PROBLEM-SOLVING TUTORIAL

5.2 Graphing: Exercises

Introduction

With these Exercises, we will practice graphing functions, but you will also learn how to format your graphs. Specially, we will talk about adding markers to a graph, changing the appearance of a curve, and adding a legend or title.

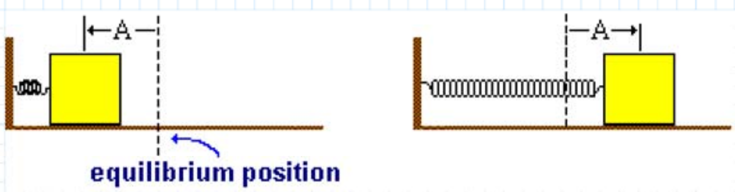
In addition, we cover some pitfalls that students run into while graphing and discuss the best way to choose the range over which to plot the function.

Question 1

A block is attached to a wall by a spring (with spring constant **k=250 N/m**).

a) Plot the potential energy stored in the spring as a function of position.

b) If the block is given **1.5 joules** of energy, what is the maximum distance that the spring compresses or expands as the block bounces back and forth (i.e. what is the **amplitude** *A* of the oscillation)?



Information given

Below, we have assigned variables to the information contained in the question.

The spring constant: $k := 250 \cdot \frac{N}{m}$

The energy given to the spring: $E_{total} := 1.5 \cdot joule$

Work and energy

To make a graph, we need to define the potential energy function and a range for the positions. If you need to review the potential energy of a spring, click on the pop-up in the margin.

Define a **function** that calculates the potential energy U in a spring for a given position x.

$$U(x) := \frac{1}{2} \cdot k \cdot x^2$$

Now we must define a range of positions. Since the block goes back and forth through zero as the spring stretches and compresses, we will want to include positive and negative numbers. But where should we begin and end the range?

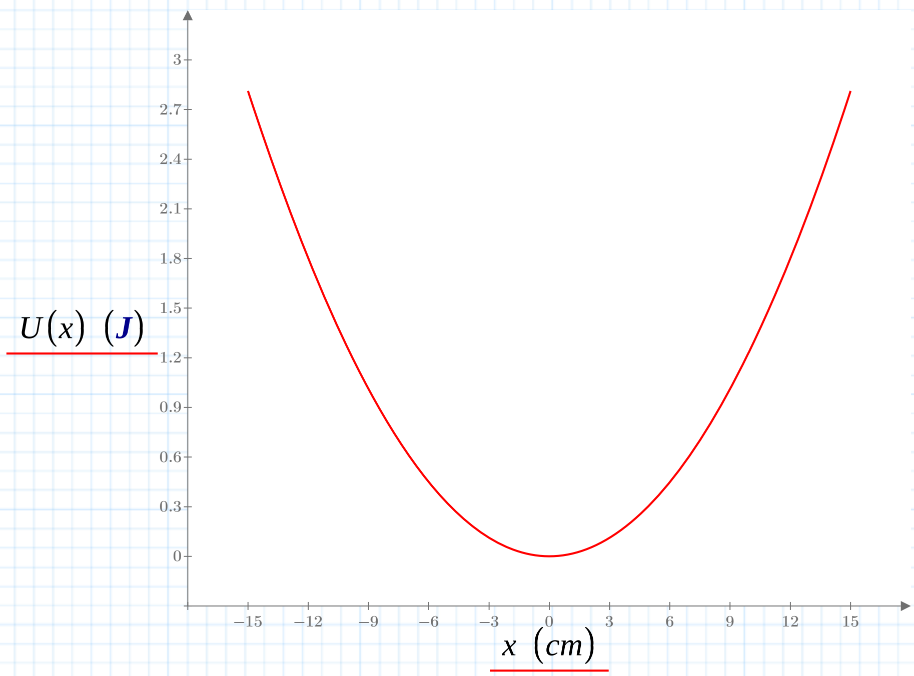
Deciding on the perfect range is not critical. If your range is too large or small, then you can always edit the numbers in the definition of the range variable and the graph will be recalculated automatically. Let's assume that the spring would not reach more than **15 cm** (we can always change it later).

