Probability Matching and Strategic Decision Making*

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September 19, 2021

Abstract

This paper examines a link between an individual's (possibly limited) strategic thinking in the 11-20 money request game and (possibly non-rational) decision-making patterns in the matching pennies games. Experimental evidence shows that subjects' strategic behavior, which used to be understood as a result of finite cognitive iterations, is closely related to their choice randomization patterns. Ignoring some individuals' choice randomization may bias the population variance of levels in cognitive iterations. Choice randomizers (which we call probability matchers) are non-rational in both the non-strategic and strategic settings, but their choice patterns are systematic—thus, rational in their way—and similar. The relationship requires attention because the assumption that individuals are rational in the decision-theoretic sense may create a sizable misinterpretation of strategic behavior.
Keywords: Level-k reasoning, Probability matching, Cognitive bound, Preference for randomization

JEL: C72, C91, C92, D81

*The previous version of this paper was titled "Mixing Propensity and Strategic Decision Making." We thank Colin Camerer and Brian Rogers for their encouragement, PJ Healy, Youngdeok Hwang, Sang-Hyun Kim, Wooyoung Lim, Kirby Nielsen, Henrik Orzen, Jörg Oechssler, Ali Özkes, Sangwon Park, Keh-Kuan Sun, and the participants in several conferences and seminars for their comments, and Kiril Lavrov for his assistance. Hee Chun Kim greatly acknowledges the financial support from Economic Theory Center at Washington University in St. Louis and Korea Information Society Development Institute.

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1 Introduction

A growing number of studies in social science consider bounded rationality both in a non-strategic setting where a single player decides in an uncertain state and in a strategic setting where he/she responds to the other agents' unknown intentions and actions. When deciding on the non-strategic setting, individuals are often cognitively limited; they may not recognize or understand all the aspects that affect their payoffs or lack the cognitive ability to draw an ideal decision as much as they need. Observations from strategic environments also seem to be inconsistent with the theoretical predictions attained under the assumption of full rationality, not only because their rationality is bounded by the limited level of cognitive ability but also because their belief about other individuals' bounded rationality varies.

This paper's primary goal is to analyze how individuals' non-strategic—and possibly non-rational—decision-making patterns over probabilistic events are related to their strategic ones. This examination is important when inferring from observations in strategic situations. Individuals who (are able to) follow the same steps of reasoning could be coded as different levels since the way they respond to their belief is different—one type best responds, and the other follow probability matching patterns.

For an analysis of the (cognitively-limited) observations in strategic situations, the main body of the literature has implicitly assumed that "individuals are rational in the decision-theoretic sense of choosing strategies that are best responses to consistent beliefs" (Crawford, 2016), which hereinafter we call *decision-theoretic rationality*. In other words, if there is a stochastically dominant action for a player given his/her belief about the other player's limited reasoning, then the literature has assumed that he/she must have chosen the stochastically dominant one all the time.

Meanwhile, experimental work shows that when subjects are asked to make repetitive decisions under uncertainty, a significant fraction (varying from 20% to 50% by study) of subjects do not make a string of decisions that maximize their expected payoff. Instead, they tend to *match* their decision frequencies to the probability of events, called *probability matching* (Rubinstein, 2002; Neimark and Shuford, 1959). For example, if people are asked to play ten rounds of Matching Pennies games wherein each game, a coin, with a 70% probability of landing on heads, will be tossed independently, some of them choose heads for seven out of the ten rounds and tails for the other three rounds to match their relative choice frequencies with the probability of events. In contrast, they should have chosen heads for all the rounds to maximize the expected payoff, regardless of their risk preferences. Although investigating why some people have such a preference for randomizing their choices is worthwhile,[1] we want to clarify that the primary purpose of this study is not to rationalize the probability-matching behavior. Rather, we take their choice patterns from the non-strategic environment as given and investigate further whether and to what extent the existence of the probability-matching—broadly speaking, choice-randomizing—players affects the analysis of the cognitive bounds inferred from the observations in strategic situations. For this purpose, we consider probability matching as preferences rather than mistakes.

We claim that when we ignore some individuals' choice-randomization behaviors, as the vast majority of the literature does, it is challenging to correctly map their strategic actions to their underlying beliefs. Two leading theories formalizing bounded rationality in strategic decision making, namely the Level-k (Lk) model (Costa-Gomes et al., 2001; Costa-Gomes and Crawford, 2006) and the Cognitive Hierarchy (CH) model (Camerer et al., 2004), share an assumption that individuals use only finite (=k) steps of iterated reasoning, and such k varies by individual. To analyze experimental observations, previous studies relying on one of those two models implicitly share an assumption that every subject is equipped with *decision-theoretic rationality*. In other words, the main body of the literature has assumed that subjects may have a limited capacity of reasoning, but within their capacity, no subject exhibits any choice randomization behavior, including probability matching.[2]

It may create a considerable misinterpretation *if the choice randomization behavior in a non-strategic situation is maintained in a strategic situation*. This is our main hypothesis. Individuals who tend to exhibit probability matching in a strategic situation may show heterogeneous choice patterns, and they might be merely regarded as less sophisticated players due to their "noisier" choices.

Is choice randomization behavior in a non-strategic setting indeed related to the behavior in a strategic setting? How could the additional observations on choice randomization help elicit the population belief distribution in the level of reasoning? To address our questions, we conducted the two following sets of within-subjects laboratory experiments: a set of two repeated matching pennies games and a set of repeated 11-20 money request games (Arad and Rubinstein, 2012). In a nutshell, from the matching pennies experiment, we distinguish individuals randomizing choices from those who consistently make stochastically dominant choices. Then, we discern how the observations from the

[1]Many studies provide models of preferences for randomization and empirical support of them. For example, Dwenger et al. (2018) provide a theory of responsibility aversion, which implies a demand for randomization. Machina (1985) and Cerreia-Vioglio et al. (2020) consider convex preferences to account for the affinity towards randomization among equally preferred options.

