PROBLEM-SOLVING TUTORIAL

3.1 Variables and Units: Tutorial

Introduction

The typical approach to problems for a novice math or science student is usually unsophisticated and, unfortunately, often unsuccessful. Such a novice's approach might have three parts:

1. Read the problem.
2. Find a relevant formula and plug in the numerical values from the question.
3. Calculate the final answer.

This novice approach provides few guideposts for how to proceed with the solution and no methods to verify the result (besides looking the answer up in the back of the book).

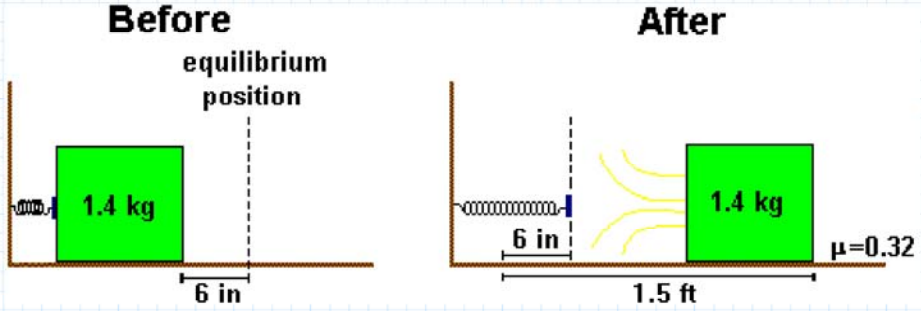
In this chapter, you will learn Mathcad skills that will totally change the way you approach problems. We will encourage you to adopt this new approach by showing you problem-solving techniques that relate to what you are learning about Mathcad. For example, when you learn about units in Mathcad we will discuss how this new skill can be used to double-check your answers.

Some students feel that the most frustrating part of a problem is calculating the final answer. Plugging values into a formula can turn a simple equation into a confusing set of numbers, where unseen errors lurk. Then, if one mistake is made while punching all the numbers into a calculator, the whole effort is wasted.

To start off this Tutorial we show why Mathcad provides an inherently better environment for solving problems than that of paper and a calculator typically used by students of math and science.

Example

A **1.4 kg** block is compressed against a spring to an initial distance of **6 inches** (spring constant of **230 N/m**). How much **energy** does the block still have left after it is released and slides on a rough surface (**μ =0.32**) for a distance of **1.5 feet**?



Solution

Begin by assigning the information that you are given in the problem to some variables. Note the use of text to **explain your definitions !**

Content TIP

Choosing Variable Names

Mass:	$mass := 1.4 \cdot kg$
Initial position:	$x_o := 6 \cdot in$
Spring Constant:	$k := 230 \cdot \frac{N}{m}$
Coefficient of friction:	$\mu := 0.32$
Distance traveled:	$d := 1.5 \cdot ft$

Once these variables have been assigned values in Mathcad, you can use them in other equations to calculate **new** variables.

For example,

Work and Energy

The potential energy from the initial compression of the spring is: $U := \frac{1}{2} \cdot k \cdot x_o^{\,2}$

The force due to friction is: $F_{friction} := \mu \cdot mass \cdot g$

After the block has slid across the table a distance d, the work done by friction is: $Work := (F_{friction}) \cdot d$

The energy remaining at that point is: $Energy := U - Work$

Mathcad then computes the values for all the new variables we just defined:

$U = 2.67\ J$ $F_{friction} = 4.39\ N$

$Work = 2.01\ J$ $Energy = 0.66\ J$

The problem is done! Defining variables and using them in place of the values they represent is a powerful approach to problem solving. The frustrating mistakes that occur even after you have figured out the physics of a problem are avoided. Understanding the equations becomes the focus and the calculations are dealt with in the background.

Notice that Mathcad also got the **units** to work out perfectly. *How ?*... Well, you will need to learn the basics of defining variables and using them in equations before we deal with units at the end of this section.

Defining Variables with Units

Let's work through another example to apply the basics of defining variables and using units.

Question

A runner runs a marathon (**26 miles** and **385 yards**) in **3 hours** and **20 minutes**. What is her speed in meters per second?

First, we will discuss how to assign the time and distance of the run to variables, so we can use these variables in an equation to calculate the answer.

The Definition Symbol

Defining a variable requires the **definition symbol** or **let equals sign** (for example $y := 3$), rather than the result equals sign. This is the same as saying let x = 1. To use the let equals symbol you must type a **colon** [**:**] Sometimes you can also type an equals sign to get a let equals. Let's try a simple example without units.

