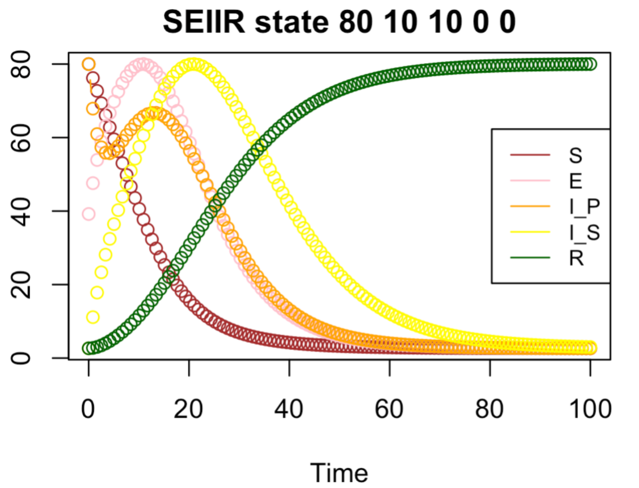


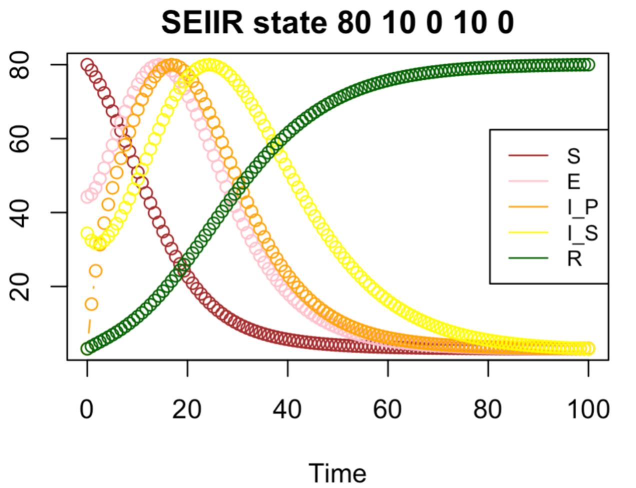


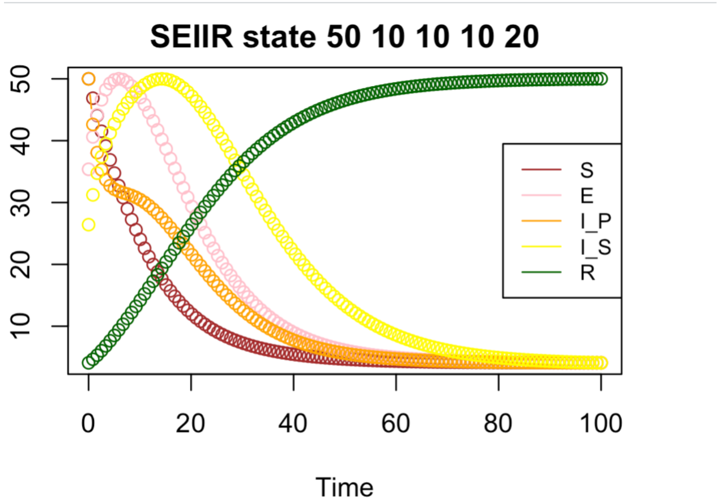


When we increase from a single individual in either E, I_P, I_S to ten individuals, the peaks of each of those states occurs earlier i.e shift to the left. In this simulation, the I_P state also has an earlier tipping point.


SEIIR state 80 20 0 0 0
S
E
I_P
I_S
R
Time


SEIIR state 80 10 10 0 0
S
E
I_P
I_S
R
Time


SEIIR state 80 10 0 10 0
S
E
I_P
I_S
R
Time



The above simulations are interesting because they illustrate how different combinations of individuals within the states can either shift or alter later interactions. Take for example the state 50-10-10-10-20. The y-axis is capped off at 50 (the number of individuals within the S state) and all other subsequent states never surpass that number. Perhaps this is an obvious statement, but it still an interesting one.

