PROBLEM-SOLVING TUTORIAL

3.2 Variables and Units: Exercises

Introduction

In the preceding tutorial you were shown how to assign values to variables, including units. In the exercises that follow you will practice using variables instead of the values they represent.

As you work, notice how this new approach allows you to concentrate on the concepts. The equations look as they do in textbooks and the calculations, even the unit conversions, are handled by Mathcad. The end result is that you can focus on understanding your material and still have the energy left over to put your answer in perspective.

In addition to practice solving problems, we will show you a technique for solving problems and double-checking your answer, called **dimensional analysis**.

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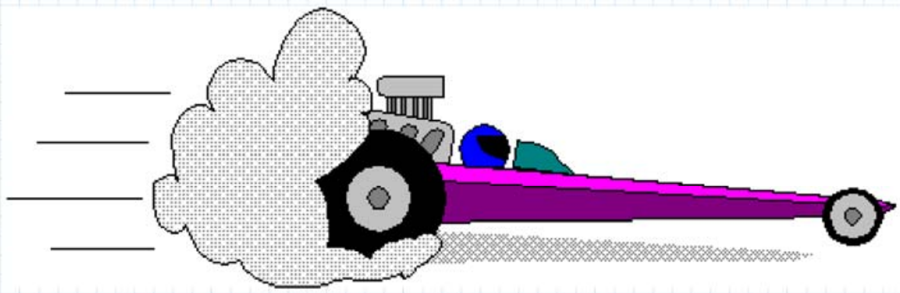
Question 1

A car that can accelerate from **zero** to **60 mph** in **10 seconds** is usually thought to be a very powerful car. How large is this acceleration when compared to the acceleration due to gravity?



Acceleration is a measure of how fast the velocity is changing in time. In our problem, the velocity increased to **60 mph** in **10 seconds**. Let's use the variables Δv and Δt for the changes in velocity and time.

Both variables use the capital Greek letter **delta** Δ.

Defining variables

Assign the variables Δv and Δt to the increase in velocity and the change in time respectively. Remember, you can find in the **Symbols** button from the **Math** tab.

$$\Delta v := 60 \cdot \boldsymbol{mph}$$

$$\Delta t := 10 \cdot \boldsymbol{s}$$

Content TIP

Acceleration

Now define the acceleration a with an equation involving Δv and Δt.

$$a := \frac{\Delta v}{\Delta t}$$

Display the answer below the equation by typing a = .

$$a = 2.682 \ \frac{\boldsymbol{m}}{\boldsymbol{s}^2}$$

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The answer is hard to interpret based on everyday experience. Let's compare it to the acceleration due to gravity g, which is already defined in Mathcad.

Redisplay the value of a in the open space and replace the $m \cdot s^{-2}$ units with the acceleration due to gravity g .

Mathcad will convert the answer to show you how many "**g**'s" the car is accelerating.

$$a = 0.274\ g$$

The car is accelerating at less than a **1/3** the rate due to gravity! You may have thought the acceleration due to gravity was not very large, so let's do a quick conversion.

Display the predefined value of g by typing g = in the space to the right.

Replace the SI units of $m \cdot sec^{-2}$ with $\frac{mph}{s}$ or $\frac{kph}{s}$.

$$g = 21.937\ \frac{mph}{s}$$

Imagine increasing speed that quickly every second! Converting answers in Mathcad lets you judge whether your answers make sense and you get a little more out of doing physics problems in the process.

The next problem is more difficult and introduces a new way to label variables.

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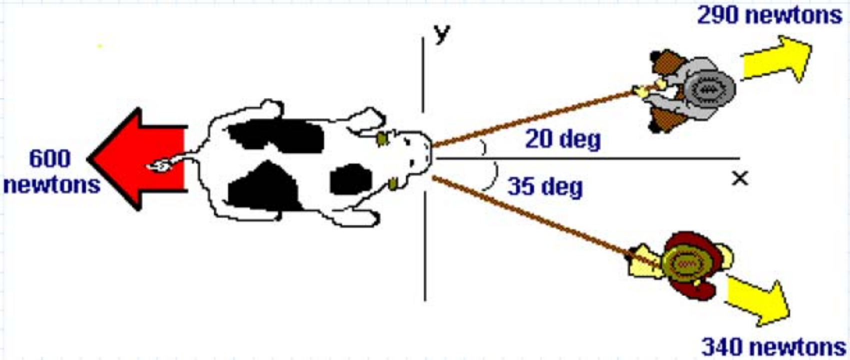
Question 2

