Positive and Negative Selection in Bargaining

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Helicopter tour first

Consider two-person bargaining. A buyer has a private value $v \sim F$. A seller makes an offer, then a buyer accepts it, takes an outside option if available, or rejects it to repeat the negotiation.

- Coase conjecture: With no outside option, the uninformed seller doesn't benefit from inter-temporal price discrimination.
- Board and Pycia (2014): When there is a commonly-known outside option, the seller enjoys the largest profit.
- We examine the validity of such a stark difference both theoretically and experimentally.
- Many experimental results go against the theoretical predictions about the difference. They are consistent with the predictions from our model with the buyer's optimism.

Coase Conjecture

- One of the most fundamental ideas in
  - Bargaining theory
  - Durable-good monopoly
  - Dynamic screening problems

    (including lemon market and sequential auctions)
- The uninformed seller eventually benefits **not at all** from inter-temporal price discrimination.
- Theoretically examined and confirmed by Fudenberg et al. (1985) and Gul et al. (1986) among others.

Negative Selection in the Demand Pool

Buyer's value is his private information.

1. Consider the value distribution $[\underline{v}, \overline{v}]$.

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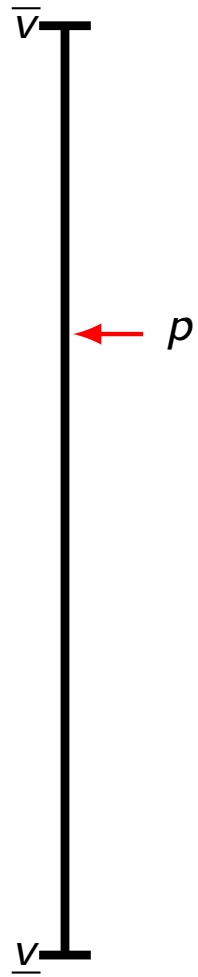

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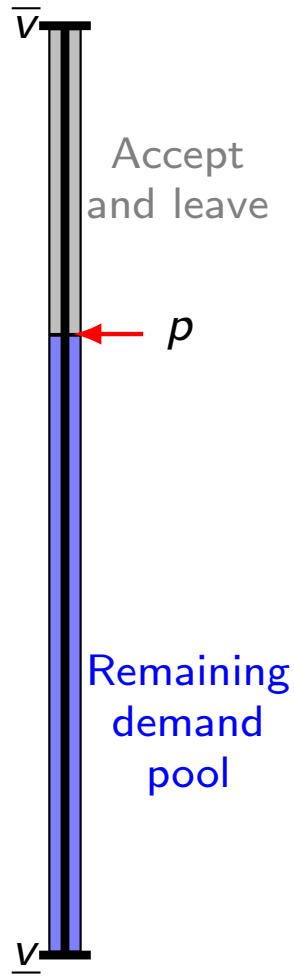

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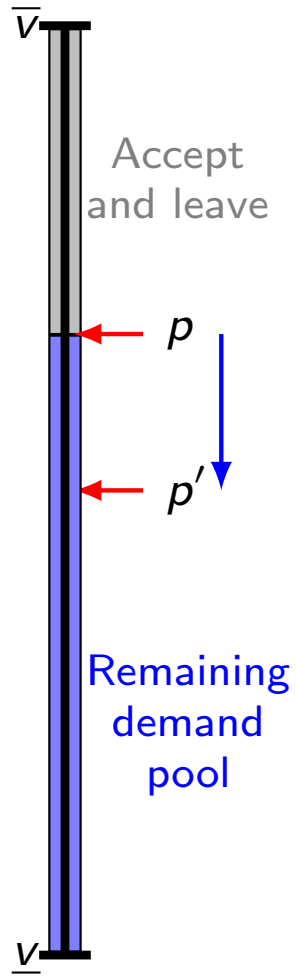

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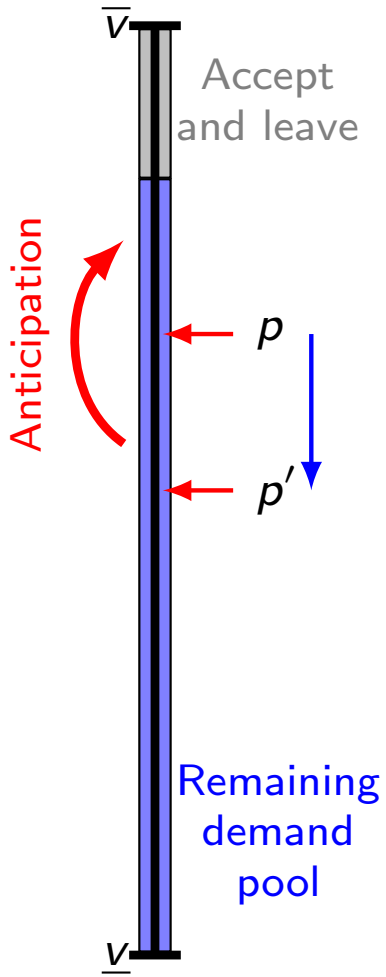

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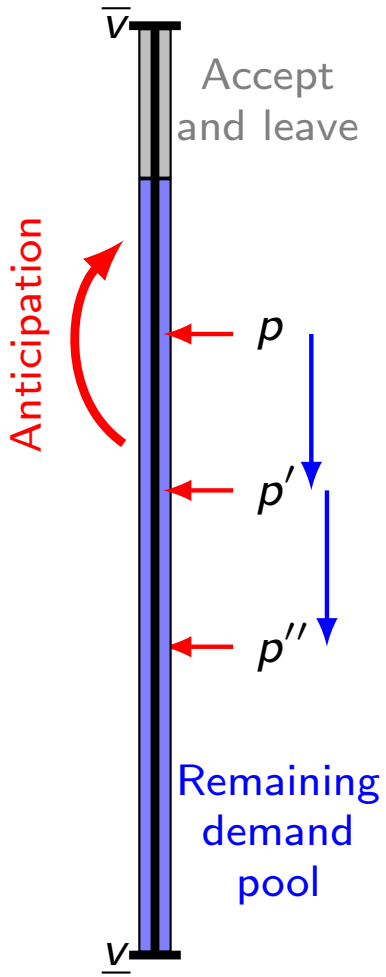

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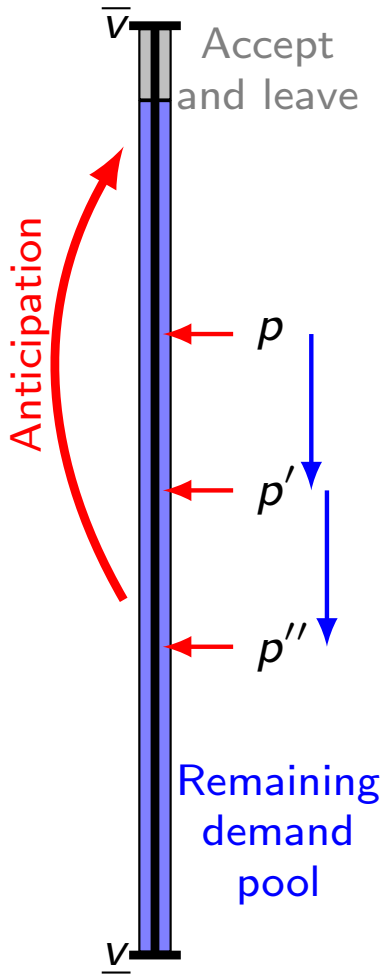

