# Multilateral Bargaining on a Loss Domain*

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Preliminary

**Abstract**

Many-player divide-the-dollar games have been a workhorse for theoretical and experimental analysis on multilateral bargaining. If we deal with a loss, that is, if we consider many-player "divide-the-penalty" games for location choices of obnoxious facilities, allocation of burdensome chores, and climate change summit to reduce carbon dioxide emissions, the theoretical predictions do not merely have flipped signs of those in the divide-the-dollar games. We show that the stationary subgame perfect equilibrium (SSPE) is no longer unique in payoffs. The most "egalitarian" equilibrium among the stationary equilibria is mirror-imaged to the unique SSPE in the Baron-Ferejohn model. That equilibrium is fragile in the sense that allocations are sensitively responding to the changes in parameters while the most "unequal" equilibrium is not affected by the changes in parameters. Experimental evidence clearly support the most unequal equilibrium: Most of approved proposals under a majority rule involve an extreme allocation of the loss to a few members. Other observations such as no delay, proposer advantage, and the acceptance rate are also consistent with the predictions based on the most unequal equilibrium.

**JEL Classification:** C78, D72, C92

**Keywords:** Multilateral bargaining, Loss division, Laboratory experiments

# 1 Introduction

Multilateral bargaining refers to a situation where a group of agents with conflicting interests try to bargain under a predetermined voting rule. Many-player divide-the-dollar (henceforth, DD) game where a group of agents reaches an agreement on a proposal dividing a dollar has served as a decent analytic tool to understand multilateral bargaining behavior (Baron and Ferejohn, 1989). We claim this model sheds light on only one side of multilateral bargaining: The other side is about the distribution of a loss or a penalty. Our contribution is twofold: to persuade that multilateral

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bargaining on the distribution of bads is theoretically[1] different from that of goods, and to provide experimental evidence that is divergent from the standard findings in the experimental multilateral bargaining literature.

Real-life situations dealing with the distribution of a loss are as often observed as those of a surplus. The climate change summit is an example of dividing a penalty in the sense that the participating countries share the global consensus to reduce carbon dioxide emission levels, but no single country wants to take the whole burden which may be harmful to their economic growth. A location choice of an obnoxious facility is another example of the allocation of a loss as the nearer area will suffer from the disutility of the facility more than the other areas. Taxation for public spending or redistribution could also be understood as a distribution of burdens. Despite its relevance to many policy issues, little attention has been paid to multilateral bargaining on a loss domain. Such inattention might be due to a naïve conjecture that the theoretical predictions of the many-player "divide-the-penalty" (henceforth, DP) game would be exactly the inverse of those of the DD game. We claim this is not the case. Our claim does not rely on any behavioral/psychological assumptions including the loss aversion.

Figuratively speaking, comparing the DD game with the DP game is not analogous to comparing an allocation of a "half-full" cup of water with that of a "half-empty" cup of water, but analogous to comparing an allocation of a full cup of "clean" water with that of a full cup of "filthy" water. The former one, though framed differently, deals with the same objective, but the latter one deals with fundamentally different objectives.

One key difference comes from a distinctive feature of loss division: No members can take advantage of being the proposer more than getting zero losses. In the DD game, a proposer exploits rent of being the proposer by forming a minimum winning coalition (MWC) to the extent that the number of "yes" votes is just enough for the proposal to be approved, and by offering the members in the MWC their continuation value so that rejecting the offer would not make them better off. Altogether, a significant amount of proposer advantage is predicted in the DD game. However, the proposer in the DP game, who will at best enjoy no losses, may not be better off than those in the MWC, who could also enjoy no losses.

The fact that the proposer cannot take more advantage than getting zero losses is a source of another theoretical difference between the DD game and the DP game. While the DD game has a unique Stationary Subgame Perfect Equilibrium (SSPE) in payoffs (Eraslan, 2002), the DP game has a continuum of Stationary Subgame Perfect Equilibria. The strategy of one SSPE (which we call the Utmost Inequality (UI) equilibrium) is for the proposer to assign all the penalty to one randomly chosen member: The other members without a penalty will accept the proposal because the continuation value (the expected payoff of moving on to the next bargaining round) would be strictly smaller than 0. At the other extreme, the strategy of another SSPE (which we call Most Egalitarian (ME) equilibrium) is for the proposer to distribute the penalty to all the members except herself to the extent that minimum winning coalition members will not be better off by rejecting the current offer. Of course, any intermediate strategy between these two extreme SSP equilibrium strategies can constitute an SSPE. Therefore, the primary goal of this project is to comprehensively

[1]As we elaborate more, we do not rely on the loss aversion or any behavioral/psychological justification.

investigate the DP game, and compare it with the DD game, both theoretically and experimentally.

Laboratory experiments have been a useful tool in the multilateral bargaining literature. We claim the use of lab experiments is more required for the DP game. Even if we narrow down our focus to stationary strategies, theory is silent in guiding us which equilibrium is more likely to be selected. Anecdotal empirical evidence might be sporadically available, but we cannot be free from the issues about measurements, endogeneity, and unobservable heterogeneities to identify a clear causal link. Moreover, it is challenging, if not impossible, for experimenting policymakers to implement different situations where an actual loss should be distributed.

We conducted experiments of four treatments which vary by two dimensions: the group size (either 3 or 5) and the voting rule (either majority or unanimity). Theoretical predictions based on the ME equilibrium were used as null hypothesis, because it resembles the essentially unique stationary subgame perfect equilibrium in the DD game, and it approaches the unique equilibrium under unanimity as the qualified number of voters for approval goes to $n$. Experimental evidence clearly reject the ME equilibrium. Instead, the UI equilibrium is primarily consistent with our experimental observations. Most of approved proposals under a majority rule involve an extreme allocation of the loss to a few members. That is, in three-member bargaining, one member receives all the losses exclusively, and in five-member bargaining, either one member receives all or two members receive a half each. The utilitarian efficiency, that is, no delay in reaching an agreement, and the proposer advantage are well observed.

The rest of this paper is organized in the following way. In the following subsection, we discuss the related literature. Section 2 presents the model of the divide-the-penalty game, and Section 3 describes theoretical properties of the model. The experimental design, hypotheses, and procedure are discussed in Section 4. We report our experimental findings in Section 5. Section 6 discusses further issues, and Section 7 concludes the paper.

## 1.1 Related Literature

This study stems from a large body of the literature on multilateral bargaining. A legislative bargaining model initiated by Baron and Ferejohn (1989) has been extended (Eraslan, 2002; Norman, 2002; Jackson and Moselle, 2002), adopted to more general models (Battaglini and Coate, 2007; Diermeier and Merlo, 2000; Volden and Wiseman, 2007; Bernheim et al., 2006; Diermeier and Fong, 2011; Ali et al., forthcoming; Kim, 2019), and experimentally tested (Diermeier and Morton, 2005; Fréchette et al., 2003, 2005; Fréchette et al., 2012; Agranov and Tergiman, 2014; Kim, 2018). Our contribution to this literature is to show that the theoretical predictions of the DP game could be significantly different due to the natural restriction of proposer advantage: In the DP game, the proposer may take no more advantage than getting no penalties.

In the sense that the fundamental idea of the model is pertinent to the allocation of bads, this study is related to chore division (Peterson and Su, 2002), a subset of envy-free fair division problems (Stromquist, 1980) in which the divided resource is undesirable. Social choice theorists are well aware of the distinctive difference between the allocation of goods and that of bads. Bogomolnaia et al. (2018) show that in the division of bads, unlike that of goods, no allocation rule dominates the other in a normative sense. While the literature on envy-free division has focused more on the algorithms or protocols that lead to the desired allocation, this paper only considers predetermined voting rules and does not focus on the design of algorithms. Another literature philosophically connected to our study is on the principle of equal sacrifice in income taxation (Young, 1988; Ok, 1995) in which the primary purpose is to justify the traditional equal sacrifice principles in taxation from a non-utilitarian perspective, by showing that the utility function that satisfies the equal sacrifice principles could be a consequence of more primitive concepts of distributive justice. Although taxation for public spending or redistribution is related to the idea of the distribution of monetary burdens, we try not to be normative in this paper.

This study also contributes to the literature documenting behavioral asymmetries between gain and loss domains. From many studies about loss aversion, we know that human behavior when dealing with losses would be different from that with gains. In this regard, Christiansen and Kagel (forthcoming) is the study philosophically related to our study. They examine how the framing changes three-player bargaining behavior. In particular, based on the model studied by Jackson and Moselle (2002), they study two treatments which are isomorphically the same in theory: One treatment is for each player to endow with 600 units and ask them to allocate 100 units of subsidy to determine a policy level. The other treatment is for each player to endow with 700 units and ask them to distribute 200 units of a tax burden to determine a policy level. Since the theoretical predictions of the two treatments are identical, their primary purpose is to observe the framing effect. Their study is rather related to the literature on the discrepancies between willingness-to-pay and willingness-to-accept. The crucial difference between our study and theirs is that we deal with the different incentive structure, so the framing does not play an important role: While the experimental design considered in Christiansen and Kagel (forthcoming) can be regarded as a 'half-full' versus 'half-empty' glass of water, figuratively speaking, ours is a full glass of clean water versus a full glass of filthy water. In the sense we compare an economic outcome on a gain domain with that on a loss domain, Gerardi et al. (2016) is another closely related study. They compare a penalty of not turning out to vote with a lottery for those who turn out, show that these two incentive structures are theoretically similar, and provide experimental evidence that voters are more likely to turn out under a lottery treatment than under a penalty treatment.