Two farmhands are trying to pull a stubborn cow into its corral. The cow is pulling backward, exerting a force of 600 Newtons in the negative x direction on the ground. Each farmhand is pulling at different angles on two separate ropes. The forces exerted on the cow by the farmhands is illustrated in the picture below. Are the farmhands able to move the cow toward the corral or the other way around?



Information Given

This problem is slightly more involved than the last problem, so let's approach it step-by-step:

The first step is to assign variables to all the information given in the question. Start by defining the angles. The usual variable for an angle is the Greek letter **theta** θ, while the second most common is **phi** ϕ. We will use θ and ϕ for the two angles in our problem.

Defining units

In the open space, assign the two angles to the variables θ and ϕ.

Don't forget your units!

Remember, if you forget the *deg* unit on an angle, then Mathcad assumes that the number is in **radians.**

$$\theta := 20\ deg$$

$$\phi := 35\ deg$$

Now assign variables to the forces. Let's keep the farmhands separate by using the variables $F1$ or $F2$ for their forces.

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Radians versus degrees

Assign the variables $F1$ and $F2$ to the forces exerted by each farmhand on the cow.

Remember to use the singular case and lower case letters for your units.

$F1 := 290\ N$

$F2 := 340\ N$

Instead of using the variable $F3$ for the cow's force, we will introduce a more detailed way of naming variables.

Literal Subscripts

Why literal subscripts?

For more descriptive variable names, you can put numbers or letters to the right and below a variable; these are called **literal subscripts**:

x_0 <-- literal subcripts

This subscript notation is often used to distinguish between values of a quantity at different times, such as the initial velocity v_o and final velocity v_f , or to clarify the definition of a variable, for example $F_{friction}$ or m_{ball} .

Defining variables

To get a literal subscript, hold ctrl and then "-" after a variable, and then type the subscript.

Let's assign the force of the cow on the ground to the variable F_{ground} by using literal subscripts. Following the instructions above, you should type:

F (ctrl+-) ground

then assign the variable its value (with the appropriate **units**).

Defining units

$F_{ground} := 600 \cdot \mathbf{\mathit{newton}}$

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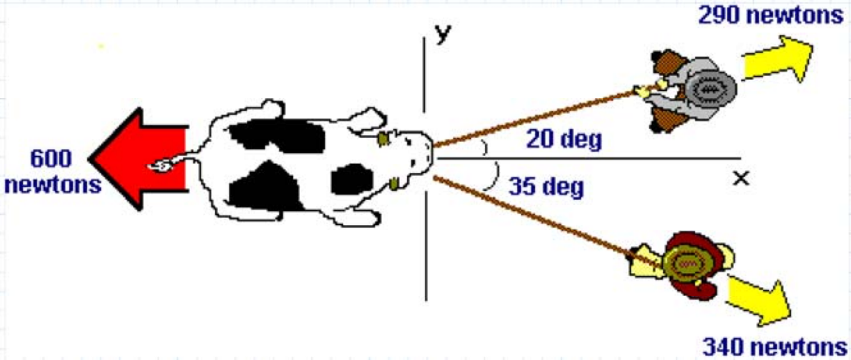
Problem SOLVING

Solve Problem

Now we are ready to think about the physics of the problem. The object of interest here is the cow. Which way is the cow going to move? The answer depends upon the net force exerted **on** the cow. There are three forces (of consequence) acting on the cow: two forces from the two farmhands and, since the cow is pushing on the ground, the ground (via Newton's third law) is exerting a 600 Newton force backwards on the cow. We must sum up these three forces to determine the net force on the cow, and thus determine whether she will move into the corral or not.