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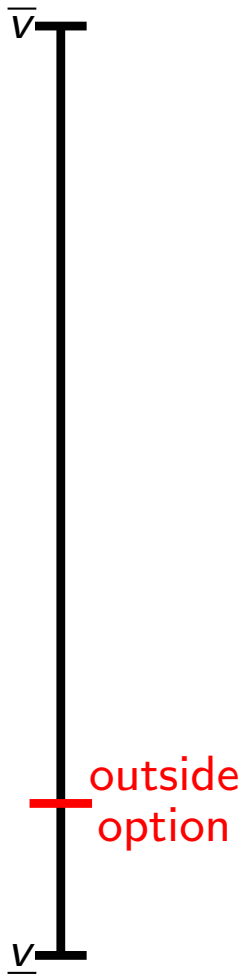
5. Pushing the seller to lower the price in the early stage even further to induce any purchase.
6. Pushing the price toward $\underline{v}$ (cf. Coase conjecture) and lead to the lowest seller profit in equilibrium.

Outside Option and Positive Selection



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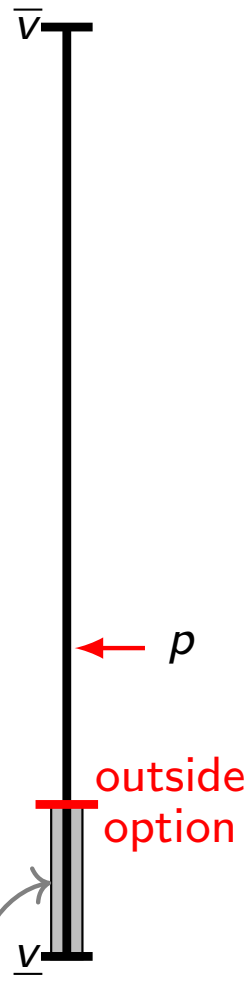

Taking the outside option is a dominant strategy

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1. Low-type buyers tend to exercise the outside option and exit the market immediately.

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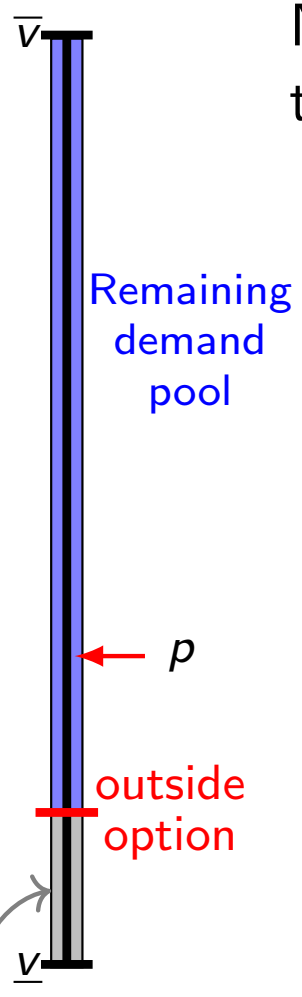

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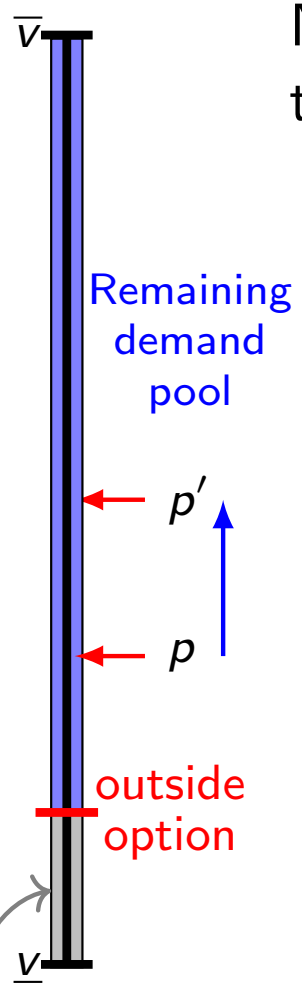

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2. The remaining demand pool consists of high-type buyers.
3. The seller responds to increase the price.

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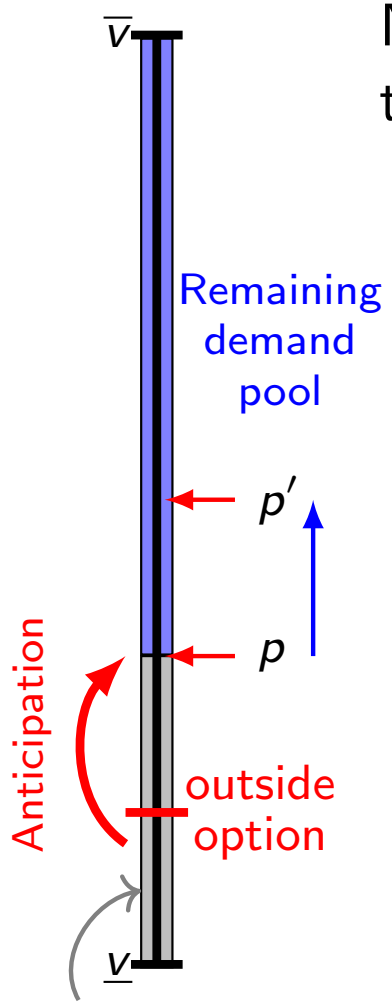

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2. The remaining demand pool consists of high-type buyers.
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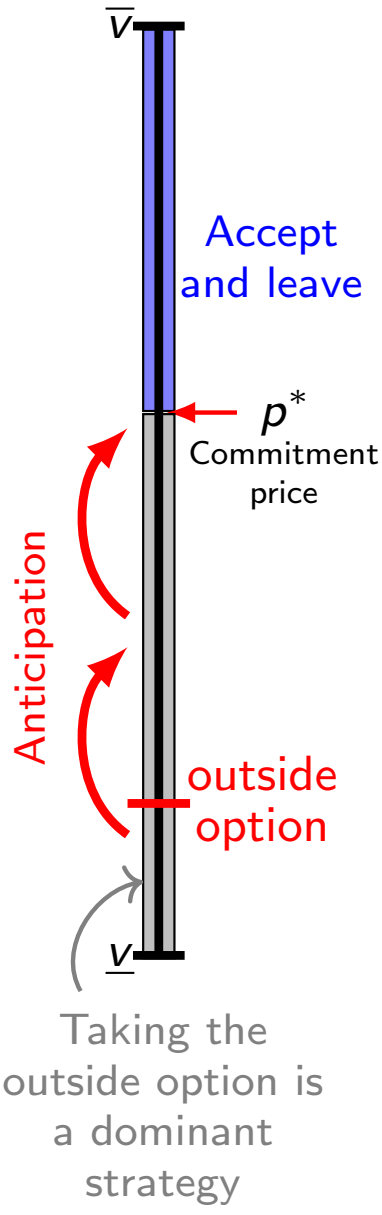
4. Anticipating such a price increase, some intermediate-type buyers tend to exercise the outside option immediately.

