PROBLEM-SOLVING TUTORIAL

4.2 Solving Equations: Exercises

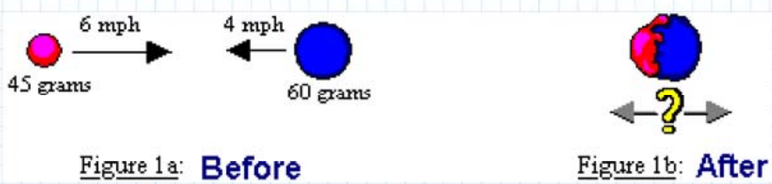
Introduction

In this exercise section you will practice solving equations with the methods you learned in the Tutorial to solve complex problems involving more than one equation. As you go through each exercise, we will also point out how the **symbolic equal sign =** allows you the flexibility to work up to the final answer, rather than try to “define the relevant equation(s)” all in one step.

Let's start out with a review of **Symbolics Ë Solve** .

Question 1

Two balls of putty collide head-on (see Figure **1a**. for details). If the two balls stick together completely (an “inelastic” collision), then what is the final velocity?



We have assigned variables to the information contained in Figure **1a**. We chose velocities going to the right to be positive and those going to the left to be negative:

Information Given

Mass of ball one: $m1 := 45\ \mathbf{gm}$

Mass of ball two: $m2 := 60\ \mathbf{gm}$

Initial velocity of ball one: $v1_i := 6\ \mathbf{mph}$

Initial velocity of ball two: $v2_i := -4\ \mathbf{mph}$

Identify Principles

Most students feel pressure to write down v_f = on the first step. Instead of focusing on the answer, start with the basic physical concepts and laws. As you go along, any equations you are considering can be jotted down with the symbolic equal sign =, since it allows for the use of variables that have not yet been defined.

Once you have the correct equation(s), the **SymbolicsË Solve** command can be used to solve for the variable you want.

Our current problem appears to involve **momentum** (mass and velocity), which should bring to mind the principle of momentum conservation.

Problem SOLVING

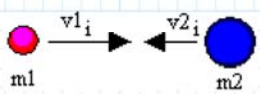
Propose Methods

Now that we have a starting place, we can ask a few more questions and see where the problem leads.

Is momentum conserved even though the balls stick together? Yes; unless an external force acts on the system (the two balls), then momentum is **always** conserved. What does it mean for momentum to be conserved?

$$Ptotal_{initial} = Ptotal_{final}$$

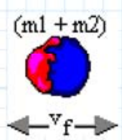
Since we know the initial momentum of each ball, let's put that into the equation:



$$P1_i + P2_i = Ptotal_{final}$$

$$m1 \cdot v1_i + m2 \cdot v2_i = Ptotal_{final}$$

What is the total momentum after the collision? Since the two balls have completely stuck together, there is just one big ball with mass $(m1 + m2)$ at some velocity v_f :



$$m1 \cdot v1_i + m2 \cdot v2_i = (m1 + m2) \cdot v_f$$

Problem SOLVING

Solve Problem

We can solve this equation for the final velocity v_f with the **SymbolicsË Solve** command on the **Math** tab.

Notice the difference in our approach to the problem: Start small and work up to the solution. If you can write down the **general** principles (like energy or momentum conservation, Newton's force law, etc.), you are halfway towards a solution instead of being halfway through your textbook looking for “the equation.”

Remember, using the **symbolic equal sign** = is a great way to rough out these problems because you can use undefined variables and still manipulate the equations.

Speaking of manipulating equations, let's solve for the final velocity v_f .

Solve for Variable

We have written our last equation in the space to the right. Solve the equation for the final velocity v_f using the **SymbolicsË Solve** command on the **Math** tab.

$$m1 \cdot v1_i + m2 \cdot v2_i = (m1 + m2) \cdot v_f$$

Then reassign the result to the final velocity v_f . Use the pop-up for help.

$$v_f := \frac{-(m1 \cdot v1_i + m2 \cdot v2_i)}{(-m1 - m2)}$$

Reassigning variables to symbolic help

Display the answer and replace the units to decide if the answer is reasonable.

$$v_f = 0.29 \ mph$$

Also, include a **text region** that explains why you think the final velocity is headed left or right.