Defining Variables

Click in the open space to position the red crosshair. Let x equals 3 or define x to be 3 by typing:

x : 3 $x := 3$

Now you can use x in any mathematical expression, and it will have the value of 3. For example, try:

4 * x ^ 2 = $4 \cdot x^2 = 36$

Next, we need to learn how to assign a number with the correct units.

Using Units

Units

To assign units to a number in Mathcad, simply type the abbreviation for the unit after the number. If you don't know the abbreviation, look in the Math tab int the Units area for the unit you would like to use. Mathcad uses implied multiplication. Therefore, if a number is next to a unit or variable name, Mathcad assumes that the unit or variable is multiplied by the number that precedes it.

In the space to the right, define the variable d to be **13 cm** by typing:

d : 13 * cm $d := 13 \cdot cm$

Mathcad units always use the singular case. Therefore, a length is typed 26 mi rather than 15 miles.

Now, we can assign time and length of the run to variables. Be careful when using certain variable names — m and g, to name two, are reserved names for units. **m** stands for meters and **g** stands for the gravitational constant. Look at what happens when you simply type g and the equals key; try it, below.

Negative exponents in answers (like sec-2)

$$g = 9.81 \frac{m}{s^2}$$

We will just assign the mass of an object to a variable called *mass*. Later, we will show you more creative and practical options.

Problem SOLVING

Using Units

In the space below, define the distance, or change in position (**Gx**), to be **26 miles 385 yards** and the time (**Gt**) to be **3 hours 20 minutes**:

D [Ctrl]g x : 26 * mi + 385 * yd $\Delta x := 26 \cdot mi + 385 \cdot yd$

D [Ctrl]g t : 3 * hr + 20 * min $\Delta t := 3 \cdot hr + 20 \cdot min$

Check to see that you have defined them properly by typing **length** or **width** and the result equals key:

$\Delta x = (4.22 \cdot 10^4)\ m$ $\Delta t = (1.2 \cdot 10^4)\ s$

Mathcad takes information in many different unit systems and converts the values to SI automatically before it calculates an answer

Defining variables and using units will soon become very familiar, just like the basic text manipulation we covered in the first chapter.

Once you have defined a few variables, you can use them in an equation to define a new variable. In our example, the speed of the runner is determined by dividing the distance by the time.

Defining Variables with Equations

In the open space below, define speed v to be distance over time by typing:

$v := \frac{\Delta x}{\Delta t}$

After pressing return, you can display the answer by typing **v =**. Notice that your solution is in the units meters per second.

$v = 3.52\ \frac{m}{s}$

Changing Units in an Answer

Let's review our solution to the problem.

Distance of the run: $\Delta x := 26 \cdot mi + 385 \cdot yd$

Time of the run: $\Delta t := 3 \cdot hr + 20 \cdot min$

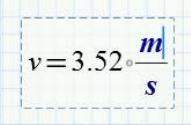
Speed of the runner: $v := \frac{\Delta x}{\Delta t}$

$v = 3.52 \ \frac{m}{s}$

The runner ran the marathon at an average speed of $3.52 \cdot m \cdot s^{-1}$. This answer maybe difficult to understand. It is often helpful to change the units of an answer to get a better feel for it, and to check whether the results are reasonable.

Here's how to change the units in an answer:

Click **once** on the units or the number in the answer. Blue editing lines will surround the answer like this:



Now delete a unit or both and you are ready to replace the units.

Replace the units in the answer with miles per hour by typing *mph* in the empty placeholder.

$v = 7.87 \ mph$

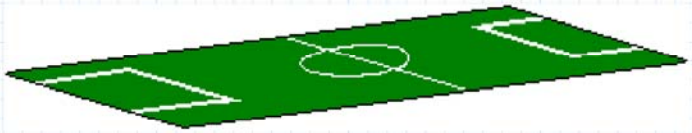
Notice that the *mph* totally replaces the $m \cdot s^{-1}$ units. This verifies that mph is, in fact, a unit of speed; but also double-checks that we defined the equations correctly. If we had inserted the wrong units or used an incorrect formula, there would be leftover units.

Verifying the correct units in your answers to double-check your work is using a technique called **Dimensional Analysis**. We will practice this more in the Exercises.