[2]This claim should not be understood that the literature has ignored the decision noises. For example, Goeree et al. (2018) provide evidence that a model with common knowledge of noise predicts the behavior of the 11-20 game well.

11-20 experiment vary by choice pattern categorized in the matching pennies experiment. We also incentivized subjects to reveal their beliefs about choice distributions at the end of those two experiments.

Our observations are summarized as follows. First, in the matching pennies games, about 75% of the subjects were classified as a rational optimizer (RO), each of whom makes consistent choices of maximizing the expected payoff, and the remaining 25% were classified as a probability matcher (PM), each of whom exhibits a choice randomization tendency.[3] Second, in the 11-20 game, the choice variance of the PMs is larger than that of the ROs, suggesting that the probability matcher's tendency of choice randomization is also maintained in the strategic situation. Third, the average level of cognitive iterations of the PMs was not statistically different from that of the ROs, which suggests that the choice randomization in the 11-20 games is not due to the lower cognitive level of the PMs. Fourth, there is a strong correlation between a subject's choice randomization in the strategic setting and his/her belief about the aggregate choice distributions.

Altogether, we find that a subject's strategic behavior observed in the 11-20 games is closely related to the decision-making patterns in the matching pennies games. When we ignore the probability matchers, the estimated population distribution of the cognitive iteration levels in strategic situations might be more dispersed than it should be. In general, our evidence of probability matching in strategic situations, which contradicts the fundamental idea of the decision-theoretic rationality, would be relevant to any model of strategic thinking.

The rest of this paper is organized as follows. In the next subsection, we review related studies. Section 2 describes the details of the experimental design and procedure. Section 3 shows the results of the experiment and discusses its implications. Section 4 concludes.

1.1 Related Literature

This study is grounded in empirical and theoretical findings of bounded rationality in strategic behavior. Two leading behavioral models—the Lk model developed by Costa-Gomes and Crawford (2006) and the CH model developed by Camerer et al. (2004)—and the follow-ups share two assumptions: (1) individuals are rational in the decision-theoretic sense as they choose strategies that are the best responses to consistent beliefs; and (2) individuals play strategies of a finite level of iterated dominance. The models

[3]The observed non-rational choice patterns may not be entirely explained by probability matching. We acknowledge the possibilities of other potential explanations, so the "probability matcher" must be regarded as a person exhibiting a broadly-defined choice-randomization pattern, including probability matching.

differ in their assumptions about subjects' beliefs regarding the strategic behavior of other players. We tackle the first maintained assumption. In the sense that we try to better understand higher-order rationality, our goal is consistent with Kneeland (2015), who proposes a more explicit design of experiments to identify higher-order rationality. Rather than adopting Kneeland's ring games of many (more than three) players, we stick to the two-person guessing-style games.[4] Since our primary objective is to find relationships between the decision-making patterns in non-strategic environments (a player vs. random events) and choices in strategic environments (a player vs. another player), we design the two experiments to be as structurally similar as possible. The 11-20 money request game introduced by Arad and Rubinstein (2012) is an excellent tool for eliciting higher-order rationality. We conduct the 11-20 game as a part of our experiment because it is simple, less arguable on the assumption about the L0 behavior, and less arguable on the interpretation of level-k behavior.

We posit that individuals may show different responses to the same belief, and this difference in decision-theoretic rationality may lead to the apparent puzzle that mixes different strategies. Examples in the non-strategic settings abound. In Rubinstein (2002), about half of the undergraduate subjects diversified their choices, and some of them exactly matched their frequency of choices to the probability of events for repetitive decision-making tasks. Thaler (2016) reports a similar result among MBA students at a leading university. Although the contexts varied, the fundamental question that the authors asked the subjects to perform was the independent repetition of the matching pennies game described above. Likewise, many studies in psychology literature find a significant propensity for mixing different strategies. Neimark and Shuford (1959) and Vulkan (2000) also provide lab-experiment observations that support probability matching behavior. If we regard this mixing propensity as preferences for the randomization of choices, the experimental evidence expands. Agranov and Ortoleva (2017) find that a vast majority of experiment participants exhibit stochastic choice when asked to answer the same questions several times in a row. Dwenger et al. (2018) report that German university applications exhibit a choice pattern consistent with a preference for randomness. If a similar probability matching behavior can also occur in strategic situations, then the underlying belief structure about the other players' cognition levels could be better revealed by the mixing strategies of different levels.

Although choice randomization has been well documented in the literature, few ex-

[4] To provide a clear identification strategy, Kneeland (2015) assumes the exclusion restriction (ER) that people do not randomize their decisions unless some decisions are indifferent, which is essentially the decision-theoretic rationality. Lim and Xiong (2016) provide evidence that the ER assumption is violated in a number of games. Jin (2020) points out that a simultaneous ring game alone is not sufficient to tell whether the observed level-k behavior is determined by belief or reasoning ability.

perimental studies have explicitly considered these behavioral patterns in the optimization process for identifying underlying belief structures in the strategic decision-making environment. Georganas et al. (2015) examine whether individuals show similar levels of iterated dominance in different forms of the game. Georganas et al. (2015) attempt to find consistency in the strategic process in different environments but do not examine the various processes regarding individual optimization patterns, which is our primary focus.[5] In a similar vein, Agranov et al. (2020) examine whether a subject's type regarding randomization behaviors is stable across different choice environments and find that randomization preferences are highly correlated across domains. Although Agranov et al. (2020) also compare the choice patterns in individual choice questions with those in games as we do, their main goal is to examine whether the individual heterogeneity is indicative of a stable distribution of types, while ours is to find the implications of the relation between the choice randomization and strategic behaviors.