Outside Option and Positive Selection



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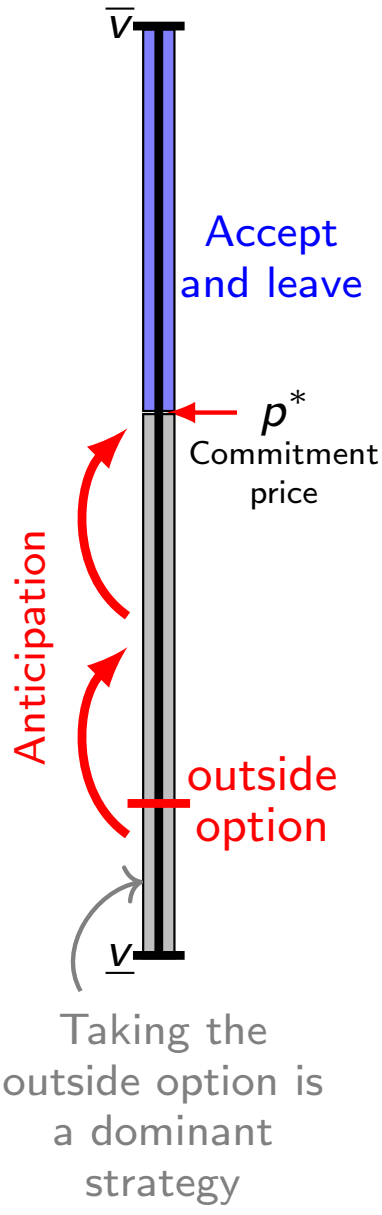

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4. Anticipating such a price increase, some intermediate-type buyers tend to exercise the outside option immediately.
5. Pushing the seller to increase the price further.
6. Leading the seller to charge the commitment price p^* and earn the largest profit in equilibrium.

Robust as long as the outside option value > 0.

Research Questions

The sharp contrast in theoretical predictions inspires our research:

1. In the absence of outside option: Negative selection results in the minimum seller profit
2. In the presence of an (arbitrarily small but positive) outside option: Positive selection leads to the maximum seller profit

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Would this stark difference be empirically valid, even when some players are not entirely rational?

- We are interested in examining the treatment effect of the outside option, but not in confirming or rejecting the Coase conjecture per se.

Related Literature

Theory

- Negative Selection: Coase (1972), Fudenberg et al. (1985), Gul et al. (1986), Ausubel and Deneckere (1989)
- Positive Selection: Board and Pycia (2014), Tirole (2016)
- Introducing Behavioral Types
  - Obstinate buyer (Myerson, 1991; Abreu and Gul, 2000)
  - Commitment type (Fanning, 2021)
  - Optimism (Li and Wong, 2009; Yildiz, 2011; Friedenberg, 2019)

Experiment

- Mixed Evidence for Negative Selection
  - Rejecting: Güth et al. (1995), Rapoport et al.(1995), Reynolds (2000), Srivastava, (2001), Cason and Reynolds (2005)
  - Supporting: Cason and Sharma (2001), Güth et al. (2004), Fanning and Kloosterman (2019)

Theory

Models

Price negotiation between a seller and a buyer:

- The seller's value is normalized to zero.
- The buyer's value is $v \in V = \{v_1, \ldots, v_N\}$ with prob $f(v_j)$.
- $\underline{v} = v_1 < v_2 < \ldots < v_N = \overline{v}$.

The buyer's outside option:

- The value of the outside option w is *type-independent*.
- Focus on the two cases: $w = -\infty$ or $w > 0$.

Net-value (gains from trade):

$$u(v) := \begin{cases} v & \text{if } w = -\infty \\ v - w & \text{if } w > 0. \end{cases}$$

Assumption: $u(v) > 0$ for all $v \in V$.

Models

Timeline:

1. In each period $n = 0, 1, 2, \ldots$, the seller offers a price $p_n \geq 0$.
2. The buyer has three options:
   - If the buyer accepts p_n, the game ends with the final payoffs
   $$V^B = e^{-rn\Delta}(v - p_n) \quad \text{and} \quad V^S = e^{-rn\Delta} p_n.$$
   - If the buyer exercises the outside option, the game ends with
   $$V^B = e^{-rn\Delta} w \quad \text{and} \quad V^S = 0.$$
   - If the buyer rejects p_n, the same protocol is repeated in the next period.

Equilibrium concept: PBE

Full Commitment Benchmark

With the full commitment power, it is optimal for the seller to commit to

$$p_n = p_w^* := \arg\max_{p \geq 0} \left\{ p \sum_{v : u(v) \geq p} f(v) \right\} \quad \forall n \geq 0.$$

The buyer accepts p_w^* in period zero iff $u(v) \geq p_w^*$.

Other buyer types are excluded (despite positive net value).

- If $w = -\infty$, these types continue to reject $p_n = p_w^*$ forever.
- If $w > 0$, these types exercise the outside option immediately.

No inter-temporal pricing and no delay.

Summary of Theoretical Predictions: $w = -\infty$

- Higher types are eager to purchase earlier (even at a high p_n).
- Low types have less to lose from haggling.
- The seller's posterior belief gets pessimistic over time.

Negative selection. The seller's posterior belief is always *a right truncation of* f after any history $h = (p_0, \ldots, p_k)$ in any PBE σ:

$$f^S(v|h) = \frac{f(v)}{F(\overline{v}^\sigma(h))} \quad \forall v \leq \overline{v}^\sigma(h).$$

$\overline{v}^\sigma(h)$: the highest v in the support of the posterior after h.

Summary of Theoretical Predictions: $w = -\infty$

Suppose that $u(\underline{v}) = \underline{v} < p^*_{-\infty}$.

No Exclusion. In any PBE, all buyer types eventually trade.

Delay of Trade. The negotiation takes over multiple periods.

Inter-temporal Pricing. The seller's offer p_n declines over time.

Coase Conjecture. The initial offer $p_0 \to \underline{v} = \min V$ as $\Delta \to 0$.

- The seller earns less than the full-commitment benchmark.
- Negative selection undermines the seller's bargaining power.

Summary of Theoretical Predictions: $w > 0$

Positive selection. After any history h in any PBE σ,

$$p_n \geq \underline{u}(\underline{v}^\sigma(h)) \quad \Longleftrightarrow \quad w \geq \underline{v}^\sigma(h) - p_n.$$

- $\underline{v}^\sigma(h)$ always prefers the outside option than haggling.
- Positive selection counters the effect of negative selection.

Theorem (Board and Pycia, 2014). The essentially unique PBE induces the full-commitment benchmark outcome.

- No delay and no inter-temporal pricing.
- Low types are excluded despite positive net value.

$\underline{v}^\sigma(h)$: the lowest v in the support of the posterior after h.

Robustness of the BP's "No Delay" result

- No delay in any period (not only period zero) after any negotiation history (both on and off the equilibrium paths)
- Even in the case that the seller trembles hands and mistakenly chooses a non-equilibrium offer.
- Predicted not only by the standard equilibrium notion but also by strongly rationalizable strategies (Cantonini, 2022)

Remarks on Positive Selection

- The main driving force of the positive selection: the market *unravels* with the low-type buyers leaving earlier.
- Unraveling may not take place perfectly if players lack
  1. first-order rationality such that some low-type buyers do not leave the market early, or
  2. higher-order rationality such that the seller is unsure about whether the lower-type buyers leave the market early.
- Any of these scenarios leads the buyer to believe that the seller's price in the subsequent rounds may decrease.
- (Later, to capture this intuition parsimoniously, we consider the bargaining game with buyer's optimism.)