2.4 Sensitivity Analysis

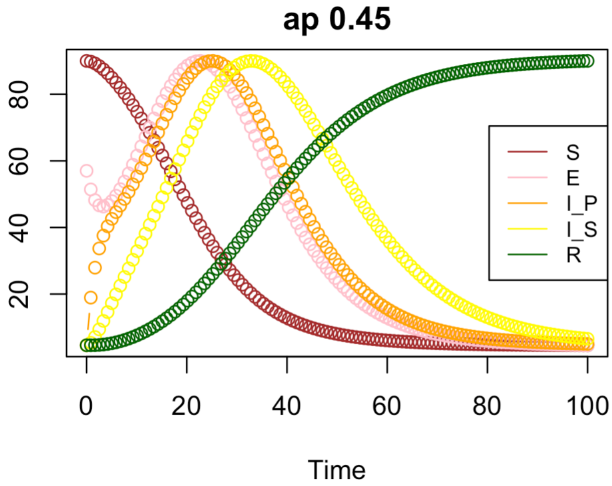
2.4.1 Local

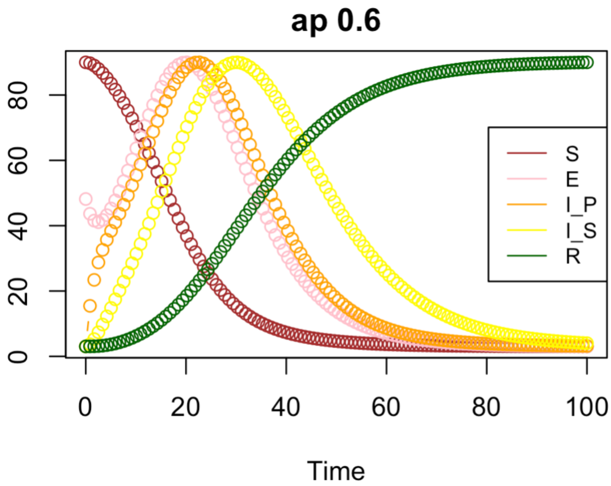
We can do a local sensitivity analysis of R_0 by taking the partial derivatives with respect to each of our parameters and multiply by the proportion of R_0 due to each parameter. This will create a timeless unit change for R_0 with respect to each parameter.

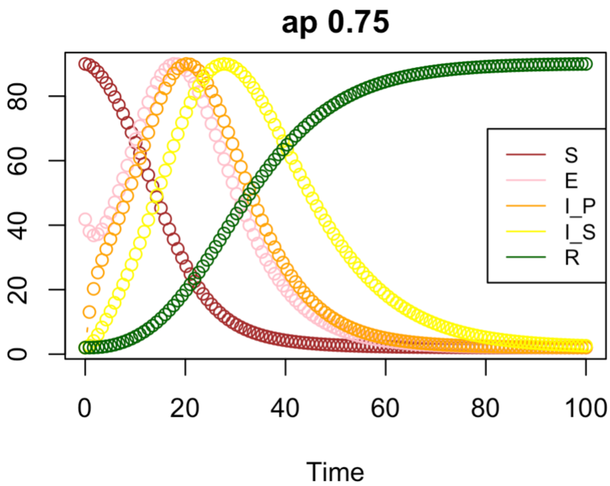
$$
\begin{aligned}
R_0 &= \frac{S_0}{N}\left(\frac{\alpha_P}{\gamma_P} + \frac{\alpha_S}{\rho}\right) \\
&\approx \left(\frac{0.6}{0.4} + \frac{0.2}{0.1}\right) = 3.5 \\
\frac{\delta R_0}{\delta \alpha_P} &= \frac{S_0}{N}\left(\frac{1}{\gamma_P}\right) \\
&\approx \frac{1}{0.4}\left(\frac{\alpha_P}{R_0}\right) = \frac{1}{0.4}\left(\frac{0.6}{3.5}\right) = 0.43 \\
\frac{\delta R_0}{\delta \alpha_S} &= \frac{S_0}{N}\left(\frac{1}{\rho}\right) \\
&\approx \frac{1}{0.1}\left(\frac{\alpha_S}{R_0}\right) = \frac{1}{0.1}\left(\frac{0.2}{3.5}\right) = 0.57 \\
\frac{\delta R_0}{\delta \gamma_P} &= \frac{S_0}{N}\left(\frac{-\alpha_P}{\gamma_P^2}\right) \\
&\approx -\frac{0.6}{0.4^2}\left(\frac{\alpha_S}{R_0}\right) = -\frac{0.6}{0.4^2}\left(\frac{0.4}{3.5}\right) = -0.43 \\
\frac{\delta R_0}{\delta \rho} &= \frac{S_0}{N}\left(\frac{-\alpha_S}{\rho^2}\right) \\
&\approx -\frac{0.2}{0.1^2}\left(\frac{\rho}{R_0}\right) = -\frac{0.2}{0.1^2}\left(\frac{0.1}{3.5}\right) = -0.57
\end{aligned}
$$