2 Experimental Design and Procedure

2.1 Design

We used a within-subject design. Each subject participated in two different games and a follow-up belief elicitation. In the Matching Pennies games, the subjects made a streak of decisions in which payoffs depend on realized (but unknown) events. In the 11-20 Token Request games, the subjects made a streak of decisions in which payoffs depend on the randomly matched subject's decisions. Afterward, they were asked to guess the distributions of entire choices made in the session, and the ones with the closest guess earned additional payoffs.

In the matching pennies games, the subjects make a total of eight choices to earn points from two single-player games. The payoff matrix in Table 1a describes the first game. A subject's options, U and D, are in the left column. A probability distribution (H, 3/4; T, 1/4) is on the top: Event H is realized with a 3/4 chance, and event T with a 1/4 chance.[6] When a subject chooses U and event H is realized, the subject earns one point. Each point earned by the subject in the matching pennies games was converted into 80 cents. A new event is independently realized before making each decision. The subjects make decisions without knowing the realized events. After making four independent

[5] The results of Georganas et al. (2015) are not contradictory to ours because they examine the consistency of strategic behaviors across different strategic tasks, but we examine the consistency between non-strategic decisions and strategic actions.

[6] We instructed what we mean by a probability distribution and how an event is independently drawn from the probability distribution in plain words.

decisions without feedback, the subjects play the second game described in Table 1b. In this game, the subject's options are U, M, and D, and the event will be L with a probability of 1/2, C with a probability of 1/4, and R with a probability of 1/4. Based on the subjects' choice patterns from the two different games, we categorize them into the two following types: the rational optimizer (RO) who chooses U (the stochastically dominant option) consistently and the probability matcher (PM) who randomizes choices close to the probability distribution.

	H, 3/4	T, 1/4
U	1	0
D	0	1

(a) Matching Pennies: Game 1

	L, 1/2	C, 1/4	R, 1/4
U	1	0	0
M	0	1	0
D	0	0	1

(b) Matching Pennies: Game 2

Table 1: Matching Pennies Games

Table 2 shows how a player with a particular type would choose actions. When a player is expected to choose an action A$\in$ {U,M,D} for n times, it is denoted by An. An RO will always play U, the choice that provides the largest expected payoff. We denote the RO's play by U4. A PM, given that he/she exactly follows a probability matching strategy, will match the frequency of his/her choices with the probability of events. Thus, in Game 1, a subject with perfect probability matching strategies will mix three Us and one D, and in Game 2, she will mix two Us, one M, and one D up to permutation. Similarly, such a play from the PM is denoted by U3D1 in Game 1 and U2M1D1 in Game 2 respectively. Considering that the mode of the choices is U, we check whether those who are classified as PMs (that is, those who randomize their choices) still choose U most frequently.

Matching Pennies	Game 1	Game 2
Rational Optimizer	U4	U4
Probability Matcher	U3D1*	U2M1D1*

$*$: up to permutation

Table 2: Predicted Behaviors of the Two Types in Matching Pennies Games

In the 11-20 games (Arad and Rubinstein, 2012), the subjects make a total of eight decisions, knowing that they are randomly matched with an anonymous participant for the first four decisions and another match for the last four decisions. In each decision round, each subject chooses one of the integers $r \in \{11,\dots,20\}$. The subject's payoff is $r+20$ tokens if the choice of the match in that round is $r+1$, and r otherwise. In other words, if the subject believes that his/her match would choose, for example, 19, then the best response is to choose 18 so that the payoff can be 38 (=18+20).[7] One of the eight rounds was randomly selected for payment.[8] The tokens earned in the selected round were converted into euros at the rate of one token to 30 cents. For counter-balancing, the 11-20 games were conducted before the matching pennies games for 24 subjects in two sessions.

The only but crucial difference from Arad and Rubinstein (2012) in the experimental design is that we ask the subjects to make repetitive decisions without feedback. Unless a subject believes that the match would play the mixed strategy of the unique equilibrium, an RO has little reason for randomizing the eight decisions. However, a PM may randomize the decisions if his/her has a non-degenerate belief about the match's strategy. If the 11-20 game were to be conducted once, the one observation might not help us to recover the subject's cognitive ability. By observing repetitive decisions, we intend to elicit differences in responses according to subjects' type.

After making a total of 16 decisions (four for the first matching pennies game, four for the second matching pennies game, and eight for the 11-20 game), the subjects were incentivized to correctly guess the aggregate choice distributions. For example, with 15 participants in a session, there are 60 choices from the first matching pennies games in aggregate. The subject whose guess (about how many of 60 choices were U and how many were D) is closest to the actual choice distribution won four extra euros.[9] Similarly, they were asked to guess the choice distributions for the second matching pennies games and the 11-20 games.[10]

2.2 Procedure

Seven sessions of laboratory experiments were conducted with a total of 106 participants at the Mannheim Laboratory for Experimental Economics (mLab) in fall 2019.The participants were drawn from the mLab subject pool. Python and its application Pygame

[7]The unique mixed-strategy Nash equilibrium is to mix 15, 16, 17, 18, 19, and 20 with probabilities 0.25, 0.25, 0.20, 0.15, 0.10, and 0.05 respectively.

[8]Azrieli et al. (2018) show that the random payment mechanism is incentive compatible in a very general environment, but they also show that the pay-all mechanism is incentive compatible as well in a slightly more restricted environment. One advantage of paying for all rounds is to minimize unnecessary effects of risk preferences and to induce risk-neutral behavior (Walker et al., 1990), so we pay all rounds in the matching pennies games.

[9]The subjects were told that the prize would be shared in case of multiple winners, but it did not happen.

