PROBLEM-SOLVING TUTORIAL

4.3 Solving Equations: Problems

Introduction

As you work on the problems in this section, try to use all the Mathcad features you have learned so far and practice the problem-solving strategies we have advised along the way. We give a brief review below.

Remember to assign the information in the question to variables, with brief phrases for explanation. Then you can investigate the problem and generate ideas that will lead towards a solution.

You may want to start by getting a feel for the concepts of the problem in everyday terms and give a rough estimate of the answer you would expect (how much?). Then think about the **general** principles at work, instead of trying to rush to the answer. If you are really stuck, you could use dimensional analysis to work backwards from the dimensions you expect the answer to have.

As you go along, you may want to define new variables with the **definition symbol** := (typed with a colon **:**) or use the **comparison equals sign =** (typed as [Ctrl] =) to just write down the equations as you rough out your ideas. The advantage of using the symbolic equals sign is that you can then manipulate those equations with the symbolic **Symbolics Ë Solve** command on the **Math tab.**

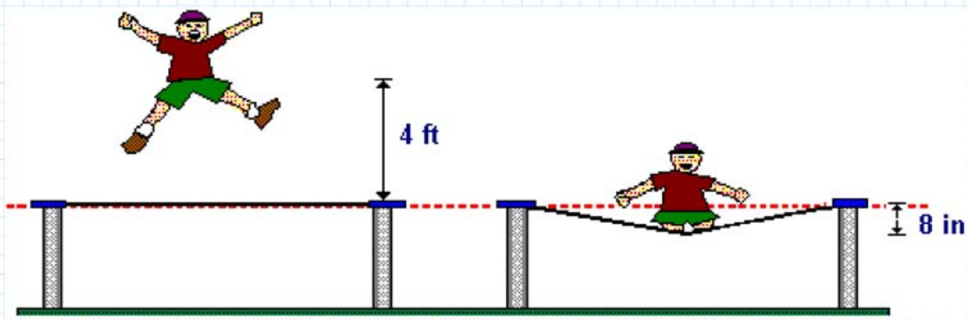
After you have derived an expression for the final answer, you will need to use the definition symbol := to assign the expression to a variable. Display your result, but don't stop there. *Always check that your answer seems reasonable.* If you cannot understand your answer in SI units, then insert some more familiar units. Remember, if you insert the units you expected for the answer and there are no “leftover” units, then that is another check of your work.

Finally, organize your solution and add appropriate comments throughout.

As we have mentioned, these are only guidelines for solving a problem; however, a structured approach often helps in solving tough problems, especially if the answer is not evident when you start the problem.

Question 1

A 30 kg boy is jumping on a trampoline. At one point the boy bounces to a height of 4 ft above the trampoline and when he comes down, the trampoline stretches so that he is 8 inches below where the trampoline surface lies normally.



Assuming that the trampoline behaves like a spring in the same the direction the boy is jumping, what is the spring constant k of the trampoline? Replace the given units with the conventional SI units in your answer.

Hint: Remember, you do not have to worry about how you are going to get an equation with k = **?**. Let the **Symbolics Ë Solve** command handle the algebra! You should think about the general laws of physics that applies best to this situation. Then you can fill in the specifics of the problem and use Mathcad to do the rest.

Problem Space

A **30 kg** boy is jumping on a trampoline. At one point the boy bounces to a height of **4 ft** above the trampoline and when he comes down, the trampoline stretches so that he is **8 inches** below where the trampoline surface lies normally.

Assuming that the trampoline behaves like a spring in the same the direction the boy is jumping, what is the spring constant k of the trampoline? Replace the given units with the conventional SI units in your answer.

Initial height: $h_i := 4 \cdot ft$

Final height: $h_f := -8 \cdot in$

Mass of boy: $mass := 30 \cdot kg$

The solution is based on application of the conservation of energy.

Conservation of energy: $E_{initial} = E_{final}$

Inserting equations for the gravitational potential energy ($PE_{grav} = mass \cdot g \cdot h$) and spring potential energy ($PE_{grav} = \frac{1}{2} k \cdot x^2$) into this expression, we have:

$$mass \cdot g \cdot h_i = mass \cdot g \cdot h_f + \frac{1}{2} \cdot k \cdot \left(h_f\right)^2$$

Solving this expression symbolically for k, we find:

$$\text{Solving for } k: \quad k := 2 \cdot \frac{\left(mass \cdot g \cdot h_i - mass \cdot g \cdot h_f\right)}{h_f^2}$$