Although it may be viewed that the public bad prevention compared with the public good provision (Andreoni, 1995) is somewhat related, the comparison between the DD game and the DP game is distinctively different from the comparison between the public good provision and the public bad prevention, because the former does not involve any form of externality.

# 2 A Model

We consider a many-player divide-the-penalty game. As the many-player divide-the-dollar game à la Baron and Ferejohn (1989) is to understand multilateral bargaining over a surplus, the divide-the-penalty game will serve as a theoretical tool to understand multilateral bargaining over a loss.

There are $n$ (an odd number greater than or equal to 3) players indexed by $i \in N = \{1, \dots, n\}$. A feasible allocation share is $p = (p_1, \dots, p_n) \in \{[-1, 0]^n | \sum_i p_i = -1\}$ and the set of feasible allocation shares is denoted as $P$. We consider $q$-quota voting rule: The consent of at least $q \leq n$ players is required for a proposal to be approved. The voting rule is called a dictatorship if $q = 1$, a (simple) majority if $q = \frac{n+1}{2}$, a unanimity if $q = n$, and a super-majority if $q \in (\frac{n+1}{2}, n)$.

The amount of the loss increases as time passes, so the delay is costly. Such a cost regarding delay is captured by the growth rate of the loss, $g \in [1,\infty)$ per delay. At the same time, delay dilutes the disutility from a penalty. If players prefer having a disutility tomorrow than the same amount of disutility today, they may want to postpone the actual allocation of the penalty as much as possible, so that the disutility from the allocation can be diluted. Let $\beta \in (0,1]$ denote such time preference. When the allocation of the penalty is made at round $t$, player $i$'s utility is $U_i^t(p) = (\beta g)^{t-1} p_i$. For notational convenience, let $\delta \equiv \beta g$, which could be larger or smaller than 1.[2] On a loss domain, these two factors, $\beta$ and $g$, lead to different incentives. When time preference dominates the growth rate of penalty, that is, $\delta < 1$, players have an incentive to postpone the actual allocation of the loss. Otherwise, players want to make a decision as quickly as possible. We focus on $\delta \geq 1$ because it could capture more pertinent situations: If the nature of bargaining drives relevant parties to postpone their agreements as much as possible, on the contrary, such bargaining may not deal with nontrivial issues.[3]

Players bargain over the loss until they reach an agreement. The timing of the game is as follows:

1. In round $t \in \mathbb{N}_+$ player $i$ is recognized as the proposer at random. The selected player proposes an allocation of $-g^{t-1}$ in terms of proportions.
2. Each player votes on the proposal. If it is approved, that is, more than $q$ players accept the proposal, the proposal is implemented, $U_i^t(p)$ is accrued, and the game ends. If not, the game moves on to round $t+1$.
3. In round $t+1$, a player is randomly recognized as the proposer. The game repeats at $t+1$.

Let $h^t$ denote the history at round $t$ that includes the identities of the previous proposers and the current proposer. Let $\{p_i^t(h^t), x_i^t(h^t)\}$ denote a feasible action for player $i$ in round $t$, where $p_i^t(h^t) \in \Delta(P)$ is the (possibly mixed) proposal offered by player $i$ as the proposer in round $t$ and $x_i^t(h^t)$ is the voting decision threshold of player $i$ as a non-proposer in round $t$, where $\Delta(P)$ is the set of probability distributions on $P$. A strategy $s_i$ is a sequence of actions $\{p_i^t(h^t), x_i^t(h^t)\}_{i=1}^n$, and a strategy profile $s$ is an $n$-tuple of strategies, one for each player.

Concerning the DD game, it is known that there are numerous stage-undominated equilibria (Baron and Kalai, 1993), and virtually all allocations can be supported as an equilibrium under majority rule (Baron and Ferejohn, 1989). A similar folk theorem can be applied to the DP game.

**Proposition 1.** *Assume $n \geq q+1 \geq 3$ and $\delta \geq 1$. For any $p \in P$, there exists an undominated subgame perfect equilibrium for which $p$ is the equilibrium outcome.*

**Proof:** See Appendix A.

[2] A discount factor in the standard dynamic models, $\delta \in [0,1)$, may be understood as the depreciation rate (the inverse of the growth rate), $1/g$, times the subjective time-discount factor, $\beta$. In this case, $\delta$ is always smaller than 1, so the distinction between the depreciation rate and time preference is not crucial. That is, on a gain domain, a discount factor $\delta \in (0,1]$ can be innocuously interpreted in two different ways: It could mean time preference, depreciation of the resource, or both.

[3] The case with $\delta < 1$ is discussed in Section 6.

The result of Proposition 1 renders a rationale for considering a refinement of the equilibria. We here focus on Stationary Subgame Perfect Equilibria (SSPE). A strategy profile is *stationary* if it consists of time- and history-independent strategies. A strategy profile is subgame perfect if any single deviation at a subgame cannot make the player better off. A strategy $s_i$ is now simplified to $\{p_i, x_i\}$. Furthermore, in this paper, we consider symmetric agents, so the strategy boils down to (1) the proposal $p$ when a member is recognized as a proposer, and (2) the voting decision threshold $x$ to accept as a nonproposer. We also restrict our focus on equilibria in which each player's strategy is symmetric.

# 3 Analysis

While the DD game has a unique SSPE in payoffs (Eraslan, 2002), the DP game has a continuum of stationary equilibria which involve different payoffs. For a brief illustration, we start with a particular case where a simple majority rule is applied, and $\delta = 1$. Perhaps the most intuitive stationary equilibrium involves allocation of the whole penalty to only one member.

**Proposition 2** (Utmost Inequality equilibrium)**.** *One Stationary Subgame Perfect Equilibrium can be described by the following strategy profile:*

- *Member $i$ being recognized as a proposer in round $t$ picks member $j \neq i$ at random, and proposes $p_j = -1$ and $p_{-j} = 0$.*
- *Members being offered to have no penalty accepts the proposal, and rejects it otherwise.*

*In this equilibrium, the proposal made by the first round proposer is approved.*

**Proof**: See Appendix A.

We call this equilibrium Utmost Inequality (UI) equilibrium because only one member will take the whole burden of the penalty. Another equilibrium is the most egalitarian among SSPE.

**Proposition 3** (Most Egalitarian equilibrium)**.** *One Stationary Subgame Perfect Equilibrium can be described by the following strategy profile:*

- *A member being recognized as a proposer in round $t$ picks $\frac{n-1}{2}$ minimum winning coalition (MWC) members at random. She proposes $p_i = -1/n$ if $i \in MWC$, $s_{-i} = -\frac{n+1}{n(n-1)}$ if $i \notin MWC$, and keeps 0 for herself.*
- *Member $i$ being offered $x \geq -1/n$ accepts the proposal, and rejects it otherwise.*

*In this equilibrium, the proposal made by the first round proposer is approved.*

**Proof**: See Appendix A.

In this Most Egalitarian (ME) equilibrium, the distribution of the penalty is spread out, least unevenly. Note that the ME equilibrium does not involve an equal split of the penalty: The allocation in the ME equilibrium is the most egalitarian in the sense that the largest share of the penalty that one member would take is the smallest among all the possible stationary equilibria.

Table 1 juxtaposes how the theoretical predictions of the DP game are different from those of the DD game under a simple majority rule when the discount factor is 1.

Table 1: Comparisons: Simple Majority, $\delta = 1$

| Game | Proposer Share | MWC Share | non-MWC Share | Proposer Advantage† |
|---|---|---|---|---|
| DD | $1 - \frac{n-1}{2n}$ | $\frac{1}{n}$ | $0$ | $\frac{n-1}{2n}$ |
| DP (UI) | $0$ | $0$ | $-1$ (one of them) | $0$ |
| DP (ME) | $0$ | $-\frac{1}{n}$ | $\frac{-n-1}{n(n-1)}$ | $\frac{1}{n}$ |

†: Proposer advantage is a difference between the payoff of the proposer and that of the MWC member.

Indeed there are other SSPE that take an intermediate form between the UI equilibrium and the ME equilibrium. For example, in one equilibrium the proposer picks $\frac{n-1}{2}$ members randomly, and offer $-\frac{2}{n-1}$ to each of them. The other $\frac{n-1}{2}$ members who got offered to have no penalty will accept the proposal. Proposition 4 and Corollary 1 describe all possible SSPE in the DP game for any $\delta \geq 1$.

