Positive Selection in Bargaining: An Experiment

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November 29, 2023
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Which is better for the seller?

Consider two-person bargaining. A buyer has a private value $v \sim F$. A seller makes an offer, then a buyer accepts it, takes an outside option if available, or rejects it to repeat the negotiation.

- Question: Which is a good situation to the seller? (A) The buyer has an outside option, and it is commonly known to both players. (B) The buyer does not have an outside option.

Which is better for the seller?

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- Question: Which is a good situation to the seller? (A) The buyer has an outside option, and it is commonly known to both players. (B) The buyer does not have an outside option.
- Board and Pycia (2014, BP henceforth): The seller enjoys the largest profit when $\exists$ a commonly-known outside option.
- This result is theoretically robust, in the sense that (i) it holds however small the value of the outside option, (ii) a key logical process works both on and off the equilibrium path, and (iii) the equilibrium strategy is the strongly rationalizable strategy (Cantonini, 2022).

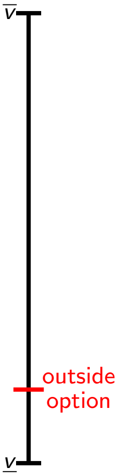
Research Questions

- BP's result has a significant implication for the market design and regulatory policy in various markets: For the consumer surplus, the designer should prevent buyers from accessing outside options.
- This implication seems contrary to the conventional wisdom that restricting monopoly power usually makes the market more competitive and increases consumer surplus.

Questions: Would the experiment participants exhibit the key logical process for the equilibrium in their belief updates? If not, *when* and *in what sense* do they fail?

Outside Option and Positive Selection

Illustration of Board and Pycia (2014)



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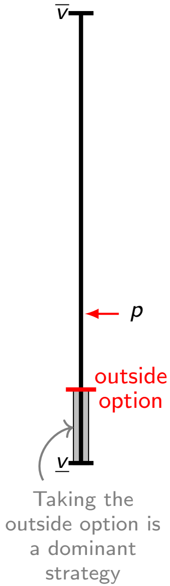

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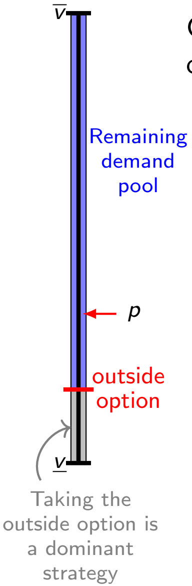

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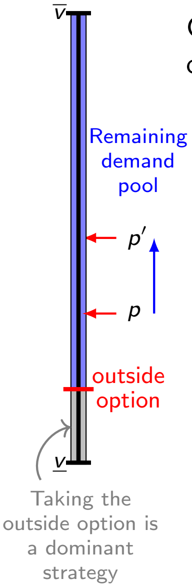

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3. The seller responds to increase the price.

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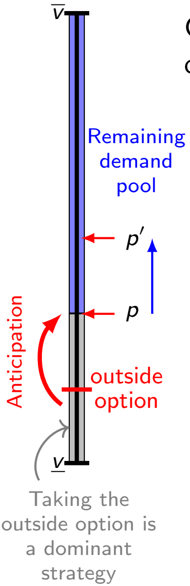

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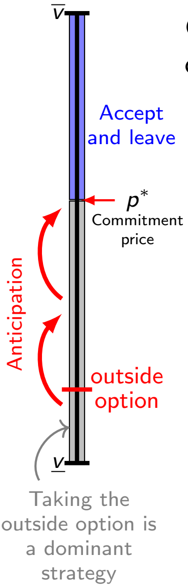

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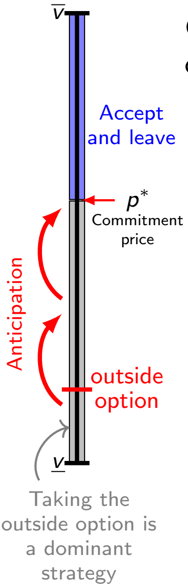

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3. The seller responds to increase the price.
4. Anticipating such a price increase, some intermediate-type buyers tend to exercise the outside option immediately.
5. Pushing the seller to increase the price further,
6. Up to the commitment price p^*. The seller earns the largest profit in equilibrium.