Summary of Theoretical Predictions

1. No Outside Option $\Rightarrow$ Negative Selection
   - Price declines over time; inter-temporal pricing.
   - Rejection (hence Delay) happens.
   - Seller profit is low.

2. Outside Option $\Rightarrow$ Positive Selection
   - No inter-temporal pricing
   - No Rejection, No Delay
   - Seller profit is high.

These differences will work as hypotheses for our experiment.

Experiment

Experimental Design

Table 1: Experimental Design

Out0 (*OutNo*)	*Out50* (*OutYes*)	*Out60* (*OutYes*)
No outside option	Outside option 50	Outside option 60

* Each participant has seven newly paired supergames (matches).
* Continuation probability to the next round is 0.8.
* Buyer's value v is drawn from $U[50, 400]$.

- Value distribution: $U[50, 400]$
- The buyer's value of the outside option $\epsilon \in \{\emptyset, 50, 60\}$
- Random termination (Roth and Murnighan, 1978) with fixed continuation probability of $c = 0.8$

Belief Reporting: Positive or Negative Selection?

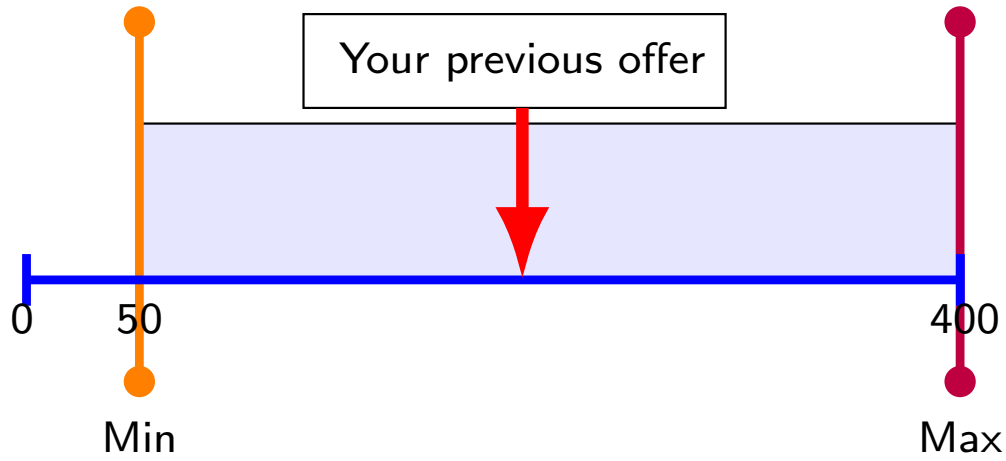

Figure 1: Reporting Beliefs in Round n

- After each rejection, the sellers are asked to report their beliefs about the buyers' value (min and max)

Belief Reporting: Positive or Negative Selection?

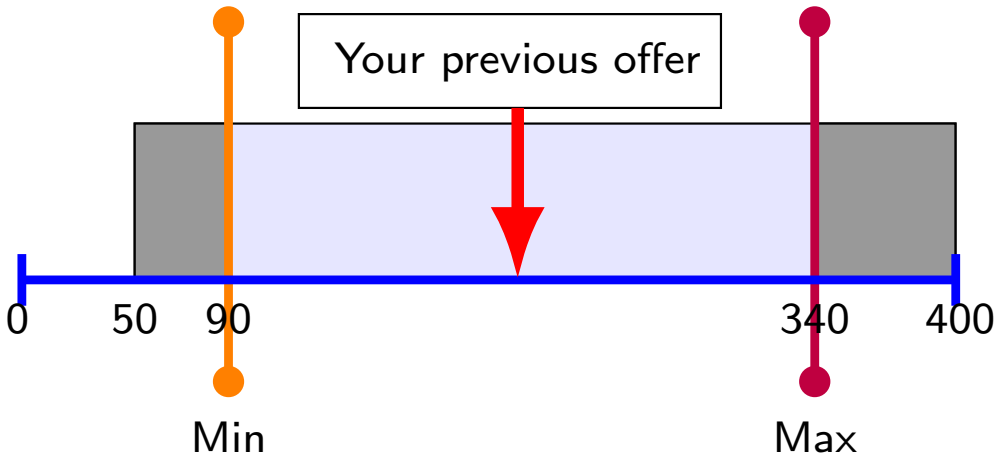

Figure 1: Reporting Beliefs in Round n

- After each rejection, the sellers are asked to report their beliefs about the buyers' value (min and max)
- We present the reported values in the decision screen of the seller in the immediate next round to aid decision-making for the new price offer.

Hypotheses

(Given no significant differences between Out50 and Out60,)

1. The average length of bargaining: OutNo $>$ 1 $=$ OutYes.
2. The seller's profits: OutNo $<$ OutYes.
3. The average offer price: OutNo $<$ OutYes.
4. Seller's reported min. buyer type | her offer being rejected: OutYes $>$ OutNo $= \underline{v}$.
5. In OutYes, low-type buyers take the outside option, and high-type buyer accepts the seller's offer (No Rejection). In OutNo, some buyers reject the price offer unless it is $\underline{v}$.

Experiment: Basic Procedure

- oTree (Chen et al, 2016) + Zoom RTO experiment
- Turning on their video was a strict requirement
- HKUST, English
- 4 sessions each for Out0 and Out50, 5 sessions for Out60
- $58 + 66 + 72 = 196$ participants
- Seven supergames (matches)
- Random matching, between-subject design
- On average, HKD 115 ($\approx$ USD 16) including HKD 40 show-up payment
- Online bank transfer via the autopay system of HKUST

Result 0

No significant differences between Out50 and Out 60

- The average length of bargaining: Out50 1.59, Out60 1.53
- The seller's average earnings: Out50 68.42, Out60 59.03
- The seller's initial offers: Out50 211.48, Out60 194.45
- Buyer's average earnings: Out50 78.72 $<$ Out60 94.82 (but recall that merely exercising the outside option in Out60 gives an additional 10 to the buyer.

$\Rightarrow$ Pool Out50 and Out60 as OutYes

Result 1: Bargaining Length

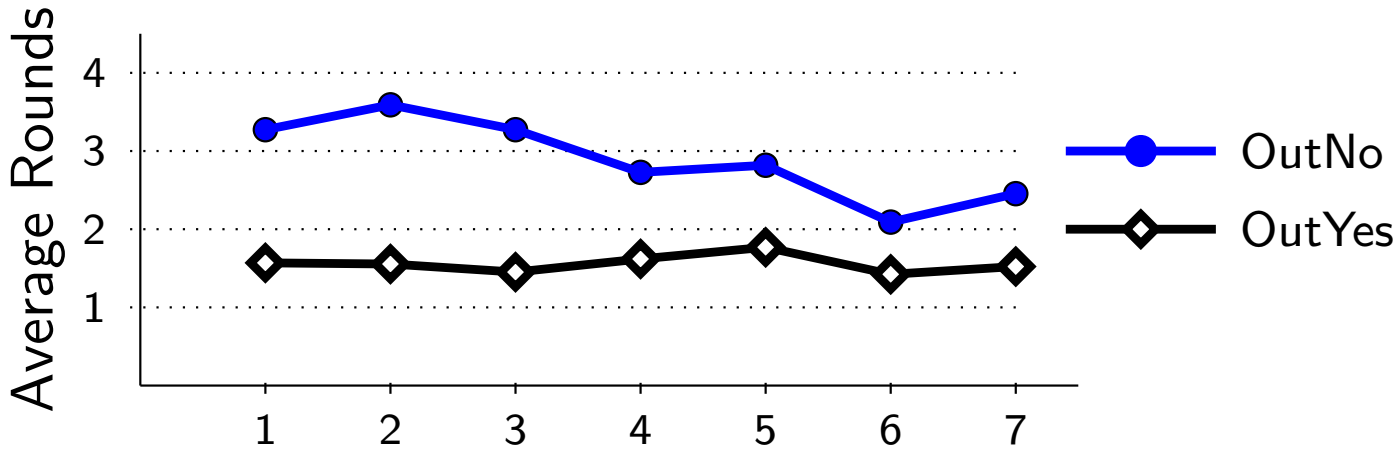

Figure 2: Average Length of Bargaining across Match

- The average # of bargaining rounds: OutNo $>$ OutYes.
- The average # of bargaining rounds in OutYes $>$ 1.

