PROBLEM-SOLVING TUTORIAL

5.3 Graphs: Problems

Introduction

Making graphs takes practice, so there are three problems in this section. Remember, you can always adjust the range variable after you have defined some rough values. In some problems, you will be able to define your range in terms of the information given in the question.

After you have worked out the bugs in your graphs, make sure they have a title and descriptions of the quantities (and units) on each axis.

Question

A girl jumps up from the ground with an initial vertical velocity of 2 m/sec. At the same instant, a boy drops down (no initial velocity) from an initial height of 2 feet.

a) Plot the height of each person separately as functions of time on the same graph. Express the height in feet, not meters. Use a marker for the ground on your graph.
b) Which person hits the ground first?
c) When are they at the same height?
d) What is the maximum height reached by the girl?

Problem Space

Below, the information from the question is assigned to variables:

Initial height of boy: $h_o := 2\ \mathbf{ft}$

Initial (upward) velocity of girl: $v_o := 2\ \dfrac{\mathbf{m}}{\mathbf{s}}$

Height of the ground: $ground := 0\ \mathbf{m}$

From the general kinematic equation for the height of an object in free fall:

$$y = y_o + v_o \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

we can define specific functions for the height of the boy and girl (based on their initial conditions):

Function for height of boy: $y_{boy}(t) := h_o - \frac{1}{2} \cdot g \cdot t^2$

Function for height of girl: $y_{girl}(t) := v_o \cdot t - \frac{1}{2} \cdot g \cdot t^2$

Using the fact that the time for the girl to return to $y = 0 \cdot m$ is $2 \cdot \frac{v_o}{g} = 0.41\ s$, we can define a range of times:

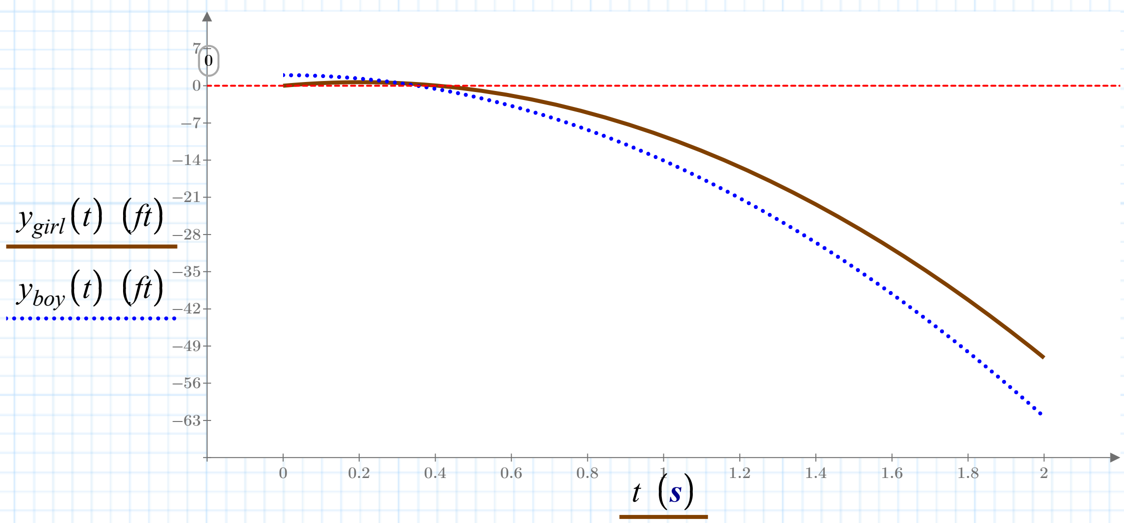
$$t := 0\ s, 0.001\ s .. 2 \cdot \frac{v_o}{g}$$

Recall each question:

a) Plot the height of each person separately as functions of time on the same graph. Express the height in feet, not meters. Use a marker for the ground on your graph.

Define range variables for both plots: $t := 0 \cdot s, 0.01 \cdot s .. 2 \cdot s$

Define some markers: $ground := 0\ ft$ $window := 20\ ft$



From the graph, we can read off the following:

b) Which person hits the ground first?