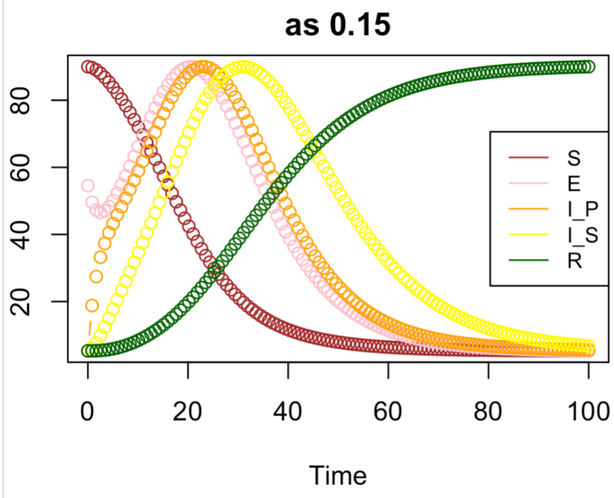
The above calculations show a clear relationship between parameters. Take the outputted value for $\frac{\delta R_0}{\delta \alpha_P} = 0.43$. We would interpret this by saying that an increase α_P by 0.43 of a unit would increase R_0 by one unit.

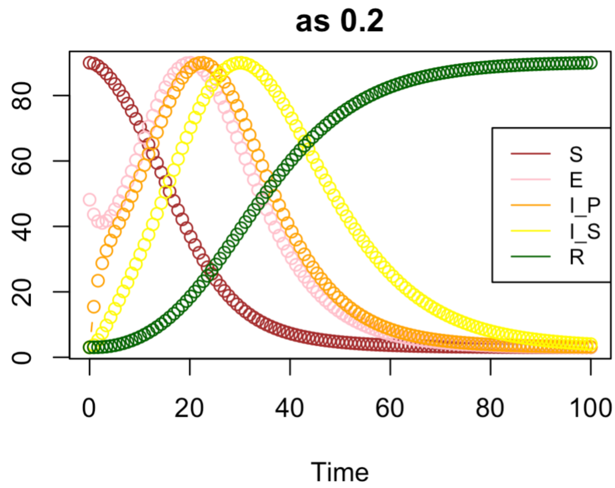
2.4.2 Global

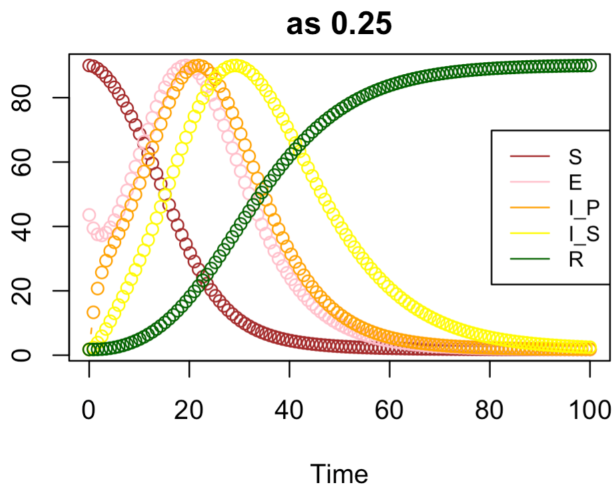


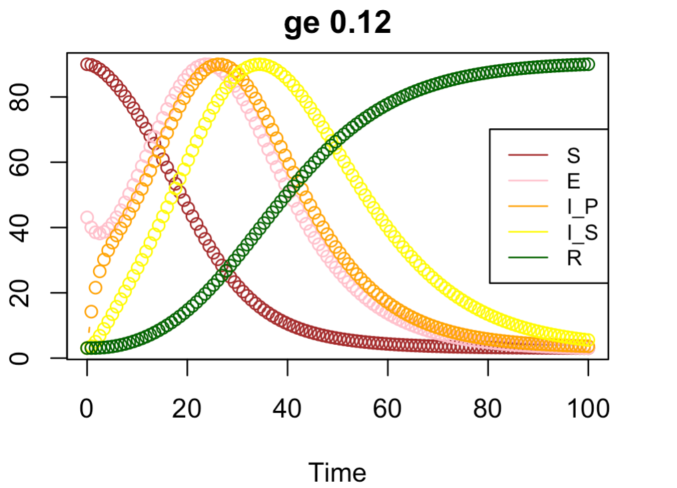