[10]The idea of comparing the guesses with actual decisions is similar to Costa-Gomes and Weizsäcker (2008), who asked subjects to state their belief in their partners' actions before or after playing a 3x3 normal-form game to test whether subjects' strategic play is the best response to their stated beliefs. In Costa-Gomes and Weizsäcker (2008), whether the elicitation procedure is before or after the actual play of actions did not show observational differences. Thus, we included the belief elicitation procedure after the actual play of actions.

were used to computerize the games and to establish a server–client platform. After the subjects were randomly assigned to separate desks equipped with a computer interface, they were asked to carefully read the instructions before taking a quiz to prove their understanding of the experiment. Except for mentioning that there are three different tasks, the instructions cover the upcoming task only. Those who failed the quiz were asked to re-read the instructions and retake the quiz until they passed. An instructor answered all questions until every participant thoroughly understood the experiment. In the 11-20 games, although new pairs were formed at the end of the fourth round, there was no physical reallocation of the subjects, and they only knew that they were randomly shuffled. They were not allowed to communicate with other participants during the experiment or look around the room. It was also emphasized to participants that their allocation decisions would be anonymous. Payments (10.63 euros on average) were made in private, and the subjects were asked not to share payment information. Each session was performed in less than an hour.

2.3 Hypotheses

Our main hypothesis is that in the 11-20 game, a probability matcher behaves "as if" he/she faces a non-strategic decision-making situation under uncertainty like the matching pennies game. For better illustration, consider a cognitive hierarchy level-3 player who exhibits probability-matching tendencies in the 11-20 games. Suppose further that player 1 believes that 75% of the population choose 19, and the other 25% choose 18.

If she behaves like a rational optimizer who best responds to her underlying belief about the match's actions in a rational manner, she should choose 18 for the entire eight rounds. In general, her choice should be a singleton regardless of the shape of his/her beliefs about others' cognitive levels.[11] On the other hand, if a probability matcher's decisions would "match" her belief distribution, we should obtain different observations from rational optimizers in the 11-20 game. More specifically, while the rational optimizers' choices would have little variances, the probability matchers' choices should reveal their underlying belief distribution, leading to more choice variations. Therefore, the variance of observed choices of the probability matchers should be larger than that of the rational optimizers.

[11] Since the 11-20 game's choice set is discrete, any mixed strategy assigns a probability to each choice option, $(p_{11}, p_{12}, \ldots, p_{20})$, where p_n is the probability of choosing n. The stochastically dominant best response to such a mixed strategy is to choose an action $k = \max\{\arg\max_{n=11,\ldots,20} p_n\} - 1$ consistently. Thus, if the players were to respond to a mixed strategy rationally, their decisions for all eight rounds should have been the same regardless of how they believed the matched player's level of cognitive iterations and his/her strategies. Note that a rational optimizer would best respond his/her underlying belief about the match's actions. Hence his/her decision should also be a singleton in the 11-20 game. Therefore, the variance of observed choices should be similar across the choice randomization types.

Hypothesis 1. *The PMs' choices in the 11-20 games vary more than the ROs' choices.*

The second hypothesis concerns the way she responds to her belief. If she merely matches the relative choice frequencies with her belief, we would observe her choosing 19 for six out of the eight rounds and 18 for the other two rounds. If she *responds* as if she faces players who choose 19 for six out of the eight rounds and players who choose 18 for the other two rounds, then she would choose 18 (which is the best response to 19) for six times and 17 (best response to 18) for two times. Although her decisions are not consistent with the best response to her belief under the decision-theoretic rationality, she does respond to her belief in a probability-matching sense. When we describe these non-rational but systematic responses, we would say that PMs "reflect" their beliefs.

Hypothesis 2. *PMs respond to their beliefs in a probability-matching manner.*

3 Experimental Findings

To examine possible effects from the order of two games, we compared the observations from the 24 subjects who first played the 11-20 games with others (81 subjects) who played the matching pennies games first. We ensure that the order of the two games did not have a meaningful impact. Two-sample Kolmogorov-Smirnov (KS) test on each of the four batches of observations—the first matching pennies games, the second matching pennies games, the first half of the 11-20 games, and the second half of the 11-20 games—does not reject the null hypothesis that two data samples come from the same distribution (combined KS p-values are 1.000, 1.000, 0.840, and 0.647 respectively). From now on, we combine two data sets for summarizing our four findings.

To recapitulate our analysis plan, we first classify subjects as ROs or PMs from the matching pennies games. Then, we examine how the classification accounts for differences in the 11-20 game. To reiterate the limit and the scope of this study, although there could be more sophisticated ways of classifying non-rational choice patterns, and it is worth investigating the reasoning of choice randomization other than probability matching, we focus on the relationships between choice randomization behavior in the non-strategic environment and the observed level of cognitive iterations in the strategic environment. Thus, the term "probability matcher" should not be misinterpreted as a person whose choice pattern is solely explained by probability matching.

In the matching pennies games, approximately 75% of the subjects (80 out of 106) were classified as ROs, each of whom consistently chose the stochastically dominant choice, U, in all decision rounds. A quarter of the subjects were classified as PMs, each of whom exhibits a probability-matching tendency. Among PMs, 58% and 42% of the subjects made decisions precisely identical to the predicted behavior of Game 1 and Game 2 in Table 2 respectively. For some PMs whose choices were not entirely consistent with the predicted behavior, their decisions cannot be regarded as evidence of pure random choices because no subjects chose stochastically dominated choices more. For example, in both matching pennies games, every PM's mode choice was U. Table 3 summarizes the aggregated relative choice frequencies.

