1. System Eq.

$$\nabla \cdot \vec{J}_g(\vec{r}) + \Sigma_{tg}(\vec{r})\, \phi_g(\vec{r})$$
$$- \sum_{g'=1}^{2} [\frac{\chi_g}{k} \nu\Sigma_{fg'}(\vec{r}) + \Sigma_{sg'g}(\vec{r})]\, \phi_{g'}(\vec{r}) = 0$$

$$\vec{J}_g(\vec{r}) + D_g(\vec{r})\nabla\phi_g(\vec{r}) = 0 \ ,$$

$$\vec{J}_g(\vec{r}) = \sum_{u=x,y,z} \hat{u} J_{ug}(\vec{r})$$

$$J_{ug}(\vec{r}) = j^{+}_{ug}(\vec{r}) - j^{-}_{ug}(\vec{r})$$

$$\vec{r} = (x,\ y,\ z)$$

$$g = 1, 2$$

Assuming homogenious medium in node (i,j,k) ,
We can write the system Eq. as

$$\nabla \cdot \vec{J}^{ijk}_g(\vec{r}) + \Sigma^{ijk}_{tg}\, \phi^{ijk}_g(\vec{r}) \tag{1.a}$$
$$- \sum_{g'=1}^{2} [\frac{\chi_g}{k} \nu\Sigma^{ijk}_{fg'} + \Sigma^{ijk}_{sg'g}]\, \phi^{ijk}_{g'}(\vec{r}) = 0$$

$$\vec{J}^{ijk}_g(\vec{r}) + D^{ijk}_g\nabla\phi^{ijk}_g(\vec{r}) = 0 \ , \tag{1.b}$$

$$\vec{J}^{ijk}_g(\vec{r}) = \sum_{u=x,y,z} \hat{u} J^{ijk}_{ug}(\vec{r})$$

$$J^{ijk}_{ug}(\vec{r}) = j^{ijk+}_{ug}(\vec{r}) - j^{ijk-}_{ug}(\vec{r})$$

$$\vec{r} = (x,\ y,\ z)$$

$$-\frac{h^{ijk}_u}{2} \le u \le +\frac{h^{ijk}_u}{2} \qquad (u = x,y,z)$$

$$g = 1, 2$$

2. Definitions of Variables

1) Volume Averaged Total Flux :

$$\overline{\phi_g^{ijk}} \equiv \frac{1}{V^{ijk}} \int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{-h_v^{ijk}/2}^{+h_v^{ijk}/2} \int_{-h_u^{ijk}/2}^{+h_u^{ijk}/2} \phi_g^{ijk}(\vec{r})\, du\, dv\, dw \ ,$$

$$where \quad V^{ijk} = h_u^{ijk} h_v^{ijk} h_w^{ijk}$$

2) Surface Averaged Partial Currents :

$$J_{urg}^{ijk\pm} \equiv \frac{1}{A_u^{ijk}} \int_{-h_v^{ijk}/2}^{+h_v^{ijk}/2} \int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} J_{ug}^{ijk\pm}(\vec{r})\Big|_{u=+h_u^{ijk}/2} dv\, dw$$

