Probability Matching and Strategic Decision Making*

Duk Gyoo Kim[†] Hee Chun Kim [‡]

January 18, 2020

Abstract

This paper examines a link between an individual's (possibly limited) strategic thinking in the 11–20 money request game and (possibly non-rational) decision-making patterns in the matching pennies games. Experimental evidence shows that subjects' strategic behavior, which used to be understood as a result of cognitive iterations, is closely related to probability-matching patterns. Ignoring some individuals' choice randomization overestimates the variance of levels in cognitive iterations. Probability matchers do not seem to have less ability of cognitive iteration in strategic decision making. The relationship requires attention because the assumption that individuals are rational in the decision-theoretic sense may create a sizable misinterpretation of strategic behavior.

Keywords: tba

JEL: tba

1 Introduction

A growing number of studies in social science consider bounded rationality both in a non-strategic environment,[1] where a single player makes a decision in an uncertain

*The previous version of this paper was titled "Mixing Propensity and Strategic Decision Making." We thank Colin Camerer and Brian Rogers for their encouragement, PJ Healy, Kirby Nielsen, Sangwon Park, Henrik Orzen, Jörg Oechssler, and participants in several conferences and seminars for their comments, and Kiril Lavrov for his assistance. Hee Chun Kim greatly acknowledges the financial support from Economic Theory Center at Washington University in St. Louis and Korea Information Society Development Institute.

[†]Department of Economics, University of Mannheim, *d.kim@uni-mannheim.de*

[‡]Korea Information Society Development Institute, *heechunkim@kisdi.re.kr*

[1]In the sense that a decision maker may use some strategies to deal with uncertainty, the non-strategic environment does not necessarily mean it involves no strategy at all. Instead, it should be understood as a situation with a single decision maker.

state and in a strategic setting, where she responds to the other agents' unknown intentions and actions. When making a decision in the non-strategic setting, individuals are often cognitively limited: They may not recognize or understand all the aspects that affect their payoffs, or they may lack the cognitive ability to draw an ideal decision as much as they needed. Observations from the strategic environment also seem to be inconsistent with the theoretical predictions attained under the assumption of full rationality, not only because their rationality is bounded, but also because their belief about other individuals' bounded rationality varies.

The primary goal of this paper is to examine how individuals' non-strategic—and possibly non-rational—decision-making patterns over probabilistic events are related to their strategic ones. To analyze strategic observations, the main body of the literature has implicitly assumed that "individuals are rational in the decision-theoretic sense of choosing strategies that are best responses to consistent beliefs" (Crawford, 2016), which hereinafter we call *decision-theoretic rationality*. However, experimental work shows that when subjects are asked to make repetitive decisions under uncertainty, a significant fraction (varying from 20% to 50% by study) of subjects do not make decisions that maximize their expected payoff. Instead, they *match* their decision frequencies to the probability of events, which is called *probability matching* (Rubinstein, 2002; Neimark and Shuford, 1959). For example, suppose that people are asked to play ten rounds of Matching Pennies (MP) games. They are informed that in each game, a coin will be tossed independently, and the coin will land heads with a 70% chance. Some of them choose Heads for seven out of the ten rounds and Tails for the other three rounds, to match their relative choice frequencies with the probability of events. To maximize the expected payoff, they should have chosen Heads for all the rounds. Although investigating why some people have such a preference for randomizing their choices is worthwhile,[2] we want to clarify that the primary purpose of this study is not to rationalize the probability-matching behavior. Rather, we take their choice patterns as a given and investigate further whether and to what extent the existence of the probability-matching, or broadly speaking, randomizing players affects the analysis of the cognitive limitation in strategic decision making.

We claim that when some individuals' choice randomization behaviors are ignored, it is challenging to correctly map the individual's strategic actions to her underlying belief. Two leading theories formalizing bounded rationality in strategic decision making,

[2] Many studies provide models of preferences for randomization. Dwenger et al. (2018) provide a theory of responsibility aversion, which implies a demand for randomization. Levitt (2016) finds that randomization (coin toss) on major life decisions positively affects happiness, which might also reflect the responsibility aversion. Machina (1985) and Cerreia-Vioglio et al. (forthcoming) consider convex preferences to account for the affinity towards randomization among equally preferred options. Richter and Rubinstein (2019) provide an axiomatic model generalizing the standard Euclidean definition of convex preferences.