Defining a range variable

Define a range of positions x that goes from **-15 cm** to +**15 cm** in **0.05 cm** steps. **Watch out:** What number should follow **-15 cm** to give the correct step size? Use the icon to the left for a review of setting a range.

$$x := -15 \cdot cm, -14.95 \cdot cm .. 15 \cdot cm$$

Having defined a function and a range to plot over, you are ready to graph.



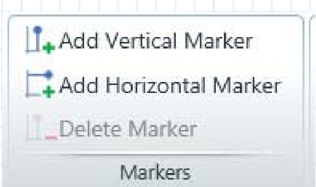
Does your graph make sense? If the spring is at its equilibrium position (**x**=**0 m**), then the potential energy is at a minimum. When the spring is compressed **or** stretched then the system gains potential energy.

Our particular block has been given **1.5 joules** of energy. Where is that on our graph? Instead of guessing, we can place a line across the graph at the total energy E_{total} .

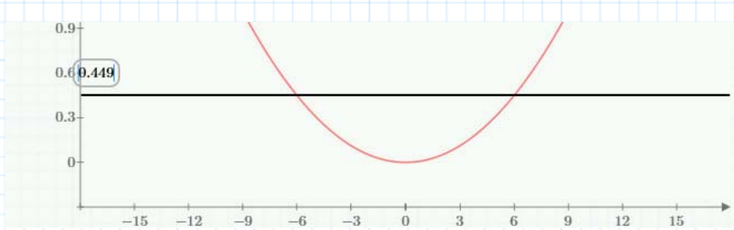
Markers

Mathcad gives you the option of displaying lines or **markers** at certain values on a graph.

If you want to insert markers you have to select your graph go to **Plots tab => Markers area.** There you have 2 options: you can add vertical or horizontal markers.

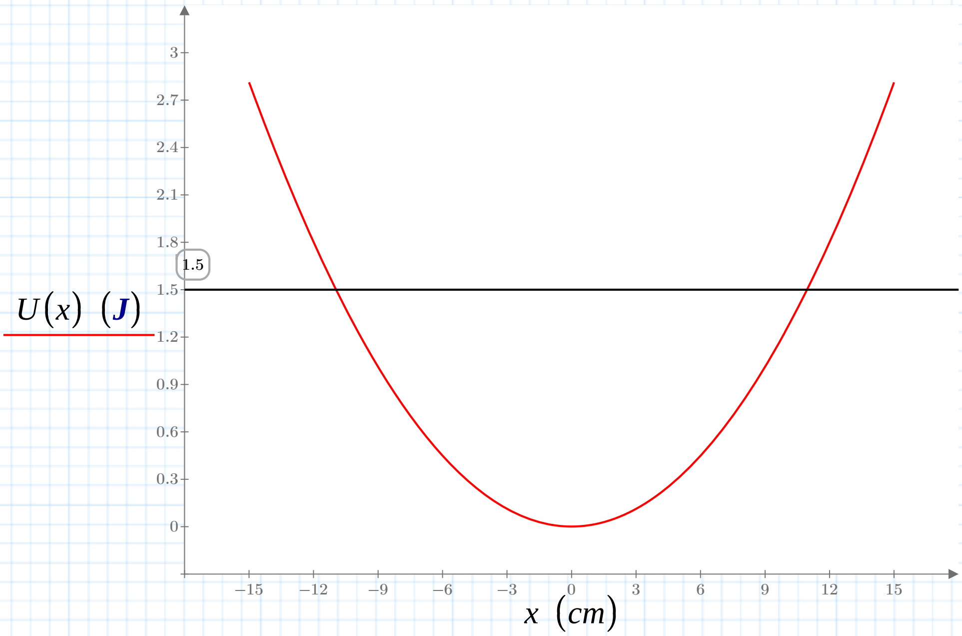


After you select the kind of marker you want it will appear on your graph. You can move to your desired position and the value of the plot in that place will be displayed at the end of it.