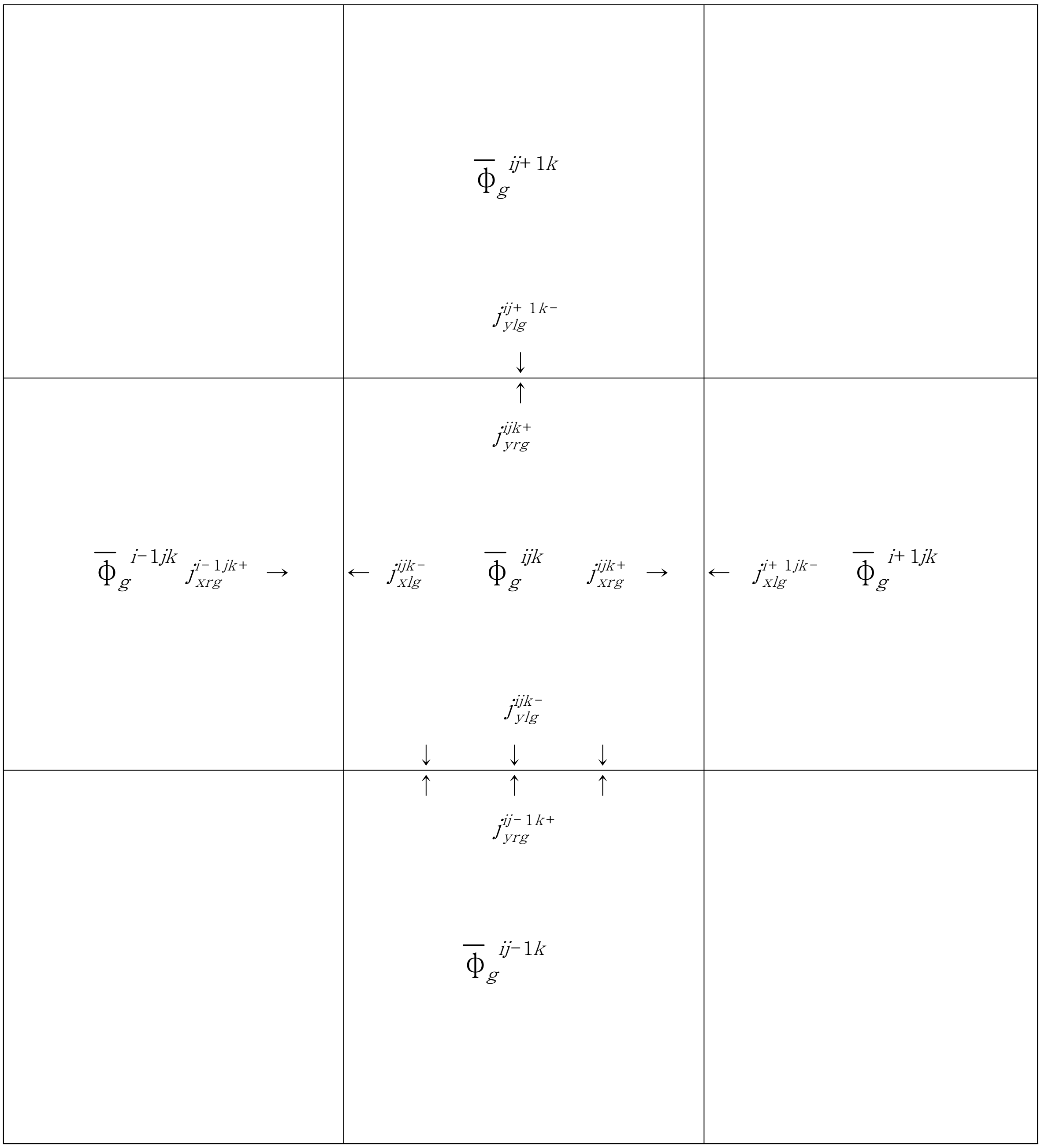
$$J_{ulg}^{ijk\pm} \equiv \frac{1}{A_u^{ijk}} \int_{-h_v^{ijk}/2}^{+h_v^{ijk}/2} \int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} J_{ug}^{ijk\pm}(\vec{r})\Big|_{u=-h_u^{ijk}/2} dv\, dw \ ,$$

$$where \quad A_u^{ijk} = h_v^{ijk} h_w^{ijk}$$

$$J_{usg}^{ijk} = J_{usg}^{ijk+} - J_{usg}^{ijk-} \quad (\ u = x, y, z\ ;\ s = r, l\ )$$

$$J_{xrg}^{ijk-} = J_{xlg}^{i+1jk-}\ ,\quad J_{yrg}^{ijk-} = J_{ylg}^{ij+1k-}\ ,\quad J_{zrg}^{ijk-} = J_{zlg}^{ijk+1-}$$

$$J_{xlg}^{ijk+} = J_{xrg}^{i-1jk+}\ ,\quad J_{ylg}^{ijk+} = J_{yrg}^{ij-1k+}\ ,\quad J_{zlg}^{ijk+} = J_{zrg}^{ijk-1+}$$


$\overline{\Phi}_g^{ij+1k}$
j_{ylg}^{ij+1k-}
j_{yrg}^{ijk+}
$\overline{\Phi}_g^{i-1jk}$
j_{xrg}^{i-1jk+}
j_{xlg}^{ijk-}
$\overline{\Phi}_g^{ijk}$
j_{xrg}^{ijk+}
j_{xlg}^{i+1jk-}
$\overline{\Phi}_g^{i+1jk}$
j_{ylg}^{ijk-}
j_{yrg}^{ij-1k+}
$\overline{\Phi}_g^{ij-1k}$

3. How to Solve the System Eq.

3.1. Volume Averaged Total Flux

$$\int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} \int_{h_u^{ijk}/2}^{+h_u^{ijk}/2} Eq.(1.a)\; du\, dv\, dw \qquad ==>$$

$$\sum_{u=x,y,z} \frac{(J_{urg}^{ijk+} - J_{urg}^{ijk-}) - (J_{ulg}^{ijk+} - J_{ulg}^{ijk-})}{h_u^{ijk}} + \Sigma_{tg}^{ijk}\, \overline{\phi_g^{ijk}}$$

$$- \sum_{g'=1}^{2} \left[\frac{\chi_g}{k} \nu\Sigma_{fg'}^{ijk} + \Sigma_{sg'g}^{ijk} \right] \overline{\phi_{g'}^{ijk}} = 0 \qquad (2.a)$$

