ECON4002 Midterm
Fall 2023

Student ID:

Name:

Instructions

1. Do NOT flip over this page until every student received this exam. The exam proctor will let you know when you can start.
2. During this closed-book exam, you cannot consult any materials except for a two-page-long cheat sheet handwritten by yourself.
3. You can write your answers either in English or in Korean. Suit yourself.
4. There are six questions. The maximum score is 100.
5. The exam proctor will not answer any questions related to the exam contents. If you believe the exam questions are incomplete or unclear, state why you think so, and assume the necessary setting on your own.
6. Should you need more spaces, use the backside of the page, with clearly indicating the relevant question number.

1. [10 points] Consider an economy with two goods, x and y. Utility function is given by $u(x, y) = x^2y^4$. Income is denoted as w, the price vector is (p_x, p_y).

(a) Check if the utility function is quasiconcave.

(b) Find the Marshallian demand $(x(p_x, p_y, m), y(p_x, p_y, m))$ and the indirect utility function $v(p_x, p_y, w)$.

(c) Verify Roy's identity in this economy.

2. [20 points] Suppose that a consumer has the following expenditure function: $e(p_1, p_2, u) = \frac{2p_1p_2}{p_1+p_2}u$.

(a) Find an indirect utility function and Marshallian demand function of this consumer.

(b) Recover a utility function $u(x_1, x_2)$ that rationalizes this consumer's demand behavior.

3. [20 points] A consumer's preferences $\succeq$ on $\mathbb{R}^L_+$ can be represented by the utility function $u : \mathbb{R}^L_+ \to \mathbb{R}_+$ with the property that for any $x \in \mathbb{R}^L_+$ and $\alpha > 0$, $u(\alpha x) = \alpha u(x)$.

(a) Show that this consumer has homothetic preferences, that is, for any $x, y \in \mathbb{R}^L_+$, $x \succeq y$ if and only if $\alpha x \succeq \alpha y$ for any $\alpha > 0$.

(b) Show that this consumer's expenditure function is such that $e(p, u) = ue(p, 1)$ for any $u > 0$ and prices p. [Hint: First show $e(p, u) = e(up, 1)$, and then use the property of the expenditure function.]

(c) Is this consumer's indirect utility function linear in wealth? Explain.

4. [15 points] Consider three revealed price-commodity pairs given by

$$\begin{aligned} p^1 &= (1,1,2), \quad x^1 = (1,0,0) \\ p^2 &= (2,1,1), \quad x^2 = (0,1,0) \\ p^3 &= (1,2,1+\varepsilon), \quad x^3 = (0,0,1) \end{aligned}$$

, where $\varepsilon > 0$ is very small.

(a) Check if it satisfies WARP.

(b) Check if it satisfies GARP.

(c) Discuss the possibility of recovering preference from those observations.

5. [15 points] Consider an economy with L goods. Dennis in this economy has a strictly increasing and strictly concave utility unction, $u : \mathbb{R}^L_+ \to \mathbb{R}$.

(a) Show that Dennis' indirect utility function $v(p, m)$ must be strictly concave with respect to income m.

(b) Dennis has two career choices, doctor and scientist. Doctor's income is drawn from distribution D and scientist's income is from distribution S. If D second-order stochastically dominates S, recommend one career choice to Dennis, and explain why he should choose the career you recommend, by using the fact in (a).

6. [20 points] Consider the following production function:

$$Q = (L^\alpha + K^\alpha)^{1/\beta},$$

where L is the input of labor and K is the input of capital. We assume that $\alpha \in (0, 1)$, $\beta \in (0, 1)$, wage is $w = 2$, and rent is $r = 1$.

(a) Find the condition on α and β for increasing/constant/decreasing return to scale production function.

(b) Find the cost function (as a function of Q).

(c) Find the condition on α and β for convex/linear/concave cost function.

(d) Compare your answers for part (a) and part (c) and explain the intuition.