Let's try this with your graph of the potential energy of a spring. If you defined $U(x)$ and x above, then the graph below will redisplay your original curve (edit the variables if an error is shown).

In our problem, the block was given **1.5 joules** of total energy and we want to mark it on the plot below.



Mathcad draws a solid, black line by default and labels the marker with the or value.

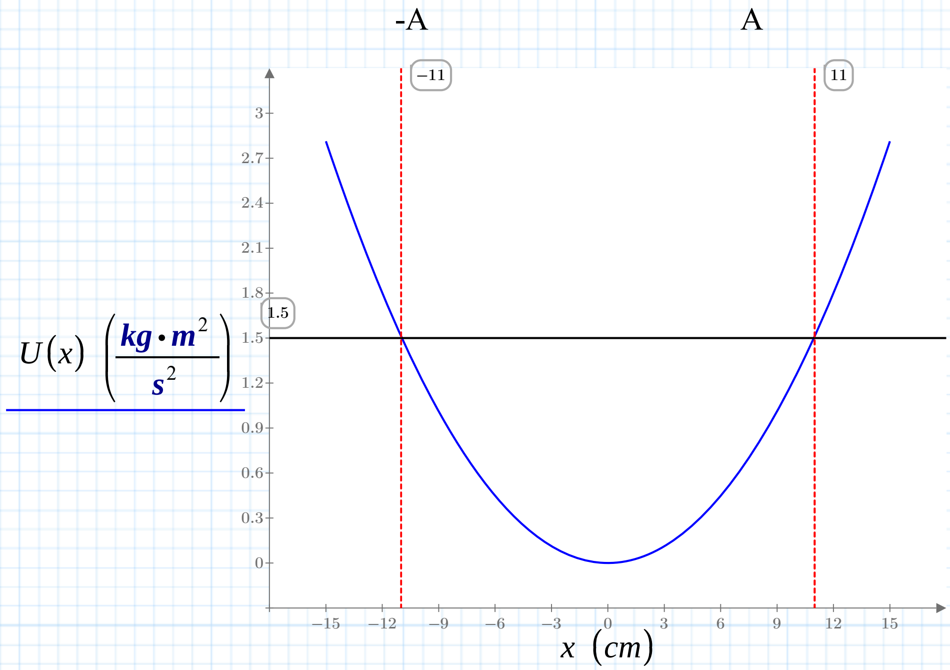
Markers are a helpful tool for identifying key information about your graph. We will use vertical lines to mark the amplitude of the motion after we find the answer.

Graphical Analysis: A Review

Now let's find the amplitude of the oscillation. At the maximum distance away from equilibrium, the block's total energy is all potential energy and the block is at rest. Instead of calculating the distance, let's use graphical analysis.

> To get a precise answer, decrease the step size in your original range to give more **x** values.

$$A := .11 \cdot \boldsymbol{m}$$



$$U(A) = 1.51\ joule$$

Graphing with Glitz

You may have noticed that the curve in our last graph was **blue.** How do you change the appearance of a graph? Within the **Plots tab** there is the area **Styles**. You can change from here things like the Trace Symbol, Line Style, Trace Color, Trace Thickness.

For more complex plots like Contour Plots you can also change things like Color Scheme or Surface Fill.

As an example, we have defined several functions and a range variable for creating a graph.

