PROBLEM-SOLVING TUTORIAL

4.4 Solving Equations: Supplemental Exercises

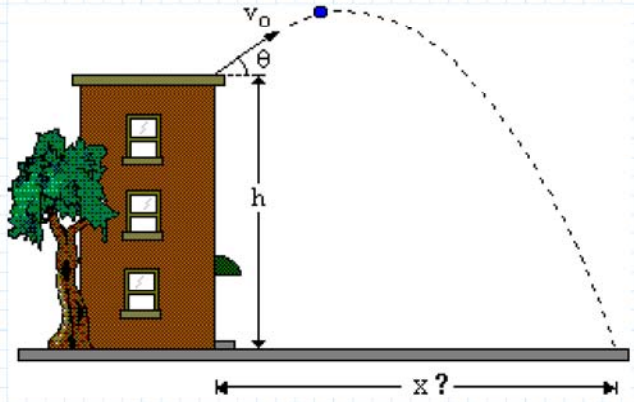
Introduction

In this supplemental section there is an exercise to provide more practice with the skills you learned about in the Tutorial and the first half of the exercises in this section.

Question 1

A student, standing on top a building that is **15 m** high, throws a ball with an initial speed of **5 mph** at an angle of **35 degrees** (with respect to horizontal).

How far horizontally away from the building does the ball land?



Problem SOLVING

Information Given

We have assigned variables to the information contained in the question.

Height of building:	$h := 15$ ***m***
Initial speed of ball:	$v_o := 5$ ***mph***
Direction of throw:	$\theta := 35$ ***deg***

Problem SOLVING

Propose methods

Remember, motion in two dimensions can be broken down into motion in the vertical (**y**) and horizontal (**x**) directions. The speed in the **y** direction determines how long the ball stays in the air and the speed in the **x** direction determines how far the ball travels in that time.

Content TIP

Components of vectors

Define v_{xo} and v_{yo} to be the components of the initial velocity. **Hint:** You may want to define one variable, then make a **copy** and edit it to get the second definition.

$$v_{xo} := v_o \cdot \cos(\theta)$$

$$v_{yo} := v_o \cdot \sin(\theta)$$

Problem SOLV ING

Analyze results

We have already displayed the values at right, so you can check whether your definitions seem to be correct (which component should be larger?)

$$v_{xo} = 4.1 \ mph$$

$$v_{yo} = 2.87 \ mph$$

The rest of the problem is to find the time the ball is in the air and use that time to compute the distance traveled in the **x** direction.

Since you may not recall the expression for the height y of a particle in a gravitational field as a function of time t, we have written it below with the symbolic equals sign.

$$y = y_o + v_{yo} \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

How long does the ball take to reach the ground ?

Problem SOLV ING

Solve Problem

Move the equation into your worksheet window and solve for the time t using the **Symbolics Solve** command on the **Math** tab.

Since time is squared, you will have to choose between the two roots. Click on the outermost operation in the expression you want to highlight, then **copy**.

WHY

Why does everything move down?

$$y = y_o + v_{yo} \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

$$\begin{bmatrix} \frac{1}{g} \cdot \left(v_{yo} + \sqrt{{v_{yo}}^2 - 2 \cdot g \cdot y + 2 \cdot g \cdot y_o}\right) \\ \frac{1}{g} \cdot \left(v_{yo} - \sqrt{{v_{yo}}^2 - 2 \cdot g \cdot y + 2 \cdot g \cdot y_o}\right) \end{bmatrix}$$

Advanced Topic

Derivation format

$$y := 0 \ m \qquad y_o := h$$

$$t := \frac{1}{g} \cdot \left(v_{yo} + \sqrt{{v_{yo}}^2 - 2 \cdot g \cdot y + 2 \cdot g \cdot y_o}\right)$$

HUH

Values for yo and y ?

$$t = 1.88 \ s$$

Redefine the result to be time t .

Note: You will see an error because the variables y and y_o have not been defined. The initial height y_o is the height of the building h . From the original equation, you can see that y is the final height (after the time t has elapsed).

Define the initial and final heights (y_o and y respectively), then locate them above the definition for the time. Use the icon in the margin for help.

Display the value for the time t and judge whether it seems reasonable. If the time is negative, then you choose the wrong root (you can edit the definition).

During the time t the ball is in the air, it is traveling in the **x** direction at the velocity v_{xo}.

Define an equation for the distance d traveled in the **x** direction during the time t.

$$d := v_{xo} \cdot t$$

Display your answer and decide whether it seems reasonable.

$$d = 11.32\ ft$$

Save your changes!

In many problems, you will want to jot down equations, edit them and solve for certain variables before you are ready to calculate an answer using the **definition** symbol. The **symbolic equals sign** = lets you write down anything you want, so you can rough out a problem before you need to think about the calculation.