the Level-k (Lk) model (Costa-Gomes et al., 2001; Costa-Gomes and Crawford, 2006) and the cognitive hierarchy (CH) model (Camerer et al., 2004), share an assumption that individuals use only finite (=k) steps of iterated dominance, and such k varies by individual.[3] To analyze experimental observations, previous studies relying on one of those two models implicitly share an assumption that every subject is equipped with *decision-theoretic rationality*. In other words, the main body of the literature has assumed that no subject exhibits any choice randomization behavior, including probability matching. It may create a sizable misinterpretation. An individual who tends to exhibit probability matching may show heterogeneous choice patterns even when she has a homogeneous belief, and she may be merely regarded as a less sophisticated player due to "noisier" choices. Questions that naturally followed include whether the failure to consider choice randomization in previous studies has compromised their elicitation of the structure of beliefs, and if so, how severe it is.

To address our questions, we conducted two sets of within-subjects laboratory experiments: a set of two matching pennies (MP) games, and a set of repeated 11–20 (ET) money request games (Arad and Rubinstein, 2012). In a nutshell, from the MP experiment, we distinguish individuals randomizing choices from those who consistently make stochastically dominant choices. Then we observe how the observations from the ET experiment vary by choice pattern categorized in the MP experiment. We also incentivized subjects to reveal their beliefs about choice distributions at the end of those two experiments.

Our observations are summarized as follows. First, in the MP games, about 75% of the subjects were classified as a rational optimizer (RO), each of whom makes consistent choices of maximizing the expected payoff, and a quarter of them were classified as a probability matcher (PM), each of whom exhibits a probability matching tendency. Second, in the ET game, the choice variance of the PM-type subjects is larger than that of the RO-type subjects, which suggests that the probability matcher's tendency of choice randomization is also maintained in the strategic situations. Third, the average number of cognitive iterations of the PM-type subjects was not statistically different from that of the RO-type subjects, which suggests that the choice randomization in the ET games is not due to the less cognitive ability of the PM-type subjects. Fourth, there is a strong correlation between a subject's choice randomization and his/her belief in the choice distributions.

Altogether, we find a subject's strategic behavior observed in the ET games is closely related to the decision-making patterns in the MP games. When we ignore the existence

[3] One notable difference is that the Lk model assumes that individuals believe that others' cognition levels are homogeneous at Lk-*1*, while the CH model assumes that individuals believe the cognition levels are distributed over L*0* to Lk-*1*, where the population distribution is assumed to be stable.

of the probability matchers, the distribution of the estimated cognitive iterations in the strategic situations would be with an overestimated variance.

The rest of this paper is organized as follows. In the following subsection, we review related studies. Section 2 describes the details of the experimental design. The statistical method for inference is described in Section 3. Section 4 shows the results of the experiment and discusses its implications. Section 5 concludes.

1.1 Related Literature

This study is grounded in empirical and theoretical findings of bounded rationality in strategic behavior. We mainly focus on two leading behavioral models: the Level-k model developed by Costa-Gomes and Crawford (2006) and the Cognitive Hierarchy model developed by Camerer et al. (2004). Both models share two assumptions: (1) individuals are rational in the decision-theoretic respect that they choose strategies that are the best responses to consistent beliefs; and (2) individuals play strategies of a finite level of iterated dominance. The models differ in their assumptions about subjects' beliefs regarding the strategic behavior of other players. The Level-k (Lk) model assumes that individuals uniformly believe that all their opponents play the same level of iterated dominant strategy. For example, the $L2$ subject assumes that all his/her opponents play a one-time iterated dominant (or $L1$) strategy. From that assumption, Lk subjects are supposed to play a certain strategy that is the best response to their uniform beliefs. In Costa-Gomes and Crawford (2006), about 55% of subjects show a level of play that indicates adoption of the Level-k model. On the other hand, some subjects explicitly mix two or more different strategies, each of which represents a different level of iterated dominance. Such a systematic pattern does not coincide with the assumption of uniform belief. Costa-Gomes and Crawford (2006) claim such a mixing propensity may be the results of learning. That is, even among individuals who start from the initial uniform belief, the experience leads subjects to shift to the higher level of iterated dominance while retaining the uniform belief structure. However, for some subjects, such mixing occurs irrespective of the time horizon. These observations demand an alternative model that explains this behavioral pattern. The Cognitive Hierarchy (CH) model allows individuals to have a heterogeneous belief structure. For example, the L2 subject assumes that his or her opponent plays both the L1 strategy and the L0 strategy; the latter is a uniform random strategy, although it is often arguable the proper assumption about the L0 behavior. Depending on his or her belief regarding the proportion of those who use the two different strategies, each subject may find a different best response. To explicitly estimate the structure of belief, Camerer et al. (2004) use observations from previous studies as well as their own experimental observations. However, even though their model allows for a heterogeneous belief structure, Camerer et al. (2004) cannot fully explain the observations of mixed choices. That is, if a subject has a heterogeneous belief about the other players, consistently choosing the interim choice which is the best response to the heterogeneous belief as a whole can be strictly better than mixing several choices, each of these respectively corresponds to the best response to a part of the heterogeneous belief.