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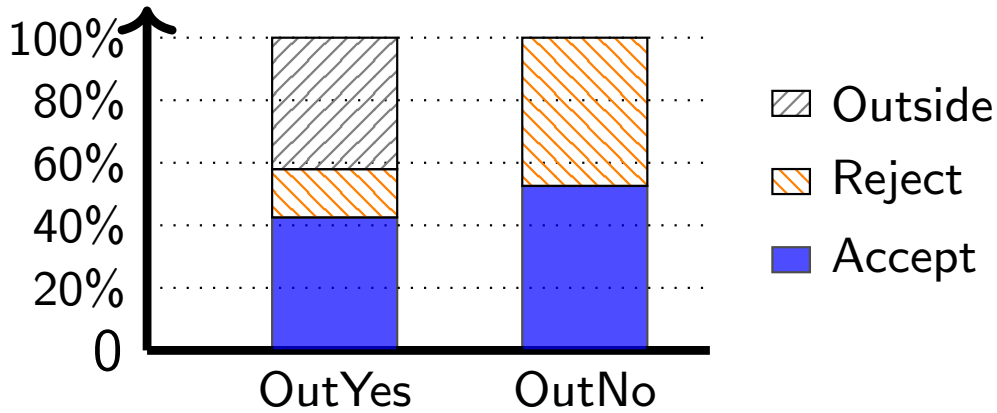


Figure 3: End-of-Bargaining States

- The average # of bargaining rounds in OutYes > 1.
- In OutYes, some fraction of the (optimistic) buyers remains by rejecting the offer, causing some delay.

Result 2: Seller Profit

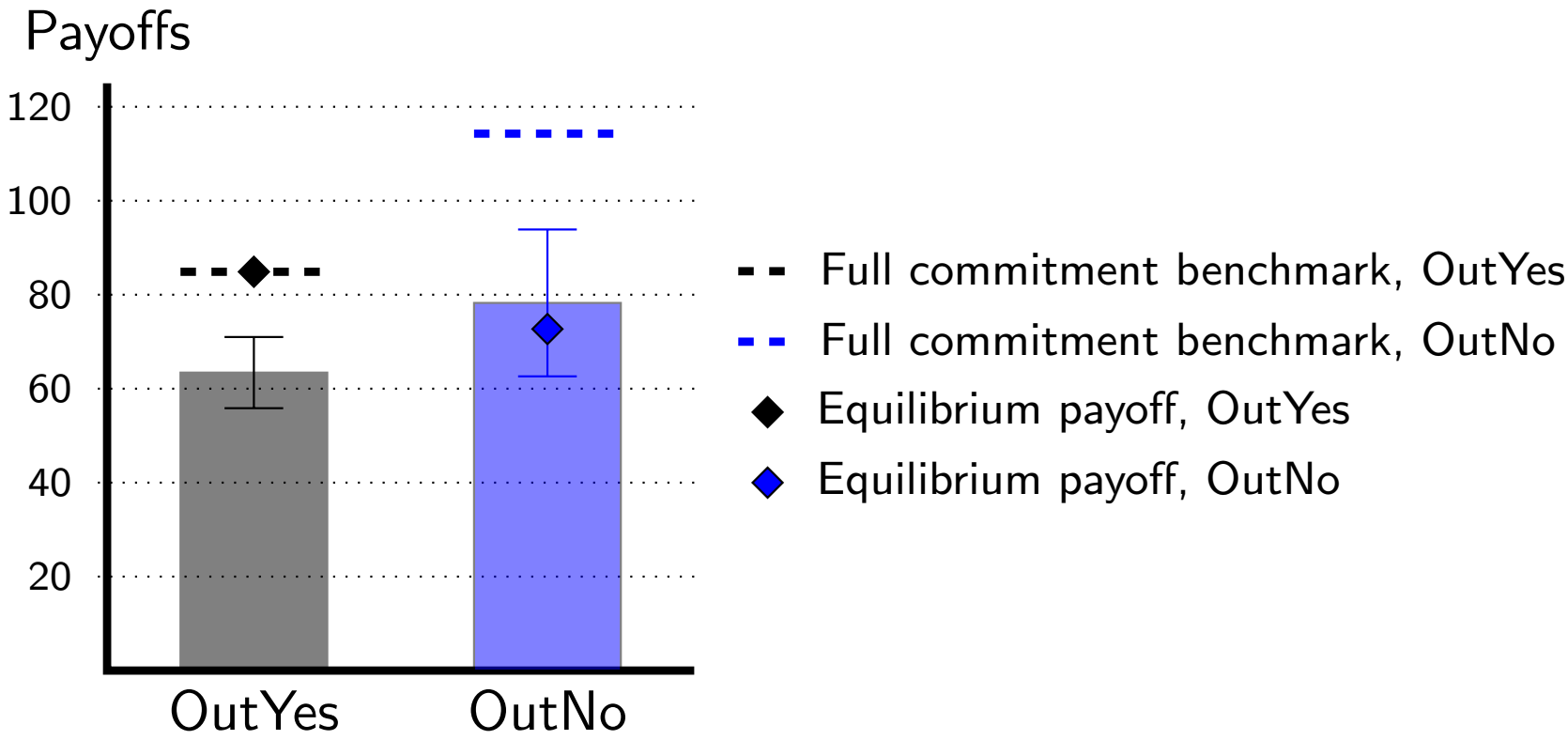


Figure 4: Seller's Earnings

- The seller's average profit: OutYes (63.43) $<$ OutNo (78.25).
- Nearly 50% of bargaining in OutNo ended with termination.

