PROBLEM-SOLVING TUTORIAL

3.3 Variables and Units of Measurement

Introduction

We begin this Problem section by doing an example to illustrate three steps you should follow when solving problems with your new Mathcad skills:

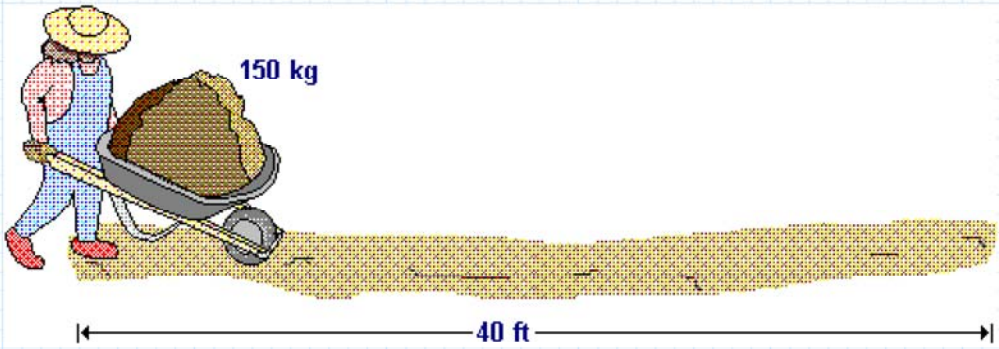
1. Assign variables to all the information in the question.
2. Define the relevant equations.
3. Display your answer, insert units, and organize the solution with text that explains your logic and comments on your answer.

While going through each step to solve the example below, we will discuss each of these steps in more detail. In doing so, we will emphasize that breaking hard problems down into smaller steps is a helpful approach to solving problems.

You may also wish to refer to the relevant stages of our structured approach to problem-solving below (some will be repeated in the margin when relevant).

Question 1

A gardener pushes a 150 kg wheelbarrow along a 40 foot path. Treating the wheelbarrow as an object sliding along the ground, the effective coefficient of kinetic friction is 0.05. How much work is done against friction?



Assign Variables

Problem SOLVING

Information Given

Since Mathcad must have all the variables defined above any equations you will use to reach the solution, your **first** step should always be to assign variables to the information contained in the problem statement. You should also write a brief phrase to explain the meaning of each variable:

Mass of the wheelbarrow: $m_{wb} := 150\ kg$

Distance traveled along the path: $d := 40\ ft$

Coefficient of kinetic friction: $\mu_k := 0.05$

In addition to saving time later, assigning variables at the start also gets you pointed in the right direction. After you finish this step, you have gone through the question carefully and have a clear definition of what you **do** know, which can often lead you to understanding what you want to know: the answer.

Content TIP

Choosing variable names

How can I choose the right variables before I know the formulas that will be involved in the solution?

Good question. You can see in the previous worksheet how you can choose good variable names.

The next step is easier said than done. "**Define the relevant equations**" does not provide detailed guidance. We have discussed how to use Mathcad to employ several methods to approach problems, including dimensional analysis, estimation of the answer, and breaking the problem into smaller steps. The next chapter introduces Mathcad's ability to do algebra, which will let you sketch out a problem and manipulate equations.

Problem SOLVING

Information Sought

For now, do not assume that the solution to the problem can be done "simply" in one step. Define a new variable for each quantity you think that you will use in your final solution and you will break the problem down into manageable steps.

Our example asks for the work done by the gardener. In general, **work** is a **force** times a **distance**. Let's define an equation for the work W :

$$W := F \cdot d$$

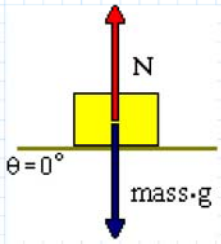
Problem SOLVING

Identify Principles

What is the **force** F? The work done by the gardener is against the force of **friction**, which is equal to the coefficient of friction μ_k times the normal force N.

$$F := \mu_k \cdot N$$

What's the **normal force** N? The normal force keeps an object from going through the surface (the ground). In this problem, the weight of gravity is pushing down on the wheelbarrow, so the normal force must be equal to the force of gravity:



$$N := m_{wb} \cdot g$$

Problem SOLVING

Propose methods

Now that we have defined all the parts, we can rearrange the equations and let Mathcad calculate the answer.

Problem SOLVING

Solve problem

Solving this problem in smaller steps was probably more reasonable than expecting to figure out a single equation that solves the problem in one step:

$$W := \mu_k \cdot m_{wb} \cdot g \cdot d$$

Problem SOLVING

Analyze results

After you have worked through a problem, insert SI units into the answer as a double-check (work should have units of **energy**). You may also want to insert more familiar units in order to judge whether the answer seems reasonable.

Problem SOLVING

State conclusions

Once you are satisfied with your answer, organize the solution and include text that explains your logic and comments on the answer. As a gauge of how much text is needed, imagine explaining the problem to someone else. Make sure to explain each variable that you define so that the reader knows what they stand for without guessing.