Obviously, the boy hits the ground first (his curve intersects the “ground” marker first in time).

c) When are they at the same height?

Since the curves intersect, both children are at the same height at ~ 0.30 seconds.

d) What is the maximum height reached by the girl?

The maximum height reached by the girl is $0.67 \cdot ft = 0.2\ m$

Question

A block that is oscillating back and forth on a spring undergoes **simple harmonic motion**. The equation that describes the motion of the block involves a trig function: $x(t) = cos(2 \cdot \pi \cdot f \cdot t)$

a) Graph several oscillations of the block, starting at 0 seconds, for a frequency f of $60 \cdot Hz$. The oscillation is assumed to have an amplitude of one.

Hint: The time for one oscillation is called the **period** T of the block. Define your range variable for time t in terms the number of periods ($0.01 \cdot T$ or $3 \cdot T$). Use the icon in the margin for help with defining the period T in terms of the frequency f.

When friction is taken into account, the oscillation becomes smaller over time. A decaying exponential term is responsible for this “damping” of the motion: $damp(t) = exp(-b \cdot t)$

The constant b is characteristic of the damping force (similar to a spring constant).

b) Include the damping term $damp(t)$ on the same graph as the oscillation. Set the value of the constant b to be $35\ Hz$.

c) Multiply the two terms together for the full expression [$x(t) \cdot damp(t)$] and include that on the graph as well. Format the graph.

Remember to use descriptive sentences to explain the solution.

Problem Space

From the question, we can define the following variables:

The frequency: $f := 60\ Hz$

The damping constant: $b := 35\ Hz$

The period: $T := \frac{1}{f}$

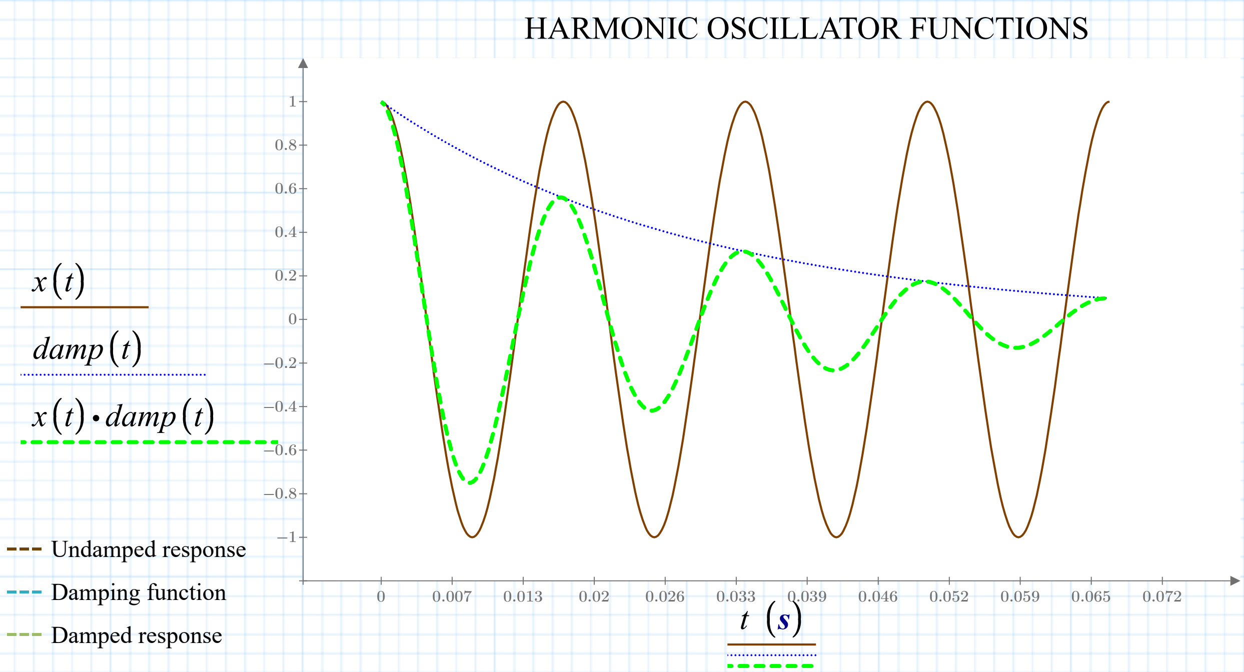
Function definitions:

Undamped response: $x(t) := \cos(2\ \pi \cdot f \cdot t)$

Damping function: $damp(t) := \exp(-b \cdot t)$

Set a range of times so that we see four oscillations: $t := 0\ s, 0.01\ T .. 4\ T$

A graph of these two functions and their product versus time is shown below.



Save your changes!

We can see from the graph that the damping function does cause the undamped response to die out.

Question 3

A **polynomial** is a mathematical expression that contains various powers of the variable in question. For example:

$$y(x) = 20 \cdot x + 7 \cdot x^2 + x^3$$

This polynomial has a **linear** term ($20 \cdot x$), a **quadratic** term ($7 \cdot x^2$), and a **cubic** term (x^3). In order to see how each term contributes to the polynomial, you will graph each term separately and then add successive terms together:

a) Define separate functions [for example, $y_1(x)$ and $y_2(x)$] for the linear and quadratic terms, then plot both over positive and negative for x. Also, add the two terms together and include that on the same graph. Describe how the linear term affects the quadratic term when they are added together.

b) Define a separate function for the cubic term and plot that on a second graph. Include the sum of the linear and quadratic terms on this second graph. Finally, include the full polynomial on the graph. Describe how the cubic term affects the final shape of the polynomial.

Hints: The behavior of the quadratic and cubic terms will begin to dominate the graph for large values of x (can you see why?). Experiment with the range of x so that you can still see how the terms add together (you may need different ranges for the two graphs). Also, remember to **Copy** and **Paste** whenever you feel it will save time.

Remember to use descriptive sentences to explain the solution.

Problem Space

A **polynomial** is a mathematical expression that contains various powers of the variable in question. For example:

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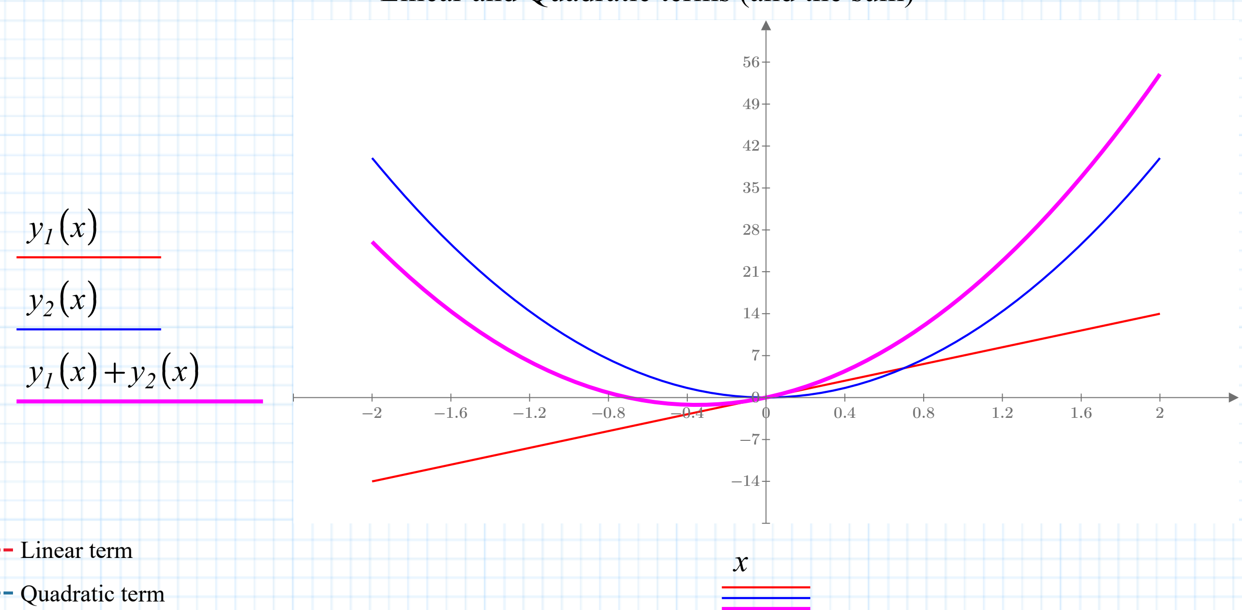
a) Define separate functions [for example, $y_1(x)$ and $y_2(x)$] for the linear and quadratic terms, then plot both over positive and negative for x. Add the two terms together and include that on the same graph. Describe how the linear term affects the quadratic term when they are added together.