Robust as long as the outside option value > 0.

Remarks on Positive Selection

- The main driving force of the positive selection: the market *unravels* with the low-type buyers leaving earlier.
- Unraveling may not take place perfectly if players lack
    1. first-order rationality: the low-type buyers leave the market early, or
    2. higher-order rationality: upon the first-order rationality, the higher-type buyers have no reason to delay.

Our challenge is to design an experiment to clearly distinguish the seller's level of higher-order rationality.

A Simpler Model (used for our experiment)

Price negotiation between a seller and a buyer:

- Indivisible good
- Two periods
- The seller's value is normalized to zero.
- The buyer's value is $v \in V = \{v_L, v_M, v_H\}$,$v_L \leq v_M \leq v_H$, with prob $q(v_j)$, $j \in \{L, M, H\}$.
- The buyer has an outside option: The value of the outside option w is *type-independent*.
- Net-value (gain from trading): $u(v) = v - w$
- Assumption: $u(v) > 0$ for all $v \in V$.

A Simpler Model (used for our experiment)

Timeline

1. In period 1, the seller offers a price $p_1 \in \mathcal{P}(\Delta) = \{u(v_L) - \Delta, u(v_M) - \Delta, u(v_H) - \Delta\}$.
2. The buyer chooses one of the three options:
   - Accepts p_1: the game ends(*) with the final payoffs
     $$V^B = v - p_1 \quad \text{and} \quad V^S = p_1.$$
   - Exercises the outside option: the game ends(*) with
     $$V^B = w \quad \text{and} \quad V^S = 0.$$
   - Rejects p_1, they move to period 2 with probability δ.
3. If moved to period 2, the seller offers $p_2 \in \mathcal{P}(\Delta)$, and the buyer either accepts it or takes the outside option.

(*): With minuscule probability ϵ, the game moves to Period 2.

A Simpler Model (used for our experiment)

Things to note: $|\mathcal{P}(\Delta)| = 3$, $\Delta > 0$, and $\epsilon > 0$

- Having only three price alternatives minimizes the (unwanted) effect of fairness concern.
- With $\Delta > 0$, type-j buyer is strictly better off by accepting p_j than taking the outside option.
- By setting $\delta(w + \Delta) < (1 - \epsilon)w + \epsilon\delta w$ or $\Delta < \frac{(1-\epsilon)(1-\delta)w}{\delta}$, rejecting the first price offer is strictly dominated by taking the outside option.
- With $\epsilon > 0$, the equilibrium prediction is almost identical to the case with $\epsilon = 0$. It helps to understand the off-the-path equilibrium, i.e., the 2nd period.

Full Commitment Benchmark

With the full commitment power, it is optimal for the seller to commit to

$$p_1 = p_2 = p_w^* := \arg\max_{p \in \mathcal{P}(\Delta)} \sum_{v : u(v) \geq p} p \cdot q(v)$$

- The buyer accepts p_w^* in period 1 iff $u(v) \geq p_w^*$.
- Other buyer types exercise the outside option immediately (despite positive net value).
- No inter-temporal pricing and no delay.

Theoretical Predictions

Proposition

There is a unique Perfect Bayesian equilibrium. Furthermore:

(i) *The seller's equilibrium offer is p_w^*.*

(ii) *The buyer accepts the seller's offer p (which may not be the equilibrium offer) in any period if and only if $u(v) \geq p$; otherwise, exercises the outside option immediately.*

(iii) *No delay occurs with probability $1 - \epsilon$ in the equilibrium both on and off the equilibrium path.*

(iv) *If ever moved to period 2, the seller's posterior belief $\hat{q}(v|p_1)$ is identical to the prior $q(v)$, so $p_2 = p_w^*$.*

Proof: Hold on. Wait for Hypotheses.