In the sense that we try to understand higher order rationality better, our goal is consistent with that of Kneeland (2015), who proposes a more explicit design of experiments to identify the higher order rationality. Rather than adopting Kneeland's ring games of many (more than three) players, we stick to the two-person guessing games. Because our primary objective is to find relationships between the decision-making patterns in non-strategic environments (a player vs. random events) and choices in strategic environments (a player vs. another player), we design the two experiments to be as structurally similar to one another as possible. The 11–20 money request game introduced by Arad and Rubinstein (2012) is an excellent tool for eliciting higher order rationality. We conduct the 11–20 game as a part of our experiment not only because it is simple but also it is less arguable on the L0 behavior. The only, but crucial difference from Arad and Rubinstein (2012) is that we ask subjects repeatedly make decisions. This repetition allows us to further examine the cognitive ability of the subject and potential misrepresentation.

We posit that individuals may show different responses to the same belief, and this difference in decision-theoretic rationality may lead to the apparent puzzle that mixes different strategies. Examples abound. In Rubinstein (2002), about half of the undergraduate subjects matched their frequency of choices to the probability of events for repetitive decision-making tasks. Thaler (2016) reports a similar result among MBA students at a top university. Though the contexts varied, the fundamental question that the authors asked subjects to perform was the independent repetition of the MP game described above.[4] Likewise, many studies in the psychology literature find a significant propensity for mixing different strategies. Neimark and Shuford (1959) and Vulkan (2000) also provide lab-experiment observations that support the existence of probability matching behavior. If we regard this mixing propensity as preferences for the randomization of choices, the experimental evidence expands. Agranov and Ortoleva

[4] Rubinstein (2002) performed the "catch the messenger" game in which a detective's task is to determine the location of a video camera each day and identify as many unknown messengers as possible while knowing the probability of catching the messenger at each location. The video camera should have been installed all the time at the location where the probability is the highest, but only a small portion of students always played the stochastic dominant action. Thaler (2016) asked MBA students to make a streak of 5 matching-pennies choices. Each choice was either Heads or Tails, and a fair coin was tossed five times after they made choices. The payoff of matching at Tails was 1.5 times higher. They should have chosen Tails all the time, which is the stochastically dominant action, but the most common observation was three Tails and two Heads, matching the ratio of the payoffs.

(2017) found that a large majority of experiment participants exhibit stochastic choice when they are asked to answer the same questions several times in a row. Dwenger et al. (2018) reported that university applications in Germany exhibit a choice pattern that is consistent with a preference for randomness. If similar probability matching behavior can also occur in strategic situations, then the underlying belief structure about the other players' cognition levels could be better revealed by the mixing strategies of different levels.

Although probability matching has been well documented in the literature, few experimental studies have explicitly considered these behavioral patterns in the optimization process for identifying underlying belief structure in the strategic decision-making environment. Georganas et al. (2015) examined whether individuals show similar levels of iterated dominance in different forms of the game. Georganas et al. (2015) had several individuals play different games: four separate non-strategic tests and a strategic decision-making session. In the strategic decision-making session, subjects played the 'undercutting game' and the 'beauty-contest game' for four and five times, respectively. While the undercutting game only allowed discrete choices, the beauty contest allowed some interim choices that do not represent any level of iterated dominance. Even in the two games that share a similar structure (requiring players to exploit an iterated dominant strategy), individuals showed almost no correlation between the levels of iterated dominance. Moreover, there was no significant connection between an individual's traits, such as IQ, and his or her level of iterated dominance. Georganas et al. (2015) attempted to find consistency in the strategic process in different environments but did not examine the process regarding individual optimization patterns.

2 Experimental Design and Procedure

We used a within-subject design. Each subject participated in two different experiments and a follow-up belief elicitation: In the Matching Pennies (MP) games, subjects made a streak of decisions in which payoffs depend on realized (but unknown) events. In the 11–20 Token Request games, subjects made a streak of decisions in which payoffs depend on the randomly matched subject's decisions. After that, they were incentivized to correctly guess the distributions of the entire choices made in the session.