Linear term: $y_1(x) := 7 \cdot x$

Quadratic term: $y_2(x) := 10 \cdot x^2$

Arbitrary range of values: $x := -2, -1.99 .. 2$

Linear and Quadratic terms (and the sum)



--- Linear term

--- Quadratic term

--- Their sum

From the graph we see that the linear term shifts the quadratic term down and to the left, but the curvature of the function is not changed.

b) Define a separate function for the cubic term and plot that on a second graph. Include the sum of the linear and quadratic terms on this second graph. Finally, include the full polynomial on the graph. Describe how the cubic term affects the final shape of the polynomial.

Cubic term: $y_3(x) := x^3$

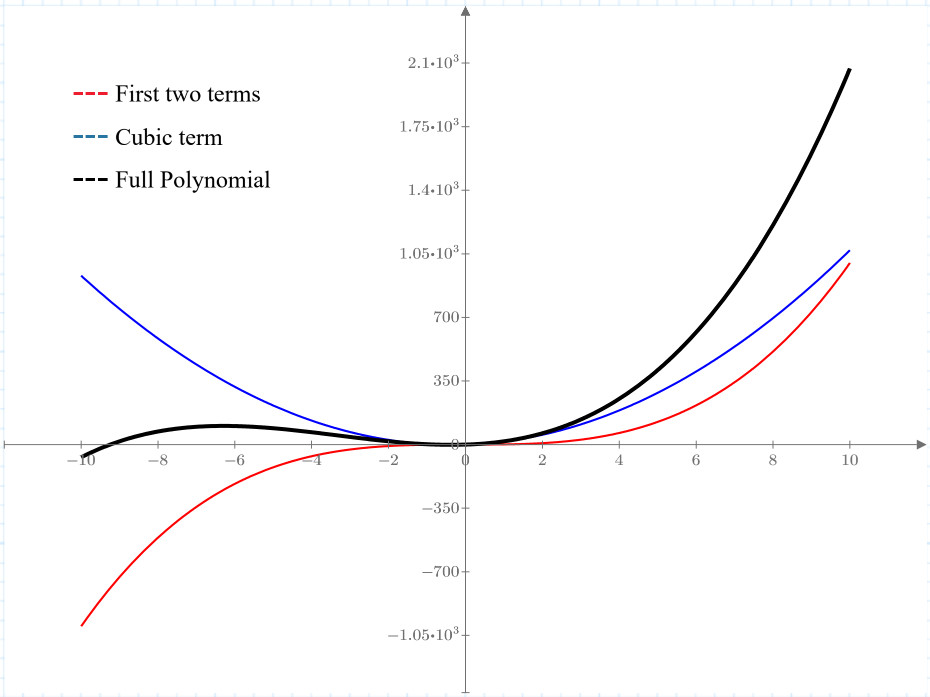
The full polynomial: $y(x) := 7 \cdot x + 10 \cdot x^2 + x^3$

Arbitrary range of values: $x := -10, -9.99 .. 10$

$y_3(x)$

$y_1(x) + y_2(x)$

$y(x)$



x

The cubic term makes the polynomial function curve up more sharply for positive x and makes the polynomial function curve downward for large enough negative x.

Although you are at the end of the chapter on graphing, we have just started using graphs. The next chapter uses graphs to look at the behavior of experimental data