**Proposition 4.** *Assume $q < n$. Every Stationary Subgame Perfect Equilibrium can be described by the following strategy profile:*

- *Member $i$ being recognized as a proposer in round $t$ selects $q-1$ MWC members at random. She proposes $p_j \geq -\delta/n$ if $j \in MWC$, proposes $p_j \leq 0$ if $j \in OTH \equiv N \setminus MWC \setminus \{i\}$ such that $\sum_{k \in OTH} p_k = -1 - \sum_{j \in MWC} p_j$, and keeps zero for herself.*
- *Member $i$ being offered $x \geq -\delta/n$ accepts the proposal, and rejects it otherwise.*

*In this equilibrium, the proposal made by the first round proposer is approved.*

**Proof:** See Appendix A.

**Corollary 1.** *Assume $q = n$. If $\delta \geq \frac{n}{n-1}$, proposer $i$ still keeps zero for herself and offer $p_j \geq -\delta/n$ for all $j \neq i$. If $\delta < \frac{n}{n-1}$, the unique stationary equilibrium is to offer $-\delta/n$ to every member and keeps $\frac{(n-1)\delta - n}{n}$.*

**Proof:** See Appendix A.

There are at least three points worth mentioning. First, the theoretical predictions of the DP game, though the structure of the game can be understood as a mirror image of the DD game, are not the inverse of the theoretical predictions of the DD game, except a particular case where $n = 3$ and $\delta = 1$.[4] By construction, the ME equilibrium corresponds to the SSPE in the DD game: The members in the minimum winning coalition are offered the smallest amount of surplus that is just

[4]When $n = 3$ and $\delta = 1$, the ME equilibrium allocation of the DP game, where the proposer keeps 0, a coalition member receives $-1/3$, and the other member received $-2/3$ look like a mirror image of the SSPE allocation of the DD game, where a proposer keeps 2/3, a coalition member receives 1/3, and the other member receives nothing.

large enough to accept the offer in the DD game, while they are offered the largest amount of losses that is just small enough to accept the offer in the DP game. We require attention to this result because one of the primary reasons why previous studies had paid little attention to the DP game is perhaps the naïve conjecture about the symmetry of the theoretical results.

Second, while the SSPE in the DD game is unique in payoffs, the ME equilibrium in the DP game, the mirror-imaged one of the equilibrium in the DD game, is fragile for many aspects. The equilibrium relies on the assumption that players vote for the proposal with probability 1 when indifferent between accepting and rejecting it, which may not hold. Even if the proposer decides to offer the loss which is "$\varepsilon$-less" than the continuation value to the minimum winning coalition members, each player's $\varepsilon$ is private information, so choosing the ME strategy may not guarantee the approval of the proposal. The ME equilibrium also requires each player to exactly calculate the continuation value, which varies by the voting rule, the size of the group, and the discount factor, and it demands a high level of rationality. Another notable observation is that when $\delta$ is sufficiently large the continuation value, the offered amount to the minimum winning coalition members, can be *smaller* than the ex-ante payoff of the other members.[5] That is, the minimum winning coalition members can be treated more badly than other members in the ME equilibrium. In such a situation, a definition of "minimum" winning coalition by itself becomes fragile as all the members receive an offer more attractive than their continuation value. Thus, the ME equilibrium, although it is more relevant to the equilibrium in the DD game, is fragile in the sense that it requires a stronger assumption about voting behavior and a high level of rationality.

Third, the existence of multiple SSPE gives rise to the equilibrium selection issue. Both the UI equilibrium and the ME equilibrium are optimal from a utilitarian perspective. Given the same level of social efficiency, which strategy the proposer would choose? On one hand, the proposer may want to choose the most egalitarian strategy because the ME equilibrium is better from a Rawlsian perspective. On the other hand, there may be an incentive for her to choose the most unequal strategy. If the proposer is uncertain about how often other players mistakenly make a wrong decision, she may want to secure strictly more votes than $q$ so that her payoff is robust to the other members' mistakes. For this purpose, she may want to allocate the penalty to the smallest number of players. Taking inequity aversion (Fehr and Schmidt, 1999) into account does not help us refine the set of the equilibria.[6] From the perspective of the members who are offered zero penalties in the UI equilibrium, although accepting the offer brings the largest disutility from the advantageous inequity, it involves the smallest disutility from the disadvantageous inequity.[7] This open question

[5]For example, consider the ME equilibrium when $n = 5$, $q = 3$, and $\delta = 1.5$. Each of the minimum winning coalition members is offered $-0.3$, while each of the other members is offered $-0.2$ on average.

[6]Similarly, taking loss aversion (Kahneman and Tversky, 2013) into account does not significantly help to refine the set of equilibria further, as we do not know the reference point of the players. If the reference point is set to be zero, the "gain domain" is never achieved, so the loss aversion does not play a role. If the reference point is set to be an equally-split loss, it implies that the reference point changes over time, which has little support. If the reference point is set to be an ex-ante expected utility in the first round, still there is a continuum of equilibria, and the set could be *larger* than what we have, depending on the loss aversion parameters. The loss aversion could encourage the coalition members (who fear about the possibility of losing more in the next round) to accept less attractive offer now.

[7]Montero (2007) showed that in the DD game, the inequity aversion might increase the proposer's share in equilibrium, and the underlying intuition is along with the same logic: From the perspective of the coalition member, the marginal disutility from the increased difference between the proposer's share and what he is offered can be smaller than the marginal

will be answered by our laboratory experiments.

# 4 Experimental Design and Procedure

## 4.1 Design and Hypotheses

We tailor laboratory experiments to examine how the distributions of the loss look like, and how people behave in a loss domain, especially regarding the choices of the winning coalition. The major treatment variables concern the group size ($n \in \{3, 5\}$) and the voting rule ($q = (n+1)/2$ or Majority; $q = n$ or Unanimity). We set the appreciation factor $\delta$ as 1.2. Table 2 presents our $2 \times 2$ treatment design. Each of those treatments is respectively called M3 (Majority rule for a group of three), M5, (Majority + five), U3 (Unanimity rule for a group of three), and U5 (Unanimity + five). M3 and M5 are collectively called the Majority treatments, and U3 and U5 are called the Unanimity treatments.

Table 2: Experimental Treatments

| | | Voting Rule | |
|---|---|---|---|
| | | Majority | Unanimity |
| Group Size | 3 | ***M3*** | ***U3*** |
| | 5 | ***M5*** | ***U5*** |

Figure 1 illustrates the theoretical predictions, which can be categorized into two qualitatively different kinds. The first type of predictions (Hypotheses 1 and 2) is those that do not depend on equilibrium selection. The second type (Hypothesis 3) is those that vary depending on which equilibrium is selected. We shall go over the first type of predictions and derive a set of experimental hypotheses.

The followings are true regardless of which equilibrium is played in all treatments. First, it is predicted that the offers are approved immediately so that full utilitarian efficiency is achieved. Second, the agreed share of the proposer is smaller than the agreed shares of non-proposers.

**Hypothesis 1** (Full Efficiency and Proposer Advantage)**.**

(a) The first round proposals are approved in all treatments.

(b) The proposer gets the smallest loss in all treatments.

The second set of hypothesis is about the distribution of loss. The agreed shares of the proposer may vary across different group sizes depending on the voting rule. First, for a given group size, the agreed share of the proposer is larger under the unanimity rule than under the majority rule. Second, given the majority rule, the agreed share of the proposer is always zero regardless of the group size. Third, given the unanimity rule, the agreed share of the proposer is larger when the

utility from the decreased difference between what he is offered and what other non-MWC members receive (zero).

group size is smaller. Accordingly, the non-proposers are offered a larger share of loss when the group size is smaller.

**Hypothesis 2** (Share of Loss)**.**

(a) The proposer keeps a smaller loss in Majority treatment than in Unanimity treatment.

(b) In Majority treatment, the proposer keeps zero regardless of the group size.

(c) The proposer keeps a larger share of loss in U3 treatment than in U5 treatment.

(d) The non-proposers are offered a larger share of loss in U3 treatment than in U5 treatment.

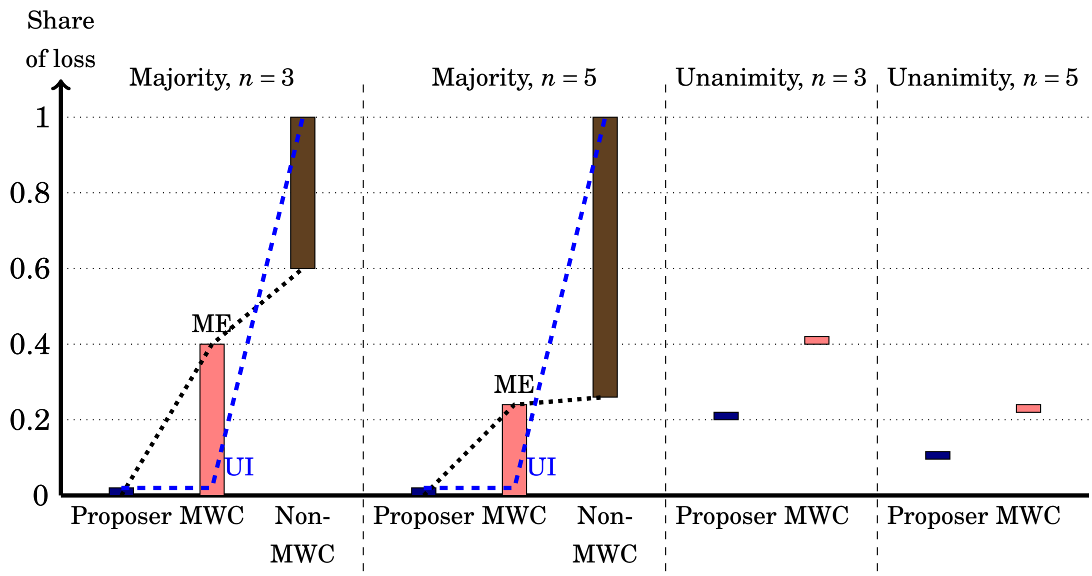



Figure 1: Hypotheses from theoretical predictions

We move on to discuss the second set of predictions that are dependent upon the equilibrium selection. First, the offers to the non-proposers vary by the choice of an equilibrium. Especially in Majority treatment, players who are not the proposer offered a share of loss ranging from zero to full penalties. Second, under the majority rule, the possible variations of what the MWC can be offered varies by the size of the group, but such theoretical variations are not allowed under the unanimity rule. Given that our main objective is to observe behaviors in the lab and falsify/select some equilibria, we shall derive our next set of *null* hypotheses based on the assumption that the ME (Most Egalitarian) equilibrium is played in the lab. We do not mean that we are selecting the ME equilibrium as the most plausible candidate. It plays a role of the benchmark for clearly stating the experimental hypotheses. There are two reasons why we take the ME equilibrium as a benchmark. First, it is the closest to the mirror image of the unique stationary equilibrium of the DD game. Second, it is the unique stationary equilibrium prediction under the unanimity (i.e., when $q = n$). By continuity, it is natural to take the same equilibrium when $q < n$. In Figure 1, the upper bound of the MWC share of loss and the lower bound of the non-MWC share constitute the ME equilibrium.