	U	D
All (100.0%)	96%	4%
RO (75.5%)	100%	0%
PM (24.5%)	86%	14%

(a) Matching Pennies: Game 1

	U	M	D
All(100.0%)	93%	6%	1%
RO (75.5%)	100%	0%	0%
PM (24.5%)	64%	22%	13%

(b) Matching Pennies: Game 2

Table 3: Relative Choice Frequencies

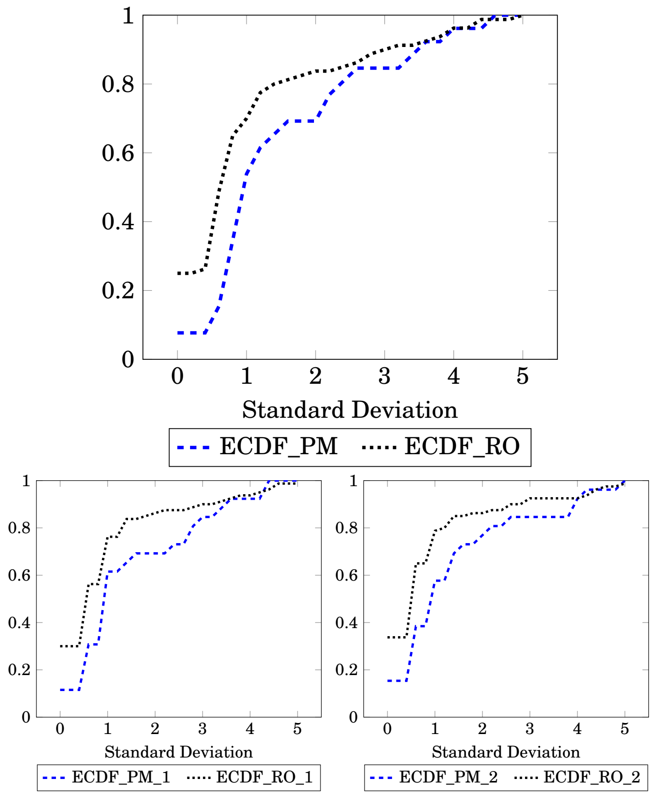
Observation 1. *A quarter of the subjects were classified as Probability Matchers.*

Second, in the 11-20 games, the choice variance of the PMs was significantly larger than that of the ROs. We claim this is our main finding which confirms Hypothesis 1. Figure 1 shows the empirical cumulative distribution functions (CDFs) of the standard deviations of individual choices in the 11-20 games, classified by type. The PMs diversify their choices more than the ROs; the two-sample KS test rejects the equality of two distributions (Combined KS=0.3558, p-value=0.014), and the one-sided t-test rejects the equality of two means (Δ = 0.4679, p-value=0.0432). A quarter of the ROs made the eight decisions constant, and 70% of the ROs had decisions with a standard deviation less than 1. These patterns are unchanged when we draw similar empirical CDFs for the choices with the first match (first four rounds) or the second match (last four rounds) only. The two sub-figures below Figure 1 show the empirical CDFs in the first match of the 11-20 games and those in the second match. The observations from the first four rounds are not statistically different from those from the last four rounds.[12] Relating to the choices in the matching pennies games, this finding suggests that the tendency of choice randomization is also maintained in strategic situations.

Observation 2. *The choices of the PMs in the 11-20 games varied more.*

[12]For example, the average level of the ROs in the first four rounds is 2.5750 is not statistically different from the average in the last four, 2.4094 (p-value of t-test=0.1656). The standard deviation of the ROs' levels in the first four is 2.4801, whose difference is not statistically different from that in the last four, 2.3431 (p-value of F-test=0.3106).

Figure 1: CDFs of the Standard Deviations of Individual Choices in the 11-20 Games



Third, neither the behavior of the ROs nor that of the PMs can be explained by Nash equilibrium. Our finding confirms the reports of Arad and Rubinstein (2012).[13] Table 4 shows the unique symmetric Nash equilibrium distribution (assuming that players maximize the expected monetary payoff) and the actual choice distributions. The choice distribution is significantly different from the Nash equilibrium (χ^2 goodness of fit test, p-value<0.0001). Only 7% of the whole actions were 15 and 16, which is significantly smaller than 50%, the mixing probability in equilibrium. The vast majority of the actions were 17, 18, and 19, and those correspond to Level-3, -2, and -1 cognitive iterations respectively.

Following Arad and Rubinstein (2012), we code $20-s$ as the level of cognitive iterations, where s is the choice of the 11-20 game. It is established upon two assumptions. First, the L0 players would choose 20. When Arad and Rubinstein (2012) introduce the 11-20 game, they emphasize that L0 behavior in this game is less arguable than the two-person guessing game and its variations: if someone does not consider others' strategic decisions, he/she will choose to acquire the largest payoff. Second, we assume that the subjects' cognitive capacity is sufficient enough to carry out the cognitive iterations. It

[13]The baseline experiment of Lindner and Sutter (2013), which replicates Arad and Rubinstein (2012), has similar results. Thus, our observed distribution in the 11-20 game is consistent with that in the literature.

Action	11	12	13	14	15	16	17	18	19	20
Equilibrium (%)					25	25	20	15	10	5
All (%)	7	1	1	2	3	4	15	27	26	13
RO (%)	8	1	1	1	3	1	13	30	26	13
PM (%)	6	2	2	4	4	7	20	19	25	11

Table 4: Equilibrium and Actual Distributions of the 11-20 Games

has been argued whether the observed behavior is mainly due to their cognitive capacity (e.g., "I choose 18 because I am unable to carry further cognitive iterations than twice.") or belief (e.g., "I choose 18 because my match would choose 19."). Arad and Rubinstein (2012) claim that the 11-20 game, thanks to its simplicity, would be less vulnerable to the issue of cognitive capacity.[14]

The average level of cognitive iterations of the PMs (2.8125) was not statistically different from that (2.4922) of the ROs (t=1.6684, p-value=0.0956). Rather, the PMs' average level is slightly higher than that of the ROs. This finding suggests that the choice randomization observed in the 11-20 games may not be due to the less cognitive ability of the PMs.

Observation 3. *Neither ROs nor PMs played the Nash equilibrium in the 11-20 game.*

So far, we report that PMs make more varied choices in the 11-20 games than the ROs. Are the PMs' varied choices in the 11-20 games due to their belief distribution being more dispersed than that of the ROs or due to their different response to the same belief of the ROs? To answer this question, we examine a difference in the guesses by type.[15]

We find a positive correlation between a subject's choice randomization behavior and his/her belief about the actual choice distributions. That is, a PM believes the choice distributions are dispersed more than what an RO believes. This finding suggests that PMs' choice randomization is not due to the different responses to the same belief of ROs.

