In [44]:
```python
import numpy as np
```

In [45]:
```python
import pandas as pd
```

$Data\,Processiong$

In [46]:
```python
#data_path = "crimedataset1,xlsx" - 결측치를 각 열의 평균으로 처리할 경우.
# 저장된 코드는 결측치를 제거한 데이터임.

data_path = "crimedataset11.xlsx"
crime= pd.read_excel(data_path)
```

In [47]:
```python
crime=crime.dropna(axis=1)
```

In [48]:
```python
crime = crime.replace("?",None)
```

In [6]:
```python
#crime=crime.fillna(crime.mean())
```

In [49]:
```python
crime=(crime-crime.mean())/crime.std()
```

In [50]:
```python
crime.head(10)
```

Out[50]:

	state_numeric	populationnumeric	householdsizenumeric	racepctblacknumeric	racePctWhitenumeric	racePct
0	-1.261380	1.043350	-0.814793	-0.629844	0.599427	
1	1.482932	-0.453823	-1.853172	-0.235276	-0.056205	
2	-0.285625	-0.453823	-0.265063	1.224624	-0.793790	
3	0.324222	-0.138629	1.872776	3.236918	-2.760686	
4	0.812100	-0.375024	0.528992	-0.629844	0.804312	
5	-1.383350	-0.296226	-1.120198	-0.472017	-0.875744	
6	0.934069	-0.375024	-0.448306	-0.708757	0.927243	
7	-1.383350	-0.375024	1.689533	-0.590387	-1.203560	
8	-0.468579	-0.217427	-0.753712	0.080378	0.353565	
9	0.019299	-0.375024	-0.387225	-0.472017	0.476496	

10 rows × 99 columns

In [71]:
```python
nX = crime.as_matrix()
nX = nX.astype(float) # 공통된 데이터

nnX = nX[:,:98] # LAsso를 위한 데이터
npX = nX[:,98] #target

nX1= np.c_[np.ones((1994,1)),nX] # Ridge를 위한 데이터
```

```
D:\anaconda\lib\site-packages\ipykernel_launcher.py:1: FutureWarning: Method .as_matrix will be re
moved in a future version. Use .values instead.
  """Entry point for launching an IPython kernel.
```

In [72]:
```python
X=nX1[:,:99]
```

In [74]:
```
nnX[:,97]
```
Out[74]:
```
array([ 0.16727414,  1.25871603, -0.61856401, ...,  0.07995879,
        0.73482392, -0.48759099])
```
In [76]:
```
npX
```
Out[76]:
```
array([-0.16301029,  1.85428781,  0.82417814, ..., -0.03424658,
       -0.20593153,  1.03878432])
```
In [62]:
```
X[:,98]
```
Out[62]:
```
array([ 0.16727414,  1.25871603, -0.61856401, ...,  0.07995879,
        0.73482392, -0.48759099])
```
In [56]:
```
predict = nX1[:,99]
```
In [75]:
```
predict
```
Out[75]:
```
array([-0.16301029,  1.85428781,  0.82417814, ..., -0.03424658,
       -0.20593153,  1.03878432])
```
In [77]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
plt.rcParams["figure.figsize"] = (10,8)
```
In [78]:
```
def Ridge(X, y,lam):
    lam_par = lam
    xtranspose = np.transpose(X)
    xtransx = np.dot(xtranspose, X)
    lamidentity = np.identity(xtransx.shape[0]) * lam_par
    matinv = np.linalg.inv(lamidentity + xtransx)
    xtransy = np.dot(xtranspose, y)
    wRR = np.dot(matinv, xtransy)
    _, S, _ = np.linalg.svd(X)
    return wRR
```
In [79]:
```
def calc_train_error(X_train, y_train):
    A= Ridge(X_train,y_train,lam)
    predictions = np.dot(X_train,A)
    mse = np.sqrt(np.sum(np.square(y_train - predictions)))
    return mse

def calc_validation_error(X_test, y_test,X_train,y_train):
    A= Ridge(X_train,y_train,lam)
    predictions = np.dot(X_test,A)
    mse = np.sqrt(np.sum(np.square(y_test - predictions)))*(len(y_train)/len(y_test))
    return mse