In the MP games, the subjects make eight choices in total to earn points from two games. The payoff matrix in Table 1a describes the first game. A subject's options, U and D, are on the left column. A probability distribution, (H, 3/4; T, 1/4), is on the top: Event H is realized with a 3/4 of chance, and event T is with a 1/4 of chance.[5] When a subject

[5]We instructed what we mean by a probability distribution and how an event is independently drawn

chooses U and event H is realized, the subject earns 1 point. Each point that the subject had won in the MP games was converted into 80 cents. A new event is independently realized before making each decision. After making four independent decisions without feedback, the subject plays the second game, described in Table 1b. In this game, the subject's options are U, M, and D, and the event will be L with a probability 1/2, C with a probability 1/4, and R with a probability 1/4. Based on the subjects' choice patterns from the two different games, we categorize them into two types: the rational optimizer (RO) who chooses U (the stochastically dominant option) consistently, and the probability matcher (PM) who randomizes choices close to the probability distribution.

	H, 3/4	T, 1/4
U	1	0
D	0	1

(a) MP: Game 1

	L, 1/2	C, 1/4	R, 1/4
U	1	0	0
M	0	1	0
D	0	0	1

(b) MP: Game 2

Table 1: MP games

Table 2 shows how a player with a particular type would choose actions. When a player is expected to choose an action A∈ {U,M,D} for n times, it is denoted by An. The RO-type subject will play U, the choice that gives the largest expected payoff, all of the time. The PM-type subject will match the frequency of her choices with the probability of events. Thus, in Game 1, she will mix three Us and one D, and in Game 2, she will mix two Us, one M, and one D, up to permutation.

MP	Game 1	Game 2
Rational Optimizer	U4	U4
Probability Matcher	U3D1*	U2M1D1*

∗: up to permutation

Table 2: Predicted Behavior of Two Types in MP Games

In the 11–20 Token Request (ET) games (Arad and Rubinstein, 2012), the subjects make eight decisions in total, with knowing that they are randomly matched with another participant for the first four decisions and another match for the last four decisions. In each decision round, each subject chooses one of the integers $r \in \{11,\ldots,20\}$. The subject's payoff is $r+20$ if the choice of the match in that round is $r+1$, and r otherwise. That

from the probability distribution in plain words.

is, if the subject believes that her match would choose, for example, 19, then the best response is to choose 18 so that the payoff can be 38 (=18+20). One of the eight rounds was randomly selected for payment. The tokens earned in the selected round were converted into euros at the rate of 1 token to 30 cents. For counter-balancing, the ET games were conducted before the MP games for 24 subjects in two sessions.

The only, but crucial difference from Arad and Rubinstein (2012) is that we ask the subjects to make repetitive decisions. As we will illustrate in the next section, unless a subject believes that the match would play the mixed strategy of the unique equilibrium, an RO-type subject has little reason for randomizing the eight decisions. However, a PM-type subject may randomize the decisions, and the first observation may not help us to recover the subject's cognitive ability.

After making 16 decisions (4 for the first MP, 4 for the second MP, and 8 for the ET) in total, the subjects were asked to correctly guess the aggregate choice distributions. For example, with N participants in a session, there are $4N$ choices from the first MP games. The subject whose guess is closest to the actual distribution of $4N$ choices won 4 extra euros. Similarly, they were asked to guess the choice distributions for the second MP games and the ET games.

Seven sessions of laboratory experiments were conducted with a total of 106 participants at the Mannheim Laboratory for Experimental Economics (mLab) in Fall 2019.[6] The participants were drawn from the mLab subject pool. Python and its application Pygame were used to computerize the games and to establish a server–client platform. After the subjects were randomly assigned to separate desks equipped with a computer interface, they were asked to carefully read the instructions before they took a quiz to prove their understanding of the experiment. Except for mentioning that there are three different tasks, the instructions cover the upcoming task only. Those who failed the quiz were asked to re-read the instructions and to retake the quiz until they passed. An instructor answered all questions until every participant thoroughly understood the experiment. In the ET games, although new pairs were formed at the end of the fourth round, there was no physical reallocation of the subjects, and they only knew that they were randomly shuffled. They were not allowed to communicate with other participants during the experiment, nor allowed to look around the room. It was also emphasized to participants that their allocation decisions would be anonymous. Payments (10.63 euros on average) were made in private, and subjects were asked not to share payment

[6]Pilot sessions with 62 subjects were conducted in the Missouri Social Science Experimental Lab at the Washington University in St. Louis in 2015, and the data are available upon request. Although the observations from the pilot experiments deliver messages similar to what we report in this paper, we did not combine the two datasets because each sample set represents a different population. The experimental design and the pay scale were different. Rather than the 11–20 games, we conducted two-person guessing games.

information. Each session ran less than an hour.