Table 5 shows how subjects guessed the actual choice distributions. In the matching pennies games, the ROs' guess was distinctively closer to the actual distributions, although they consistently chose U. This observation suggests that the ROs have a

[14]Another implicit assumption is that the level of reasoning does not exceed 10. Otherwise, playing 20 could mean the player is level 0 or level $10n$, $n \in \mathbb{N}_+$. We believe this assumption is innocuous as the literature has not reported such high levels of reasoning, to the best of our knowledge.

[15]Rubinstein and Salant (2016) report the possibility that the ex-post elicited belief is affected by what subjects actually did. Although we incentivize the subjects to guess the choice distribution correctly, we admit that the ex-post belief elicitation could be affected by their will to justify their previous decisions. With considering the potential limitations of the ex-post belief elicitation, the analyses on the beliefs should be interpreted with caveats.

	U	D
Actual (%)	96.0	4.0
RO (%)	93.5	6.5
PM (%)	80.2	19.8

(a) Matching Pennies: Game 1

	U	M	D
Actual (%)	93.0	6.0	1.0
RO (%)	89.1	5.7	5.2
PM (%)	64.4	19.9	16.8

(b) Matching Pennies: Game 2

	11	12	13	14	15	16	17	18	19	20
Actual (%)	7	1	1	2	3	4	15	27	26	13
RO (%)	10	2	2	2	3	4	9	16	25	27
PM (%)	13	3	2	5	7	6	12	18	19	16

(c) 11-20 Games

Table 5: Relative Choice Frequencies and Guesses by Type

good sense that a small fraction of the whole subjects would exhibit a sort of choice-randomization tendency. Meanwhile, the PMs seem to believe that the entire subjects would behave like themselves. Their guesses are close to their actual choice distributions in Table 1. For example, in the second matching pennies games, 64%, 22%, and 13% of the PMs' choices were U, M, and D, respectively, and they guessed the actual distributions would be around 64%, 20%, and 17% respectively. Analogous to the aggregate relationship, the individual level's correlation between a PM's choice frequencies of U and his/her belief of U is 0.5248, which is statistically significant (p-value<0.0001).

In the 11-20 games, the ROs' guess was closer to the actual distribution, although they overly weighted the L0 behavior. The average level of cognitive iterations based on their guesses is 2.47, which exactly corresponds to their actual average level of cognitive iterations, 2.49. Besides, 69% of the ROs' choices were 17, 18, and 19, and 68% of their guesses were 18, 19, and 20. Therefore, ROs were, albeit not perfectly, best-responding to their beliefs. Combined with the point that the ROs choices are not varied much, we can roughly summarize that the ROs who believe that the others would choose 18(, 19, and 20, respectively) best respond by choosing 17(, 18, and 19) constantly.

The PMs, who randomized their decisions more than the ROs, guessed that the actual choice distribution would be more dispersed than what they chose. Similar to what we observe for the ROs, PMs diversified their choices in the 11-20 games to "reflect" what they believe. 71% of the PMs' choices were between 16 and 19, and 65% of their guesses were between 17 and 20. Combined with the point that the PMs' choices are varied much, we can roughly summarize that the PMs who believe that others would choose one in the range of 17 and 20 best respond to each of the possible realizations, leading to their varies choices in the range of 16 and 19.

We want to clarify that PMs "reflecting" their beliefs do not mean PMs become rational in the decision-theoretic sense. PMs do not best respond to their beliefs in a rational manner, but they respond to their beliefs in a probability-matching manner. It is worth reiterating that no matter how one person forms a dispersed belief about other's choices, the best response to that belief must be a *singleton*. Thus, PMs' varied choices are not the result of the best response to their beliefs. By reflecting beliefs, we mean that PMs best respond to *each of the events* that could be realized in their beliefs.

We further examine how much each individual reflects their guesses to choices. We develop a standardized measure capturing the degree of discrepancies between an individual's choices and guesses. The discrepancy measure is the sum of the squared differences between the individual-level belief distribution induced by the individual's choices and the self-reported belief distribution. To be more illustrative, we took subject 1 in our first session as an example. In the 11-20 games, her choices in rounds one to eight were (18, 18, 18, 18, 17, 17, 17, 17). When asked to guess the choice frequencies over 11 to 20 for the whole 160 choices in a 20-subject session, her guesses were (0, 0, 0, 0, 0, 0, 40, 90, 20, 10). Her discrepancy measure is calculated in the following way. First, we standardize the ratio of the guesses induced by an individual's choices. Due to the structure of the 11-20 game, her choices correspond to her guesses. For example, the choice of 18 corresponds to the guess of 19 since 18 is the best response to the match's action of 19. Then, the induced proportion of the guess of 19 is 0.5, which is four (a choice frequency of 18) divided by eight (the number of choices in the 11-20 game). Thus, her standardized belief distribution induced by her choices is (0, 0, 0, 0, 0, 0, 0, 0.5, 0.5, 0), which matches the proportion of each level of her choices. Second, we standardize the ratio of the self-reported guesses. Her standardized self-reported distribution of guesses was (0, 0, 0, 0, 0, 0, 0.25, 0.5625, 0.125, 0.0625) which also matches the proportion of each level of her guesses. For example, her proportional distribution for the guess of "20" is 10/160 = 0.0625. From these two standardized distributions, we calculate the measure of the individual discrepancy as the sum of the squared differences between the two distributions for each level, which is $6(0-0)^2+(0-0.25)^2+(0.5-0.5625)^2+(0.5-0.125)^2+(0-0.0625)^2 = 0.2109$. We can calculate the discrepancy measure for every subject in the same way.