def calc_metrics(X_train, y_train, X_test, y_test,lam):
    #model.fit(X_train, y_train)
    train_error = calc_train_error(X_train, y_train)
    validation_error = calc_validation_error(X_test, y_test,X_train,y_train)
    return train_error, validation_error
```

In [80]:
```
Tr1 = nX1[:200,:99] #결측치를 평균으로 처리할 경우 99->118
Tv1 = nX1[:200,99]
Tr2 = nX1[200:400,:99]
Tv2 = nX1[200:400,99]
Tr3 = nX1[400:600,:99]
Tv3 = nX1[400:600,99]
Tr4 = nX1[600:800,:99]
Tv4 = nX1[600:800,99]
Tr5 =  nX1[800:1000,:99]
Tv5 = nX1[800:1000,99]
Tr6 = nX1[1000:1200,:99]
Tv6 = nX1[1000:1200,99]
Tr7 = nX1[1200:1400,:99]
Tv7 = nX1[1200:1400,99]
Tr8 = nX1[1400:1600,:99]
Tv8 = nX1[1400:1600,99]
Tr9 = nX1[1600:1800,:99]
Tv9 = nX1[1600:1800,99]
Tr10 = nX1[1800:,:99]
Tv10 = nX1[1800:,99]
```

In [81]:
```
Train1 = np.delete(X, [i for i in range(200)], axis=0)
Tval1 = np.delete(predict, [i for i in range(200)], axis=0)
Train2 = np.delete(X, [i for i in range(200,400)], axis=0)
Tval2 = np.delete(predict, [i for i in range(200,400)], axis=0)
Train3 = np.delete(X, [i for i in range(400,600)], axis=0)
Tval3 = np.delete(predict, [i for i in range(400,600)], axis=0)
Train4 = np.delete(X, [i for i in range(600,800)], axis=0)
Tval4 = np.delete(predict, [i for i in range(600,800)], axis=0)
Train5 = np.delete(X, [i for i in range(800,1000)], axis=0)
Tval5 = np.delete(predict, [i for i in range(800,1000)], axis=0)
Train6 = np.delete(X, [i for i in range(1000,1200)], axis=0)
Tval6 = np.delete(predict, [i for i in range(1000,1200)], axis=0)
Train7 = np.delete(X, [i for i in range(1200,1400)], axis=0)
Tval7 = np.delete(predict, [i for i in range(1200,1400)], axis=0)
Train8 =np.delete(X, [i for i in range(1400,1600)], axis=0)
Tval8 =np.delete(predict, [i for i in range(1400,1600)], axis=0)
Train9 = np.delete(X, [i for i in range(1600,1800)], axis=0)
Tval9 = np.delete(predict, [i for i in range(1600,1800)], axis=0)
Train10 = np.delete(X, [i for i in range(1800,len(predict))], axis=0)
Tval10 = np.delete(predict, [i for i in range(1800,len(predict))], axis=0)