**Hypothesis 3** (Winning Coalition and Non-proposers' Shares under Majority)**.**

In the Majority treatments,

(a) The number of non-proposers who accept the proposal is $(n-1)/2$. That is, one member rejects the proposal in M3 treatment, and two members do in M5.

(b) The agreed share of the non-proposers who accept the proposal is larger than that of the proposer.

(c) The agreed share of the non-proposers who accept the proposal is larger in M3 than in M5.

As we have emphasized already, predictions summarized in Hypothesis 3 do not hold with the UI equilibrium. While the ME equilibrium predicts that there are two members who reject the proposal in M5 treatment (Hypothesis 3 (a)), the UI equilibrium predicts that only one member will reject the proposal. Contrary to Hypothesis 3 (b), the share of the MWC members is the same with that of the proposer in the UI equilibrium. Also, the share of accepting non-proposers is the same as zero in both majority treatments. Thus, testing these hypotheses using the observed behaviors in the lab would enable us to justify one of stationary equilibria. Given that the observed behaviors can be rationalized, we could conclude which equilibrium will be more likely to be selected.

## 4.2 Experimental Procedure

All the experimental sessions were conducted in English at the experimental laboratory of the Hong Kong University of Science and Technology in November, 2018. The participants were drawn from undergraduate population of the university. Four sessions were conducted for each treatment. A total of 271 subjects participated in one of the 16 ($= 4 \times 4$) sessions. Python and its application Pygame were used to computerize the games and to establish a server-client platform. After the subjects were randomly assigned to separate desks equipped with a computer interface, the instructor read the instructions for the experiment out loud. Subjects were also asked to carefully read the instructions before they took a quiz to prove their understanding of the experiment. Those who failed the quiz were asked to reread the instructions and to retake the quiz until they passed. An instructor answered all questions until every participant thoroughly understood the experiment. Whenever a private question is raised, the instructor repeated the question out loud and answered it so that every subject was equally informed.

We conducted many-person divide-the-penalty experiments. Concerning the structure, the game is a mirror-imaged one of a typical many-person divide-the-dollar game, and it proceeds as follows: At the beginning of each bargaining period (called a 'Day' in the experiment), each bargainer is endowed with 400 tokens, where a token is a currency unit used in the laboratory. In each bargaining round (called a 'Meeting' in the experiment) one randomly selected player proposes a division of $-50 * n$ tokens, where $n$ is the number of players in each group. The proposal is immediately voted on. If the proposal gets $q$ or more votes, the bargaining period ends, and according to the proposal their endowment is deducted. Otherwise, the bargaining proceeds to the second round, where the penalty increases by 20 percent. That is, in the second round, the players deal with an allocation of $-60 * n$ tokens. A new proposer is randomly selected, and the new proposal is voted on. This process is repeated indefinitely until a proposal is passed.

Since the subjects were informed that they eventually earn at least a show-up payment of HKD 30 (≈ USD 4), we implicitly limit the largest possible losses out of the equilibrium. As long as the largest out-of-equilibrium losses is sufficiently large, in particular, if it is larger than $\delta * 50 * n$, any stationary equilibrium is neither restricted nor ruled out. Thus, the theoretical analysis still serves as benchmarks for out experiments.

Subjects in U3 and M3 treatments participated in 12 bargaining Days and those who were in U5 and M5 treatments participated in 15 bargaining Days.[8] We used the random matching protocol and between-subject design. Although new groups were formed every bargaining Day, there was no physical reallocation of the subjects, and they only knew that they were randomly shuffled. They were not allowed to communicate with other participants during the experiment, nor allowed to look around the room. It was also emphasized to participants that their allocation decisions would be anonymous. The experimental instructions for M5 treatment are presented in Appendix B.

At the end of the experiment, they were asked to fill out a survey asking their gender and age as well as their degree of familiarity with the experiment. The subjects risk preferences were also measured by the dynamically optimized sequential experimentation (DOSE) method (Wang et al., 2010). The amount of tokens that each subject earned at one randomly selected period (Azrieli et al., 2018) was converted into HKD at the rate of 2 tokens = 1 HKD. The average payment was HKD 202.7 (≈ USD 26) including HKD 30 guaranteed show-up fee. The payments were made in private, and subjects were asked not to share their payment information. Each session lasted 1.5 hours on average.

# 5 Experimental Results

Before presenting the test results of the hypotheses posed in the previous section one by one, we provide a summary of main findings as follows:

1. In Majority treatments, experimental evidence clearly rejects the ME equilibrium and supports the UI equilibrium.
2. In Unanimity treatments, the allocations in the approved proposals are consistent with theoretical predictions.
3. Most of the proposals are approved in the first round.
4. In Majority treatments, the proposers form the winning coalition to minimize their losses.
5. Risk preferences, familiarity with the game, and comprehensibility were not significant factors affecting the outcomes of the experiments. Females tend to take a slightly more share of the loss than males, and older subjects tend to accept the proposal.

[8] The number of bargaining Days varies to make sure that every participant plays a role as a proposer at least twice. If there were 12 bargaining Days in treatments with $n = 5$, each subject could have been recognized as a proposer 2.4 times on average, which is not sufficiently large to see observational variations by individual. We did not use the strategy method (i.e., asking all subjects to submit their proposals with knowing that one of which is randomly selected for voting afterwards) because we were unsure whether the strategy method, in this particular context of the DP game, works the same as the standard method. Brandts and Charness (2011) report that 15 out of 29 existing comparisons between two methods show either significant differences or some mixed evidence.

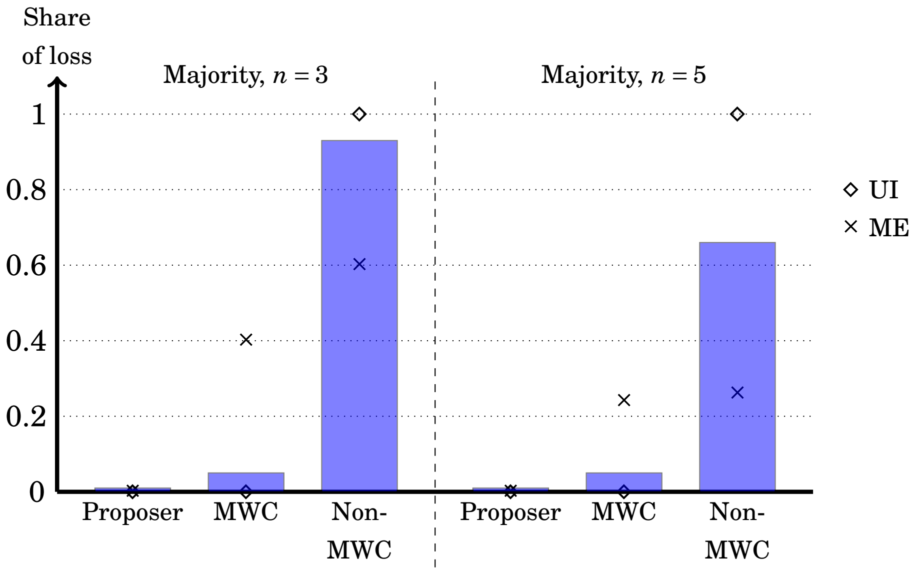



Figure 2: Proposed Shares, Majority
Approved proposals in the last 5 Days

Figure 2, which juxtaposes the equilibrium predictions for Majority treatments and the observed average allocation of the loss from the approved proposals in the last five Days, represents the main finding. The share of loss in the UI equilibrium is marked with $\diamond$, and that in the ME equilibrium is with $\times$. In M3 treatment, it is clear that one (non-MWC) member is offered almost all the loss, and such an allocation is distinctively different from the ME equilibrium prediction (Mann-Whitney test, $p < 0.001$, standard errors cluster-adjusted at the session level, aggregated across all individuals in the last 5 Days.)[9] We find a similar observation in M5 treatment. A proposer keeps nothing for herself, which is consistent with a theoretical prediction (Hypothesis 2 (b)), offers at least two members almost nothing, and allocates almost all the loss to at least one of the remaining two members. In the sense that the loss is exclusively allocated to the non-MWC member(s), the observations from both M3 and M5 treatments reject the null hypothesis (Hypothesis 3) based on the ME equilibrium, while those support the UI equilibrium. Specifically, we reject Hypothesis 3 (b) as the average share of the MWC members is at most marginally different from the share of the proposer in Majority treatments (Mann-Whitney test, $p = 0.1292$ in M3 and $p = 0.0814$ in M5). In M5 treatment, roughly speaking, a third of the approved proposals are similar to $(0,0,0,0,1)$ up to permutation, as the most share of the loss is allocated to one member, and the other two thirds are similar to $(0,0,0,0.5,0.5)$ up to permutation, as the most share of the loss is evenly distributed to two members. Thus, the average non-MWC share of loss is about 0.66, which rejects Hypothesis 3 (a) and (c). More detailed test results are followed.