3.2. Surface Averaged Partial Currents

$$\int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} Eq.(1.b) \Big|_{u=+h_u^{ijk}/2} dv\, dw$$

$$==>$$

$$\int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} Eq.(1.b) \Big|_{u=-h_u^{ijk}/2} dv\, dw$$

$$J_{urg}^{ijk} = J_{urg}^{ijk+} - J_{urg}^{ijk-} =$$

$$-D_g^{ijk} \frac{d}{du} \left[\frac{1}{A_u^{ijk}} \int_{h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} \phi_g^{ijk}(\vec{r})\; dv\, dw \right] \Big|_{u=+h_u^{ijk}/2} \qquad (2.b-1)$$

$$J_{ulg}^{ijk} = J_{ulg}^{ijk+} - J_{ulg}^{ijk-} =$$

$$-D_g^{ijk} \frac{d}{du} \left[\frac{1}{A_u^{ijk}} \int_{h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} \phi_g^{ijk}(\vec{r})\; dv\, dw \right] \Big|_{u=-h_u^{ijk}/2} \qquad (2.b-2)$$

3.3. Iterative Scheme

$$(J_{urg}^{ijk-})^{(t-1)},\ (J_{ulg}^{ijk+})^{(t-1)} \quad ==> \quad (2.a) \quad ==> \quad (\overline{\phi_g^{ijk}})^{(t)}$$

$$(\overline{\phi_g^{ijk}})^{(t)} \quad ==> \quad (2.b-1,2) \quad ==> \quad (J_{urg}^{ijk+})^{(t)},\ (J_{ulg}^{ijk-})^{(t)}$$

How can we obtain $\overline{\phi_g^{ijk}}$ in terms of incoming partial currents(J_{urg}^{ijk-}, J_{ulg}^{ijk+}) ? ==> See section 5.

How can we obtain outgoing partial currents(J_{urg}^{ijk+}, J_{ulg}^{ijk-}) in terms of $\overline{\phi_g^{ijk}}$? ==> See section 4.

4. How to Obtain j_{urg}^{ijk+} , j_{ulg}^{ijk-} from $\overline{\phi_{g}^{ijk}}$

4.1. Definition of 1-D Flux and 1-D Eq.

Defining 1-D flux as

$$\phi_{ug}^{ijk}(u) \equiv \frac{1}{A_{u}^{ijk}} \int_{h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} \phi_{g}^{ijk}(\vec{r})\ dv\,dw$$

We can convert Eq(1.a) to 1-D Eq.

$$-D_{g}^{ijk}\frac{d^{2}}{du^{2}}\phi_{ug}^{ijk}(u) + D_{g}^{ijk}L_{ug}^{ijk}(u) + \Sigma_{tg}^{ijk}\phi_{ug}^{ijk}(u) - \sum_{g'=1}^{2}\left[\frac{\chi_{g}}{k}\nu\Sigma_{fg'}^{ijk} + \Sigma_{sg'g}^{ijk}\right]\phi_{ug'}^{ijk}(u) = 0\ , \tag{3}$$

where

$L_{ug}^{ijk}(u)$

$$= -\frac{1}{A_{u}^{ijk}} \int_{h_w^{ijk}/2}^{+h_w^{ijk}/2} \int_{h_v^{ijk}/2}^{+h_v^{ijk}/2} \left[\frac{\partial^{2}}{\partial v^{2}} + \frac{\partial^{2}}{\partial w^{2}}\right]\phi_{g}^{ijk}(\vec{r})\ dv\,dw$$

$$= -\frac{1}{A_{u}^{ijk}}\left[\int_{-h_w^{ijk}/2}^{+h_w^{ijk}/2} \frac{\partial\phi_{g}^{ijk}(\vec{r})}{\partial v}\Big|_{v=+h_v^{ijk}/2} - \frac{\partial\phi_{g}^{ijk}(\vec{r})}{\partial v}\Big|_{v=-h_v^{ijk}/2}\ dw + \int_{-h_v^{ijk}/2}^{+h_v^{ijk}/2} \frac{\partial\phi_{g}^{ijk}(\vec{r})}{\partial w}\Big|_{w=+h_w^{ijk}/2} - \frac{\partial\phi_{g}^{ijk}(\vec{r})}{\partial w}\Big|_{w=-h_w^{ijk}/2}\ dv\right]$$

$D_{g}^{ijk}L_{ug}^{ijk}(u)$: Transverse Leakage.