Arbitrary mass: $mass := 1 \cdot \mathbf{kg}$

Function definitions: $U_{grav}(z) := mass \cdot \mathbf{g} \cdot z$

$$U_{spring}(z) := \frac{1}{2} \cdot k \cdot z^2$$

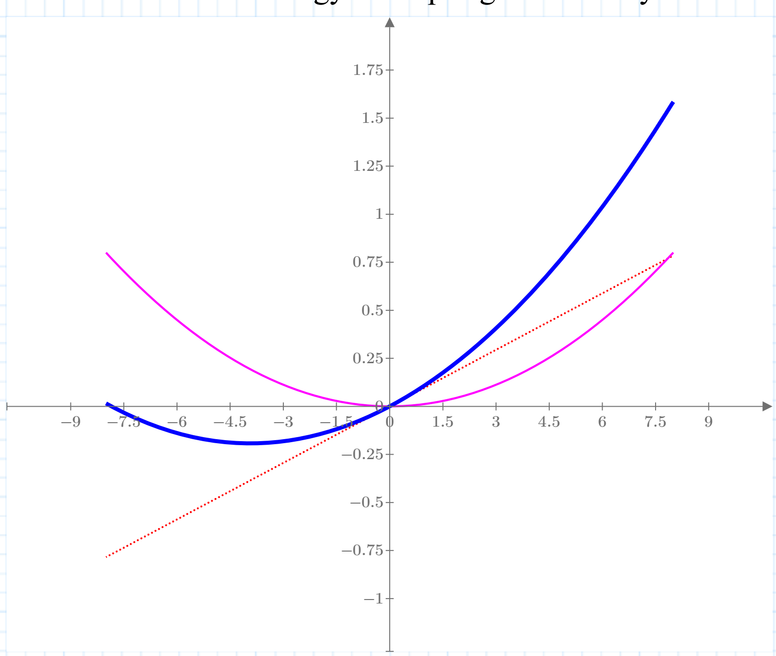
$$U_{total}(z) := U_{grav}(z) + U_{spring}(z)$$

Range of distances: $z := -8 \cdot \mathbf{cm}, -7.95 \cdot \mathbf{cm} .. 8 \cdot \mathbf{cm}$

Resizing graphs

Next, we have plotted the three functions on the same graph and formatted the results. **Note:** We made the graph larger by selecting the graph and dragging the bottom right corner, just as you would do to resize a text region.

Potential Energy of a Spring and Gravity



Whenever you include a graph in your worksheet, the reader should be able to tell what is being plotted and the units along each axis... without having to understand how Mathcad works!

Question: Boyle's Law

The volume V of a gas varies inversely as the pressure P upon it. The volume of a gas is 200 cm^3 under pressure of 32 $\frac{kg}{cm^2}$. What will its volume be under a pressure of 40 $\frac{kg}{cm^2}$? If the volume of the gas is 500 cm^3, what is the pressure on that gas?

Problem Space

To make a graph (and then use that graph to confirm the results we find numerically), we need to find the constant of variation. Remember that volume varies inversely with pressure. Write an equation to express this relationship using k as the constant.

$$V = \frac{k}{P}$$

Now, to find the value of the constant, use 200 cm^3 for volume and 32 for kg . Then, solve for k.

$$V := 200 \cdot \mathbf{cm}^3$$

Altering the symbolic equation

$$P := 32 \cdot \frac{\mathbf{kg}}{\mathbf{cm}^2}$$

$$V = \frac{k}{P}$$ has solution(s)

Solve for Variable

$$V \cdot P = \left(6.4 \cdot 10^3\right) \mathbf{kg} \cdot \mathbf{cm}$$

Using the constant k, rewrite the relationship between V and P in function form. Since we are trying to find the volume of the gas at a particular pressure, let pressure be the independent variable and volume be the dependent variable.