In Unanimity treatments, the allocation in the approved proposals are weakly consistent with the unique SSPE predictions. Figure 3 shows the theoretical predictions and the average share of loss. On average, the proposers keep a larger share of loss than the equilibrium level in both U3

[9]All aggregate data reported and used for statistical testings are from the last 5 Days. Using data from the last 5 Days allows us to give more weight to converged behavior. However, the qualitative aspects of our findings remain unchanged if we use, for example, data from the last 8 or 10 Days.

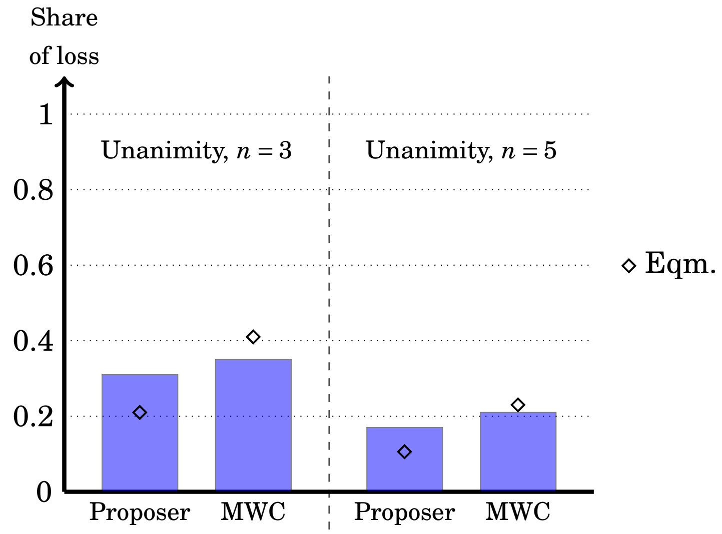
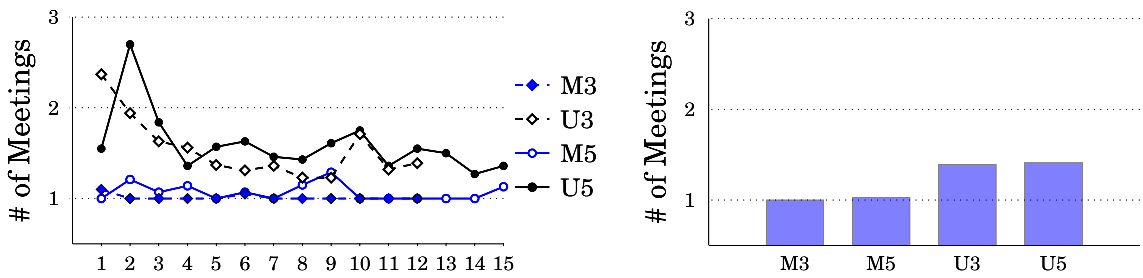



Figure 3: Proposed Shares, Unanimity
Approved proposals in the last 5 Days

and U5 treatments, and offer a smaller share of loss to non-proposers compared with the equilibrium level. The observation that the proposer keeps a larger loss in Unanimity treatment than in Majority treatment is consistent with Hypothesis 2 (a). Although statistically significant only in U5 treatment, the proposers keep a smaller share of loss than what the other members are offered in both U3 and U5 treatments (Mann-Whitney tests, $p = 0.2482$ in U3 and $p = 0.0209$ in U5.) Altogether with Majority treatments, we find the observations are consistent with Hypothesis 1 (b).

Figure 4 shows the average number of Meetings by Day. In the Majority treatments, nearly all of the proposals are approved in the first Meeting, which is consistent with a theoretical prediction (Hypothesis 1 (a)). Even in the Unanimity treatments, although the first three Days are rather variant (Figure 4 (a)), the average number of Meetings of the last five Days is less than 1.5 (Figure 4 (b)). Efficiency loss under a unanimity rule is one of the common findings in the multilateral bargaining experiments as in Kagel et al. (2010), Miller and Vanberg (2013), and Kim (2018), to name a few.

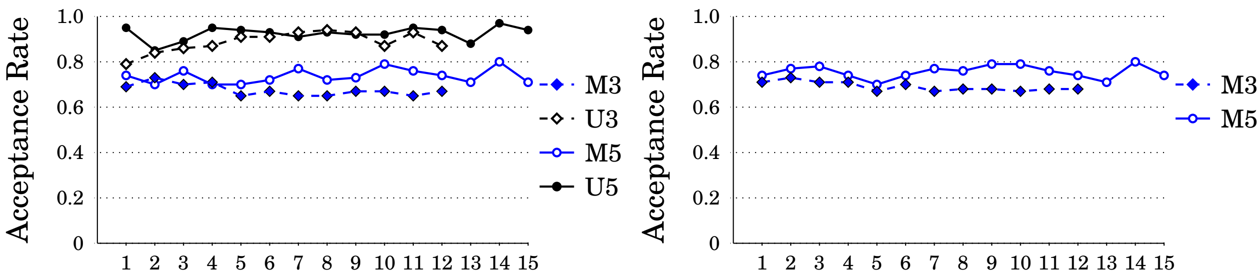



(a) Time Trend

(b) Last 5 Days

Figure 4: Average Number of Meetings

Figure 5 shows the average proportion of subjects who accept a given proposal. In M3 treatment for which all the stationary equilibria make the same prediction about the size of winning coalition, two thirds of the subjects, or two out of the three members, accept the proposal. It is consistent with the theoretical prediction. However, in M5 treatment, nearly 80% of the subjects, that is, around four out of the five members, accept the proposal, which clearly rejects Hypothesis 3 (a) that there are two members who reject the proposal ($t$−test, $p = 0.002$, $n = 350$, standard errors cluster-adjusted at the session level).




(a) All Proposals

(b) Approved Proposals

Figure 5: Average Acceptance Rate

Table 3 reports some regression results to examine whether individual characteristics have any impact on the outcomes of the experiments. To make a summary first, we did not find any strong impact of individual characteristics. The dependent variable in the first three regressions is the proposer's own share, and the dependent variable in the last two regressions is the non-proposer's voting decision. Some explanatory variables are from the post-experiment survey. We collected self-reported gender and age. The subjects risk preferences were measured by at most two survey questions. The second question is dynamically adjusted based on the answer to the first question asking to compare a simple lottery with a sure payment. This method enables us to categorize a subject into one of seven types regarding risk preference. Familiarity is a subjective assessment of how familiar the subject was with the underlying game for the experiment. QuizFailed is the dummy variable indicating whether the subject had to re-take the quiz after failing to pass, which would serve as a proxy of comprehensibility of the experiment. As control variables, we include treatment dummies and a time trend (labeled as Day) for regressions on the proposer's own share. In addition, we also include the offered share and the standard deviation of the proposal for regressions on the non-proposer's voting decision. The standard deviation of the proposal is added to examine whether the shape of the proposal matters for the subject to vote.[10] In all regressions, M3 is set to be the baseline treatment. We focus one the approved proposals in Meeting 1 only. Since the individual choices are positively correlated across Days, standard errors are cluster-adjusted at the individual level.

Risk preference, familiarity, and comprehensibility of the experiment did not make any significant impact on the proposer's decisions and the non-proposer's voting decisions. We found that females are slightly (about 1.52% to 1.66%, varying by model specification) more allocating losses to themselves. Older subjects tend to accept the proposals more, but the statistical significance is weak and the age variance is not substantially large, as in many typical laboratory experiments.

[10]For example, consider two proposals (0.2, 0, 0.8) and (0.2, 0.4, 0.4). As for member 1, these two proposals offer the same amount of losses, 0.2, while the distribution of the proposal varies. The standard deviation of the proposal would capture the impact of the distribution of the proposal if subjects' voting decision is indeed affected by it.