4.2. Expansion of the Flux

Expanding the flux in polynomial

$$\phi_g^{ijk}(\vec{r}) = \sum_{l,m,n}^{l+m+n\le 4} C_{lmng}^{ijk} f_l(\tau_x^{ijk})\, f_m(\tau_y^{ijk})\, f_n(\tau_z^{ijk}) \ ,$$

where

$$f_0(\tau) = 1$$
$$f_1(\tau) = 2\tau$$
$$f_2(\tau) = 6\tau^2 - 1/2$$
$$f_3(\tau) = 2\tau(\ \tau - 1/2\ )(\ \tau + 1/2\ )$$
$$f_4(\tau) = (\ 5\tau^2 - 1/4\ )(\ \tau - 1/2\ )(\ \tau + 1/2\ )$$
$$\tau_u^{ijk} = \frac{u}{h_u^{ijk}}$$

We can show that $\phi_{ug}^{ijk}(u)$ and $L_{ug}^{ijk}(u)$ can be reduced to

$$\phi_{ug}^{ijk}(u) = \sum_{m=0}^{4} C_{mug}^{ijk} f_m(\tau_u^{ijk}) \qquad (4.a)$$

$$L_{ug}^{ijk}(u) = \sum_{m=0}^{2} K_{mgu}^{ijk} f_m(\tau_u^{ijk}) \ , \qquad (4.b)$$

where

$$C_{mxg}^{ijk} = C_{m00g}^{ijk}$$
$$C_{myg}^{ijk} = C_{0m0g}^{ijk}$$
$$C_{mzg}^{ijk} = C_{00mg}^{ijk}$$

4.3. Determination of C_{0ug}^{ijk}, C_{1ug}^{ijk} and C_{2ug}^{ijk}

From the definition of 1-D flux $\phi_{ug}^{ijk}(u)$ and $\overline{\phi_g^{ijk}}$,

We can find

$$\overline{\phi_g^{ijk}} = \frac{1}{h_u^{ijk}} \int_{-h_u^{ijk}/2}^{+h_u^{ijk}/2} \phi_{ug}^{ijk}(u)\ du = C_{0ug}^{ijk}$$

Using $\phi_{ulg}^{ijk} = \phi_{ug}^{ijk}(-\frac{h_u^{ijk}}{2}) = C_{0ug}^{ijk} - C_{1ug}^{ijk} + C_{2ug}^{ijk}$

and $\phi_{urg}^{ijk} = \phi_{ug}^{ijk}(+\frac{h_u^{ijk}}{2}) = C_{0ug}^{ijk} + C_{1ug}^{ijk} + C_{2ug}^{ijk}$,

We can find

$$C_{1ug}^{ijk} = \frac{1}{2}(\phi_{urg}^{ijk} - \phi_{ulg}^{ijk})$$

$$C_{2ug}^{ijk} = \frac{1}{2}(\phi_{urg}^{ijk} + \phi_{ulg}^{ijk}) - C_{0ug}^{ijk}$$

According to P_1 approximation, ϕ_{urg}^{ijk} and ϕ_{ulg}^{ijk} can be expressed as

$$\boxed{\phi_{usg}^{ijk} = 2\,(\,j_{usg}^{ijk+} + j_{usg}^{ijk-}\,) \qquad (\ s = r, l\ )}$$

therefore,

$$\boxed{\begin{aligned} C_{0ug}^{ijk} &= \overline{\phi_g^{ijk}} \\ C_{1ug}^{ijk} &= (\,j_{urg}^{ijk+} + j_{urg}^{ijk-}\,) - (\,j_{ulg}^{ijk+} + j_{ulg}^{ijk-}\,) \\ C_{2ug}^{ijk} &= (\,j_{urg}^{ijk+} + j_{urg}^{ijk-}\,) + (\,j_{ulg}^{ijk+} + j_{ulg}^{ijk-}\,) - \overline{\phi_g^{ijk}} \end{aligned}}$$