TrainSet = [Train1,Train2,Train3,Train4,Train5,Train6,Train7,Train8,Train9,Train10]
TvalSet=[Tval1,Tval2,Tval3,Tval4,Tval5,Tval6,Tval7,Tval8,Tval9,Tval10]
```

In [82]:
```
TargetSet=[Tr1,Tr2,Tr3,Tr4,Tr5,Tr6,Tr7,Tr8,Tr9,Tr10]
TavalSet=[Tv1,Tv2,Tv3,Tv4,Tv5,Tv6,Tv7,Tv8,Tv9,Tv10]
```

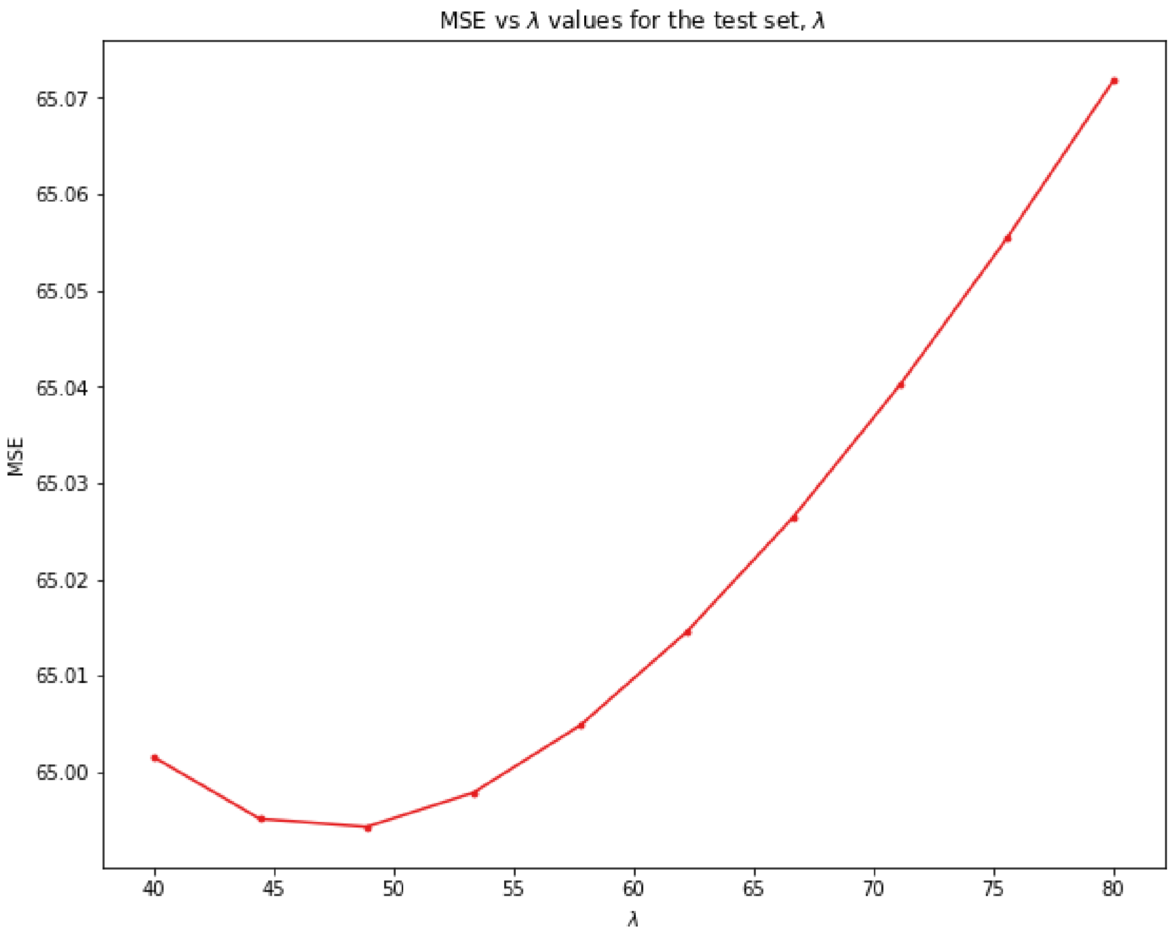
$10\ fold\ Validation$

In [86]:

```
#lamdas = np.logspace(0,30,50)

# 변경가능

lamdas = np.linspace(40,80,10)

MSE_list = []
for lam in lamdas:
    train_errors = []
    validation_errors = []
    for i in range(10):


        X_train, X_val = TrainSet[i], TargetSet[i]
        y_train, y_val = TvalSet[i], TavalSet[i]


        train_error, val_error = calc_metrics(X_train, y_train, X_val, y_val,lam)

        # append to appropriate list
        train_errors.append(train_error)
        validation_errors.append(val_error)

    MSE_list.append(min(validation_errors))
    # generate report
    print('lam: {:6} | min(train_error): {:7} | min(val_error): {}'.
        format(lam,
               round(np.min(train_errors),4),
               round(np.min(validation_errors),4)))
```