Table 3: Individual Characteristics

| Dep.Var. | Proposer's Own Share (1) | (2) | (3) | Non-proposer's Vote (LPM) | (Logit) |
|---|---|---|---|---|---|
| M5 | 0.0170 | 0.0174$^{*}$ | 0.0159 | $-0.0834^{***}$ | $-1.6503^{***}$ |
| | (0.0103) | (0.0100) | (0.0100) | (0.0222) | (0.4879) |
| U3 | 0.2768$^{***}$ | 0.2720$^{***}$ | 0.2720$^{***}$ | | |
| | (0.0116) | (0.0091) | (0.0092) | | |
| U5 | 0.1522$^{***}$ | 0.1523$^{***}$ | 0.1503$^{***}$ | | |
| | (0.0073) | (0.0071) | (0.0073) | | |
| Share | | | | $-0.9659^{***}$ | $-7.7228^{***}$ |
| | | | | (0.0249) | (0.7866) |
| St.Dev | | | | 0.0006 | 0.0077 |
| | | | | (0.0004) | (0.0047) |
| Day | $-0.0035^{***}$ | $-0.0039^{***}$ | $-0.0039^{***}$ | $-0.0019$ | $-0.0186$ |
| | (0.0008) | (0.0008) | (0.0008) | (0.0022) | (0.0263) |
| Gender | | 0.0166$^{**}$ | 0.0152$^{**}$ | $-0.0171$ | $-0.2057$ |
| | | (0.0067) | (0.0068) | (0.0273) | (0.3802) |
| Age | | | $-0.0063$ | 0.0535$^{**}$ | 0.7272$^{*}$ |
| | | | (0.0088) | (0.0262) | (0.3712) |
| RiskAversion | | | 0.0020 | 0.0066 | 0.0894 |
| | | | (0.0018) | (0.0063) | (0.0802) |
| Familiarity | | | $-0.0001$ | $-0.0556$ | $-0.9365$ |
| | | | (0.0072) | (0.0347) | (0.4394) |
| QuizFailed | | | $-0.0050$ | 0.0209 | 0.4068 |
| | | | (0.0071) | (0.0305) | (0.4615) |
| _Cons. | 0.0478$^{***}$ | 0.0425$^{***}$ | 0.0440$^{***}$ | 0.9039$^{***}$ | 3.0777$^{***}$ |
| | (0.0108) | (0.0099) | (0.0150) | (0.0555) | 0.7724) |
| $R^2$ | 0.7058 | 0.7392 | 0.7434 | 0.6875 | 0.6360 |
| N | 781 | 735 | 728 | 1239 | 1239 |

Only approved proposals in Meeting 1 are considered. In parentheses are standard errors cluster adjusted at the individual level. *, **, and *** indicate statistical significance at the 10% level, 5% level, and 1% level, respectively.

In summary, the observed patterns of our experimental data are primarily consistent with theoretical predictions based on the UI equilibrium, and individual characteristics do not lead to noticeably different outcomes of the experiments.

# 6 Discussions

In this section we discuss some theoretical deviations to which we paid less attention.

## 6.1 Incentive Compatibility of Participation

On a gain domain, the ex-ante expected payoff in the SSPE is $1/n$. Thus it is always incentive-compatible to participate in bargaining. Therefore, adding a pre-stage for agents to make a participation decision does not lead to any theoretical differences. This pre-stage decision, however, matters in multilateral bargaining on a loss domain: If the members know that they are about to divide losses, and the ex-ante expected loss in any stationary equilibria is $-1/n$, simply quitting the bargaining process would certainly be better. We implicitly assume here that a certain form of enforcement for participation exists. Dealing with inevitable issues, such as an allocation of tax burdens to different socioeconomic groups, and the international agreement on the greenhouse gas emission abatement, are relevant in the sense that members cannot easily choose to opt out the country or the planet. Even if the issue is avoidable, there are many ways to implement full participation of the members. For example, collectively agreeing that all the losses go to some of those who do not participate in bargaining would prevent every member from doing so: Given that other members agree on this protocol, one would get the entire loss by not participating. In this case, agreeing to participate make the one better off.

## 6.2 Voting Rules Other Than Unanimity

Another issue may be the choice of the voting rules other than unanimity. Since the UI equilibrium involves an extreme allocation of the loss to a few members, some risk-averse agents may demand nothing but unanimity. However, unanimity is not suitable for every situation. Implementation of a new policy would be one important example where a majority rule is applied. For example, the Tax Cuts and Jobs Act of 2017 in the United States was passed by the Senate on December 20, 2017 in a 51–48 vote. Assume for simplicity that a government wants to reform a tax policy to cope with a budget deficit, and there are only three equally-populated types of citizens: the rich, the poor, and the middle-income class. In this case, victimizing one out of the three distinctive groups by allocating tax burdens to the group may be implemented, but we do not claim that we should change the voting rule to unanimity due to that possibility. In addition, although the stability of the voting rule is beyond our concerns in this paper, studies including Barbera and Jackson (2004) characterize a self-stable majority voting rule with a persuasive argument that the general trend is away from unanimity. Moreover, as our experimental evidence and many other similar experimental studies, a unanimity rule accompanies efficiency loss due to delay.[11] Risk-neutral agents who negotiate over a loss repeatedly may want to avoid unanimity that might be eventually harmful to every agent.

### 6.3 Bargaining When Delay is Socially Desirable

We assume $\delta = \beta g \geq 1$ so that no one has an incentive to postpone their bargaining decisions. However, in the situations where $\delta < 1$, or $\beta$ (the subjective discount factor of a future payoff) is sufficiently smaller than $1/g$ (the inverse of the growth rate of the penalty,) the Pareto optimal allocation is for everyone to reject any form of proposals for any round $t$ so that everyone could eventually have zero losses. Still in this situation the stationary subgame perfect equilibria can be sustained, as long as we maintain the assumptions that each individual is self-interested and subgame-perfect strategies are considered. For example, when a proposal of allocating all the losses to one member is put on the vote, a member who receives an offer of zero losses would accept the proposal, because the continuation value of the next bargaining round is at least weakly smaller than the zero losses. If the qualified number of votes for approval is less than $n$, the proposal would be accepted immediately. Similar to the public goods game situation, the Pareto-optimal collective behavior is distinctively different from the equilibrium behavior.

We have paid less attention to the case with $\delta < 1$ for several reasons. First, we try to make the structure of the DP game as similar to that of the DD game as possible. In the DD game delay is discouraged, so is in the DP game with $\delta \geq 1$. Second, the experimental evidence may be confounded, because each subject's internalized social norms may be heterogeneous and unobservable (Kimbrough and Vostroknutov, 2016). If the primary purpose of this study were to observe how subjects differently behave when the Pareto-optimal behavior and the equilibrium behavior diverge, a typical linear public goods game would have been more pertinent. Third, since it is unusual to have losses that will be gone away as time goes by without doing anything, we claim $\delta < 1$ is less relevant to real-life situations.

## 7 Concluding Remarks

We examine the divide-the-penalty (DP) game to better understand multilateral bargaining when agents deal with a distribution of a loss. Although the literature on multilateral bargaining is substantial, both theoretically and experimentally, multilateral bargaining on a loss domain has been less attended. It is perhaps because of a naïve conjecture that theoretical properties of the DP game are mirror-imaged to those of the divide-the-dollar (DD) game due to their structural resemblance. We theoretically show that there are fundamental differences. The Stationary Subgame Perfect Equilibria in the DP game are no longer unique in payoffs, unlike in the DD game. One extreme among the continuum of SSPE, which we call the Most Egalitarian (ME) equilibrium, is characterized similarly to the unique SSPE in the DD game. The other extreme equilibrium,

[11] Bouton et al. (2018) discourage to use unanimity rule for a different reason, by showing that unanimity is Pareto-inferior to majority rules with veto power.

which we call the Utmost Inequality (UI) equilibrium, predicts that the proposer concentrates the penalty to a few members. Although the ME equilibrium shares many properties with the SSPE in the DD game, experimental evidence is primarily consistent with the predictions based on the UI equilibrium.

Our results have at least two implications. First, multilateral bargaining on a loss domain should not be understood through the lens of the typical DD game because both theoretical properties and experimental evidence deviate from those in the DD game. Second, many interesting studies in multilateral bargaining on a gain domain are worth being revisited. This direction of research should distinguish simple behavioral/psychological framing effects from the fundamental differences.

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# A Appendix - Proofs

**Proof of Proposition 1:** This is analogous to the proof of Proposition 2 in Baron and Ferejohn (1989). Fix a strategy profile with the following statements.

1. For all $i \in N$ and $t \in \mathbb{N}_+$, if $i$ is recognized in $t$, $i$ proposes $p^{it} = p$, and all individuals vote for $p$.
2. If $p$ is rejected under the $q-$quota rule in $t$, $j \in N$ recognized in $t+1$ proposes $p^{j(t+1)} = p$.
3. If, in any period $t$, the chosen proposer $i$ offers an alternative other than $p$, say $p^{it} = y \neq p$, then
   (3.a) a set $M(y)$ of at least $q$ individuals rejects $y$;
   (3.b) the period $t+1$ proposer, say $j$, offers an allocation $z$ such that $z_i = -1$ and all individuals in $M(y)$ vote for $z$ against $y$.
4. If, in (3.b), the period $t+1$ the proposer $j$ offers some alternative $y' \neq z$, repeat (3) with $y'$ replacing $y$ and $j$ replacing $i$.

Statement 1 specifies what happens along the equilibrium path. Statements 2, 3, and 4 describe off-the-equilibrium path behavior. That is, those jointly specify the consequences of any deviation from the behavior specified in 1.