4.4. Determination of C_{3ug}^{ijk} and C_{4ug}^{ijk}

Insering Eq.(4.a) and Eq.(4.b) into Eq.(3),
We obtain

$$
\begin{aligned}
& -\frac{D_g^{ijk}}{(h_u^{ijk})^2}\ [\ C_{2ug}^{ijk} f_2''(\tau_u^{ijk}) + C_{3ug}^{ijk} f_3''(\tau_u^{ijk}) + C_{4ug}^{ijk} f_4''(\tau_u^{ijk})\ ] \\
& + D_g^{ijk}\ [\ K_{0ug}^{ijk} f_0(\tau_u^{ijk}) + K_{1ug}^{ijk} f_1(\tau_u^{ijk}) + K_{2ug}^{ijk} f_2(\tau_u^{ijk})\ ] \\
& + \Sigma_{tg}^{ijk}\ [\ C_{0ug}^{ijk} f_0(\tau_u^{ijk}) + C_{1ug}^{ijk} f_1(\tau_u^{ijk}) \\
& \qquad + C_{2ug}^{ijk} f_2(\tau_u^{ijk}) + C_{3ug}^{ijk} f_3(\tau_u^{ijk}) + C_{4ug}^{ijk} f_4(\tau_u^{ijk})\ ] \\
& - \sum_{g'=1}^{2}\ [\ \frac{\chi_g}{k} \nu\Sigma_{fg'}^{ijk} + \Sigma_{sg'g}^{ijk}\ ]\ [\ C_{0ug'}^{ijk} f_0(\tau_u^{ijk}) + C_{1ug'}^{ijk} f_1(\tau_u^{ijk}) \\
& \qquad + C_{2ug'}^{ijk} f_2(\tau_u^{ijk}) + C_{3ug'}^{ijk} f_3(\tau_u^{ijk}) + C_{4ug'}^{ijk} f_4(\tau_u^{ijk})\ ] \\
& = 0
\end{aligned}
\tag{5}
$$

Applying WRM to Eq.(5) ;

$$\int_{-1/2}^{+1/2} W_m(\tau_u^{ijk})\ [\ Eq.(5)\ ]\ d\tau_u^{ijk}\ ,$$

where $W_m(\tau)$ *is arbitrary function of* τ $(\ m=1,2\ )$

We can determine C_{3ug}^{ijk} and C_{4ug}^{ijk} .

4.4.1. Determination of C_{3ug}^{ijk}

$$W_1(\tau) = f_1(\tau)$$

Eq.(5) ==>

$$10 D_g^{ijk} h_u^{ijk} K_{1ug}^{ijk} - [\ 60\beta_{ug}^{ijk} + \ \Sigma_{tg}^{ijk} h_u^{ijk}\ ]\ C_{3ug}^{ijk} + \ 10\Sigma_{tg}^{ijk} h_u^{ijk} C_{1ug}^{ijk}$$

$$- \sum_{g'=1}^{2} [\ \frac{\chi_g}{k} \nu\Sigma_{fg'}^{ijk} h_u^{ijk} + \ \Sigma_{sg'g}^{ijk} h_u^{ijk}\ ]\ [\ 10C_{1ug'}^{ijk} - \ C_{3ug'}^{ijk}\ ]$$

$$= 0\ ,$$

$$where \quad \beta_{ug}^{ijk} = \frac{D_g^{ijk}}{h_u^{ijk}}$$

4.4.2. Determination of C_{4ug}^{ijk}

$$W_2(\tau) = f_2(\tau)$$

Eq.(5) ==>

$$14 D_g^{ijk} h_u^{ijk} K_{2ug}^{ijk} - [\ 140\beta_{ug}^{ijk} + \ \Sigma_{tg}^{ijk} h_u^{ijk}\ ]\ C_{4ug}^{ijk} + \ 14\Sigma_{tg}^{ijk} h_u^{ijk} C_{2ug}^{ijk}$$

$$- \sum_{g'=1}^{2} [\ \frac{\chi_g}{k} \nu\Sigma_{fg'}^{ijk} h_u^{ijk} + \ \Sigma_{sg'g}^{ijk} h_u^{ijk}\ ]\ [\ 14C_{2ug'}^{ijk} - \ C_{4ug'}^{ijk}\ ]$$

$$= 0$$

4.4.3. Explicit Form of C^{ijk}_{3ug} and C^{ijk}_{4ug}

$$\underline{C}_3 = 10\ \underline{R}_1^{-1}\ \underline{M}_1$$

$$\underline{C}_4 = 14\ \underline{R}_2^{-1}\ \underline{M}_2\ ,$$

where

$$\underline{C}_n = \begin{pmatrix} C^{ijk}_{nu1} \\ C^{ijk}_{nu2} \end{pmatrix}$$

$$\underline{R}_1 = \underline{N} + \frac{60}{h^{ijk}_u}\ \underline{B}$$

$$\underline{R}_2 = \underline{N} + \frac{140}{h^{ijk}_u}\ \underline{B}$$

$$\underline{N} = \begin{pmatrix} \Sigma^{ijk}_{a1} + \Sigma^{ijk}_{s} - \frac{\nu\Sigma^{ijk}_{f1}}{k} & -\frac{\nu\Sigma^{ijk}_{f2}}{k} \\ -\Sigma^{ijk}_{s} & \Sigma_{a2} \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} \beta^{ijk}_{u1} & 0 \\ 0 & \beta^{ijk}_{u2} \end{pmatrix}$$