3 An Illustration

Before we report the experimental observations, we illustrate how the assumption about decision-theoretic rationality could yield a sizable distortion about the inference of the experimental evidence on strategic behavior, when a substantial fraction of the population are randomizing their choices as if they do in the decision-theoretic situations.

Only for this section, we maintain two assumptions. First, all players believe that people are cognitively limited. Second, a probability matcher, if one exists, matches the choice frequencies with the underlying belief about other's cognitive levels.

Consider the following thought experiments. There are several decision makers (DMs) whose level of cognitive iteration is known to be less than three. Suppose that a DM makes a streak of ten decisions against one anonymous match to maximize her payoffs, and the optimal decisions simply depend on the matched player's level of cognitive iteration. Along with the ET games, suppose that the DM's optimal strategy to Level-k player is to perform $a_k \in \mathbb{N}$, for $k = 0, 1, 2$. That is, if the DM believes that the match is a $L1$ type for sure, then she must choose a_2. Suppose further that the choices from several DMs are categorized into one of the six equally-populated patterns illustrated in Table 3.

Patterns	1	2	3	4	5	6	7	8	9	10	Type
DM1	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	RO
DM2	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	RO
DM3	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	RO
DM4	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	PM
DM5	a_1	a_1	a_1	a_2	a_1	a_1	a_2	a_1	a_1	a_2	PM
DM6	a_2	a_2	a_1	a_2	a_2	a_2	a_1	a_2	a_1	a_2	PM

Table 3: Illustration: Representative Choice Patterns

Suppose that an econometrician, who observes the choices of the DMs but does not know whether the DM is RO or PM, tries to estimate their level of cognitive ability and the belief distribution. DM1, DM3, and DM4 will be coded as Level-2 players, and DM2 will be coded as Level-1 for sure. DM5 will be coded as Level-1, and DM6 will be coded as Level-2, although there are three "noisy" decisions for each of them.

If the econometrician knows the type of the DMs, then substantially different conclusions can be drawn. DM5 is Level-2 and believes that 70% of the population is Level-0, and 30% of it is Level-1. DM6 is also Level-2 but believes that the population distribution consists of 30% of Level-0 and 70% of Level-1. After learning this, we are no longer sure if DM2 is Level-1: She might have the same belief of DM5 but make stochastically dominant choices to maximize her expected payoff by best-responding to Level-0 players' action, the most likely one. If we assume that the belief distributions of the ROs are similar to those of the PMs, then we could conclude that DM1 and DM4, and DM3 and DM6 are sharing the same belief. Thus, DM2 might be sharing the same belief with DM5 and be coded as Level-2. Table 4 summarizes our arguments.

	When types are ignored	When types are informed
DM1	L2 who believes 100% of L1	L2 who believes 100% of L1
DM2	L1 who believes 100% of L0	L2 who believes 30% of L1 and 70% of L0
DM3	L2 who believes 100% of L1	L2 who believes 70% of L1 and 30% of L0
DM4	L2 who believes 100% of L1	L2 who believes 100% of L1
DM5	L1 who believes 100% of L0	L2 who believes 30% of L1 and 70% of L0
DM3	L2 who believes 100% of L1	L2 who believes 70% of L1 and 30% of L0

Table 4: Caption

From this illustration, we want to emphasize how recognizing probability matchers is important for two reasons. First, the assumption of decision-theoretic rationality leads to a substantial overestimation of the variance of the cognitive levels. When the DM's types are considered, the entire population is coded as L2. However, without taking the types into account, a third of the population are regarded as L1. Second, repeated choices from the PMs can be helpful to better understand the DM's true belief about the cognitive ability of the population. Of course, we drew the second claim with the assumption that the PMs match the choice frequencies with their underlying belief, so it must be examined with the experimental data.

4 Experimental Findings

We ensure that the order of the two games did not have a meaningful impact. We compare the observations from the 24 subjects who played the ET games first with the others who played the MP games first. Two-sample Kolmogorov-Smirnov (KS) test on each of the four batches of observations—the first MP, the second MP, the first half of the ET, and the second half of the ET—does not reject the null hypothesis that two data samples come from the same distribution. We combine two data sets for summarizing our four findings.