For notational simplicity, relabel players in a way that player $n$ is the period $t$ proposer who offers $p^{nt} = y \neq p$ where $p_n < 0$, and $y_j \leq y_{j+1}$ for all $j = 1, \ldots, n-2$. If $p_n = 0$, there is no way for player $n$ to be better off, so $p_n < 0$ is reasonable without loss of generality. It is trivial that $y_n > -1$, because player $n$ does not have an incentive to deviate from $p$ to keep the all the loss from the beginning. Under (3.a) and (3.b), players in $M(y)$ reject $y$, and the next proposer offers an alternative proposal $z$ with $z_n = -1$ such that $M(y)$ approves. Such a distribution $z$ for which (3.a) and (3.b) describe best response behavior to $y$. We divide situations into two cases: Assume first that the proposer conditional on $y$ being rejected is some individual $j \neq n$. Let $M^*(y) = \{1, \ldots, q\}$, let $Y^* = \sum_{i \in M^*(y)} y_i$, and let $m^* = |\{i \in M^*(y) : y_i < 0\}|$. By construction of $M^*(y)$, $Y^* < 0$ and $m^* > 0$. $Y^* = 0$ (and hence $m^* = 0$) implies that $p_n = -1$ and thus $p = z$. If $y_i < 0$, then $z$ is strictly preferred because $\delta z_i = 0 > y_i$. If $y_i = 0$, then $z$ is as preferred as $y$ because the payoff of $i$ is unaffected. If $z$ is rejected, then under strategy statement 4, it will simply become the next proposal and so on.

Now we assume that player $n$ is again recognized as a proposer in the next period. Our goal is to show that player $n$ cannot benefit from proposing any allocation other than $p$. In such a case, (3.b) specifies that player $n$ proposes the allocation $z$, which "punishes" herself for her initial deviation. Apparently, she should fail to do this and instead propose some $y' \neq z$, strategy statement 4 requires a $q$-majority to reject $y'$ and the period $t+2$ agenda-setter to offer $z$, which then passes. Therefore, the only circumstance under which the period $t$ proposer $n$ can avoid having $z$ proposed and accepted in response to an initial deviation to $y \neq p$ is when player $n$ is chosen in every period as the proposer. Such probability $(1/n)^t$ approaches zero, and the size of the penalty for the deviation is non-decreasing. Therefore, player $n$ is not better off by deviating than she is proposing $p$ as required, with hoping that she could eventually attain a higher payoff than proposing $p$ and accepting $p_n$. $\square$

**Proof of Proposition 2:** Suppose for every round players have an identical stationary strategy described above. A member who received an offer of zero penalties this round will accept the proposal if moving on to the next round does not make him better off. In the next round, with probability $(n-1)/n$, he will be a proposer or a member who receives no penalty. With probability $1/n$, he will be randomly selected by a proposer in that round and take all the penalty. Certainly, the utility from the current offer (zero) is strictly larger than the continuation value $(-\delta^{t-1})$, he will accept the offer. The proposer, who keeps no penalty for herself, cannot be better off by any other proposal. Thus, everyone would not be better off by deviating from this stationary strategy profile for any round. □

**Proof of Proposition 3:** Consider player $i$ who received an offer of $-1/n$. If the game moves on to the next round, his expected payoff is

$$\frac{1}{n}0-\frac{n-1}{2n}\frac{1}{n}-\frac{n-1}{2n}\frac{n+1}{n(n-1)}=-\frac{n-1}{2n^2}-\frac{n+1}{2n^2}=-\left(\frac{2n}{2n^2}\right)=-\frac{1}{n}.$$

Therefore, he will not be better off by rejecting the current offer. From the perspective of the current proposer, there is no strategy to make her better off than receiving zero penalties. □

**Proof of Proposition 4:** First we show that unless unanimity, there is no stationary equilibrium where the proposer keeps strictly negative payoff.

**Lemma 1.** *For any $q < n$, the proposer's share in the proposal of any of SSPE is zero.*

**Proof:** Without loss of generality, relabel that member 1 is the proposer in the first round, and $p_i \geq p_{i+1}$ for $i = 1,\dots,n-1$. Suppose for the contradiction $p_1 < 0$. There could be at most $n-q$ members who vote against the proposal. Define $M(p)$ as a set of members who vote against the proposal. If $M(p)$ is nonempty, consider an alternative proposal $p'$ that subtracting $p_1$ from $p$ and adding $p_n$ to one randomly selected member in $M(p)$. $p'$ would make the proposer better off, while the members who vote for the proposal are not affected, because the continuation value under a stationary proposal $p'$ is identical to that under $p$, that is,

$$\delta\sum_{i=1}^{n}\frac{1}{n}p_i=-\frac{\delta}{n}=\delta\frac{1}{n}\sum_{i=1}^{n}\frac{1}{n}p'_i.$$

Therefore, the proposer has an incentive to deviate the equilibrium proposal, which contradict the supposition of subgame perfection. □

Next, for any stationary strategies, the continuation value is $-\frac{\delta}{n}$. Suppose that a proposer in the current round offers $(p_1,\dots,p_n)$. For any player $i$, the expected payoff of moving on to the next round is:

$$\delta\left(\frac{1}{n}p_1+\cdots+\frac{1}{n}p_n\right)=\frac{\delta}{n}\sum_{i=1}^{n}p_i=-\frac{\delta}{n}.$$

Therefore, players offered a share more than $-\frac{\delta}{n}$ are willing to accept the current proposal. Since the proposer, who keeps zero (Lemma 1), wants her proposal to be approved, must offer more than $-\frac{\delta}{n}$ to $q-1$ players. The allocation of the remaining losses, $-1-\sum_{j\in MWC}p_j$ must be allocated to the other members who are not included as a minimum winning coalition. □

**Proof of Corollary 1:** As long as players use stationary strategy, the continuation value is $-\frac{\delta}{n}$. If the proposer offers $-\frac{\delta}{n}$ to every player, then $-1+\frac{(n-1)\delta}{n}$ is the remaining loss that she would take. If $\delta \geq \frac{n}{n-1}$, then $-1+\frac{(n-1)\delta}{n} > 0$. That is, the proposer still has a room to keep zero for herself, and allocate the losses unevenly to other players as long as what other players are offered is greater than or equal to $-\frac{\delta}{n}$. If $\delta < \frac{n}{n-1}$, however, the proposer must keep $\frac{(n-1)\delta}{n} - 1$ for herself and offer $-\frac{\delta}{n}$ to other members.

# B Appendix - Experimental Instructions (M5 )

Welcome to this experiment. Please read these instructions carefully. The cash payment you will receive at the end of the experiment will depend on the decisions you make as well as the decisions other participants make. The currency in this experiment is called "tokens."

**Overview**

The experiment consists of 15 "Days." In each Day, every participant will be endowed with 400 tokens, and you will be randomly matched with four other participants to form a group of five. The five group members need to decide how to split a DEDUCTION of (at least) 250 tokens from group members' endowments.

**How the groups are formed**

In each Day, all participants will be randomly assigned to groups of five members. Each member of a group is assigned an ID number (from 1 to 5), which will be displayed on the top of the screen. In a given Day, once your group is formed, the five group members will not change. Your ID is fixed throughout the Day.

Once the Day is over, you will be randomly re-assigned to a new group of five, and you will be assigned a new ID. Check your ID number when making your decisions.

You will not learn the identity of the participants you are matched with, nor will those participants learn your identity. Identities remain anonymous even after the end of the experiment.

**How a deduction of tokens is divided**

In each Day, you and your group members will decide how to split a deduction of (at least) 250 tokens across group members. Each Day may consist of several 'Meetings.'

In Meeting 1, one of the five members in your group will be randomly chosen to make a proposal to split the deduction of 250 tokens as follows.

| | Member 1 | Member 2 | Member 3 | Member 4 | Member 5 |
|---|---|---|---|---|---|
| # of Tokens Deducted: | ____________ | ____________ | ____________ | ____________ | ____________ |

The number of tokens deducted from each member must be between 0 and 250. The total number of tokens must add up to 250 tokens.

Each member has the same chance of being chosen to be the proposer. After the proposer has made his/her proposal, the proposal will be voted up or down by all members of the group. Each member, including the proposer, has one and only one vote.

- If the proposal gets three or more votes, it is approved. The tokens allocated to you are DEDUCTED from your endowment and then the day ends.
- Otherwise, the proposal is rejected and your group moves to Meeting 2.

In Meeting 2, one member will be randomly selected to be a proposer. Every member, including the proposer in Meeting 1, has an equal chance to be a proposer. <u>The total amount of tokens to be deducted will increase by 20% of that in the previous Meeting.</u> That is, the five members in Meeting

2 need to decide how to split a deduction of 300 tokens. After the proposer proposes how to split the deduction of 300 tokens, it will be voted up or down by all members of the group. If this new proposal is rejected in Meeting 2, then in Meeting 3, another randomly selected member proposes to how to split a deduction of 360 tokens (20% more of 300 tokens), and so on. Your group will repeat the process until a proposal is approved. The following table shows the size of the deduction of tokens for each meeting.

| Meeting | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ··· |
|---|---|---|---|---|---|---|---|---|
| Deduction (in Tokens) | 250 | 300 | 360 | 431 | 518 | 622 | 746 | 20% Larger |

The amount of tokens you need to deduct is growing

To summarize, if you are selected as a proposer, make a proposal of splitting the deduction of the current number of tokens, and move to the voting stage. If you are not a proposer, wait until the proposer makes a proposal, examine it and decide whether to accept or reject it. Previous proposers can be a proposer again. If a proposal is approved, the number of tokens offered to you will be DEDUCTED from your endowment.