```
lam:   40.0 | min(train_error): 23.0868 | min(val_error): 65.0016
lam: 44.44444444444444 | min(train_error): 23.1081 | min(val_error): 64.9951
lam: 48.888888888888886 | min(train_error): 23.1284 | min(val_error): 64.9943
lam: 53.333333333333336 | min(train_error): 23.1478 | min(val_error): 64.9978
lam: 57.77777777777778 | min(train_error): 23.1664 | min(val_error): 65.0048
lam: 62.22222222222222 | min(train_error): 23.1843 | min(val_error): 65.0145
lam: 66.66666666666667 | min(train_error): 23.2014 | min(val_error): 65.0265
lam: 71.11111111111111 | min(train_error): 23.2179 | min(val_error): 65.0402
lam: 75.55555555555556 | min(train_error): 23.2337 | min(val_error): 65.0554
lam:   80.0 | min(train_error): 23.2491 | min(val_error): 65.0718
```

In [87]:
```
def plotRMSEValue(lamdas,RMSE_list):
    colors = ['#e41a1c']
    plt.plot(lamdas, MSE_list, color = colors[0])
    plt.scatter(lamdas, MSE_list, color = colors[0] , s = 8)
    # df(lambda)
    plt.xlabel("$₩lambda$")
    plt.ylabel("MSE")
    # and a legend
    plt.title(r"MSE vs $₩lambda$ values for the test set, $₩lambda$ ")
plt.figure()
plotRMSEValue(lamdas,MSE_list)
plt.show()
```



적절한 규제가 있을때, MSE가 최소가 된다. ($RMSE$이지만 편의상 MSE로 쓰겠음)

따라서 주어진 데이터는 상당한 $multi-colinearility$를 가짐을 알 수 있다.

또한 훈련데이터에 대한 MSE와 검증데이터에 대한 MSE는 약 3배가 차이가 난다.

이것은 $validation$을 위해 데이터 세트를 k겹으로 나눌때 발생하는 결과이다.

적절한 k를 찾아서 이 차이를 줄일 수 있을 것이다.

.

.

Lasso

In [88]:
```
def soft_threshold(rho,lamda):
    '''Soft threshold function used for normalized data and lasso regression'''
    if rho < - lamda:
        return (rho + lamda)
    elif rho >  lamda:
        return (rho - lamda)
    else:
        return 0


def lasso(theta,X,y,lamda = .01, num_iters=50, intercept = False):
    '''Coordinate gradient descent for lasso regression - for normalized data.
    The intercept parameter allows to specify whether or not we regularize theta_0'''

    m,n = X.shape
    X = X / (np.linalg.norm(X,axis = 0)) #normalizing X in case it was not done before

    y = y.reshape(-1,1)

    for i in range(num_iters):

        for j in range(n):

            X_j = X[:,j].reshape(-1,1)
            y_pred = np.dot(X,theta)
            rho = np.dot(X_j.T, (y - y_pred  + theta[j]*X_j))

            if intercept == True:
                if j == 0:
                    theta[j] =  rho
                else:
                    theta[j] =  soft_threshold(rho, lamda)

            if intercept == False:
                theta[j] =  soft_threshold(rho, lamda)