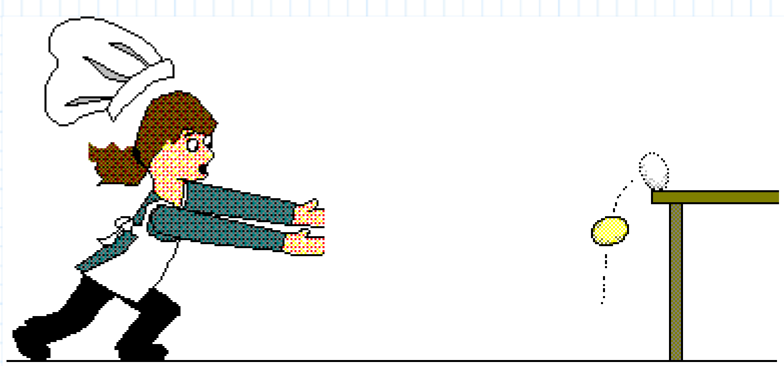
Numerically, k is:

$$k = \left(2 \cdot 10^4\right) \frac{kg}{s^2} \qquad \text{or} \qquad k = \left(2 \cdot 10^4\right) \frac{N}{m}$$

Using conservation of energy and the definitions of gravitational potential energy and spring potential energy, we found k to be $2.03 \cdot 10^4 \ \frac{N}{m}$ from only the mass and displacements of the boy.

Question 2

A chef is **6 feet** away when she notices an egg roll off the edge of a **1 m** kitchen table. If she moves with an average speed of **10 mph**, will the egg hit the floor before she reaches it?



Problem Space

A chef is **6 feet** away when she notices an egg roll off the edge of a **1 m** kitchen table. If she moves with an average speed of **10 mph**, will the egg hit the floor before she reaches it?

We are given in the problem that:

Distance away from table	$d := 6 \cdot ft$
Height of table:	$h := 1 \cdot m$
Speed of chef:	$v := 10 \cdot mph$

We can easily write down equations for the distance traveled by the chef and the egg as some time t:

Chef: $d = v \cdot t$

Egg: $y = h - \frac{1}{2} \cdot g \cdot t^2$

Solving the first equation symbolically for the time it takes the chef to reach the egg:

$$t := \frac{d}{v} \qquad t = \left(4 \cdot 10^{-1}\right)\ s$$

Using the symbolic **Symbolics Ë Substitute** command to insert this time in the second equation, we find the distance the egg falls before the chef arrives is:

$$y := h - \frac{1}{2} \cdot g \cdot \left(\frac{d}{v}\right)^2 \qquad y = 7\ in$$

Therefore, we conclude that the chef reaches the egg before it hits the floor and, assuming she can catch it, the egg is saved.

Try solving this next problem with a **solve block**. You may find the icons below helpful.

Question 3

A ball of mass M_1 = 1.25 kg, moving to the right, collides head-on with another ball of mass M_2 = 5.65 kg. The initial velocity of the first ball $v1_i$ is 3.9 m/sec, while that of the other ball $v2_i$ is -1.8 m/sec. If the collision is perfectly elastic, what are the final velocities of each ball?

Problem Space

Question

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Solution

The initial velocities and masses are given in the problem as:

$$v1_i := 3.9\ \frac{m}{s} \qquad v2_i := -1.8\ \frac{m}{s} \qquad M_1 := 1.25 \cdot kg$$

Since the collision is head-on, all motion takes place along a straight line (one-dimensional). Momentum is conserved in a collision, so we can write:

Momentum before = momentum after

$$M_1 \cdot v1_i + M_2 \cdot v2_i = M_1 \cdot v1_f + M_2 \cdot v2_f$$

where $v1_f$ and $v2_f$ are final velocities.

Insert or delete blank lines

Because the collision is assumed to be perfectly elastic, kinetic energy is also conserved. Thus,

Kinetic energy before = Kinetic energy after

$$\frac{1}{2} \cdot M_1 \cdot v1_i^{\,2} + \frac{1}{2} \cdot M_2 \cdot v2_i^{\,2} = \frac{1}{2} \cdot M_1 \cdot v1_f^{\,2} + \frac{1}{2} \cdot M_2 \cdot v2_f^{\,2}$$

Solve block

Next, we define the solve block. In this problem, we must satisfy the momentum and kinetic energy equations:

추측값

$$v1_f := -1 \cdot \frac{m}{s} \qquad v2_f := 1 \cdot \frac{m}{s}$$

제약 조건

$$M_1 \cdot v1_i + M_2 \cdot v2_i = M_1 \cdot v1_f + M_2 \cdot v2_f$$

$$\frac{1}{2} \cdot M_1 \cdot v1_i^{\,2} + \frac{1}{2} \cdot M_2 \cdot v2_i^{\,2} = \frac{1}{2} \cdot M_1 \cdot v1_f^{\,2} + \frac{1}{2} \cdot M_2 \cdot v2_f^{\,2}$$

The “**Find**” function produces the solutions

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$$\mathbf{Find}\left(v1_f, v2_f\right) = \begin{bmatrix} -5 \\ 3 \cdot 10^{-1} \end{bmatrix} \frac{m}{s}$$

Have you saved your changes?

Now that you have tackled Mathcad's important symbolic commands and the **solve block**, you have the tools to solve even the most difficult problems that you may face in your classes.

If you have mastered these new equation solving skills, then proceed to **Chapter 5: Graphs**, where you will learn how graphs can help you understand more about each problem you encounter.