Experimental Design

Table 1: Experimental Design

M90	*M240*	*M420*
$v \in \{70, 90, 500\}$	$v \in \{70, 240, 500\}$	$v \in \{70, 420, 500\}$

- Each participant has ten newly paired matches (periods).
- $w = 50$, $\Delta = 10$
- Continuation prob. to the next round upon rejection is 0.8.
- Buyer's value v is uniformly drawn from $\{70, v_M, 500\}$. $q(v_L) = q(v_M) = q(v_H) = 1/3$
- $\epsilon = 0.001$, instructing participants that this probability is to *theoretically* guarantee the possibility of moving to round 2, so it should be negligible.

Belief Reporting

Part of instructions in *M240*

<u>Your Task as a Seller in Round 2</u>: Before submitting a new price offer, report how you believe the buyer's value, by filling out the following sentence.

I believe that the value of the buyer paired in this match is
70 with a (____)% of chance,
240 with a (____)% of chance,
500 with a (____)% of chance.

The three numbers must sum up to 100. **The reported probabilities will appear in your decision screen but will not be shared with the buyer.**

Hypotheses (1/5)

Each step of the proof of proposition 1 will be associated with a testable hypothesis.

- The "minimal" rationality: The low type should never delay. (Taking the outside option now = 50. Rejecting the first-round offer with hoping that the second round offer is most favorable = $\delta(v_L - (v_L - w - \Delta)) = 0.8(70 - (70 - 50 - 10)) = 48.$)
- If the game moves on to round 2, then the seller must believe that the low type remains because of ϵ. This leads to

Hypothesis (First-order positive selection)

No low-type buyers choose to delay. If ever moved to the second round, the posterior belief that a buyer is a high/middle type is weakly greater than the posterior belief that a buyer is a low type, for any price offer in round 1. That is, $\hat{q}(v_L|p_1) \leq \min\{\hat{q}(v_M|p_1), \hat{q}(v_H|p_1)\}$.

Hypotheses (2/5)

- Given the first-order positive selection, a rational seller will never offer $p_L := u(v_L) - \Delta$ in round 2.
  (The seller's expected payoff from $p_2 = p_L$ is 10. The seller's expected payoff from $p_2 = p_M$ is $[u(v_M) - \Delta][\hat{q}(v_M) + \hat{q}(v_L)]$, where $\hat{q}(v_M) + \hat{q}(v_L) \geq 2/3$. Thus the latter one is greater than the former.)
- Then, by following similar reasoning for the no-delay of the low type, the middle type also finds it strictly suboptimal to delay the negotiation to round 2.

Hypothesis (Second-order positive selection)

No middle-type buyers choose to delay. If ever moved to the second round, the posterior belief that a buyer is a high type is weakly greater than the posterior belief that a buyer is a low/middle type, for any price offer in round 1. That is, $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) \leq \hat{q}(v_H|p_1)$.

Hypothesis (3/5)

When $v_M = 90$ or 240

Consider the case $v_M = 90$ or 240 where the full commitment price $p^* = u(v_H) - \Delta = 440$.

- Given the first- and second-order positive selections, the posterior belief $\hat{q}$ weakly FOSD the prior belief q.
- The seller's round 2 (unrestricted) optimal price offer must be greater than p^*, but we have only three price alternatives. $p_2 = p_H$, implying that the high type has no reason to delay.

Hypothesis (Third-order positive selection, $v_M = 90$ or 240)

Suppose $v_M = 90$ or 240 so that $p^* = u(v_H) - \Delta := p_H$. No high-type buyers choose to delay. If ever moved to the second round, the posterior belief is the same as prior, $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) = \hat{q}(v_H|p_1)$.