    return theta.flatten()
```

Lasso code explanation

The Lasso cost function is following :

$$
\begin{aligned}
RSS^{lasso}(\theta) &= RSS^{OLS}(\theta) + \lambda||\theta||_1 \\
&= \frac{1}{2}\sum_{i=1}^{m}\left[y^{(i)} - \sum_{j=0}^{n}\theta_j x_j^{(i)}\right]^2 + \lambda\sum_{j=0}^{n}|\theta_j|
\end{aligned}
$$

Then for j,

$$\lambda\sum_{j=0}^{n}|\theta_j| = \lambda|\theta_j| + \lambda\sum_{k\neq j}^{n}|\theta_k|$$

$$\partial_{\theta_j}\lambda\sum_{j=0}^{n}|\theta_j| = \partial_{\theta_j}\lambda|\theta_j| = \begin{cases} \{-\lambda\} & \text{if } \theta_j < 0 \\ [-\lambda, \lambda] & \text{if } \theta_j = 0 \\ \{\lambda\} & \text{if } \theta_j > 0 \end{cases}$$

$$\begin{aligned} \frac{\partial}{\partial\theta_j}RSS^{OLS}(\theta) &= -\sum_{i=1}^{m} x_j^{(i)}\left[y^{(i)} - \sum_{j=0}^{n}\theta_j x_j^{(i)}\right] \\ &= -\sum_{i=1}^{m} x_j^{(i)}\left[y^{(i)} - \sum_{k\neq j}^{n}\theta_k x_k^{(i)} - \theta_j x_j^{(i)}\right] \\ &= -\sum_{i=1}^{m} x_j^{(i)}\left[y^{(i)} - \sum_{k\neq j}^{n}\theta_k x_k^{(i)}\right] + \theta_j\sum_{i=1}^{m}(x_j^{(i)})^2 \\ &\triangleq -\rho_j + \theta_j z_j \end{aligned}$$

$To\ obtain\ the\ minimum(convex),\ each\ partial\ derivative\ must\ be\ zero.$

$$\begin{aligned} \partial_{\theta_j}RSS^{lasso}(\theta) &= \partial_{\theta_j}RSS^{OLS}(\theta) + \partial_{\theta_j}\lambda||\theta||_1 \\ 0 &= -\rho_j + \theta_j z_j + \partial_{\theta_j}\lambda||\theta_j|| \\ 0 &= \begin{cases} -\rho_j + \theta_j z_j - \lambda & \text{if } \theta_j < 0 \\ [-\rho_j - \lambda, -\rho_j + \lambda] & \text{if } \theta_j = 0 \\ -\rho_j + \theta_j z_j + \lambda & \text{if } \theta_j > 0 \end{cases} \end{aligned}$$

$Then\ we\ obtain\ the\ following:$

$$\begin{cases} \theta_j = \frac{\rho_j+\lambda}{z_j} & \text{for } \rho_j < -\lambda \\ \theta_j = 0 & \text{for } -\lambda \leq \rho_j \leq \lambda \\ \theta_j = \frac{\rho_j-\lambda}{z_j} & \text{for } \rho_j > \lambda \end{cases}$$

$Note\ that\ \theta_j\ can\ be\ zero.$

$In\ short, if\ we\ iterate\ this\ process$

$we\ can\ know\ whether\ \theta_j\ has\ a\ meaning\ for\ crime\ or\ not\ for\ each\ j.$

In [89]:
```
def calc_train_error1(X_train, y_train,initial_theta):
    A= lasso(initial_theta,X_train,y_train,lam,num_iters=50)
    predictions = np.dot(X_train,A)
    mse = np.sqrt(np.sum(np.square(y_train - predictions)))
    return mse

def calc_validation_error1(X_test, y_test,X_train,y_train,initial_theta):
    A= lasso(initial_theta,X_train,y_train,lam,num_iters=50)
    predictions = np.dot(X_test,A)
    mse = np.sqrt(np.sum(np.square(y_test - predictions)))*(len(y_train)/len(y_test))
    return mse

def calc_metrics1(X_train, y_train, X_test, y_test,lam,initial_theta):
    #model.fit(X_train, y_train)
    train_error = calc_train_error1(X_train, y_train,initial_theta)
    validation_error = calc_validation_error1(X_test, y_test,X_train,y_train,initial_theta)
    return train_error, validation_error
```

In [90]:
```
#nX를 가져와야함
#평균으로 처리할 경우 98->117, 99->118