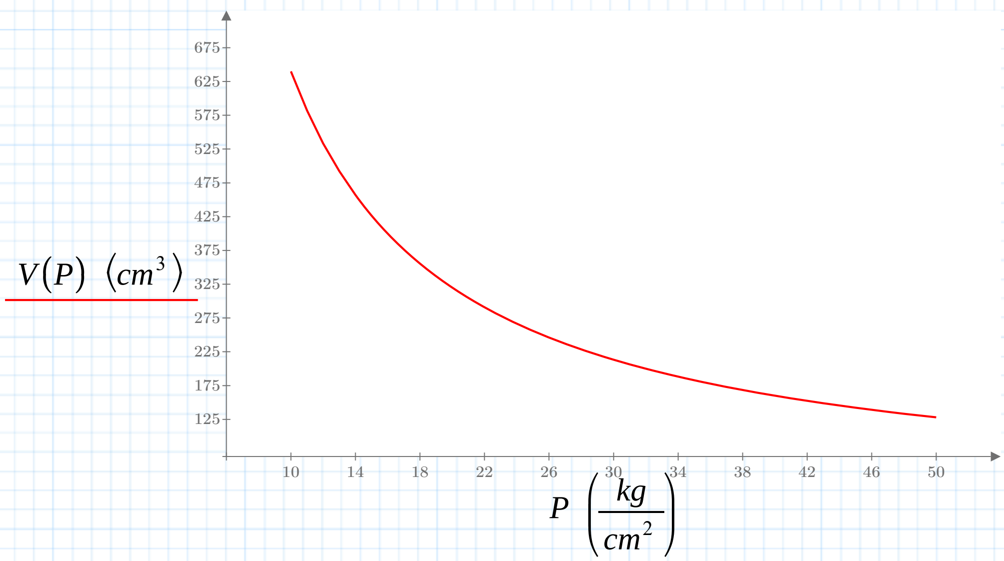
$$k := 6400 \cdot \mathbf{kg} \cdot \mathbf{cm} \qquad V(P) := \frac{k}{P}$$

Defining a range variable

Be sure to define a range variable.

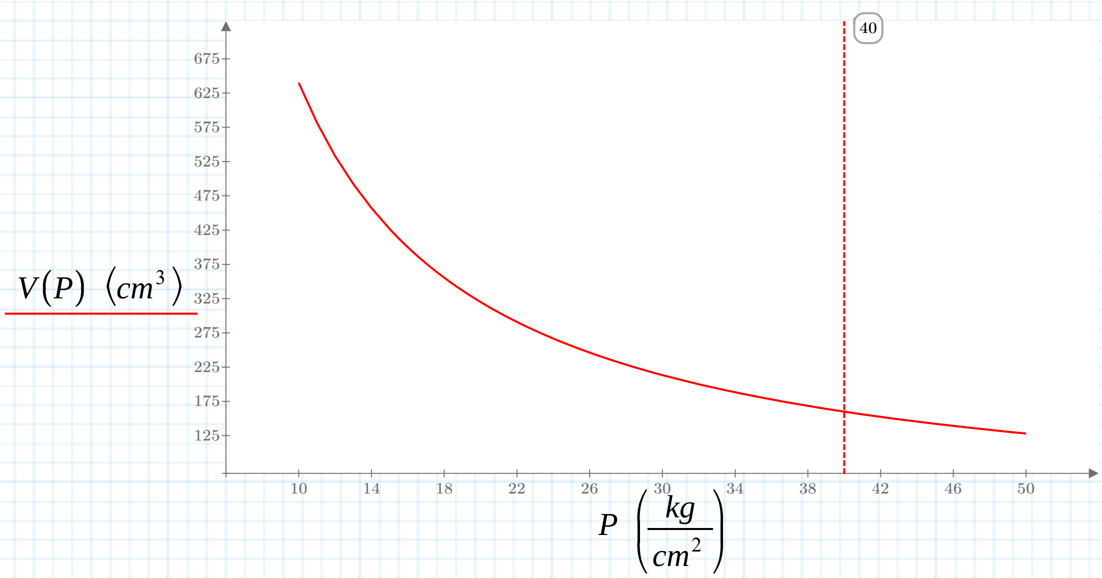
$$P := 10 \cdot \frac{kg}{cm^2}, 11 \cdot \frac{kg}{cm^2} .. 50 \cdot \frac{kg}{cm^2}$$

Now, you're ready to graph the function.



The curved shape of the graph is characteristic of an inverse relationship.

To answer the question posed above, place a marker at 40 and make an estimate of the volume at that point.