$$\underline{M}_m = \begin{pmatrix} D^{ijk}_1 K^{ijk}_{mu1} + N^{ijk}_{11} C^{ijk}_{mu1} + N^{ijk}_{12} C^{ijk}_{mu2} \\ D^{ijk}_2 K^{ijk}_{mu2} + N^{ijk}_{21} C^{ijk}_{mu1} + N^{ijk}_{22} C^{ijk}_{mu2} \end{pmatrix}$$

$$\Sigma^{ijk}_{s} = \Sigma^{ijk}_{s12}$$

4.5. Determination of K^{ijk}_{0ug}, K^{ijk}_{1ug} and K^{ijk}_{2ug}

Similarly,

$$\overline{L^{ijk}_{ug}} = \frac{1}{h^{ijk}_u}\int_{h^{ijk}_u/2}^{+h^{ijk}_u/2} L^{ijk}_{ug}(u)\ du = K^{ijk}_{0ug}$$

$$L^{ijk}_{ulg} = L^{ijk}_{ug}(-\frac{h^{ijk}_u}{2}) = K^{ijk}_{0ug} - K^{ijk}_{1ug} + K^{ijk}_{2ug}$$

$$L^{ijk}_{urg} = L^{ijk}_{ug}(+\frac{h^{ijk}_u}{2}) = K^{ijk}_{0ug} + K^{ijk}_{1ug} + K^{ijk}_{2ug} \quad ,$$

On the other hand,

Using the definition of $L^{ijk}_{ug}(u)$ in Eq.(3),

$\overline{L^{ijk}_{ug}}$ can be calculated.

Using the conditions

$$L^{ijk}_{xlg} = L^{i-1jk}_{xrg},\ \ L^{ijk}_{ylg} = L^{ij-1k}_{yrg},\ \ L^{ijk}_{zlg} = L^{ijk-1}_{zrg}$$

$$L^{ijk}_{xrg} = L^{i+1jk}_{xlg},\ \ L^{ijk}_{yrg} = L^{ij+1k}_{ylg},\ \ L^{ijk}_{zrg} = L^{ijk+1}_{zlg}$$

$$D^{ijk}_g \frac{dL^{ijk}_{xg}(x)}{dx}\,\Big|_{\,x=-h^{ijk}_x/2} = D^{i-1jk}_g \frac{dL^{i-1jk}_{xg}(x)}{dx}\,\Big|_{\,x=+h^{i-1jk}_x/2}$$

$$D^{ijk}_g \frac{dL^{ijk}_{yg}(y)}{dy}\,\Big|_{\,y=-h^{ijk}_y/2} = D^{ij-1k}_g \frac{dL^{ij-1k}_{yg}(y)}{dy}\,\Big|_{\,y=+h^{ij-1k}_y/2}$$

$$D^{ijk}_g \frac{dL^{ijk}_{zg}(z)}{dz}\,\Big|_{\,z=-h^{ijk}_z/2} = D^{ijk-1}_g \frac{dL^{ijk-1}_{zg}(z)}{dz}\,\Big|_{\,z=+h^{ijk-1}_z/2}$$

$$D^{ijk}_g \frac{dL^{ijk}_{xg}(x)}{dx}\,\Big|_{\,x=+h^{ijk}_x/2} = D^{i+1jk}_g \frac{dL^{i+1jk}_{xg}(x)}{dx}\,\Big|_{\,x=-h^{i+1jk}_x/2}$$

$$D^{ijk}_g \frac{dL^{ijk}_{yg}(y)}{dy}\,\Big|_{\,y=+h^{ijk}_y/2} = D^{ij+1k}_g \frac{dL^{ij+1k}_{yg}(y)}{dy}\,\Big|_{\,y=-h^{ij+1k}_y/2}$$

$$D^{ijk}_g \frac{dL^{ijk}_{zg}(z)}{dz}\,\Big|_{\,z=+h^{ijk}_z/2} = D^{ijk+1}_g \frac{dL^{ijk+1}_{zg}(z)}{dz}\,\Big|_{\,z=-h^{ijk+1}_z/2}$$

L_{usg}^{ijk} can also be calculated.