Hypothesis (3/5)

When $v_M = 420$

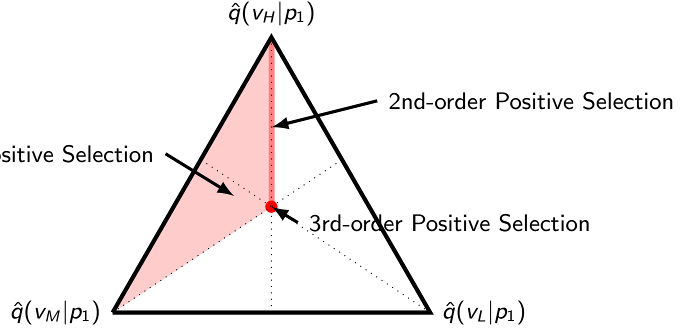
When $v_M = 420$, the full commitment price $p^* = u(v_M) - \Delta = 360 := p_M$.

- The similar logic, with two subcases.
- If $p_1 < p_H$, the same reasoning applies. The high-type buyer does not choose to delay.
- If $p_1 = p_H$, the high-type buyer mixes to accept and reject p_1. (The expected payoff in round 2 is greater than accepting p_1 with prob 1. Rejecting p_1 with prob. 1 will lead $p_2 = p_H$ (because of the first- and second- positive selection), rendering less payoff than accepting p_1.)
- After some algebra, we get

Hypothesis (Third-order positive selection, $v_M = 420$)

Suppose $v_M = 420$ so that $p^* = u(v_M) - \Delta$. (i) If $p_1 = p_L$ or p_M, no high-type buyers choose to delay. Posterior=prior. (ii) If $p_1 = p_H$, high-type buyers rarely choose to delay. $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) < \hat{q}(v_H|p_1)$.

Summarizing the first three hypotheses



Positive Selections and Posterior Beliefs

- The earlier hypothesis *nests* the later one.
- If our experimental data do not support the theoretical predictions, these three hypotheses can provide us clear identification from where subjects fail.

Hypotheses (4/5)

This regards the seller's rationality.

- Given any posterior beliefs $\hat{q}(\cdot)$ the seller reported, the seller faces a static profit maximization problem.

$$\max_{p_2 \in \mathcal{P}(\Delta)} \sum_{v:u(v) \geq p_1} p_2 \cdot \hat{q}(v|p_1) \quad \forall p_1 \in \mathcal{P}(\Delta). \tag{1}$$

- We expect that the seller is at least best responding to her own (perhaps incorrect) belief.

Hypothesis

Given the seller's reported posterior belief $\hat{q}(\cdot)$, the price offered in the second round maximizes the seller's expected payoff.

Hypotheses (5/5)

This regards the buyer's rationality.

- When $v_M \in \{90, 240\}$, H1–H3 state that no buyers would choose to delay. This is the case when the buyer expects:

$$E[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\} \quad \forall v, p_1,$$

- which means that some buyers may reject p_1, based on her subjective (perhaps incorrect) belief.
- We have only three types and three price alternatives, so we check in which case the above inequality *can* be violated.

Hypotheses (5/5)

$$E[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\}?$$

$v \setminus p_1$	p_L	p_M	p_H
v_L	always hold	always hold	always hold
v_M	always hold	can be violated if $p_2 = p_L$	can be violated if $p_2 \leq p_M$
v_H	always hold	is expected too much	is expected too much

Validity of not expecting "too low price" in Round 2

In words, if our experimental data shows some "rejections," then it is most likely in the $v - p_1$ pair shaded in red, somewhat likely in the pair shaded in blue, and not likely in other pairs. This leads

Hypothesis

Buyers with v_M or v_H are more likely to reject p_H, somewhat likely to reject p_M, and not likely to reject p_L. Buyers with v_L are not likely to reject any price offer in round 1.

Experiment: Basic Procedure

- oTree (Chen et al, 2016) + Zoom RTO experiment
- Turning on their video was a strict requirement
- HKUST, English
- 5 sessions each for *M90*, *M240*, and *M420*.
- $106 + 100 + 88 = 294$ participants
- Ten matches
- Random matching, between-subject design
- On average, HKD 115 ($\approx$ USD 16) including HKD 40 show-up payment
- Online bank transfer via the autopay system of HKUST