First, in the MP games, about 75% of the subjects (80 out of 106) were classified as ROs, each of whom consistently chose the stochastically dominant choice, U, in all decision rounds. A quarter of the subjects were classified as PMs, each of whom exhibits a probability-matching tendency. No subjects chose stochastically dominated choices more: For example, in the second MP games, the mode choice of every PM was U. Table 5 summarizes the aggregated relative choice frequencies.

	U	D
All	0.96	0.04
RO (75.5%)	1.00	0.00
PM (24.5%)	0.86	0.14

(a) MP: Game 1

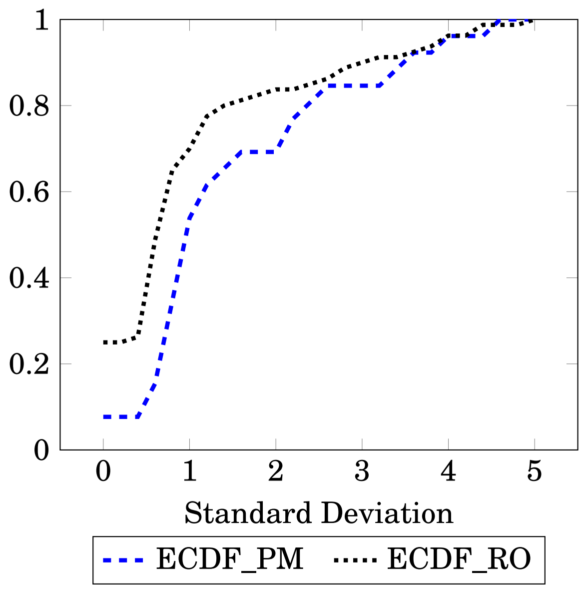
	U	M	D
All	0.93	0.06	0.01
RO (75.5%)	1.00	0.00	0.00
PM (24.5%)	0.64	0.22	0.13

(b) MP: Game 2

Table 5: Relative Choice Frequencies

Second, in the ET games, the choice variance of the PM-type subjects was larger than that of the RO-type subjects. Figure 1 shows the empirical CDFs of the standard deviations of individual choices in the ET games, grouped by the type. The PM-type subjects diversify their choices more than the RO-type subjects (KS test=1.4769, p-value=0.0255). A quarter of the RO-type subjects made the eight decisions constant. These patterns are unchanged when we draw similar empirical CDFs for the choices with the first match or the second match only. Relating to the choices in the MP games, this finding suggests that the PM's tendency of choice randomization is also maintained in strategic situations.

Figure 1: CDFs of the Standard Deviations of Individual Choices in the ET Games



Third, neither the RO- or the PM-type subjects' behavior can be explained by Nash equilibrium. Our finding confirms the reports of Arad and Rubinstein (2012). Table 6 shows the unique symmetric Nash equilibrium distribution (with assuming that players maximize the expected monetary payoff) and the actual choice distributions. The choice distribution is significantly different from the Nash equilibrium (χ^2 goodness of fit test, p-value<0.0001). Only 7 percent of the actions were 15 and 16, which is significantly smaller than 50 percent. The vast majority of the actions were 17, 18, and 19, and those are corresponding to 3, 2, and 1 level of cognitive iterations, respectively.

Action	11	12	13	14	15	16	17	18	19	20
Equilibrium (%)					25	25	20	15	10	5
All (%)	7	1	1	2	3	4	15	27	26	13
RO (%)	8	1	1	1	3	1	13	30	26	13
PM (%)	6	2	2	4	4	7	20	19	25	11

Table 6: Equilibrium and Actual Distributions of the ET Games

The average level of cognitive iterations of the PM-type subjects (2.8125) was not statistically different from that (2.4922) of the RO-type subjects (t=1.6684, p-value=0.0956). Rather, the PMs' average level is slightly higher than that of the ROs. This observation suggests that the choice randomization in the ET games is not due to the less cognitive ability of the PM-type subjects.

Fourth, there is a strong correlation between a subject's choice randomization behavior and his/her belief of the actual choice distributions. That is, a PM-type subject believes the choice distributions are dispersed more than what an RO-type subject does.