The result is

$$K_{0ug}^{ijk} = \overline{L_{ug}^{ijk}}$$

$$K_{1ug}^{ijk} = \frac{1}{2}\left(L_{urg}^{ijk} - L_{ulg}^{ijk} \right)$$

$$K_{2ug}^{ijk} = \frac{1}{2}\left(L_{urg}^{ijk} + L_{ulg}^{ijk} \right) - \overline{L_{ug}^{ijk}} \ ,$$

where

$$\overline{L_{ug}^{ijk}} = \frac{1}{D_g^{ijk}}\left[\frac{j_{vrg}^{ijk+} - j_{vrg}^{ijk-} - j_{vlg}^{ijk+} + j_{vlg}^{ijk-}}{h_v^{ijk}} + \frac{j_{wrg}^{ijk+} - j_{wrg}^{ijk-} - j_{wlg}^{ijk+} + j_{wlg}^{ijk+}}{h_w^{ijk}} \right]$$

$$L_{xlg}^{ijk} = \left[\frac{h_x^{i-1jk}}{h_x^{ijk}}\overline{L_{xg}^{ijk}} + \frac{D_g^{i-1jk}}{D_g^{ijk}}\overline{L_{xg}^{i-1jk}} \right] / \left[\frac{h_x^{i-1jk}}{h_x^{ijk}} + \frac{D_g^{i-1jk}}{D_g^{ijk}} \right]$$

$$L_{xrg}^{ijk} = \left[\frac{h_x^{i+1jk}}{h_x^{ijk}}\overline{L_{xg}^{ijk}} + \frac{D_g^{i+1jk}}{D_g^{ijk}}\overline{L_{xg}^{i+1jk}} \right] / \left[\frac{h_x^{i+1jk}}{h_x^{ijk}} + \frac{D_g^{i+1jk}}{D_g^{ijk}} \right]$$

$$L_{ylg}^{ijk} = \left[\frac{h_y^{ij-1k}}{h_y^{ijk}}\overline{L_{yg}^{ijk}} + \frac{D_g^{ij-1k}}{D_g^{ijk}}\overline{L_{yg}^{ij-1k}} \right] / \left[\frac{h_y^{ij-1k}}{h_y^{ijk}} + \frac{D_g^{ij-1k}}{D_g^{ijk}} \right]$$

$$L_{yrg}^{ijk} = \left[\frac{h_y^{ij+1k}}{h_y^{ijk}}\overline{L_{yg}^{ijk}} + \frac{D_g^{ij+1k}}{D_g^{ijk}}\overline{L_{yg}^{ij+1k}} \right] / \left[\frac{h_y^{ij+1k}}{h_y^{ijk}} + \frac{D_g^{ij+1k}}{D_g^{ijk}} \right]$$

$$L_{zlg}^{ijk} = \left[\frac{h_z^{ijk-1}}{h_z^{ijk}}\overline{L_{zg}^{ijk}} + \frac{D_g^{ijk-1}}{D_g^{ijk}}\overline{L_{zg}^{ijk-1}} \right] / \left[\frac{h_z^{ijk-1}}{h_z^{ijk}} + \frac{D_g^{ijk-1}}{D_g^{ijk}} \right]$$

$$L_{zrg}^{ijk} = \left[\frac{h_z^{ijk+1}}{h_z^{ijk}}\overline{L_{zg}^{ijk}} + \frac{D_g^{ijk+1}}{D_g^{ijk}}\overline{L_{zg}^{ijk+1}} \right] / \left[\frac{h_z^{ijk+1}}{h_z^{ijk}} + \frac{D_g^{ijk+1}}{D_g^{ijk}} \right]$$

4.6. Relationship between j_{urg}^{ijk+} , j_{ulg}^{ijk-} and $\overline{\phi_g^{ijk}}$

Recalling the definition of 1-D flux $\phi_{ug}^{ijk}(u)$,

Eq.(2.b-1) and Eq.(2.b-2) can be interpreted as,

$$J_{urg}^{ijk} = j_{urg}^{ijk+} - j_{urg}^{ijk-} = -D_g^{ijk} \frac{d\phi_{ug}^{ijk}(u)}{du} \Big|_{u=+h_u^{ijk}/2}$$

$$J_{ulg}^{ijk} = j_{ulg}^{ijk+} - j_{ulg}^{ijk-} = -D_g^{ijk} \frac{d\phi_{ug}^{ijk}(u)}{du} \Big|_{u=-h_u^{ijk}/2}$$