We have reorganized our example in the space below, using these guidelines.

Solution

A gardener pushes a 150 kg wheelbarrow along a 40 foot path. Treating the wheelbarrow as an object sliding along the ground, the effective coefficient of kinetic friction is 0.05. How much work is done against friction?

From the question, we know the following:

Mass of the wheelbarrow: $m_{wb} := 150 \cdot kg$

Distance traveled along the path: $d := 40 \cdot ft$

Coefficient of kinetic friction: $\mu_k := 0.05$

The work is done against the frictional force. To account for the wheelbarrow staying above ground, the normal force must be equal to the force of gravity:

$$N := m_{wb} \cdot g$$

The frictional force is $F := \mu_k \cdot N$

and so the work done over the distance d is:

$$W := F \cdot d \qquad W = 896.72\ J$$

The SI unit of energy is a *joule* : $W = 896.72\ J$

Although the number of joules may seem high, recall that the **joule** is a small unit of energy in comparison to the **kilocalorie**, which is equivalent to the "dietary calorie" used to discuss food.

The work expressed in kilocalories: $W = 0.21\ kcal$

Question 2

Make the following "conversions":

a) Your mass in kilograms (from pounds).
b) Your height in meters (from feet and inches).
c) The speed of a car on a highway (55 mph) in meters per second.
d) The volume of a milk jug (half of a gallon) in liters.
e) The pressure in a tire (35 pounds per square inch) in atmospheres.
f) The area of an average plot of land for a home (a third of an acre) in square feet.

Units

Hint: You do **not** need to know the conversion factors. Recall that Mathcad "thinks" in SI units, and you can always change the units in an answer (what happens when you do that?). If you want to know the abbreviation for any unit, use the **Units** dialog box (the icon in the margin explains how).

a) Your mass in kilograms (from pounds).

$$165 \cdot lb = 74.84\ kg$$

b) Your height in meters (from feet and inches).

$$6 \cdot ft + 1 \cdot in = 1.85\ m$$

c) The speed of a car on a highway (55 mph) in meters per second.

$$55 \cdot mph = 24.59\ \frac{m}{s}$$

d) The volume of a milk jug (half of a gallon) in liters.

$$0.5 \cdot gal = 1.89\ L$$

e) The pressure in a tire (35 **p**ounds per **s**quare **i**nch) in atmospheres.

$$35 \cdot psi = 2.38\ atm$$

f) The area of an average plot of land for a home (a third of an acre) in square feet.

$$\frac{1}{3} \cdot acre = \left(1.45 \cdot 10^4\right)\ ft^2$$

Have you saved your changes?

Question 3

A typical dieter loses 10 pounds a month. How much mass (in milligrams) does he lose a second?

Note: Mathcad does not recognize a month by default.

Define Variables

Months is not recognized by Mathcad, so the unit must be defined:

$$month := 30 \cdot day$$

Calculate Solution

The answer is just a simple matter of conversion

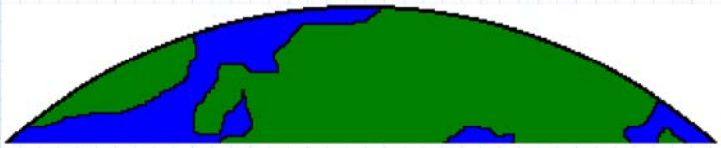
$$rate := 10 \cdot \frac{lb}{month} \qquad rate = \left(1.75 \cdot 10^{-6}\right) \frac{kg}{s}$$

Conclusion The dieter loses almost **0.002 mg per second**.

Question 4

a) If you walk 12 miles every day, how long would it take you to travel around the earth (radius of $6.4 \cdot 10^6$ meters)? Display your answer in meaningful units (not seconds).

b) What is your average speed in mph?



From the problem statement we have:

Radius of the earth: $R_e := 6.38 \cdot 10^6 \cdot m$

Walking speed: $speed := 12 \cdot \frac{mi}{day}$

a) The distance traveled is the Earth's circumference:

$circumference := 2 \cdot \pi \cdot R_e$

The time to travel around the Earth is the distance traveled divided by the speed.

$time := \frac{circumference}{speed}$

$time = \left(2.08 \cdot 10^3\right)\ day$ $\quad$ $time = 5.68\ yr$

b) Converting the units of speed, we find that

$speed = 0.5\ mph$

The speed is low because we have taken an average speed over a **whole** day, while many of those hours we may not have been walking.

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Now that you have successfully completed this chapter, you have developed the Mathcad tools and problem-solving skills necessary to solve the majority of problems you will encounter in you will encounter in your math and science classes..

You can now proceed to Chapter 4, where we explore the methods offered in Mathcad for solving equations. These equation solving methods will make it much easier to solve more difficult problems that require multiple steps and the manipulation of equations.

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