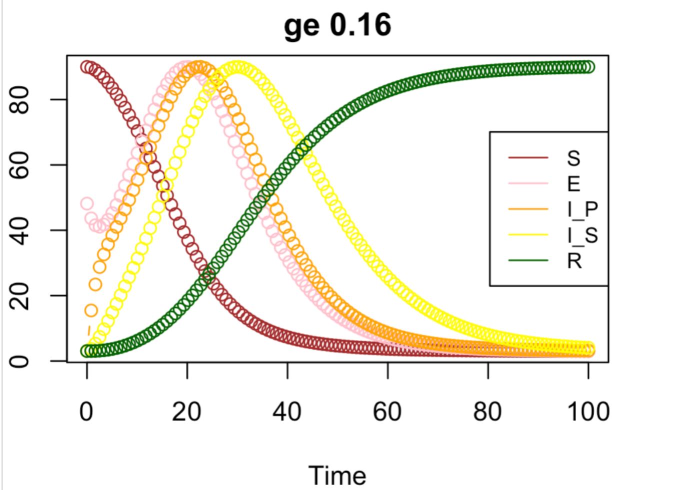


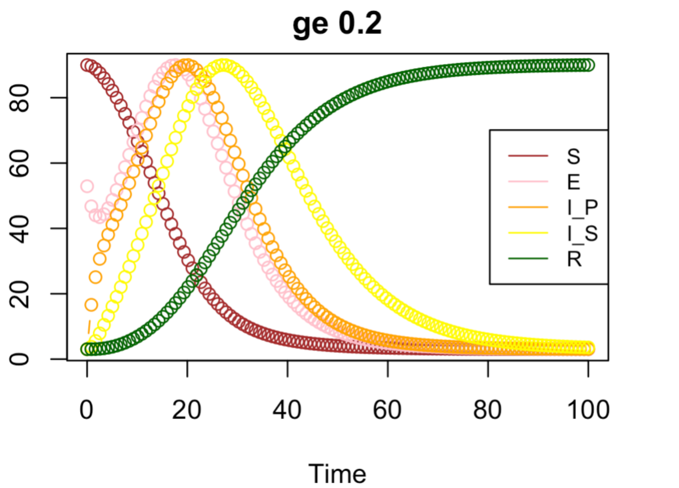

as 0.15
20
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R

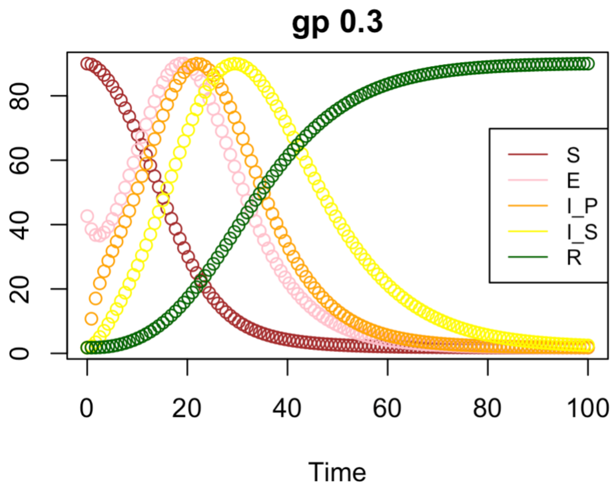

as 0.2
0
20
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R


as 0.25
0
20
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R


ge 0.12
S
E
I_P
I_S
R
Time


ge 0.16
S
E
I_P
I_S
R
Time


ge 0.2
S
E
I_P
I_S
R