	U	D
Actual	0.96	0.04
RO	0.935	0.065
PM	0.802	0.198

(a) MP: Game 1

	U	M	D
Actual	0.93	0.06	0.01
RO	0.891	0.057	0.052
PM	0.644	0.199	0.168

(b) MP: Game 2

	11	12	13	14	15	16	17	18	19	20
Actual	7	1	1	2	3	4	15	27	26	13
RO	10	2	2	2	3	4	9	16	25	27
PM	13	3	2	5	7	6	12	18	19	16

(c) ET Games

Table 7: Relative Choice Frequencies and Guesses by Type

Table 7 shows how subjects guessed the actual choice distributions. In the MP games, the RO-type subjects' guess was distinctively closer to the actual distributions, although they consistently chose U. This observation suggests that the RO-type subjects have a good sense that a fraction of the whole subjects would exhibit probability-matching tendency. Meanwhile, the PM-type subjects seem to believe that the entire subjects would behave like themselves. Their guesses are close to their actual choice distributions in Table 1. For example, in the second MP games, 64%, 22%, and 13% of the choices were U, M, and D, respectively, and they guessed the actual distributions would be around 64%, 20%, and 17%.

In the ET games, the RO-type subjects' guess was closer to the actual distribution, although they overly weighted L0 behavior. The average level of cognitive iterations, according to their guess is 2.47, which is exactly corresponding to their actual average level of cognitive iterations, 2.49. 69% of the RO-type's choices were 17, 18, and 19, and 68% of their guesses were 18, 19, and 20. Therefore, they are indeed best-responding to their belief. The PM-type subjects, who randomized their decisions more than the RO-type ones, guessed that the actual choice distribution would be more dispersed than what they chose. It may suggest that if they were allowed to make more decisions, then their choice distributions would be more dispersed. One common observation is that both RO- and PM-type subjects diversified their choices in the ET games, to best respond to what they believe.

5 Concluding remarks

In this paper, we examine how an individual's (possibly non-rational) choice patterns are related to their strategic decision-making patterns. We consider that each individual who faces a probabilistic event has a different way of making decisions, and we categorize the subjects into two different types: the rational optimizer (RO), and the probability matcher (PM) by the observed choices in the MP games. We found more than a quarter of the subjects show choice patterns other than rational optimization. Our main observation is that when asked to make strategic decisions in the ET games, each type's choice patterns were different. PM-type subjects, in particular, diversified their actions to multiple levels of cognitive iterations in the ET games in a similar way of diversifying their decisions in the ET games.

The relationship between the decision-making patterns in the MP games and the ET games suggests that the literature may have overestimated the variance of the level of cognitive iterations. If every subject were the PM type, the belief structure estimated by the Level-k theory must be downward biased. If every subject were the RO type, the belief distribution estimated by the Cognitive Hierarchy model would underestimate the variance of the distribution and even the level of the RO type subject's reasoning.

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Appendix. Experiment Instructions

Welcome.

You will perform three different tasks. In each task, you will input your decisions to the computer interface. Your payment will be based on the decisions you make, the decisions that other participants make, and some luck. Your payment will be informed at the end of all three tasks.

Everyone starts the same task at the same time. If you finish some part of the experiment earlier than others, you will be asked to wait.

In each task, you will read the instructions first and, if necessary, will take a quiz to check your understanding of the instructions. Please read the instructions carefully.

Task 1: Decision-Making under Uncertainty

Important preliminary: "...randomly drawn from a probability distribution"?

We want you to understand what we mean by "an event is randomly drawn from a probability distribution." A probability distribution is a description of possible events and their chances.

For example, if you toss a fair coin, with a 50% of chance, it will land heads (H) and with another 50% of chance it will land tails (T). Here the possible events are the faces of a coin, H and T, and the corresponding chances are 1/2 each. Then, the probability distribution of coin tossing can be described as (H, 1/2; T, 1/2).

When we say "an event (here, the face of a coin) is randomly drawn from (H, 1/2; T, 1/2)," it means that we toss a coin, and either H or T is realized with an equal probability, but we will not disclose what the actual realization is. Also, note that when each event is independently drawn, that event has nothing to do with the previous events whatsoever.

Another example: "an event is randomly drawn from (L, 0.2; C, 0.5; R, 0.3)" means the following three. (1) An event L (, C or R) will be drawn with a 20% (, 50% or 30%) of chance. (2) One among L, C, and R is realized according to their chances. (3) We will not disclose what the realization is.

During this task, you will frequently read "an event is randomly drawn from a probability distribution" in various contexts. We will assume now that you completely understand the meaning of the sentence.