Inserting Eq.(4.a) into above Equations, finally we obtain

$$j_{urg}^{ijk+} = [\, 6\,\overline{\phi_g^{ijk}} - C_{4ug}^{ijk}\,]\, Q_{0ug}^{ijk} - 8\, j_{ulg}^{ijk+}\, Q_{1ug}^{ijk} + j_{urg}^{ijk-}\, Q_{2ug}^{ijk} - C_{3ug}^{ijk}\, Q_{3ug}^{ijk} \qquad (6.b-1)$$

$$j_{ulg}^{ijk-} = [\, 6\,\overline{\phi_g^{ijk}} - C_{4ug}^{ijk}\,]\, Q_{0ug}^{ijk} - 8\, j_{urg}^{ijk-}\, Q_{1ug}^{ijk} + j_{ulg}^{ijk+}\, Q_{2ug}^{ijk} + C_{3ug}^{ijk} Q_{3ug}^{ijk} \ , \qquad (6.b-2)$$

where

$$Q_{0ug}^{ijk} = \frac{\beta_{ug}^{ijk}}{1 + 12\,\beta_{ug}^{ijk}}$$

$$Q_{1ug}^{ijk} = \frac{\beta_{ug}^{ijk}}{(1 + 4\,\beta_{ug}^{ijk})(1 + 12\,\beta_{ug}^{ijk})}$$

$$Q_{2ug}^{ijk} = \frac{1 - 48\,(\beta_{ug}^{ijk})^2}{(1 + 4\,\beta_{ug}^{ijk})(1 + 12\,\beta_{ug}^{ijk})}$$

$$Q_{3ug}^{ijk} = \frac{\beta_{ug}^{ijk}}{1 + 4\,\beta_{ug}^{ijk}}$$

5. $\overline{\phi}_g^{ijk}$ from j_{urg}^{ijk-}, j_{ulg}^{ijk+} How to Obtain

Inserting Eq.(6.b-1) and Eq.(6.b-2) into Eq.(2.a),
we obtain

$$\begin{pmatrix} \overline{\phi}_1^{ijk} \\ \overline{\phi}_2^{ijk} \end{pmatrix} = (\underline{N} + 12\underline{Q})^{-1} \begin{pmatrix} S_1^{ijk} \\ S_2^{ijk} \end{pmatrix} ,$$

where

$$\underline{Q} = \begin{pmatrix} \sum_{u=x,y,z} \frac{Q_{0u1}^{ijk}}{h_u^{ijk}} & 0 \\ 0 & \sum_{u=x,y,z} \frac{Q_{0u2}^{ijk}}{h_u^{ijk}} \end{pmatrix}$$

$$S_g^{ijk} = \sum_{u=x,y,z} \frac{2Q_{0ug}^{ijk} C_{4ug}^{ijk} + (j_{urg}^{ijk-} + j_{ulg}^{ijk+})(1 + 8Q_{1ug}^{ijk} - Q_{2ug}^{ijk})}{h_u^{ijk}}$$

Appendix A.

Properties of $f_m(\tau)$

$f_0(-1/2) = +1 \qquad f_0(+1/2) = +1$
$f_1(-1/2) = -1 \qquad f_1(+1/2) = +1$
$f_2(-1/2) = +1 \qquad f_2(+1/2) = +1$
$f_3(-1/2) = 0 \qquad f_3(+1/2) = 0$
$f_4(-1/2) = 0 \qquad f_4(+1/2) = 0$

$f_0'(-1/2) = 0 \qquad f_0'(+1/2) = 0$
$f_1'(-1/2) = +2 \qquad f_1'(+1/2) = +2$
$f_2'(-1/2) = -6 \qquad f_2'(+1/2) = +6$
$f_3'(-1/2) = +1 \qquad f_3'(+1/2) = +1$
$f_4'(-1/2) = -1 \qquad f_4'(+1/2) = +1$

$< f_0 > = 1 \qquad < f_1 > = 0$
$< f_2 > = 0 \qquad < f_3 > = 0$
$< f_4 > = 0$

$< f_1 f_0 > = 0 \qquad < f_1 f_1 > = 1/3$
$< f_1 f_2 > = 0 \qquad < f_1 f_3 > = -1/30$
$< f_1 f_4 > = 0$

$< f_2 f_0 > = 0 \qquad < f_2 f_1 > = 0$
$< f_2 f_2 > = 1/5 \qquad < f_2 f_3 > = 0$
$< f_2 f_4 > = -1/70$

$< f_1 f_0'' > = 0 \qquad < f_1 f_1'' > = 0$
$< f_1 f_2'' > = 0 \qquad < f_1 f_3'' > = 2$
$< f_1 f_4'' > = 0$

$< f_2 f_0'' > = 0 \qquad < f_2 f_1'' > = 0$
$< f_2 f_2'' > = 0 \qquad < f_2 f_3'' > = 0$
$< f_2 f_4'' > = 2$,

where $< f_i > \equiv \int_{-1/2}^{+1/2} f_i(\tau)\, d\tau$, $\quad f_i f_j \equiv f_i(\tau)\, f_j(\tau)$

Appendix B.

Physical Meaning of Transverse Leakage