Time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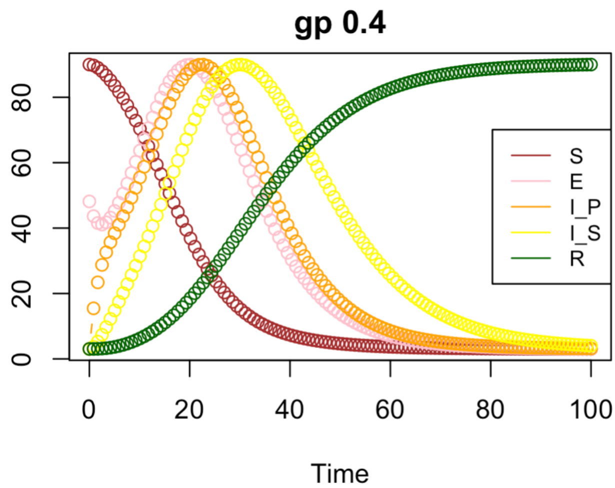
0
20
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R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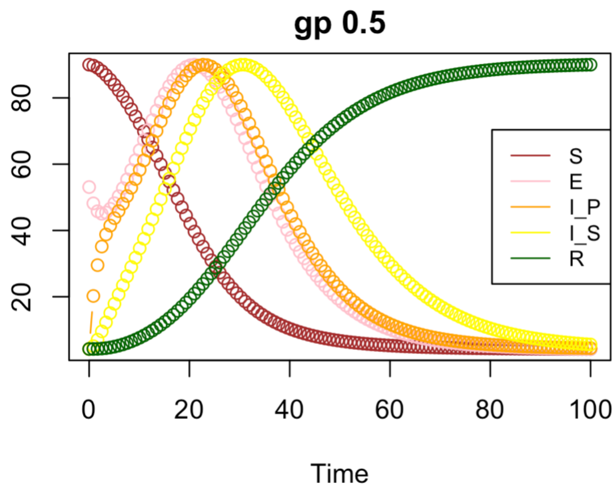
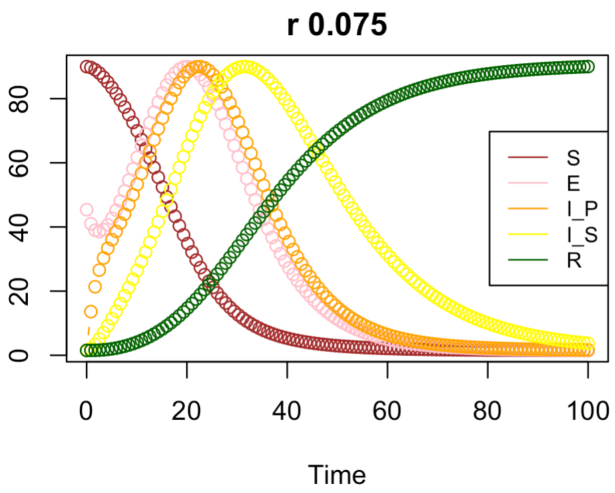
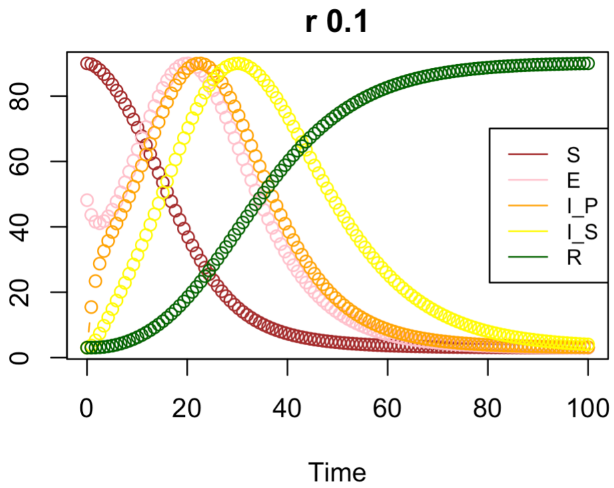
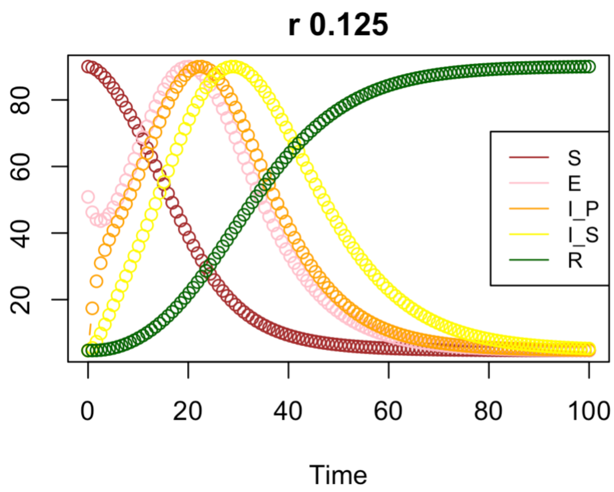
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R


