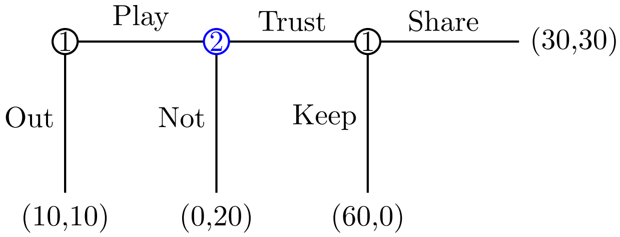
Game Theory: In-class Quiz 4
Fall 2023

1. Consider the following trust game:



(a) Suppose this game is played once. Represent the game in a normal form, and find all pure-strategy Nash equilibria. [Hint: There are two.]

⇒ ((Out,Share),(Not)) and ((Out,Keep),(Not)) are two pure-strategy Nash equilibria.

P1\P2	Trust	Not
Play,Share	30,30	0,20
Play,Keep	60,0	0,20
Out,Share	10, 10	10, 10
Out,Keep	10, 10	10, 10

(b) Suppose the game is repeated indefinitely. Both players discount future payoffs by $\delta \in [0, 1]$. Under what range of δ, {(Play, Share), Trust} is sustainable as a subgame perfect equilibrium by a grim-trigger strategy? Here the grim-trigger strategy is P1 playing (Out, Keep) and P2 playing (Not) forever after observing a deviation from {(Play, Share), Trust}.

⇒ P1's payoff when staying {(Play, Share),Trust}: $\frac{30}{1-\beta}$.

P1's payoff when deviating to (Play,Keep): $60 + \frac{10\beta}{1-\beta}$

Repeatedly playing {(Play, Share), Trust} is a SPE if $\frac{30}{1-\beta} > 60 + \frac{10\beta}{1-\beta}$, or $\beta > 3/5$.

2. Consider the Battle of Sexes with incomplete information: Player 2(P2) has one of the two possible types ("Meet"(M) and "Avoid"(A)).

- M-type P2 wishes to meet P1 at the concert playing Bach.
- A-type P2 wishes to avoid meeting at the concert playing Bach.
- P2 is type M with probability p and type A with probability $1 - p$.
- P2 knows her type. P1 only knows the prior probability distribution of P2's type.

They simultaneously choose Bach(B) or Stravinsky(S). Payoffs are shown in the matrices below.

P1 \ M-type P2	B	S
B	2,1	0,0
S	0,1	1,0

P1 \ A-type P2	B	S
B	2,0	0,2
S	0,1	2,2

Find a range of p such that (B; B, S) is a pure strategy Bayesian equilibrium. (The strategy is described in the form of (P1's strategy; M-type P2's strategy, A-type P2's strategy).)

Sol: $p > 1/3$

- M-type P2 won't deviate from B because S is strictly dominated.
- A-type P2 won't deviate from S given P1 plays B. (or, A-type P2 won't deviate from S because B is strictly dominated.)
- So we only need to check P1's behavior. The expected payoff of P1 playing B is $2p + 0(1-p) = 2p$, and the expected payoff of P1 playing S is $0P + 2(1-p) = 2 - 2p$. Thus, if $2p > 2 - 2p$, or $p > 1/2$, playing B is P1's best response.

3. Consider the Cournot competition with incomplete information. Market demand is $P = 12 - q_1 - q_2$. It is commonly known that firm 1's production cost is zero. Firm 2's marginal production cost is either 2 (type L) or 4 (type H) with equal probability of 1/2. Denote type-L firm 2's production quantity by q_2^L, and type-H firm 2's quantity by q_2^H.

(a) Find firm 2's type-dependent best responses to q_1.

⇒ H-type maximizes $(12 - q_1 - q_2^H)q_2^H - 2q_2^H$, thus $q_2^H = \frac{10-q_1}{2}$. L-type maximizes $(12 - q_1 - q_2^L)q_2^L - 4q_2^L$, thus $q_2^L = \frac{8-q_1}{2}$.

(b) Find firm 1's best response to maximize the expected profit.

⇒ Firm 1 maximizes $\frac{1}{2}\{(12 - q_1 - q_2^H)q_1\} + \frac{1}{2}\{(12 - q_1 - q_2^L)q_1\}$. Thus, $q_1 = \frac{12-(q_2^H+q_2^L)/2}{2}$

(c) Find the Bayesian Nash equilibrium of this game.

⇒ $q_1^* = 6 - \frac{1}{4}(q_2^{H*} + q_2^{L*}) = 6 - \frac{1}{4}(5 - q_1^*/2 + 4 - q_1^*/2) = 15/4 + q_1^*/4$. Thus, $3q_1^*/4 = 15/4$, or $q_1^* = 5$. Plug in q_1^* to $q_2^H =$ and q_2^L. The Bayesian equilibrium is (5, (2.5, 1.5)).