Please note that the discrepancy measure does not involve any type-dependent information since it uses each individual's guesses and choices. After we have such (individuals') discrepancy measures, we examine whether there are differences in their type from the matching pennies games. Also, note that regardless of their type, if the subjects' beliefs are precisely matched to their choices and if the distribution of their reported guesses and that inferred by their choices are the same, then the measure of discrepancies will be zero.[16] In other words, as their distribution of guesses is closer to the one-level shift of their choice distribution, the measure of discrepancies will be closer to zero. Figure 2 shows the box and whisker plots of the individual discrepancies by type.

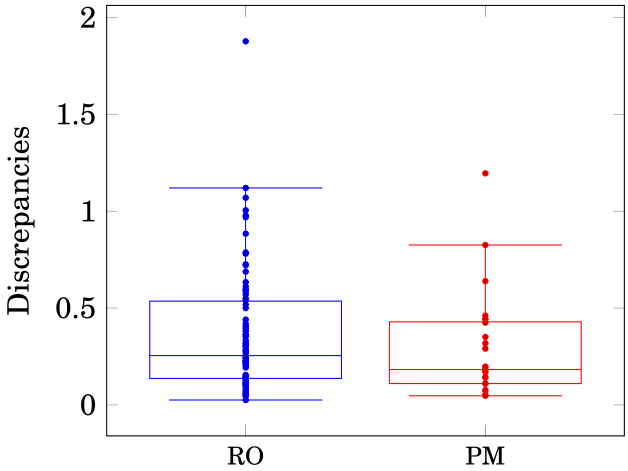


Figure 2: Box Plots of Individual Discrepancies by Type

Figure 2 shows that the two distributions by type seem to have no significant differences. The mean discrepancy of the ROs is 0.3647, and that of the PMs is 0.2885. We can neither reject the null hypothesis that two distributions have the same mean in discrepancy (t-test, $p = 0.276$) nor reject the null hypothesis that two distributions are from the same population distribution (combined KS p-value=0.097.) Along with the points that the PMs' choices and guesses are more dispersed than those of the ROs, this result confirms that both the ROs and the PMs reflect their beliefs to a similar degree *in their way*. Note that those discrepancies do not merely reflect the noises or mistakes in the guesses. If so, we must have observed more discrepancies from the ROs because their actions vary less. The following summarizes our findings.

Observation 4. *Choices of both ROs and PMs in the 11-20 game "reflected" their beliefs.*

[16]We illustrate two hypothetical cases with zero discrepancies, inspired by two actual subjects. First, suppose that Subject 18 is PM and has guessed her match's choices over 11 to 20 as follows: (0, 0, 0, 0, 0, 0, 0, 0.5, 0.25, 0.25). If Subject 2 perfectly "reflects" her belief in her choices as a PM type, then her choices would be four 17s (best response to 18), two 18s (best response to 19), and two 19s (best response to 20) up to permutation. In that case, her measure of discrepancy would be 0 ($= 7(0-0)^2 + (0.5-0.5)^2 + 4(0.25-0.25)^2$). Second, suppose that Subject 3 is RO and has guessed her match's choices as follows: (0, 0, 0, 0, 0, 0, 0, 0, 1, 0). It means that Subject 3 believes that all her matches will choose 19 and perfectly fits an RO-type belief. If Subject 3 perfectly reflects her belief in choices as an RO type, her choices will consistently be 18. In such a case, the measure of discrepancy for Subject 3 will be 0 ($= 9(0-0)^2 + (1-1)^2$) as well. In these two examples, the discrepancy measures are calculated in the same manner, but those become zero for different reasons.

4 Concluding Remarks

This paper examines how an individual's (possibly non-rational but systematic) choice patterns are related to his/her strategic decision-making patterns. We consider that each individual who faces a probabilistic event has a different way of making decisions. We categorize the subjects into two types: the rational optimizer (RO) and the probability matcher (PM) by the observed choices in the matching pennies games. We found that a quarter of the subjects show choice patterns other than rational optimization. Our main observation is that when asked to make strategic decisions in the 11-20 games, choice patterns differed by type. In particular, PMs diversified their actions to multiple levels of cognitive iterations in the 11-20 games in a similar way of diversifying their decisions in the matching pennies games.

The relationship between the decision-making patterns in the matching pennies games and the 11-20 games suggests that the literature, which has assumed the decision-theoretic rationality, may have underestimated the level of cognitive iterations and overestimated the variance of the population distributions of it. If every subject were to have a non-degenerate belief distribution, the level of cognitive iterations estimated through the lens of the Lk theory could be downward biased.[17] On the other hand, if every subject were the RO, the belief distribution estimated by the CH model would underestimate the variance of the distribution because repetitive decisions from some ROs with a non-degenerate belief distribution will be observationally equivalent to those from ROs with a degenerate belief.

The main implication of our findings is twofold. First, an individual's decisions under probabilistic events can deviate from the rational-theoretic predictions, but such deviations can be predictable. Second, we can improve recovering the underlying belief of decision makers once we confirm that such prediction for deviation remains valid in strategic situations. We observed a statistical correlation between non-strategic and strategic decision making by exploiting our within-subject design that bridges a subject's type of choice randomization and strategic decision making. Such an observation might suggest that, by making use of the PMs' choices, we can recover underlying beliefs better, which calls further examination.

Since we show the relationship—not causality—between the observations, more studies should follow. Adding more structures to classify non-strategic decision-making patterns more finely would help investigate our findings further. It would also be worth examining the predictability of the choice randomization patterns to the other strategic

[17]This claim holds if there exists a bijection between the actions and the corresponding levels, that is, if there is no way to interpret an action as an outcome of two or more different levels of cognitive iterations. It is not possible to overestimate the player's level.

environments and their interactions.

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Appendix. Experiment Instructions

Welcome.

You will perform three different tasks. In each task, you will input your decisions to the computer interface. Your payment will be based on the decisions you make, the decisions that other participants make, and some luck. Your payment will be informed at the end of all three tasks.