gp 0.5
20
40
60
80
0
20
40
60
80
100
Time
S
E
I_P
I_S
R







3 Code

3.1 Solving Equations

```
library(deSolve)
state<-c(S=90,
         E=10,
```

```
            IP=0,
            IS=0,
            R=0)
N=100
# define parameter values for equation
parameters<-c(ap=0.6,
              as=0.2,
              ge=0.16,
              gp=0.4,
              rho=0.1)

# define DE function or system of DEs
LV<-function(t, state, parameters) {
  with(as.list(c(state, parameters)),{
    # rate of change
    dS <- -(ap*IP+as*IS)*S/N
    dE <- (ap*IP+as*IS)*S/N-ge*E
    dIP <- ge*E -gp*IP
    dIS <-gp*IP -rho*IS
    dR <- rho*IS
    # return the rate of change
    list(c(dS,dE, dIP, dIS,dR))
  })
}
# provide times (10 years, monthly points)
times<-seq(0,100,length.out=120)
# run and store output
out <- ode(y = state, times = times, func = LV, parms = parameters)
# look at output head(out)
head(out)

# plot output from solver
par(mar = c(5,2,2,5)) # Widen's the margin
plot(out[,1],out[,2],type="b",col="brown",ylab="S",xlab="Time",main=paste("SEIIR
par(new = T) #allows plot overlay
plot(out[,1],out[,3], type="b",axes=F, xlab=NA, ylab=NA, col="pink") #plots E
par(new = T)
plot(out[,1],out[,4], type="b",axes=F, xlab=NA, ylab=NA, col="orange") #plots I_
par(new = T)
plot(out[,1],out[,5], type="b",axes=F, xlab=NA, ylab=NA, col="yellow") #plots I_
par(new = T)
plot(out[,1],out[,6], type="b",axes=F, xlab=NA, ylab=NA, col="darkgreen") #plots
legend("right", legend=c("S", "E", "I_P", "I_S", "R"),
       col=c("brown", "pink", "orange", "yellow", "darkgreen"), lty=1, cex=0.8)
```