7.2 Advanced Problem Solving: Exercises

Introduction

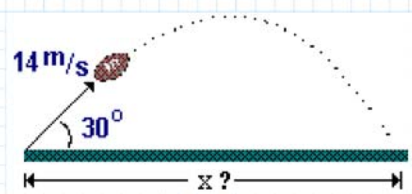
The exercise in this section provides additional practice with all the problem solving techniques that were in the Tutorial:

i. **What if ?** analysis.
ii. Graphs of solutions and families of curves.
iii. Limiting cases.

The exercise that follows involves more difficult physics than we have covered in previous sections, and you will notice that the depth of our analysis increases as well. Remember, the point of these new skills is to get more out of each problem.

Question 1

A football is throw with an initial velocity of **14 m/sec** at an angle of **30 degrees** (with respect to the ground).



a) Find the distance the football travels before hitting the ground.

b) What angle gives the maximum distance?

Defining Variables

In order to avoid confusion during our discussion, we have assigned variables to the information given in the question.

Initial velocity of the football: $v_o := 14 \cdot \frac{m}{s}$

Angle with respect to ground: $\theta := 30 \cdot deg$

Solution

Since the football was thrown at an angle, the initial velocity v_o will have a vertical and a horizontal component (v_{yo} and v_{xo} respectively). Decomposing a **vector** into its components involves using the $sin(\theta)$

and $cos(\theta)$ functions. As in the Tutorial, you can decide what function to use with each component by considering the limiting cases for the angle. We have redisplayed the definition of the angle below:

Angle with respect to ground: $\theta := 30 \cdot deg$

Define the vertical and horizontal components of the initial velocity.

$$v_{yo} := v_o \cdot \sin(\theta)$$

$$v_{xo} := v_o \cdot \cos(\theta)$$

Double-check your definitions by making sure they have the correct behavior when the football is thrown horizontally ($\theta = 0$ deg).

The values of v_{yo} and v_{xo} will be displayed at right when you define the components of the initial velocity.

$$v_{yo} = 7 \ \frac{m}{s}$$

$$v_{xo} = 12.12 \ \frac{m}{s}$$

We redefine the angle ($\theta := 30 \cdot deg$), just in case you did not above.

The position of the football is a **vector** equation as well:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \vec{a} t^2$$

Decomposing this equation into components gives two equations that describe the vertical and horizontal position.

Vertical position (height): $y = v_{yo} \cdot t - \frac{1}{2} \cdot g \cdot t^2$

Horizontal position (distance): $x = v_{xo} \cdot t$

Instead of deriving an exact solution for the distance, let's graph the trajectory of the football and read the answer from the graph. We will also be able to find the angle that gives the maximum distance just by experimenting with the graph.

In the past, we have always graphed the height as a function of time. For the actual trajectory, you will have to graph the height against the horizontal distance, but both are ultimately functions of time t .

X-Y Axes section

Copy and edit the two symbolic equations for the position of the football so that they are both defined as **functions** of time t .

$$y(t) := v_{yo} \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

Note: If any variables are shown to be undefined, then make their names consistent with your definitions above.

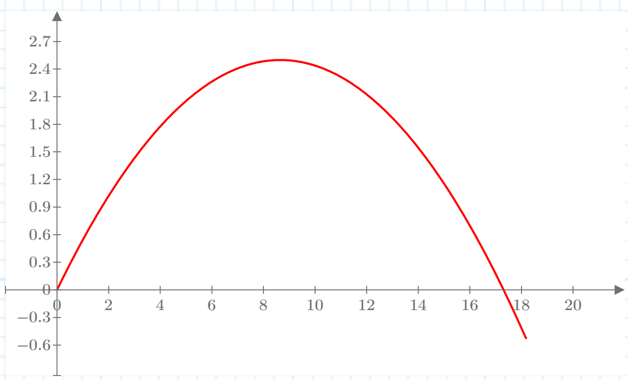
$$x(t) := v_{xo} \cdot t$$

Define a range of times t . You can always come back and adjust the size and steps of the range.

$$t := 0 \cdot s, 0.001 \cdot s .. 1.5 \cdot s$$

Graph the height $y(t)$ against the position $x(t)$ in the open space. Change the axes style from “boxed” to “crossed” in the **X-Y Axes** section of the **Format** dialog box. Notice that the "crossed" axes style shows where the ground is without a marker.

Trajectory of the Football



$y(t)$ (m)

$x(t)$ (m)

Trace dialog box

Use the **X-Y Trace** dialog box to read off the distance where the football hits the ground.

Hint: You may need to decrease the step size (which increases the number of points plotted) if you want a precise answer.

Your answer:

$$range := 17.3 \cdot m$$

$$range = 56.76 \ ft$$

$$range = 18.92 \ yd$$

With the first part worked out in Mathcad, you could easily see what happens to the trajectory when you change the angle at which the football was thrown. We will take that idea one step further by graphing a family of trajectories and comparing them.