Numbers in physics problems often have units which convey the physical meaning of the numbers. Mathcad can handle units, so you can avoid many of the common mistakes and concentrate on the physics. Let's use a couple of examples here to illustrate.

Question

Is it possible to run down a soccer field (~**100 yards**) in **30 seconds**?



Solution

The solution below is an example of how the Mathcad skills you have learned can be used effectively in solving a problem.

Negative exponents in answers (like s^{-2})

1.Assign variables to the information in the problem:

Length of the field: $d := 100 \cdot yd$

Time: $t := 30 \cdot s$

2.Define the relevant formula(s):

The formula for speed is given by $v := \frac{d}{t}$

3.Display the answer with some text for explanation:

Your average speed would have to be $v = 3.05\ m \cdot s^{-1}$ to get across a soccer field in 30 seconds.

Now, most students do not have a good feel for speeds in **m/sec**. You may be able to judge the speed better if the answer is in **miles/hour** *mph* or in **kilometer/hour** *kph*; the units of speed found on a car speedometer.

Click on the answer and replace $m \cdot s^{-1}$ with miles per hour $\frac{mi}{hr}$ or *mph* ; or with $\frac{km}{hr}$ or *kph*.

$v = 6.82\ mph$

6b4ydue1p fg{ ;

An answer of about $7 \cdot mph$ or $11 \cdot kph$ may be a little more familiar. If not, try driving this speed the next time you are in a parking lot. It is pretty slow for running. The next question is "can you run that fast for **30 seconds**"? Hmmm . . .

The Units Dialog Box

You may feel intimidated about having to know the correct units for your classes, and *now* you also have to remember Mathcad's abbreviations!! Well, you need not worry. There is a dialog box that you can refer to for information about units. On the **Math** tab there is a button that brings up the **Units list.**

Note: Mathcad's default system is SI. It also has the **USCS** and **CGS** systems of units.

Difference between dimensions and units

A list of dimensions is on the top and a list of the various units for the highlighted dimension is on the bottom. In the picture above, we have **Time** area displayed also and you can see some of the possible units of time (hr,min,s. . .). You can also use this window to find which units Mathcad knows and their corresponding abbreviations. You close the window by clicking somewhere else on the window.

Just for practice, open the Units dialog box and see how many different units of **pressure** Mathcad recognizes.

6b4yduelp fg{

Defining Your Own Units

What if Mathcad does not know the units that you want? You can always define new units just like we define new variables.

Whenever Mathcad does not know a unit that you want to use, just define the unit with the definition symbol. You can keep a collection of units and physical constants that you frequently use at the beginning of your homework done with Mathcad.

Undefined Variables

You may have noticed that the definition of a new variable does not need to be in the viewing window for Mathcad to know the value. For example, Mathcad still knows the time of the marathon runner that we defined in the problem several pages above:

$$\Delta t = \left(1.2 \cdot 10^4\right)\ s$$

Do variable definitions apply to the whole document? What if we want to do several problems in the same document, but they all involve different lengths?

Variable definitions only apply to equations that are **below** the location where the variable is defined.

If we redefine *length* (say $length := 5 \cdot ft$), the new definition will **not** affect the equations above the new definition, only those below. This way, you can do several problems that use the same variable names (e.g. length,t,v,x . . .) and Mathcad will know which values apply to each problem.

This also means that Mathcad will not know how to calculate the answer for an equation that uses variables which have not been previously defined above the equation. Here are some examples:

Equations with undefined variables ⇨ $3 \cdot z + \sqrt{z} = ?$

⇩

$6 \cdot z + z^2 = ?$

$z := 2$ <- Line where variable is defined

Variable is now defined and can be used in the equation **below** the definition.

⇨ $3 - z = 1$

The variables in the first two equations are highlighted in red to show they are undefined because the equations are **above** the location where z is defined.

Drag the variable definitions that are in the space to the right to your worksheet window, and arrange them so that Mathcad calculates and displays the answer.

$a := 14 \cdot c$ $\quad answer = ?$

$c := 3$ $\quad b := a^3$

Notice that new variables which rely on an undefined variable are also undefined.

$$answer := \frac{c + \sqrt{b}}{a}$$

It is not a disaster to have undefined variables. If you want to write an equation first and then define the variables, you can always move the definitions above the equation later.

Having been introduced to the way Mathcad handles units and how you can define your own variables, you are now ready to approach the exercises in the next section with these more effective problem-solving tools.