Results: Overview

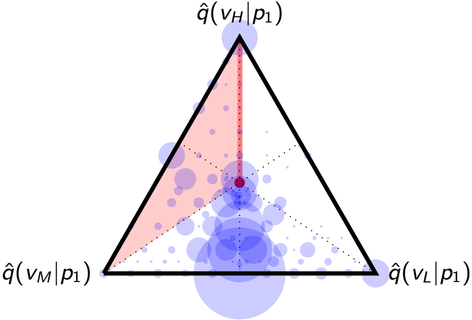
	$M90$	$M240$	$M420$
Avg.Offer (Theory)	346.25 (440)	295.86 (440)	378.50 (360)
%Reject_v_L (Theory)	39 (0)	35 (0)	32 (0)
%Reject_v_M (Theory)	51 (0)	60 (0)	74 (0)
%Reject_v_H (Theory)	63 (0)	50 (0)	59 (0)
Avg.SellerPayoffs (Theory)	54.43 (146.67)	87.94 (146.67)	165.16 (240)
Avg.BuyerPayoffs (Theory)	111.60 (53.33)	119.20 (53.33)	86.25 (83.33)

Table 2: Summary of Experimental Findings

Substantial differences:

- Avg.Offer is largest in $M420$. Theory predicts the opposite.
- Avg.Offer in $M90$ is significantly larger than that in $M240$. The equilibrium price offers are the same.
- Avg.SellerPayoffs is much smaller than the equilibrium payoff.
- Avg.BuyerPayoffs in $M420$ was the smallest, opposing theory.

Result 1



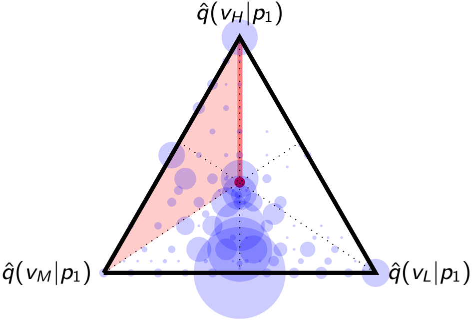
Observed posterior beliefs

Result

41.77% (614 out of 1470) of the first-round price offers were rejected. 36.64% of the posterior beliefs are rationalized in the first-order positive selection.

Note: we interpret our observations in the most "favorable" way.

Result 2

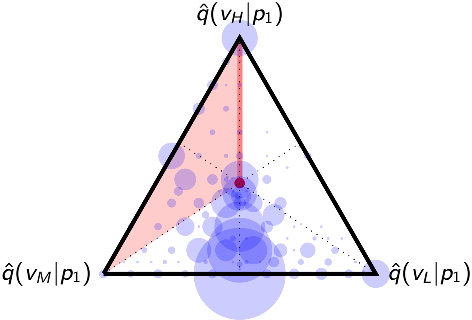


Observed posterior beliefs

Result

24.59% of the posterior beliefs (151 out of 614) are rationalized in the second-order positive selection.

Result 3



Observed posterior beliefs

Result

Only 7.49% of the posterior beliefs (46 out of 614) are rationalized in the third-order positive selection.

Caveat: The third-order positive selection leads the posterior belief to be identical to the prior belief. They might be completely naïve.

Result 4

- 66.94% (411 out of 614) of p_2s were optimal from their subjective beliefs.
- Among sellers on the 1st-order positive selection, 70.22% of p_2s were optimal. Among sellers on the 2nd-order positive selection, 76.82% of p_2s were optimal.
- Among sellers on the 3rd-order positive selection, only 47.83% of p_2s were optimal. More than half of them seem to be the most naïve ones who kept the prior.

Result

Majority (66.94%) of the second-round price offers were optimal in the sense that the offer maximizes the expected profit calculated with their subjective beliefs. Higher-order reasoning on positive selection is positively associated with pricing optimality.