The force from the ground is pulling straight backwards (along the **x**-axis), while the farmhands are pulling slightly outward. We have to find the component of each farmhand's force that directly opposes the ground's force. Here is where the literal subscripts are useful again. We can label the x components $F1_x$ and $F2_x$

Define the variable $F1_x$ to be the component of the $F1$ force along the **x**-axis. You will have to multiply $F1$ by the appropriate trig function. Be sure to use the angle that goes with $F1$. Type i#@ or t@ to recall your definitions if necessary.

Instead of typing everything again for $F2_x$, you could make a **copy** of the definition and change the $F1$ to $F2$ and the angle ϕ to θ (or vice-versa).

$F1_x := F1 \cdot \cos(\theta)$

$F2_x := F2 \cdot \cos(\phi)$

$F1_x = 272.511\ \boldsymbol{N}$

$F2_x = 278.512\ \boldsymbol{N}$

We can now find out whether the cow is moving towards the corral or not by adding up the x components of the net force on the cow. Remember, force is a **vector**. The variables we have defined are magnitudes, but you must be careful how you add them all together (i.e. what is positive or negative direction).

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Content TIP

The sign of the forces

Define the variable F_{total} to be the x component of the **vector** sum of the components $F1_x$ and $F2_x$ and the ground's force $-F_{ground}$. **Note:** the negative sign appears here because the force of the ground on the cow is opposite to the force of the cow on the ground F_{ground}.

Display the answer F_{total} and replace the units in the answer with the **SI** unit of force.

$$F_{total} := \left(F1_x + F2_x\right) - F_{ground}$$

$$F_{total} = -48.977\ N$$

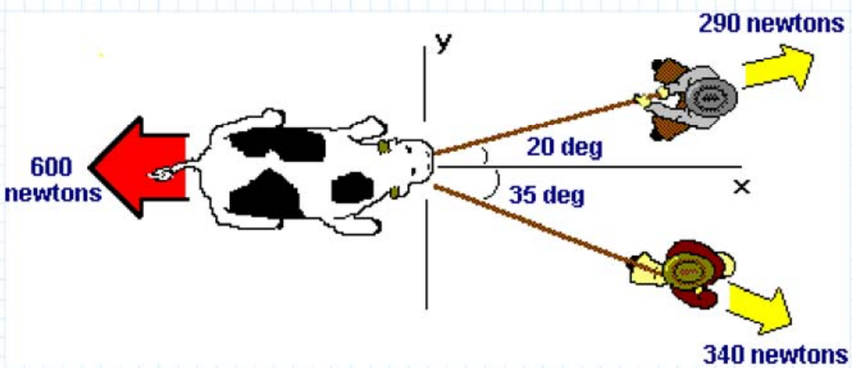
Problem SOLVING

State conclusions

Collect everything together into the Problem Space provided below: **Copy** the question and the picture, explain your variable definitions and the equations used to get the answer. The answer to the question comes from interpreting the sign (±) correctly. You should write some text to explain who is winning the tug-of-war and why.

Problem Space

Two farmhands are trying to pull a stubborn cow into its corral. The cow is pulling backward, exerting a force of 600 Newtons in the negative x direction on the ground. Each farmhand is pulling at different angles on two separate ropes. The forces exerted on the cow by the farmhands is illustrated in the picture below. Are the farmhands able to move the cow toward the corral or the other way around?



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Solution

Define the problem variables:

Angles: $\theta := 20\ deg$ $\phi := 35\ deg$

Cowhand forces: $F1 := 290\ N$ $F2 := 340\ N$

Force of ground: $F_{ground} := 600\ N$

The direction of the net force (the sum of the forces exerted by the farmhands and the cow) along the x-direction will determine the direction of the net acceleration and, hence, the direction of motion.

Components of forces along x-axis: $F1_x := F1 \cdot \cos(\theta)$ $F1_x = 272.511\ N$

$F2_x := F2 \cdot \cos(\phi)$ $F2_x = 278.512\ N$

Net force along the x-axis: $F_{total} := (F1_x + F2_x) - F_{ground}$

$F_{total} = -48.977\ N$

Incompatible Units Error

The cow is winning the tug of war because the net force on the cow is backwards.

Mathcad gives you the upper hand on units. Eventually you will learn to anticipate the units you expect the answer to have and you will begin to recognize the basic combinations of SI units for force, energy, etc.

