Machine Learning HW 1

21300126 Seungwoo Kim

MNIST mean, variance and eigenvector graph

 

**Figure 1.** Mean of train\_x **Figure 2.** Variance of train\_x



**Figure 3**. First 100 eigenvalues



**Figure 4**. Eigenvector 1 **Figure 5**. Eigenvector 2 **Figure 6**. Eigenvector3

**Figure 7**. Eigenvector 4 **Figure 8**. Eigenvector 5 **Figure 9**. Eigenvector 6



**Figure 10**. Eigenvector 7 **Figure 11**. Eigenvector 8 **Figure 12**. Eigenvector 9



**Figure 13**. Eigenvector 10

Bishop

1.5 Using the definition (1.38) show that satisfies (1.39).

1.6 Show that if two variables and are independent, then their covariance is zero.

if two variables are independent, then . Therefore,

1.9 Show that the mode (i.e. the maximum) of the Gaussian distribution (1.46) is given by . Similarly, show that the mode of multivariate Gaussian (1.52) is given by .

In order to calculate the mode, or the maximum value, of the Gaussian distribution, the simplest way is to take a derivative of it and set it to 0. It is well-known that the Gaussian distribution is bell-shaped, so that it is not necessary to check whether the second derivative is less than 0. Without concerning the constants, the derivative of single Gaussian function is

For the derivative to be equal to 0, or . Therefore, the mode (the maximum) of the Gaussian distribution is equal to .

Similary, for the multivariate Gaussian distribution,

Since is symmetry, multiplying both side with leads to . Therefore, the mode of the multivariate Gaussian distribution is equal to .

1.10 Suppose that the two variables and are statistically independent. Show that the mean and the variance of their sum satisfies

Suppose that and are discrete random variable,

Since and are independent, by problem 1.6. Therefore,

1.11 By setting the derivatives of the log likelihood function (1.54) with respect to and equal to zero, verify the results (1.55) and (1.56).

When taking derivative of this log function with respect to ,

To satisfy the equation,

1.13 Suppose that the variance of a Gaussian is estimated using the result (1.56) but with the maximum likelihood estimates replaced with the true value of the mean. Show that this estimator has the property that its expectation is given by the true variance

Expectation of after replacing with is