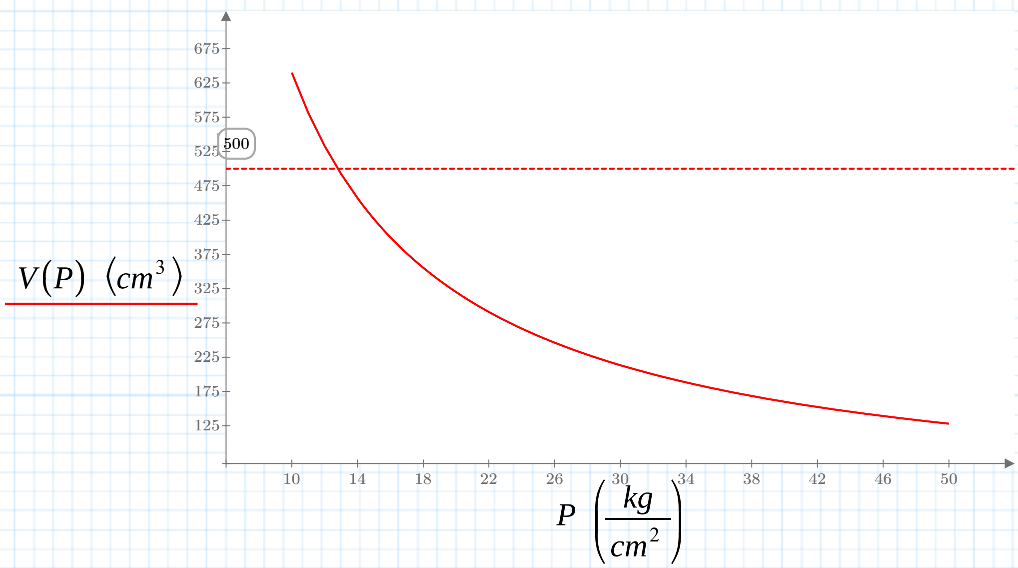


At 40 $\frac{kg}{cm^2}$ pressure, the gas has less than 200 cm^3 of volume. Use the function to find the exact value of the volume at 40 $\frac{kg}{cm^2}$ pressure.

$$V\left(40 \cdot \frac{kg}{cm^2}\right) = 160\ cm^3$$

Notice that as the pressure decreases, the volume increases, and as the pressure increases, the volume decreases. Such is the nature of an inverse relationship.

Now, what about the second question: If the volume of the gas is 500 cm^3 , what is the pressure on that gas? How do you solve for the independent variable, P, when the dependent variable quantity (V=500 cm^3) is known? Try using the graph to make a guess.



Add a horizontal Marker on the plot and place it add the value 500. It looks like the pressure on the gas is about 13 $\frac{kg}{cm^2}$.Let's test that quantity by plugging it into the function V(P).

$$V\left(13 \cdot \frac{kg}{cm^2}\right) = 492.31\ cm^3$$

This value is pretty close, but to find out for sure either write a new function P(V) and evaluate that function at 500 cm^2 or use the **Symbolics Ë Solve** method.

Remember....

$$V(P) := \frac{k}{P}$$

$$V := 500 \cdot cm^3 \qquad\qquad k = \left(6.4 \cdot 10^3\right)\ kg \cdot cm$$

Solve for variable, by selecting P in the equation below and choosing **Symbolics Ë Solve** under the **Math tab**.

$$V = \frac{k}{P} \qquad \text{has solution(s)} \qquad \frac{k}{V} = 12.8\ \frac{kg}{cm^2}$$

To write a new function method, note that pressure and volume vary inversely. So you can simply switch P and V and rewrite in function form.

Save your changes!

$$P(V) := \frac{k}{V} \qquad \text{Evaluate at } 500\ cm^3\ .$$

$$P\left(500 \cdot cm^3\right) = 12.8\ \frac{kg}{cm^2}$$

You have learned how to format your graphs, and we hope the last exercise gave you some experience in graphing functions on your own. Try to remember these lessons as you work through the problems in the last section.