Try another problem to gain more experience with variables and units in Mathcad.

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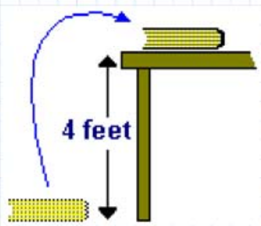
Question 3

A student picks up a **0.8 kg** book off the floor and puts it on a desk that is **4 feet** high. How much **work** does the student do to pick up the book?



Information Given

Assign variables to the information contained in the problem statement.

Defining units

Assign the variables M and d to the mass and distance given in the question.

If your units end up highlighted in black, use the icon to the left for help on units.

$M := 0.8\ kg$

$d := 4\ ft$

Does the variable M affect the definition of the meter m ? **No**: Mathcad treats lowercase and capitalized letters as different variables. In fact, Mathcad even treats letters in different colors and fonts as separate variables.

Solve Problem

The **work** done by the student is the **force** of gravity multiplied by the **distance**. Work either takes energy away or gives energy to a system. The work done by the student against the force of gravity gives the book potential energy due to its new height.

Define F to be the force of gravity on the book. Remember, the acceleration due to gravity g is already defined by Mathcad.

Then define the work W to be the force F times the distance d that the book was raised. Display the answer by typing W= .

$F := M \cdot g$

$W := F \cdot d$

$W = 9.565\ J$

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Analyze Results

According to our explanation above, work should have the units of energy. A good double-check of your work is to try inserting the specific SI unit you expected for the answer. Let's try that here with energy.

Replace the basic units in the answer with the SI unit of energy. If you do not know the correct unit, use the **Units** button from the **Math tab**

If your work was correct and you chose the right SI unit for energy, then the original SI units will be totally replaced **and** there will be no conversion of the number.

For example, say $answer := 150\ J$:

The original, basic SI units:	$answer = 150\ kg \cdot m^2 \cdot s^{-2}$
The correct SI unit of energy:	$answer = 150\ joule$
Choosing a non-SI unit of energy converts the number:	$answer = 0.036\ kcal$
Choosing the **wrong dimension** (force instead of energy) leaves extra units (an extra m):	$answer = 150\ m \cdot N$

Using Units

Mathcad gives you the upper hand on units. Eventually you will learn to anticipate the units you expect the answer to have and you will begin to recognize the basic combinations of SI units for force, energy, etc.

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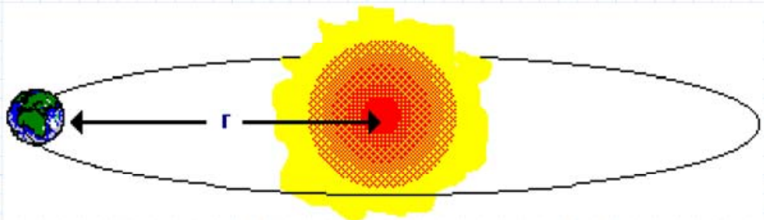
Question 4

The radius of the earth's orbit is $1.5 \cdot 10^{11}$ **meters** and it travels around the sun **once every year**. How fast is the earth traveling?



As with every problem, let's assign variables to the information contained in the question. Let's take them one at a time:

Information Given

In the open space, define the variable r_E to be the radius of the earth's orbit. Notice that we have used a literal subscript.

$$r_E := 1.5 \cdot 10^{11} \cdot m$$

Literal subscripts

The second piece of information may or may not make sense: "*...it travels around the sun* ***once every year***." Unless you recognize that this refers to the **frequency** of the earth's orbit, then you may get stuck. Is there any way to solve the problem without knowing what that part of the question means? Or what if you cannot remember the formula that relates the frequency to the speed?

Information Sought

Solving physics problems is not about memorizing every formula in the book or knowing how to solve every problem in one step. You can learn techniques to help you figure out a problem without having all the information at the start. Let's try an approach called **dimensional analysis**.

Dimensions versus units

Forget about trying to remember the right formula and just focus on the quantity you are trying to find. Our answer will be a **speed**. What are the **dimensions** of speed?