LTr1 = nX[:200,:98]
LTv1 = nX1[:200,99]
LTr2 = nX[200:400,:98]
LTv2 = nX1[200:400,99]
LTr3 = nX[400:600,:98]
LTv3 = nX1[400:600,99]
LTr4 = nX[600:800,:98]
LTv4 = nX1[600:800,99]
LTr5 =  nX[800:1000,:98]
LTv5 = nX1[800:1000,99]
LTr6 = nX[1000:1200,:98]
LTv6 = nX1[1000:1200,99]
LTr7 = nX[1200:1400,:98]
LTv7 = nX1[1200:1400,99]
LTr8 = nX[1400:1600,:98]
LTv8 = nX1[1400:1600,99]
LTr9 = nX[1600:1800,:98]
LTv9 = nX1[1600:1800,99]
LTr10 = nX[1800:,:98]
LTv10 = nX1[1800:,99]
```

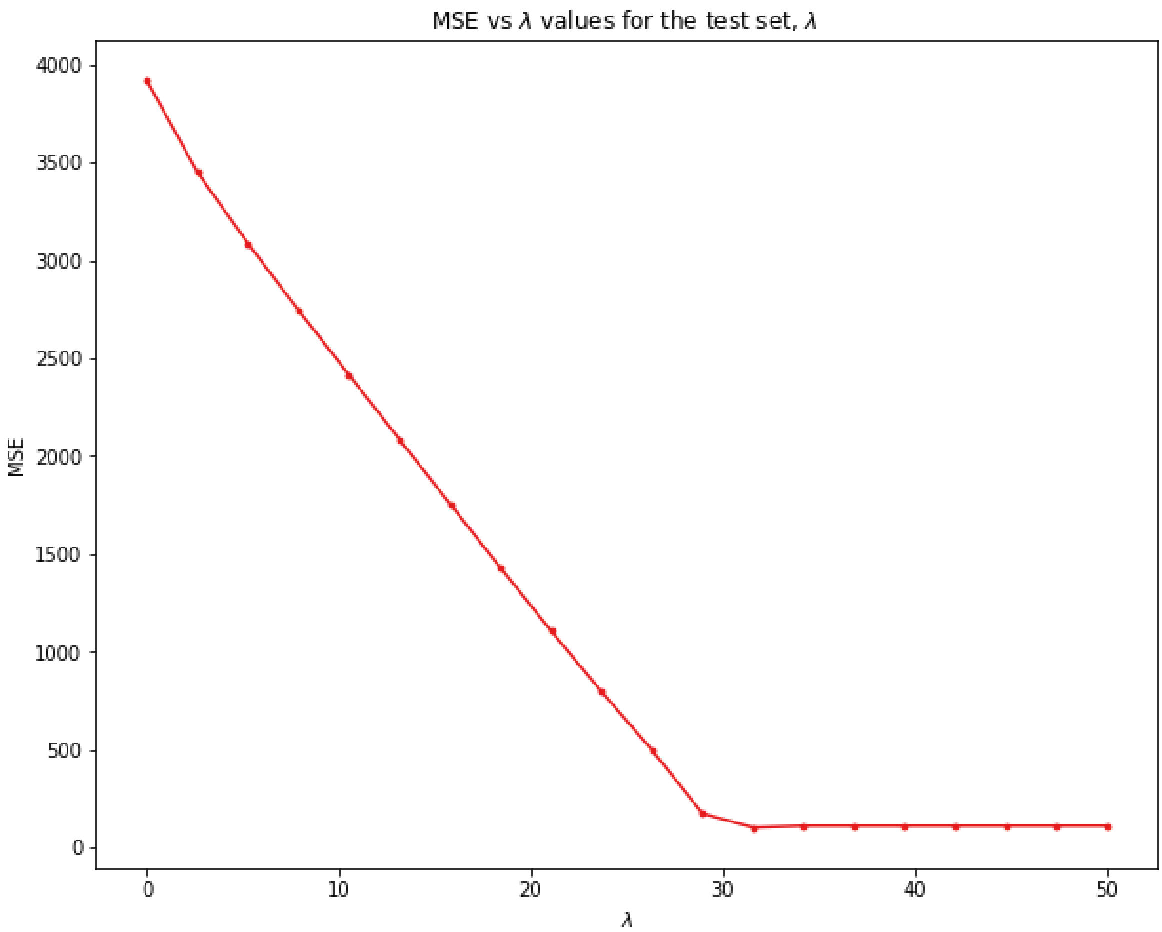
In [91]:
```
LTrain1 = np.delete(nnX, [i for i in range(200)], axis=0)
LTval1 = np.delete(predict, [i for i in range(200)], axis=0)
LTrain2 = np.delete(nnX, [i for i in range(200,400)], axis=0)
LTval2 = np.delete(predict, [i for i in range(200,400)], axis=0)
LTrain3 = np.delete(nnX, [i for i in range(400,600)], axis=0)
LTval3 = np.delete(predict, [i for i in range(400,600)], axis=0)
LTrain4 = np.delete(nnX, [i for i in range(600,800)], axis=0)
LTval4 = np.delete(predict, [i for i in range(600,800)], axis=0)
LTrain5 = np.delete(nnX, [i for i in range(800,1000)], axis=0)
LTval5 = np.delete(predict, [i for i in range(800,1000)], axis=0)
