Game Theory: In-class Midterm
Fall 2023

Student ID:

Name:

Instructions

1. Do NOT flip over this page until every student received this exam. Your TA will let you know when you can start.
2. During this closed-book exam, you cannot consult any materials.
3. If you are unable to explain your reasoning in English, it is okay to write in Korean.
4. Should you need more spaces, use the backside of the page, with clearly indicating the relevant question number.
5. There are five questions, worth 70 points in total. Allocate your time wisely.

Honor Code: Cheating on exams or quizzes, plagiarizing someone else's answers as one's own, or any other instance of academic dishonesty violates the standards of academic integrity.

Confidentiality Code: Sharing the information of the exam or quiz contents with other students in any form and medium is strongly prohibited, as it raises information inequity. **Violation of this code will be regarded as academic misconduct.**

I, ______________, consent to the Honor Code and the Confidentiality Code.
(write your name)

1. [12 points] Consider the following normal-form game:

P1 \ P2	x	y	z
A	4, 2	0, 0	5, 0
B	0, 0	2, 4	0, 3
~~C~~	~~1, 3~~	~~0, 2~~	~~2, 2~~

0.4 > 0
3.2 > 3.

(a) Examine if there are strictly dominated strategies. If so, explain how such strategies are dominated.

(b) Is the game dominance solvable?

(c) Check if there are pure strategy Nash equilibria. If so, describe them.

(d) Find a mixed-strategy Nash equilibrium.

(a) C is strictly dominated by $(A, \frac{1}{2}; B, \frac{1}{2}; C, 0)$.

z is strictly dominated by, say, $(x, 0.2; y, 0.8; z, 0)$.

any number in (0.75, 1) should be fine.

(b) No.

(c) (A, x) and (B, y) are pure strategy Nash equilibria.

(d)

		x (q)	y ($1-q$)
p	A	4, 2	0, 0
$1-p$	B	0, 0	2, 4

P1's expected payoff of playing $A = 4q$

" " $B = 2-2q$

$4q = 2-2q \Rightarrow \boxed{q = \frac{1}{3}}$

P2's expected payoff of playing $x = 2p$

" $y = 4-4p$

$\Rightarrow \boxed{p = \frac{2}{3}}$

$\therefore \{(A, \frac{2}{3}; B, \frac{1}{3}; C, 0), (x, \frac{1}{3}; y, \frac{2}{3}; z, 0)\}$ is a MSNE.

No penalty for not describing this.

2. [12 points] Consider the following game:

	P1 \ P2	L (q)	R ($1-q$)
p	U	3, 3	0, 2
$1-p$	D	1, 2	2, 2

Find all pure-strategy and mixed-strategy Nash equilibria.

$\Longrightarrow$ (U,L), (D,R) are PSNE.

P1's exp. payoff of playing $U = 3q$

〃 $D = q+2-2q = 2-q$ $\Rightarrow$ $\boxed{q=\frac{1}{2}}$

P2's 〃 $L = 3p+2-2p = 2+p$ $\Rightarrow$ $\boxed{p=0.}$

〃 $R = 2$

$\rightarrow$ $\{(U,0\,;\,D,1),\ (L,\tfrac{1}{2}\,;\,R,\tfrac{1}{2})\}$ is a MSNE.

3. [15 points] Two firms compete by choosing quantity produced in a market. The demand function is given by $P(q_1, q_2) = 12 - q_1 - q_2$, where q_1 and q_2 are quantity produced by firm 1 and firm 2. Firm 1 has a cost function $C_1(q_1) = 4q_1$ and Firm 2 has a cost function $C_2(q_2) = 2q_2$.

(a) Describe each firm's profit as a function of q_1 and q_2.

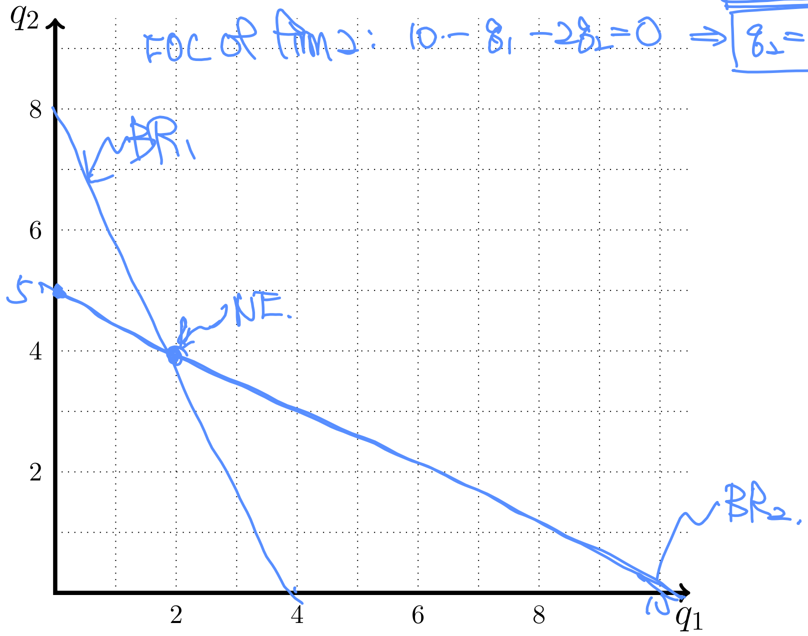
$\pi_1(q_1; q_2) = (12 - q_1 - q_2)q_1 - 4q_1$

$\pi_2(q_2; q_1) = (12 - q_1 - q_2)q_2 - 2q_2$

(b) Draw Best Response functions below.