The SI *units* are $\dfrac{m}{s}$, so the *dimensions* of speed must be $\dfrac{distance}{time}$.

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What is the point? Instead of thumbing through a textbook, we now know that the speed is given by dividing a **distance** by a **time**. Instead of answering one difficult question, we have broken the problem down into answering two less threatening questions:

What **distance** are we talking about? What is the **time**?

Defining units

The time is one year. Define the variable *time* in the open space.

$time := 1\ yr$

The distance is more subtle. In one year, the earth travels in a circular path around the sun, which is the circumference: $2 \cdot \pi \cdot r_E$. The Greek letter **pi**, $\pi = 3.142$, is a predefined variable in Mathcad (like g) and it is used so often that it can be entered directly with the keystrokes p and then [Ctrl] +G.

In the open space, define the speed v to be the distance $2 \cdot \pi \cdot r_E$ divided by the *time*.

$$v := \frac{2 \cdot \pi \cdot r_E}{time}$$

Display the answer and replace the SI units with any you feel are appropriate.

$$v = \left(2.987 \cdot 10^4\right) \frac{m}{s}$$

$$v = \left(6.681 \cdot 10^4\right)\ mph$$

Dimensional analysis cannot give you factors like the $2 \cdot \pi$, but the technique is an alternative approach when you get stumped by a problem or forget a formula.

Incompatible Units Error

Now that you know a little more about dimensions, we can explain how Mathcad automatically double-checks your work. When you write an equation, Mathcad checks to see if the dimensions are consistent. If you try to add a distance and a time together, for example, then Mathcad will display an error message (see icon for general information on error messages) as shown below:

$10 \cdot ft + \boxed{9 \cdot s} = ?$

These units are not compatible.

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Error messages

You will also get this error message if you forgot to include units in the assignment statement of one of the quantities you are adding. You can, for example, add hours and minutes because Mathcad converts everything into the SI system to calculate the answer:

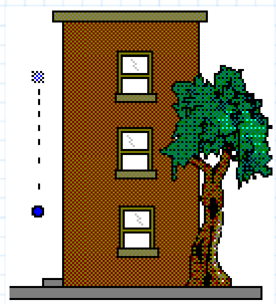
$$8 \cdot hr + 35 \cdot min = \left(3.09 \cdot 10^{4}\right)\ \mathbf{s}$$

How can this error message help you? Let's take a look at an example.

Question 5

A ball is dropped from a building. The ball takes **2 seconds** to reach the pavement.

How tall is the building?



Someone has already attempted this problem for you below, but Mathcad has found dimensional problems with the formula.

Take a look:

Time of flight: $t := 2 \cdot \mathbf{s}$

Height when the ball hits the ground: $y_o := 0$

Acceleration: $a := 32 \cdot \dfrac{\mathbf{ft}}{\mathbf{s}}$

Equation for the height of the ball: $y := y_o + \boxed{v_o} \cdot t + \dfrac{1}{2} \cdot a \cdot t^{3}$

Answer : $\boxed{y} = ?$

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Every term in the formula for the height y should have dimensions of length. There are actually **four** mistakes that cause the error message. Here is a list:

1. The formula is wrong. The third term should be a t^2 instead of a t^3 . The formula is not adding together only distances because this error creates an extra factor of **time**.

2. The units on the acceleration a are entered incorrectly. The dimensions are a speed $\dfrac{length}{time}$, rather than for an acceleration $\dfrac{length}{time^2}$. The acceleration needs the time squared to cancel the times from the t^2 term and leave distance.

3. The variable y_o does not have dimensions. Even though a quantity is zero, you must use the appropriate dimensions.

4. The variable v_o is undefined, which causes the $v_o \cdot t$ term to only have dimensions of time and not distance.

The incompatible units error message could result from any of these four errors. If you make a mistake, Mathcad will often find your error because of a dimensional inconsistency in one of your formulas.

Go back to the problem above and correct the **four** mistakes until Mathcad can calculate the answer. Make sure you understand the reason for the error.

STOP

Have you saved your changes?

That is the end of these exercises. Use the hyperlink below to go to the problems in the next section and try out these new tools and problem-solving skills on your own.

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