Everyone starts the same task at the same time. If you finish some part of the experiment earlier than others, you will be asked to wait.

In each task, you will read the instructions first and, if necessary, will take a quiz to check your understanding of the instructions. Please read the instructions carefully.

Task 1: Decision-Making under Uncertainty

Important preliminary: "...randomly drawn from a probability distribution"?

We want you to understand what we mean by "an event is randomly drawn from a probability distribution." A probability distribution is a description of possible events and their chances.

For example, if you toss a fair coin, with a 50% of chance, it will land heads (H) and with another 50% of chance it will land tails (T). Here the possible events are the faces of a coin, H and T, and the corresponding chances are 1/2 each. Then, the probability distribution of coin tossing can be described as (H, 1/2; T, 1/2).

When we say "an event (here, the face of a coin) is randomly drawn from (H, 1/2; T, 1/2)," it means that we toss a coin, and either H or T is realized with an equal probability, but we will not disclose what the actual realization is. Also, note that when each event is independently drawn, that event has nothing to do with the previous events whatsoever.

Another example: "an event is randomly drawn from (L, 0.2; C, 0.5; R, 0.3)" means the following three. (1) An event L (, C or R) will be drawn with a 20% (, 50% or 30%) of chance. (2) One among L, C, and R is realized according to their chances. (3) We will not disclose what the realization is.

During this task, you will frequently read "an event is randomly drawn from a probability distribution" in various contexts. We will assume now that you completely understand the meaning of the sentence.

For further explanation, please ask the experimenter at any time.

Your Task:

Your task is to make eight choices in total, to earn points from two games. The following payoff matrix describes the first game.

	H, 3/4	T, 1/4
U	1	0
D	0	1

Your options—U and D—are shown on the left. A probability distribution is on the top; (H, 3/4; T, 1/4): H happens in a 3/4 of chance, and T happens in a 1/4 of chance. The matrix shows your payoff. For example, if you choose U, and an event H is randomly drawn from the probability distribution, you will earn 1 point. If you choose D, when an event T is drawn, you will also earn 1 point.

You make four choices. For each choice, an event is randomly and independently drawn from the probability distribution.

The following payoff matrix describes the second game.

	L, 1/2	C, 1/4	R, 1/4
U	1	0	0
M	0	1	0
D	0	0	1

In this game, your options are U, M, and D, and the event will be L with a probability 1/2, C with a probability 1/4, and R with a probability 1/4. For example, if you choose D when the event L is drawn, you earn 0 points. If you choose M when the event C is drawn, you earn 1 point.

For each choice, an event is randomly and independently drawn from the probability distribution.

Payment:

All the points that you have earned in Task 1 will be converted into euros at the rate of 1 point = 80 cents.

Quiz:

Before you make your choices, you will answer two multiple-choice questions to check your understanding of this instruction. You can proceed only with all correct answers. You may ask an experimenter for help.

Q1 Suppose that the following matrix describes a certain game. Which of the following is true?

	H, 0.7	T, 0.3
U	1	0
D	0	1

(1) You can choose either H or T. (2) If you choose D, when an event T is drawn, you earn 0 points. (3) At the time you choose one option, you will not know your payoff. (4) The event will be H with a 50% of chance.

Q2 Suppose a probability distribution over events is (L,0.3; C,0.5; R,0.2). Which of the following is NOT TRUE? (1) When the realized event was L in the previous round, it is more likely to have an event R. (2) It is possible to face an event C both in the previous round and a current round. (3) In each round, a new event is randomly drawn from the probability distribution. (4) It is possible to face an event L in the previous round and face an event R now.

Task 2: 11-20 Token Request

Your Task:

Your task is to make four decisions with a randomly matched participant and make another four decisions with another randomly matched participant. In total, you make eight decisions. You will not know who your matches are, and they will not know you either.

In each decision round, choose one of the integers between 11 and 20, including 11 and 20. You will get the tokens (the currency in this task) corresponding to the integer you chose. Also, if your choice is one token less than your match's choice in that round, you will earn 20 extra tokens.

Payment:

One of the eight rounds will be randomly selected, and the earnings of that round will be paid. The tokens you have earned in that round will be converted into euros at the rate of 1 token=30 cents. Each round is equally possible to be selected, so it is of your best interest to consider every round equally seriously.

Quiz:

Before you make your choices, you will answer two multiple-choice questions to check your understanding of this instruction. You can proceed only with all correct answers. You may ask an experimenter for help.

Q1 Which of the followings is NOT TRUE? (1) You choose an integer between 11 and 20 in each of eight rounds. (2) In each round, you are randomly matched with a new participant. (3) In a particular round, if you choose 18, and your match chooses 17, you earn 18 tokens. (4) In a particular round, if you choose 19, and your match chooses 20, you earn 39 tokens.

Q2 Which of the followings is TRUE? (1) Your match will always choose the same integer for all four rounds. (2) Three randomly selected rounds out of the eight rounds will be paid. (3) You will know who your matches are. (4) At the end of the fourth rounds, you will be randomly matched with a new participant.

Task 3: Bonus Prizes

In this session are N participants, including you. You will win bonus prizes if you correctly guess how N participants answered in the previous two tasks in aggregate.

Bonus 1: In the first game of Task 1, each made four choices of either U or D. The probability distribution was (L: 3/4, R: 1/4). In total, there are $4 * N$ choices. Guess the total choice frequencies of U and D. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)

Bonus 2: In the second game of Task 1, each made four choices among U, M, and D. The probability distribution was (L: 1/2, C: 1/4, R: 1/4). In total, there are $4 * N$ choices. Guess the total choice frequencies of U, M, and D. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)

Bonus 3: In Task 2, every participant submitted eight integers between 11 and 20. In total, there are $8 * N$ choices. Guess the total choice frequencies of each integer. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)