LTrain6 = np.delete(nnX, [i for i in range(1000,1200)], axis=0)
LTval6 = np.delete(predict, [i for i in range(1000,1200)], axis=0)
LTrain7 = np.delete(nnX, [i for i in range(1200,1400)], axis=0)
LTval7 = np.delete(predict, [i for i in range(1200,1400)], axis=0)
LTrain8 =np.delete(nnX, [i for i in range(1400,1600)], axis=0)
LTval8 =np.delete(predict, [i for i in range(1400,1600)], axis=0)
LTrain9 = np.delete(nnX, [i for i in range(1600,1800)], axis=0)
LTval9 = np.delete(predict, [i for i in range(1600,1800)], axis=0)
LTrain10 = np.delete(nnX, [i for i in range(1800,len(predict))], axis=0)
LTval10 = np.delete(predict, [i for i in range(1800,len(predict))], axis=0)
```

In [92]:
```python
LTrainSet = [LTrain1,LTrain2,LTrain3,LTrain4,LTrain5,LTrain6,LTrain7,LTrain8,LTrain9,LTrain10]
LTvalSet=[LTval1,LTval2,LTval3,LTval4,LTval5,LTval6,LTval7,LTval8,LTval9,LTval10]
LTargetSet=[LTr1,LTr2,LTr3,LTr4,LTr5,LTr6,LTr7,LTr8,LTr9,LTr10]
LTavalSet=[LTv1,LTv2,LTv3,LTv4,LTv5,LTv6,LTv7,LTv8,LTv9,LTv10]
```

In [93]:
```python
#lamdas = np.logspace(0,30,50) # 20은 변경가능
lamdas = np.linspace(0,50,20)