Result 3: Price Offers

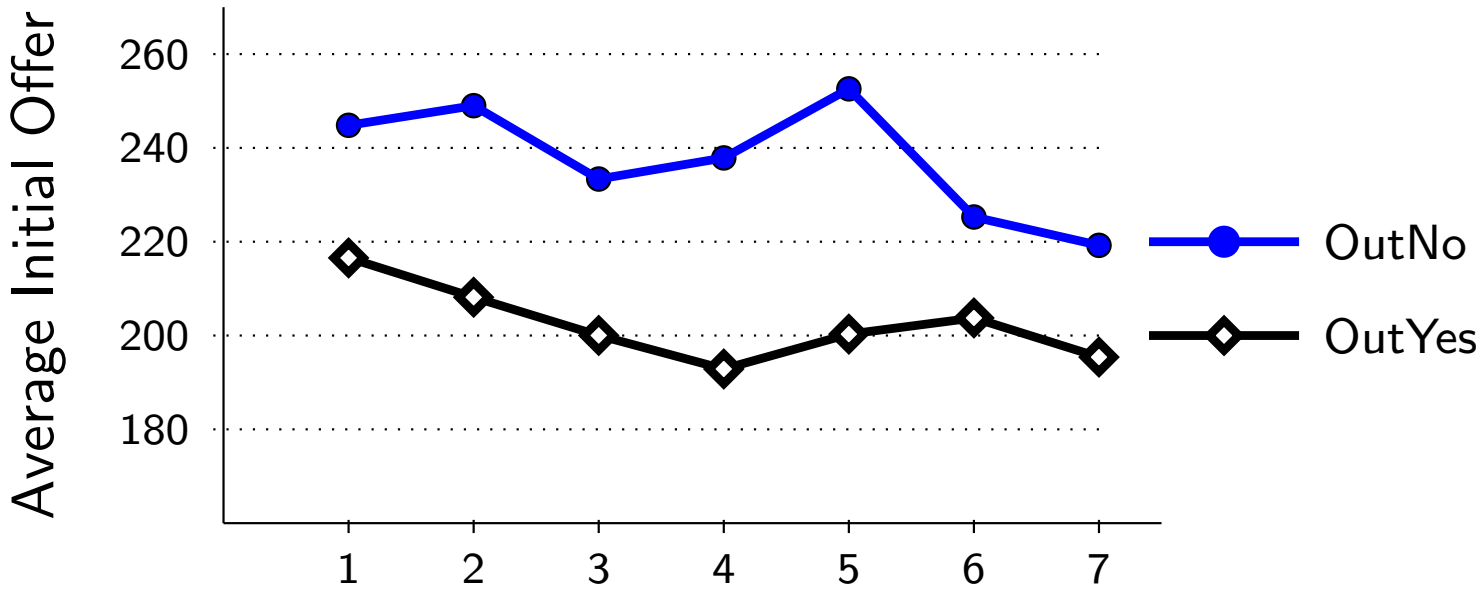


Figure 5: Round 1 Offer across Match

- The seller's initial offer: OutYes $<$ OutNo.
- The mild negative trends observed in OutNo and OutYes are not significantly different from each other.

Result 4: Average Minimum Belief

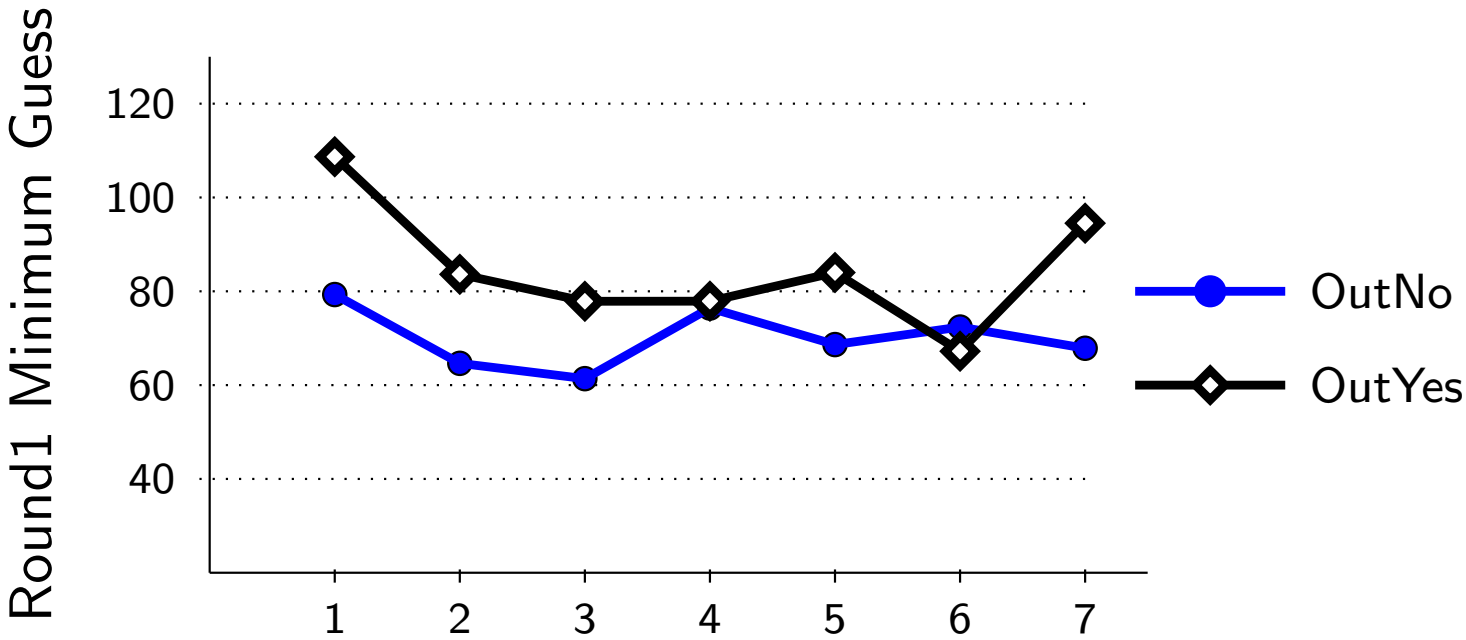
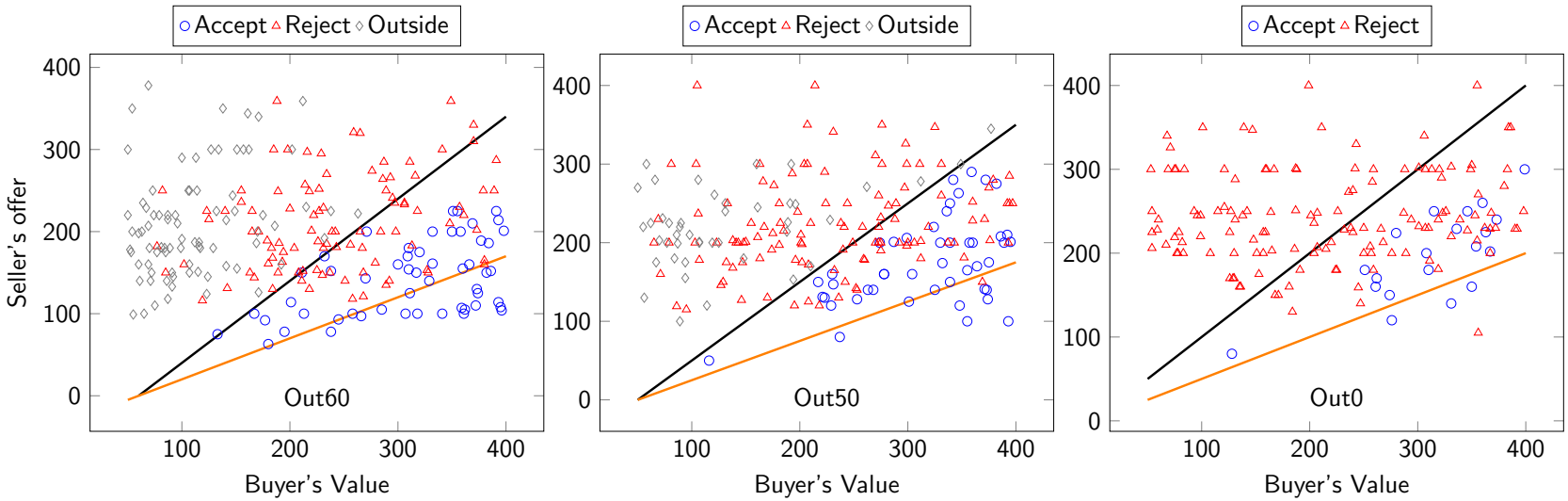


Figure 6: Minimum of the Guess after Round 1 Rejection across Match

- The min of the guess in OutYes is larger than that in OutNo (p=0.059), but a substantial fraction of the min guesses are below w/c, the lower bound of the guess.
- The individual-level reports on the min of the guess in OutYes and OutNo are, by and large, the same (KS test, p=0.328).