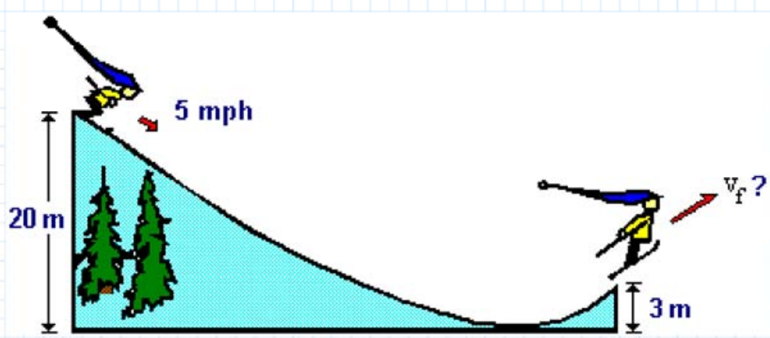
State Conclusions

Since we defined positive velocities to be going to the right, then a positive answer means that the mass is headed to the **right**.

Let's try another problem using the same approach: Start by writing down a general principle that you think applies (in symbolic form), then deal with the specific details of the problem.

Question 2

A skier pushes off from the top of a **20 m** high ski jump ramp, giving herself an initial speed of **5 mph**. After traveling down the slope, she launches off the lip of the jump, which is **3 m** high. Assuming there is no friction, what is her speed just as she leaves the jump?



Information Given

We have assigned variables to the information contained in the question.

Initial and final height of the skier: $h_i := 20 \cdot m$ $h_f := 3 \cdot m$

Initial speed of the skier: $v_i := 5 \cdot mph$

Identify Principles

There are two distinct principles in mechanics: **forces** and **conservation of energy**. Since there is no mention of forces or acceleration, let's think about energy. What does it mean for energy to be conserved?

$$E_{initial} = E_{final}$$

What are the specific types of energy involved in our problem? The skier has energy due to her height (gravitational potential energy U) and her speed (kinetic energy KE). So our conservation of energy equation becomes:

$$\left(U_i + KE_i\right) = \left(U_f + KE_f\right)$$

Propose Methods

Now you will need to put in the way that U depends on height and KE depends on speed. Do you remember the expressions? Use the icon to the left for review if necessary.

Work and Energy

Write the energy equation again, but use the explicit forms for the potential and kinetic energies. Use literal subscripts to distinguish between initial and final states.

$$m \cdot g \cdot h_i + \frac{1}{2} \cdot m \cdot v_i^{2} = m \cdot g \cdot h_f + \frac{1}{2} \cdot m \cdot v_f^{2}$$

Worried about the **mass**?

Remember, the symbolic equal sign lets you use any variables in an equation, so just use m for mass.

Once you have the full equation for the conservation of energy, you can solve for the skier's speed v_f as she leaves the jump.

Propose Methods

Why does everything move down?

Derivation Format

Move or copy the equation into your worksheet window. Use the **Solve** command to find the skier's speed when she jumps.

$$m \cdot g \cdot h_i + \frac{1}{2} \cdot m \cdot {v_i}^2 = m \cdot g \cdot h_f + \frac{1}{2} \cdot m \cdot {v_f}^2$$

Since the speed is squared in the formula for KE, you should get **two** roots for the speed. Notice that the mass cancels out!