Family of Curves

To compare trajectories with different angles θ, you will have to change the functions to include both time t and angle θ as arguments. There is a slight complication in making this change, so will use the horizontal position as an example for you:

The original function: $x(t) := v_{xo} \cdot t$

Including the angle θ as an argument $x(t,\theta) := v_{xo} \cdot t$

The way that the distance $x(t)$ depends on the angle θ is hidden in the definition of the initial horizontal velocity v_{xo} . When defining a function, Mathcad needs the dependence to be explicitly shown in the formula; as follows:

Substituting the definition of v_{xo} $x(t,\theta) := v_o \cdot \cos(\theta) \cdot t$

If you recall, we ran into the same problem in the Tutorial. Mathcad needs to see the variable in the definition of a function. Now it's your turn with the vertical position.

Copy and edit your original function for the vertical position to include the angle θ as an argument.

Also replace v_{yo} with its definition in terms of θ.

$$y(t,\theta) := v_o \cdot \sin(\theta) \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

By choosing several different angles (θ_1, θ_2...), we get have several different trajectories.

Define three angles $\theta_1, \theta_2, \theta_3$ whose trajectories you are interested in seeing. Try to imagine the trajectory you would expect for each value (imagine the limiting cases).

$\theta_1 := 85 \cdot deg$

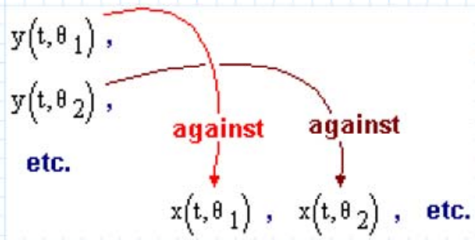
$\theta_2 := 45 \cdot deg$

$\theta_3 := 10 \cdot deg$

We are ready to graph... almost. You have graphed multiple functions before, but always against a single range variable on the horizontal axis. We now want to plot several functions for height against several functions for distance. Here is how that is done:

You can list the functions for distances along the horizontal axis by separating each with a **comma** (just like putting several functions along the vertical axis).

Note: You have to make sure to pair together each set of functions in the proper order. The first item listed along the vertical axis is plotted against the first item listed along the horizontal axis. There is a picture below that shows what is plotted against what.



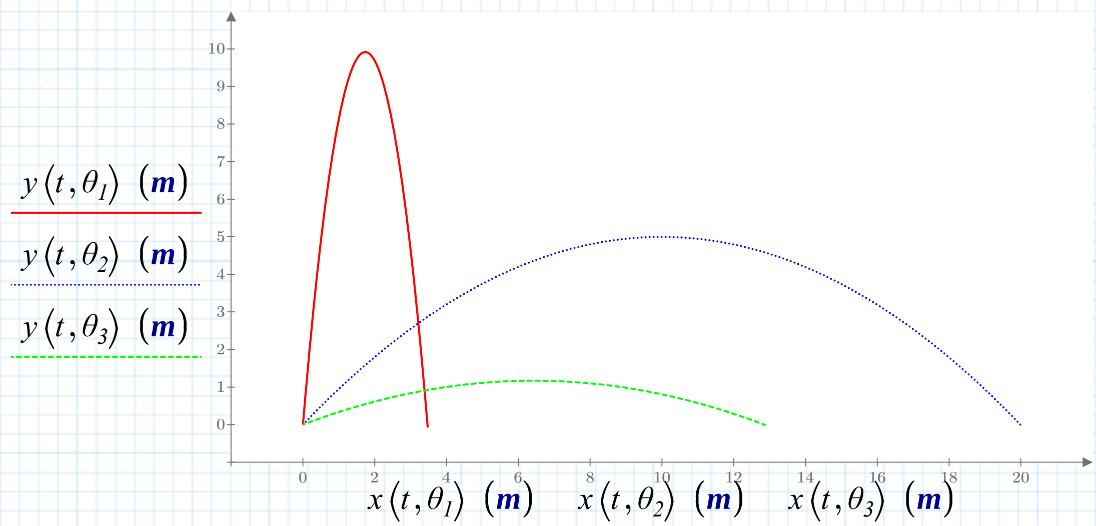
Now you are ready to make your graph.

Graph three trajectories in the space below, using the angles $\theta_1, \theta_2, \theta_3$.

Note: Copy the range of times and three angle variables directly above your graph so you can change them and watch the results. Remember, copy and edit as much as possible to save time (e.g. copy your old graph).

$$t := 0 \cdot \mathbf{s}, 0.01 \cdot \mathbf{s} .. 2 \cdot \frac{v_o}{g}$$

$\theta_1 := 85 \cdot \mathbf{deg}$ $\theta_2 := 45 \cdot \mathbf{deg}$ $\theta_3 := 20 \cdot \mathbf{deg}$



We are not interested in the trajectories after the football hits the ground. You can edit the **axis limits** to confine the **y**-axis above zero and limit the **x**-axis as well, "zooming in" on the part of the graph that is important.

Click on the lower limit on the **y**-axis, hit the backspace key, and change the limit to **zero**. Modify the upper limit on the **x**-axis so that the graph stops after the furthest distance.

Note: You only need to backspace once to delete the default numbers.

Save your changes!

By changing the definitions of your angle variables, you will be able to find the angle that gives the maximum range for the football. Why doesn't the football go further for angles that are greater or lesser than this angle? (test out the limiting cases in this situation).