Result 5: Outcome

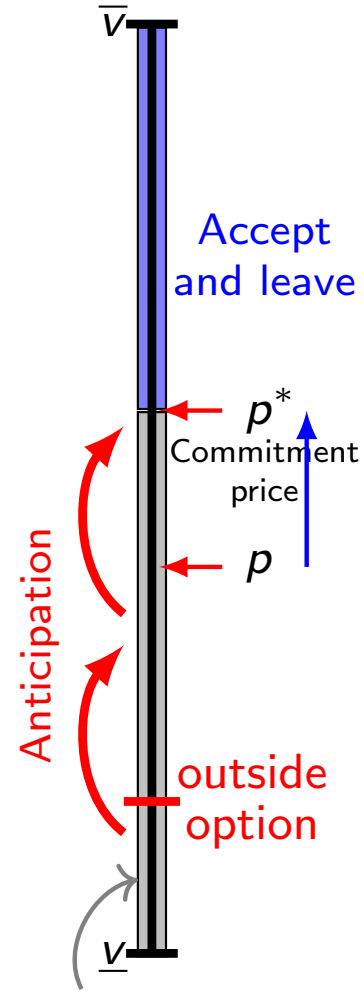


- On the right-hand side of the **black 45-degree line** $(v - w)$, accepting the offer is strictly better than taking the outside option.
- The orange line $(\frac{v-w}{2})$ equally splits the gains from trade between the buyer and the seller.

Figure 7: Buyer's Action in Round 1

- Rejections are pervasive in both Out60 and Out50.
- Inequity aversion (Fehr and Schmidt, 1999) does not help explain the pervasive rejections.

The Main Driver



Taking the outside option is a dominant strategy

The belief that **some low-type buyers may remain in the market** $\Rightarrow$ Failure of unraveling. Our setup with optimism as a parsimonious workhorse model visualizing it.

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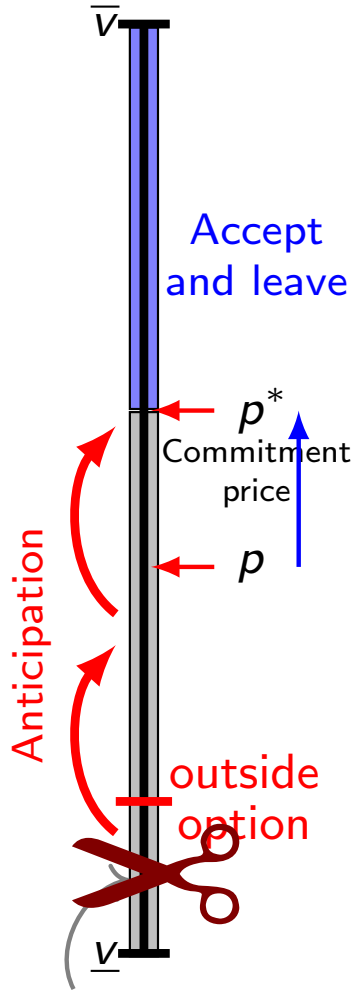

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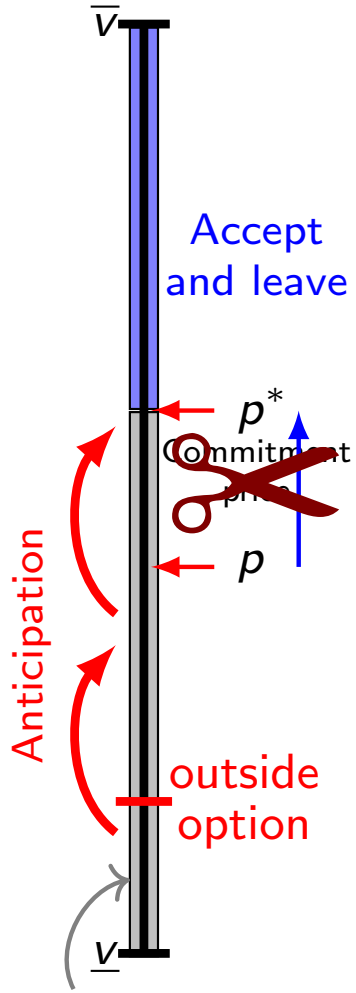

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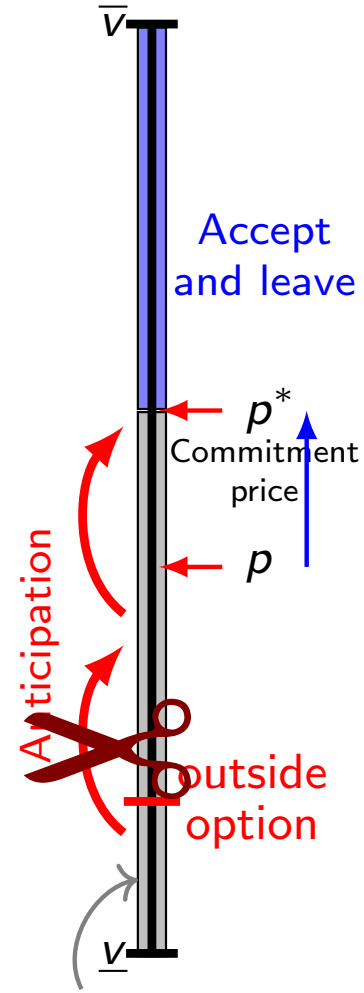
1. Lack of 1st-order rationality (our appendix)
2. Lack of 2nd-order rationality
3. Behavioral, obstinate buyer type (Myerson, 1991; Abreu and Gul, 2000)
4. Commitment types to a fixed price (Fanning, 2021)

The Main Driver



Taking the outside option is a dominant strategy

The belief that **some low-type buyers may remain in the market** ⇒ Failure of unraveling. Our setup with optimism as a parsimonious workhorse model visualizing it.