FOC of firm 1: $8 - 2q_1 - q_2 = 0 \Rightarrow \boxed{q_1 = 4 - \frac{1}{2}q_2}$

FOC of firm 2: $10 - q_1 - 2q_2 = 0 \Rightarrow \boxed{q_2 = 5 - \frac{1}{2}q_1}$



(c) Find a Nash equilibrium.

$q_2^* = 5 - \frac{1}{2}q_1^* = 5 - \frac{1}{2}(4 - \frac{1}{2}q_2^*) = 3 + \frac{1}{4}q_2^*$

$\Rightarrow \frac{3}{4}q_2^* = 3 \Rightarrow \boxed{q_2^* = 4}$

$q_1^* = 4 - \frac{1}{2}q_2^* = 4 - \frac{1}{2}4 = 2 \quad \boxed{q_1^* = 2}$

$\therefore$ $(q_1^*, q_2^*) = (2, 4)$ is a NE.

4. [15 points] Suppose there are N bystanders who observe an emergency. If no one calls 911, all bystanders get a payoff of 0. If at least one person calls 911, the emergency is soon resolved, and every bystander earns a payoff of 1. However, the bystanders who called 911 must spend some extra cost $c \in (0, 1)$, so their payoff is $1 - c$.

(a) Suppose $N = 2$. Describe the game among bystanders on a payoff matrix form.

(b) Find a symmetric mixed-strategy Nash equilibrium, and in that equilibrium, calculate the probability that no one calls 911.

(c) Now suppose $N = 3$. Find a symmetric mixed-strategy Nash equilibrium. (Hint: Don't draw a payoff matrix. By symmetry, all players will call 911 with the same probability.) Compare the probability that no one calls 911 when $N = 3$ with your answer in part (b).

(d) Describe one correlated equilibrium in this game.

(a)

P1 \ P2	Not	Call
Not	0, 0	1, 1-c
Call	1-c, 1	1-c, 1-c

(b) $Pr(\text{Not call}) = p$

Exp. payoff of call $= 1-c$

" Not $= (1-p)1 + p \cdot 0$ $\Rightarrow \boxed{p = c}$

$\therefore$ $\{(\text{Call}, 1-c \,;\, \text{Not}, c)$ for all players$\}$ is a MSNE.

In that equilibrium, $Prob(\text{No one calls}) = c^2$.

(c) $Pr(\text{Not}) = p$ Exp payoff of call $= 1-c$

Not $= (1-p^2)1 + p^2 0$ $\Big) = 1-c = 1-p^2$.

$\boxed{p = \sqrt{c}}$

$\therefore$ $\{(\text{Call}, 1-\sqrt{c} \,;\, \text{Not}, \sqrt{c})$ for all players$\}$ is a MSNE.

In that equilibrium, $Prob(\text{No one calls}) = c^{\frac{3}{2}}$

(d) One randomly selected person is asked to call 911, and no one else calls 911.

any randomization device is fine.

5. [16 points] An airline carrier lost bags of passengers X and Y. They do not know each other, but coincidentally, their bags (and the items inside) are identical. The airline manager tries to compensate their losses in the following manner:

- Two passengers simultaneously report the bag's value. For simplicity, assume there are only three options: \$100, \$200, and \$300.
- The claimed value will be paid. Also, if one person claims \$100 **lower than** the other person, then that person will be additionally paid \$150.
- For example, if passenger X claimed \$300, and Y claimed \$100, they will be paid as they claimed. If passenger X claimed \$300, and Y claimed \$200, then X will be paid \$300, and Y will be paid \$350 (=200 + 150).

(a) Describe the game on a payoff matrix form.

(b) Examine if there are pure strategy Nash equilibria. If so, describe them.

(c) Find a mixed-strategy Nash equilibrium.

(a)

X\Y	100	200	300
100	100,100	250,200	100,300
200	200,250	200,200	350,300
300	300,100	300,350	300,300

(b) (200,300) and (300,200) are PSNE.

(c) observe 100 is strictly dominated by 300.

X\Y	200 (q)	300 ($1-q$)
200	200,200	350,300
300	300,350	300,300

$200q + 350 - 350q = 350 - 150q$

300

$\Rightarrow \boxed{q = \frac{1}{3}}$

$\therefore \{(100, 0; 200, \frac{1}{3}; 300, \frac{2}{3}), (100, 0; 200, \frac{1}{3}; 300, \frac{2}{3})\}$ is a MSNE.