For further explanation, please ask the experimenter at any time.

Your Task:

Your task is to make eight choices in total, to earn points from two games. The following payoff matrix describes the first game.

	H, 3/4	T, 1/4
U	1	0
D	0	1

Your options—U and D—are shown on the left. A probability distribution is on the top; (H, 3/4; T, 1/4): H happens in a 3/4 of chance, and T happens in a 1/4 of chance. The matrix shows your payoff. For example, if you choose U, and an event H is randomly drawn from the probability distribution, you will earn 1 point. If you choose D, when an event T is drawn, you will also earn 1 point.

You make four choices. For each choice, an event is randomly and independently drawn from the probability distribution.

The following payoff matrix describes the second game.

	L, 1/2	C, 1/4	R, 1/4
U	1	0	0
M	0	1	0
D	0	0	1

In this game, your options are U, M, and D, and the event will be L with a probability 1/2, C with a probability 1/4, and R with a probability 1/4. For example, if you choose D when the event L is drawn, you earn 0 points. If you choose M when the event C is drawn, you earn 1 point.

For each choice, an event is randomly and independently drawn from the probability distribution.

Payment:

All the points that you have earned in Task 1 will be converted into euros at the rate of 1 point = 80 cents.

Quiz:

Before you make your choices, you will answer two multiple-choice questions to check your understanding of this instruction. You can proceed only with all correct answers. You may ask an experimenter for help.

Q1 Suppose that the following matrix describes a certain game. Which of the following is true?

	H, 0.7	T, 0.3
U	1	0
D	0	1

(1) You can choose either H or T. (2) If you choose D, when an event T is drawn, you earn 0 points. (3) At the time you choose one option, you will not know your payoff. (4) The event will be H with a 50% of chance.

Q2 Suppose a probability distribution over events is (L,0.3; C,0.5; R,0.2). Which of the following is NOT TRUE? (1) When the realized event was L in the previous round, it is more likely to have an event R. (2) It is possible to face an event C both in the previous round and a current round. (3) In each round, a new event is randomly drawn from the probability distribution. (4) It is possible to face an event L in the previous round and face an event R now.

Task 2: 11–20 Token Request

Your Task:

Your task is to make four decisions with a randomly matched participant and make another four decisions with another randomly matched participant. In total, you make eight decisions. You will not know who your matches are, and they will not know you either.

In each decision round, choose one of the integers between 11 and 20, including 11 and 20. You will get the tokens (the currency in this task) corresponding to the integer you chose. Also, if your choice is one token less than your match's choice in that round, you will earn 20 extra tokens.

Payment:

One of the eight rounds will be randomly selected, and the earnings of that round will be paid. The tokens you have earned in that round will be converted into euros at the rate of 1 token=30 cents. Each round is equally possible to be selected, so it is of your best interest to consider every round equally seriously.

Quiz:

Before you make your choices, you will answer two multiple-choice questions to check your understanding of this instruction. You can proceed only with all correct answers. You may ask an experimenter for help.

Q1 Which of the followings is NOT TRUE? (1) You choose an integer between 11 and 20 in each of eight rounds. (2) In each round, you are randomly matched with a new participant. (3) In a particular round, if you choose 18, and your match chooses 17, you earn 18 tokens. (4) In a particular round, if you choose 19, and your match chooses 20, you earn 39 tokens.

Q2 Which of the followings is TRUE? (1) Your match will always choose the same integer for all four rounds. (2) Three randomly selected rounds out of the eight rounds will be paid. (3) You will know who your matches are. (4) At the end of the fourth rounds, you will be randomly matched with a new participant.

Task 3: Bonus Prizes

In this session are N participants, including you. You will win bonus prizes if you correctly guess how N participants answered in the previous two tasks in aggregate.

Bonus 1: In the first game of Task 1, each made four choices of either U or D. The probability distribution was (L: 3/4, R: 1/4). In total, there are $4 * N$ choices. Guess the total choice frequencies of U and D. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)

Bonus 2: In the second game of Task 1, each made four choices among U, M, and D. The probability distribution was (L: 1/2, C: 1/4, R: 1/4). In total, there are $4 * N$ choices. Guess the total choice frequencies of U, M, and D. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)

Bonus 3: In Task 2, every participant submitted eight integers between 11 and 20. In total, there are $8 * N$ choices. Guess the total choice frequencies of each integer. If your guess is closest to the actual choice frequencies, you will win 4 extra euros. (When multiple winners, the prize will be split.)