MSE_list = []
for lam in lamdas:
    train_errors = []
    validation_errors = []
    for i in range(10):


        X_train, X_val = LTrainSet[i], LTargetSet[i]
        y_train, y_val = LTvalSet[i], LTavalSet[i]
        t,s = LTrainSet[i].shape
        initial_theta = np.ones((s,1))


        train_error, val_error = calc_metrics1(X_train, y_train, X_val, y_val,lam,initial_theta)

        # append to appropriate list
        train_errors.append(train_error)
        validation_errors.append(val_error)

    MSE_list.append(min(validation_errors))
    # generate report
    print('lam: {:6} | min(train_error): {:7} | min(val_error): {}'.
        format(lam,
               round(np.min(train_errors),4),
               round(np.min(validation_errors),4)))
```

```
lam:    0.0 | min(train_error): 1436.64 | min(val_error): 3919.0516
lam: 2.6315789473684212 | min(train_error): 1236.3153 | min(val_error): 3450.1463
lam: 5.2631578947368425 | min(train_error): 1102.8961 | min(val_error): 3086.3807
lam: 7.894736842105264 | min(train_error): 977.7253 | min(val_error): 2745.7607
lam: 10.526315789473685 | min(train_error): 857.102 | min(val_error): 2415.3729
lam: 13.157894736842106 | min(train_error): 738.5364 | min(val_error): 2084.8802
lam: 15.789473684210527 | min(train_error): 620.3362 | min(val_error): 1755.8539
lam: 18.42105263157895 | min(train_error): 502.8699 | min(val_error): 1429.3071
lam: 21.05263157894737 | min(train_error): 387.106 | min(val_error): 1107.4352
lam: 23.68421052631579 | min(train_error): 275.1279 | min(val_error): 795.9303
lam: 26.315789473684212 | min(train_error): 165.1992 | min(val_error): 498.2547
lam: 28.947368421052634 | min(train_error): 59.2058 | min(val_error): 173.4421
lam: 31.578947368421055 | min(train_error): 37.8856 | min(val_error): 102.8379
lam: 34.21052631578948 | min(train_error): 41.7191 | min(val_error): 111.5371
lam: 36.8421052631579 | min(train_error): 41.7191 | min(val_error): 111.5371
lam: 39.473684210526315 | min(train_error): 41.7191 | min(val_error): 111.5371
lam: 42.10526315789474 | min(train_error): 41.7191 | min(val_error): 111.5371
lam: 44.736842105263165 | min(train_error): 41.7191 | min(val_error): 111.5371
lam: 47.36842105263158 | min(train_error): 41.7191 | min(val_error): 111.5371
lam:   50.0 | min(train_error): 41.7191 | min(val_error): 111.5371
```

In [94]:
```python
def plotRMSEValue1(lamdas,RMSE_list):
    colors = ['#e41a1c']
    plt.plot(lamdas, MSE_list, color = colors[0])
    plt.scatter(lamdas, MSE_list, color = colors[0] , s = 8)
    # df(lambda)
    plt.xlabel("$₩lambda$")
    plt.ylabel("MSE")
    # and a legend
    plt.title(r"MSE vs $₩lambda$ values for the test set, $₩lambda$ ")
plt.figure()
plotRMSEValue1(lamdas,MSE_list)
plt.show()
```



$Lasso\ Regression$ 에서는

적절한 $gradient$ 구할 수 없기 때문에 $Optimization\ Algorithm$을 이용했다.

따라서 $Optimziation$을 위한 $Hyperparameter$가 필요하기 때문에

$Ridge\ Regression$의 경우보다 더 계산량이 많아진다는 단점이 있다.

또 $Hyperparameter$를 적절히 튜닝을 해야하는 문제가 있다.

그러나 $Bayesian$의 관점에서 보면 주어진 데이터에 적절한 회귀분석법은 $Lasso$인듯 하다.

$Lasso$는 변수들이 $Laplace$ 분포를 따른다고 판단될 때 적절한 분석법이다.

범죄율 데이터 분석을 위한 변수에 관한 적절한 가정은 $Normal$분포가 아니라 $Laplace$분포일 것이다.

또 범죄율에 영향을 주는 변수들은 범죄율에 영향을 주거나 아니면 전혀 주지 않는다고

전제하는것은 모델링을 위한 적절한 가정인 듯 하다.

따라서 회귀계수가 0이 될 수 있다고 생각해야 하므로 $Lasso$가 $Ridge$ 보다 더 적절할수있다.

구체적인 근거는 다음과 같다.

결측치를 가진 데이터 열을 평균치로 처리한 경우와 결측치 데이터를 가진 열을 제거한경우와

$Lasso$분석은 같은 결과를 내놓았다. 제거된 변수들은 범죄율에 큰 영향이 없음을 시사한다.

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$Elastic\ Net$

$Consider\ J(\beta) = \|y - X\beta\|^2 + \lambda\|\beta\|^2 + (1-\lambda)|\beta|$

$Let\ X_\lambda = \frac{1}{(1+\lambda^2)}\begin{pmatrix} X \\ \sqrt{\lambda}I \end{pmatrix},\ y_\lambda = \begin{pmatrix} y \\ 0_\lambda \end{pmatrix}$

$Then\ J(\frac{1}{(1+\lambda^2)}\beta) = \|y_\lambda - X_\lambda\beta\|^2 + \frac{1-\lambda}{1+\lambda^2}|\beta|$

$It\ implies\ solving\ the\ modified\ lasso\ problem\ can\ give\ rescaled\ solution\ for\ original\ probl$

$Lasso$가 적절히 작용하는 데이터 이기에 결과는 거의 같을것이다.