**Information Feedback**
At the end of each Meeting, you will be provided with a summary of what happened in the Meeting, including the proposed split of the deduction, the proposer's ID, and the voting outcome. At the end of each Day, you will learn the approved proposal and your earning from the Day.

**Payment**
In each Day, your earning is

[400 tokens − the number of tokens offered to you in the approved proposal]

The server computer will randomly select one Day and your earning in that Day will be paid. Each day has an equal chance to be selected for the final cash payment. So it is in your best interest to take each Day equally seriously. Your total cash payment at the end of the experiment will be the number of tokens you earn in the selected Day converted into HKD at the exchange rate of 2 tokens = 1 HKD plus 30 HKD guaranteed show-up fee.

**Summary of the process**

1. The experiment will consist of 15 Days. There may be several Meetings in each Day.
2. Prior to each Day, every participant is endowed with 400 tokens and will be randomly matched with four other participants to form a group of five. Each member of the group is assigned an ID number.
3. At the beginning of each Day, one member of the group will be randomly selected to propose how to split a deduction of (at least) 250 tokens.

4. If three or more members in the group accept the proposal, the proposal is approved, and tokens offered to you will be DEDUCTED from your endowment.
5. If the proposal is rejected, then the group proceeds to the next Meeting of the Day and a proposer will be randomly selected.
6. The volume of the tokens that need to be deducted increases by 20% following each rejection of a proposal in a given Meeting.

Remember that tokens offered to you in the approved proposal are DEDUCTED, not added.

**Quiz and Practice Day**

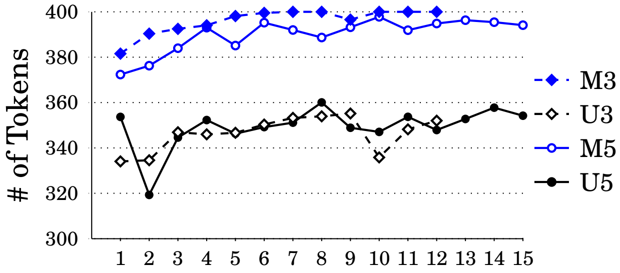
To ensure your understanding of the instructions, we will provide you with a quiz below. After the quiz, you will participate in a Practice Day. The Practice Day is part of the instructions and is not relevant to your cash payment. Its objective is to get you familiar with the computer interface and the flow of the decisions in each Meeting. Once the Practice Day is over, the computer will tell you when the official Days begin.

**Quiz**

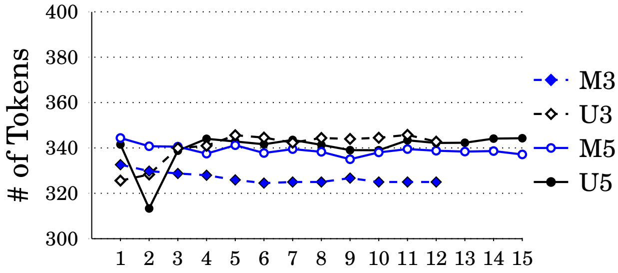
To ensure your understanding of the instructions, we ask that you complete a short quiz before we move on to the experiment. This quiz is only intended to check your understanding of the written instructions. It will not affect your earnings. We will discuss the answers after you work on the quiz.

# C Appendix - Supplementary Figures

Some figures placed in Appendix.




(a) Proposers




(b) Non-proposers

Figure 6: Average Earnings

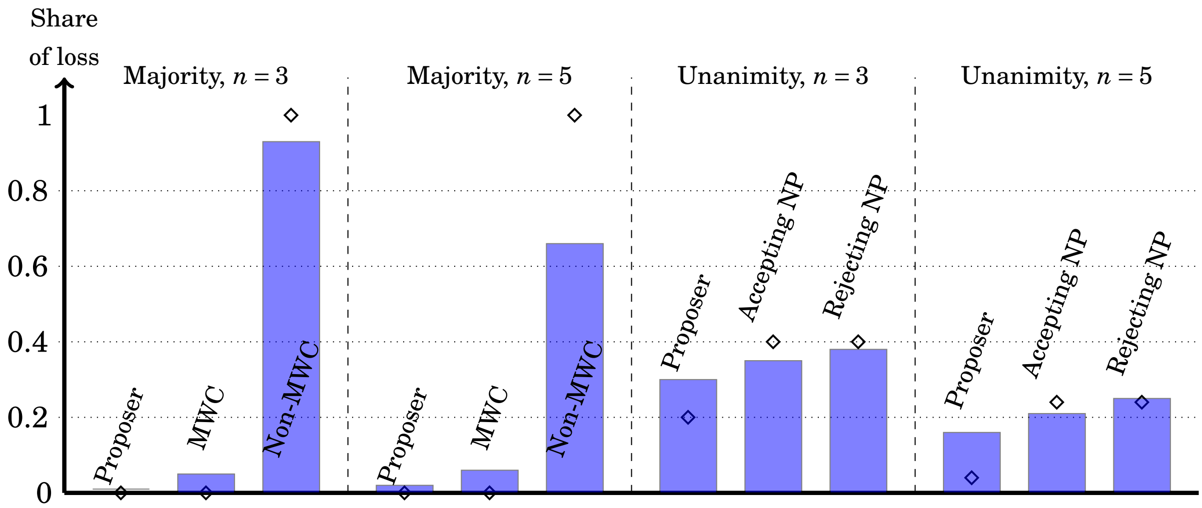
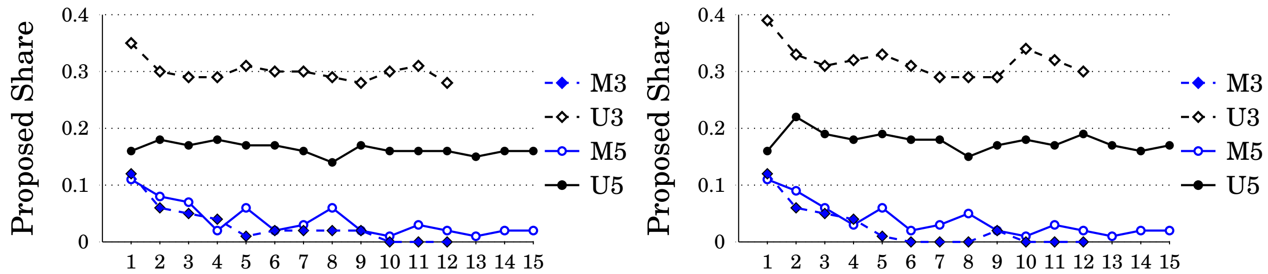


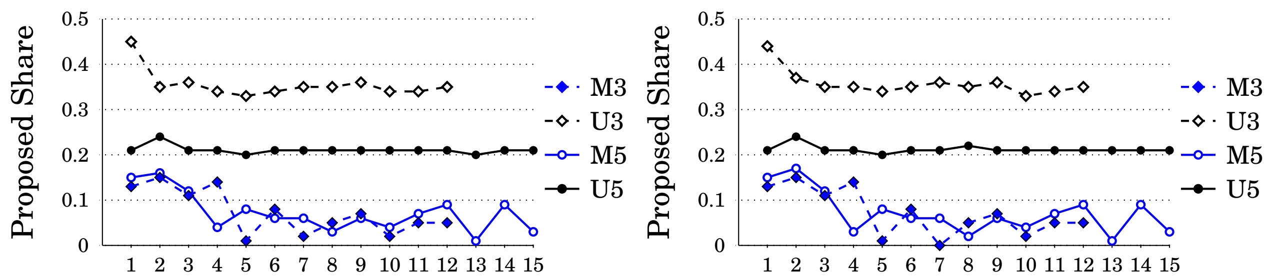

Figure 7: Proposed Shares
All (including rejected) proposals in the whole periods




(a) All Proposals

(b) Approved Proposals

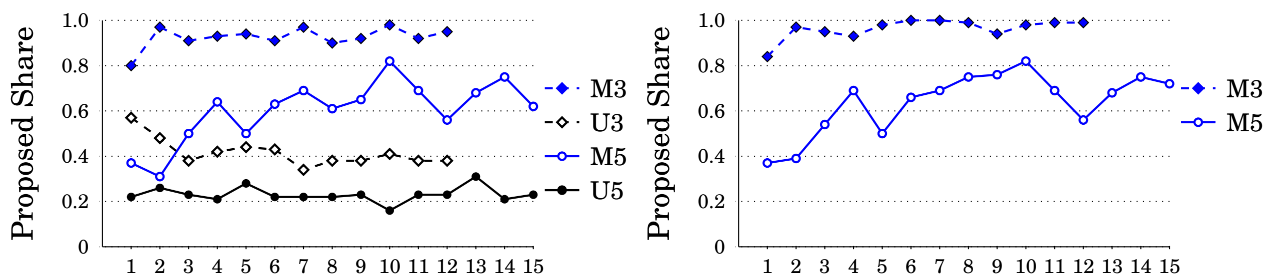
Figure 8: Average Proposed Share - Proposer




(a) All Proposals

(b) Approved Proposals

Figure 9: Average Proposed Share - Accepting Non-proposer




(a) All Proposals

(b) Approved Proposals

Figure 10: Average Proposed Share - Rejecting Non-proposer