Result 5

$v \setminus p_1$	p_L	p_M	p_H
v_L	9% (1/11)	36% (76/211)	36% (102/280)
v_M	0% (0/3)	57% (137/240)	67% (151/227)
v_H	0% (0/7)	31% (72/231)	82% (214/260)

Proportions of Rejecting the First-Round Offer

Result

Some high- and middle-type buyers reject p_1 *with expecting that* p_2 *would be more favorable to them. Some low-type buyers also reject* p_1 *where they could have been better off by exercising the outside option.*

Take-away Messages

- BP's prediction is robust in theory, in many ways.
- It builds upon many layers of rational belief updating, positive selection of the remaining demand pool.
- We found that a substantial fraction (41.77%) of p_1s are rejected, which shouldn't be observed in theory.
- Only about 7% of the sellers report the beliefs based on the 3rd-(or higher-)order positive selection.
- About half of them were naïve, meaning that few thought in an "equilibrium" way.
- Our contribution is not only checking the validity of BP but also presenting and utilizing a way to decipher which level of positive selection reasoning fails.

Related Literature

Theory

- Board and Pycia (2014), Tirole (2016)

Experiment

- Kneeland (2015), our companion paper Advertise

Research Questions

The sharp contrast in theoretical predictions inspires our research:

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2. In the presence of an (arbitrarily small but positive) outside option: Positive selection leads to the maximum seller profit

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Would this stark difference be empirically valid, even when some players might not be entirely rational?

- We are interested in examining the treatment effect of the outside option, but not in confirming or rejecting the Coase conjecture per se.

Coase Conjecture

- One of the most fundamental ideas in
  - Bargaining theory
  - Durable-good monopoly
  - Dynamic screening problems

    (including lemon market and sequential auctions)
- The uninformed seller eventually benefits **not at all** from inter-temporal price discrimination.
- Theoretically examined and confirmed by Fudenberg et al. (1985) and Gul et al. (1986) among others.

Negative Selection in the Demand Pool

Buyer's value is his private information.

1. Consider the value distribution $[\underline{v}, \overline{v}]$.

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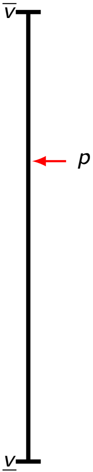

Buyer's value is his private information.

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2. For any price offer, the remaining buyers are more likely to be of low types.

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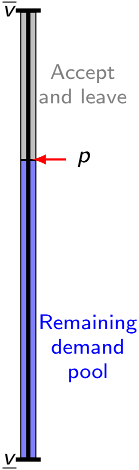

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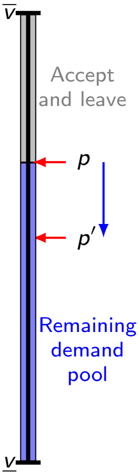

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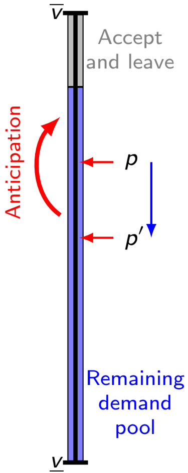

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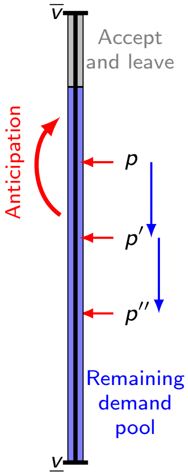

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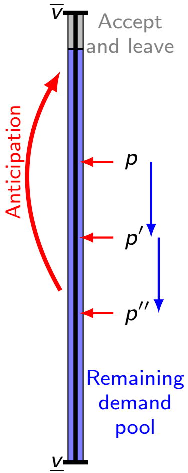
4. Anticipating such a price cut, even a high-type buyer tends to delay her purchase.
5. Pushing the seller to lower the price in the early stage even further to induce any purchase.

Negative Selection in the Demand Pool



Buyer's value is his private information.

1. Consider the value distribution $[\underline{v}, \overline{v}]$.
2. For any price offer, the remaining buyers are more likely to be of low types.
3. The seller responds to cut the price over time.
4. Anticipating such a price cut, even a high-type buyer tends to delay her purchase.
5. Pushing the seller to lower the price in the early stage even further to induce any purchase.
6. Pushing the price toward $\underline{v}$ (cf. Coase conjecture) and lead to the lowest seller profit in equilibrium.