Choose the positive root and use the **definition symbol** to reassign that expression to the speed of the skier. **Note:** If any variables become highlighted, you will need to make them consistent with our original definitions from above.

$$\begin{bmatrix} -\sqrt{2 \cdot g \cdot h_i + {v_i}^2 - 2 \cdot g \cdot h_f} \\ \sqrt{2 \cdot g \cdot h_i + {v_i}^2 - 2 \cdot g \cdot h_f} \end{bmatrix}$$

$$v_f := \sqrt{2 \cdot g \cdot h_i + {v_i}^2 - 2 \cdot g \cdot h_f}$$

$$v_f = 41.15 \ mph$$

Analyze Results

Display your answer and decide whether the answer is reasonable.

The speed is positive (moving to the right) and seems reasonable considering that the drop $h_i = 65.62 \ ft$ is six stories up!

In many problems, you will want to jot down equations, edit them and solve for certain variables before you are ready to define an answer using the **definition** symbol. The **comparison symbolic equal sign** allows you to write down any equation(s) you think of during the initial stages of solving a problem. The freedom to sketch out your ideas may help you think more creatively, which is needed when you are starting a tough problem.

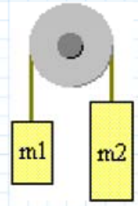
The next problem emphasizes the use of Mathcad's **solve block**. If you want more practice with the skills we used for the last two exercises, there is another exercise in **Section 4.4: Supplemental Exercises** . Use the hyperlink if you want additional practice now, otherwise there will be another hyperlink at the end of this section.

Question 3

Two blocks, initially at rest, are connected over a frictionless pulley by a rope (see figure for details). What is the resulting **acceleration** and the **tension** on the rope?

Mass of block one: $m1 := 10\ kg$

Mass of block two: $m2 := 35\ kg$



Why two equations

Since we need to find two variables, there must be two equations. Here is another helpful hint for approaching these complicated problems:

Before you worry about the details, just look at the situation and describe what you think will happen (roughly).

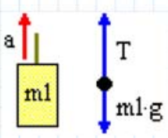
For example: Which way will the masses go?

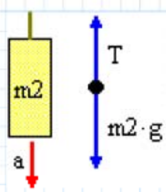
Well, the heavier mass will go down, pulling the lighter mass upwards.

By testing your intuition before the problem starts, you know what to expect for the answer. Also, you may stumble across some important physics without trying hard.

In the current problem, we have decided on a direction to choose for the acceleration of each mass. The vector sum of the forces on $m1$ results in an upward acceleration and the sum of forces on $m2$ results in a downward acceleration.

In general: $\sum_{sum} Forces = M \cdot A$





$T - m1 \cdot g = m1 \cdot a$

$T - m2 \cdot g = m2 \cdot (-a)$

(Notice that we took **up** to be positive and **down** to be negative).

Remember, try to test your intuition before you bother with the detailed physics. Now that we have the relevant equations, you need to guess the answers for the solve block.

Units

Define guesses for the tension T and acceleration a. The units are different for each quantity! Use the icon for help.

Note: Since the directions have been taken into account through the equations, both quantities should be positive.

Guesses:

$T := 1 \cdot N$

$a := 1 \cdot g$

Now you have to write the solve block. Remember, Given a set of equations, Find( ) the unknown variables. You can **copy** the equations from above.

Write a solve block for the tension T and acceleration a in the space below.

제약조건

$$T - m1 \cdot g = m1 \cdot a$$

$$T - m2 \cdot g = m2 \cdot (-a)$$

풀이

$$sol := \mathbf{Find}(T, a) = \begin{bmatrix} 152.55 \dfrac{kg \cdot m}{s^2} \\ 5.45 \dfrac{m}{s^2} \end{bmatrix}$$

$T := sol_1 = 152.55\ N$ $\qquad$ $a := sol_2 = 0.56\ g$

Solve Blocks

FAQs about Solve Blocks

The solve block is a powerful tool for solving systems of equations. We have looked at systems of two equations, but Mathcad's solve block can solve up to **50** simultaneous equations! The solve block can also be used to get approximate solutions when an exact answer cannot be found.