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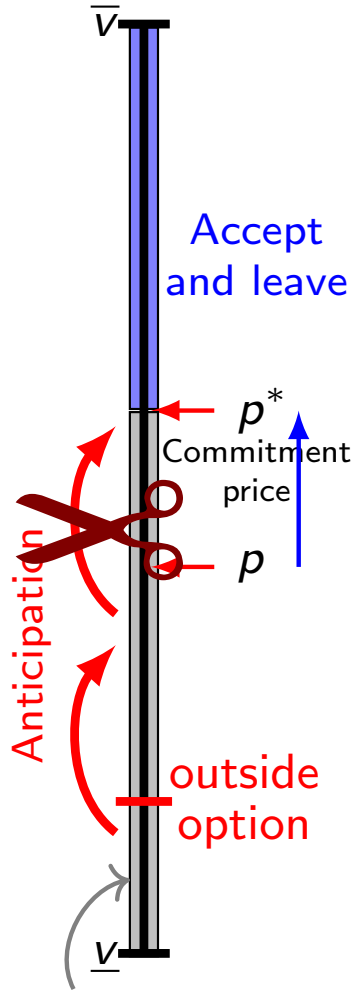

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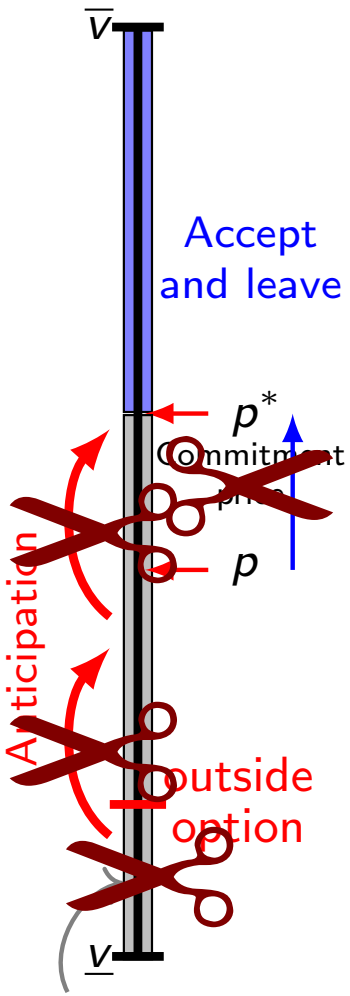

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Any of them results in a failure of the inductive process of unraveling and the positive selection.

Summary of Theoretical Predictions: Optimism

Expand the buyer's type spaces to $\Theta := V \times \{\tau_o, \tau_r\}$.

Rational types with $\tau = \tau_r$ understand the equilibrium correctly.

Optimistic types with $\tau = \tau_o$ *incorrectly* believes

- the seller plays the equilibrium strategy with prob $1 - \eta$;
- the seller offers $p^\dagger < p_w^*$ with prob $\eta \in (0, 1)$.
- $p^\dagger$ and η are common knowledge.

$v \perp\!\!\!\perp \tau$ with $\tau = \tau_o$ with probability ϕ.

Summary of Theoretical Predictions: Optimism

Equilibrium concept: (tighter than the standard) ϵ-PBE.

- The seller's strategy is
  - exactly optimal after any h with a price offer that does not induce a delay
  - *ϵ-optimal* after any h with a delay-inducing price offer.
- The seller's posterior belief is consistent after any h.
- The buyer's strategy is exactly optimal after any h.

Proposition. Suppose $w > 0$ and $\phi = 0$.

- For any $\epsilon > 0$, there is a unique ϵ-PBE.
- The ϵ-PBE induces the full-commitment benchmark outcome.

Summary of Theoretical Predictions: Optimism

Definition An ϵ-PBE is called *quasi-Coasean* if the following holds on its equilibrium path with some $v^* \in V$.

- Delay. Negotiation takes multiple periods w/ positive prob.
- Inter-temporal Pricing. p_n declines over time on the path.
- Positive Selection. Any type with $\tau = \tau_r$ and $v < v^*$ exercise the outside option immediately.
- Negative Selection. Any type with $\tau = \tau_r$ and $v \geq v^*$ trades (possibly after a delay).

Summary of Theoretical Predictions: Optimism

$$\Pi_r(p) := p \sum_{v:u(v)\geq p} f(v,\tau_r) = \left(\begin{array}{l} \text{static profit generated} \\ \text{from rational buyer types} \end{array} \right)$$

Assumption. $\Pi_r(p)$ is "effectively" single-peaked at $p = p_w^* > \underline{v} - w$.

Assumption. $0 \leq p^\dagger < p_w^*$.

Provided that $\phi > 0$ is small:

- The presence of optimistic types doesn't alter the benchmark.
- With full commitment, the seller insists on $p_n \equiv p_w^*$.

Proposition. Fix $w > 0$ and $\epsilon > 0$.

$\exists \bar{\phi}(\epsilon) > 0$ such that the following holds with $0 < \phi < \bar{\phi}(\epsilon)$:

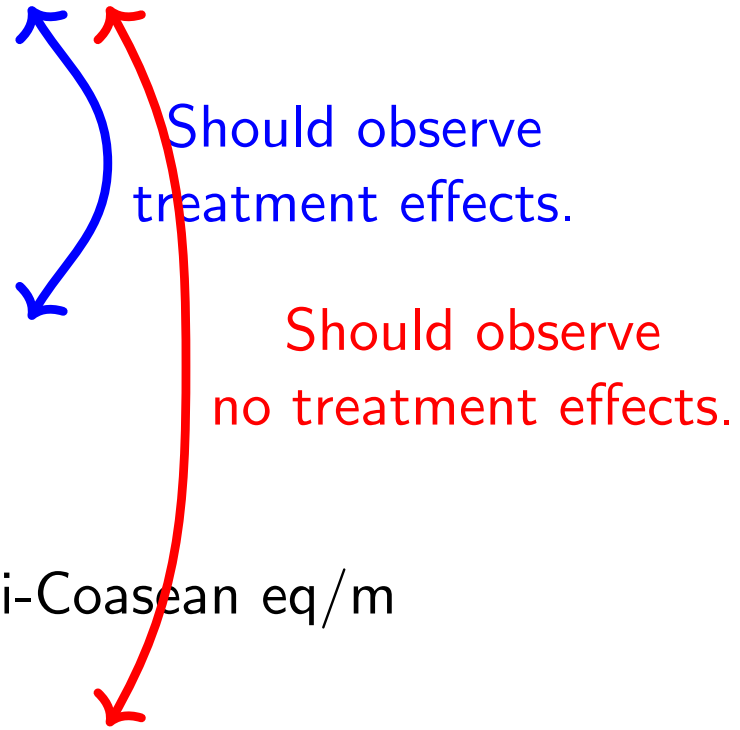
A quasi-Coasean ϵ-PBE exists whenever Δ is sufficiently small.

Summary of Theoretical Predictions, again

1. No Outside Option $\Rightarrow$ Negative Selection
   - Price declines over time; inter-temporal pricing.
   - Rejection (hence Delay) happens.
   - Seller profit is low.

2. Outside Option $\Rightarrow$ Positive Selection
   - No inter-temporal pricing
   - No Rejection, No Delay
   - Seller profit is high.

3. Outside Option + Optimism $\Rightarrow$ Quasi-Coasean eq/m
   - Price declines over time.
   - Rejection (hence Delay) happens.
   - Some buyers exercise the outside option immediately.



Take-away Messages

- The absence/presence of an outside option
  $\Rightarrow$ the stark theoretical difference
  $\not\Rightarrow$ our experimental data.

- Most of our experimental results are
  - inconsistent with the predictions from the standard model with positive selection.
  - consistent with the predictions from the model with buyer's optimism.

- We found supporting evidence that
  - some buyers reject the current-round offers,
  - optimistically believing a more favorable next offer.