Dynamic integrative bargaining

Mehmet Eckmekci and Jack Fanning*

Boston College and Brown University

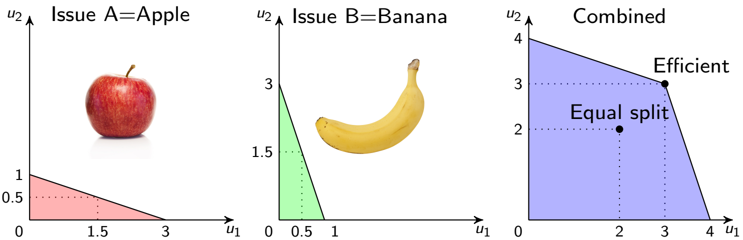
May 14, 2025

Original motivation: US congress fails to pass popular bills

- Popular measures often delayed if parties differ on intensity of support
  - e.g. Legal protection for "Dreamers": 84% Democrats and 69% Republicans support vs 8% and 24% opposed.
- Held up by low value party for leverage in some future "grand bargain"
  - Logrolling: Traded in exchange for support on other issues

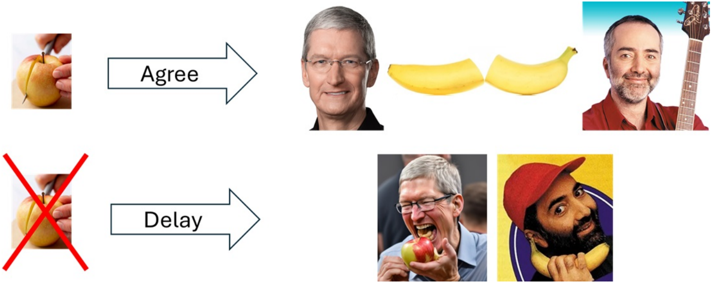
Logrolling: form of integrative bargaining

- Static benefits from negotiating different issues together vs separately
  - Give both sides more of what they value: *"I'll roll your log if you roll mine"*
- 2 issues - units of "pie" (or fruit) for which players have different values
- Issue A=Apple. Issue B=Banana (1 unit of each)
  - P1 get 3 utils/unit of Apple, 1 util/unit of Banana
  - P2 get 1 utils/unit of Apple, 3 util/unit of Banana
- *Logrolling:* all Apple for P1, all Banana for P2 $\succ_i$ equal split by issue (1/2 unit of each fruit)



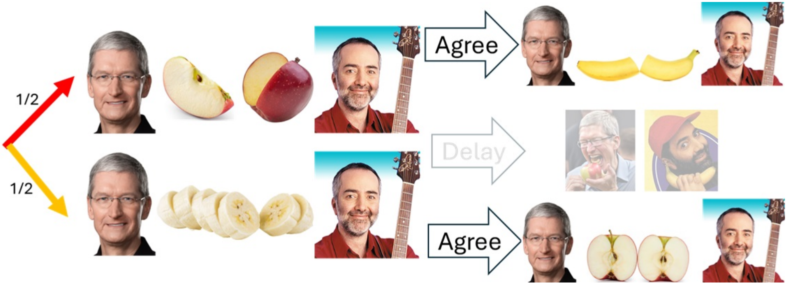
But incentives for delay?

- New issues arrive (stochastically) over time...
- P2 may refuse to divide available Apple until Banana arrives
  - Apple is useful leverage: o/w P1 might later demand 1/2 Banana
  - Can't commit to future divisions



Inefficiency even without delay

- An immediate division of the Apple must compensate players for giving up benefit of delay (leverage)
  - P2 must receive more than half *because* he values it less!
- If 1^{st} issue to arrive equally likely to be A or B then P1+P2 may lose in expectation
  - *Efficiency:* player 1 gets all of any Apple *whenever* it arrives, while player 2 gets all of any Banana



Can we do better?

- Are alternative institutional arrangements limiting logrolling more efficient?
  - **Independent committees:** dissimilar issues negotiated separately
  - **Stop searching:** for new issues until reach agreement on current issue
    - Contrary to typical negotiation advice: always search for more surplus “grow the pie”
    - What if search is endogenous choice?

Contributions/results

1. Develop general model of integrative bargaining and search
2. Explain delay: hold up issues opponent cares about for future leverage
3. Highlight important source of inefficiency: get small shares on valuable issues
    - Expanding issues' utility sets or making new issues arrive faster can decrease everyone's payoff
4. Justification for independent committees (prevent logrolling): higher payoffs if new issues arrives slowly
5. Stopping search during negotiations even better: can increase payoffs (even if new issues arrive fast): contrary to typical advice
6. Endogenous search: strong incentives for all to search even if bad equilibrium payoff effects

Literature:

- **Partial agreements with delayed arrival:** *Most related* Acharya and Ortner (2013)
- **Agenda setting:** *Simultaneous, Separate, Sequential* Busch and Horstmann (1997), Inderst (2000)
- **Logrolling relaxes informational constraints:** Jackson et al. (2024)
- **Vote trading and storage:** Casella (2005), Casella and Macé (2021)
- **Other perspectives on integration:** Fisher and Ury (1981), Raiffa (1982), Chang et al. (2024)

Model: more general in paper

- Players $i = 1, 2$ interact in periods $n = 0, 1, 2, \ldots$, period length Δ
- Finite set of issue types Θ
  - Issue of type θ associated with compact, convex utility possibility set U^θ

Model: more general in paper

- Players $i = 1, 2$ interact in periods $n = 0, 1, 2, ...$, period length Δ
- Finite set of issue types Θ
  - Issue of type θ associated with compact, convex utility possibility set U^θ
- In *finite issue* game state $\omega \in \Omega$ records number of currently available issues of each type, and number of past issues
  - Game starts in ω_0 with no past or current issues

Model: more general in paper

- **Bargaining Stage:** at start of period (of length Δ), each player selected as proposer w. prob $1/2$ iid
  - *Simple offer:* subset C of currently available issues and feasible utilities u
  - Other player accepts or rejects
    - $u \in \sum_{c \in C} U^{\theta_c}$ where θ_c is type of issue c
    - No transferable utility or contingent contracts
    - *Randomized offer:* randomize over simple offers if accepted
  - If issue set S agreed then transition to new state $\underline{\omega} = T(\omega, S)$

Model: more general in paper

- **Bargaining Stage:** at start of period (of length Δ), each player selected as proposer w. prob $1/2$ iid
  - *Simple offer:* subset C of currently available issues and feasible utilities u
  - Other player accepts or rejects
    - $u \in \sum_{c \in C} U^{\theta_c}$ where θ_c is type of issue c
    - No transferable utility or contingent contracts
    - *Randomized offer:* randomize over simple offers if accepted
  - If issue set S agreed then transition to new state $\underline{\omega} = T(\omega, S)$
- Then **Arrivals Stage:** at most one new issue arrives stochastically:
  - State transitions from $\overline{\omega}$ to $\underline{\omega}$ with prob. $q^{\underline{\omega},\overline{\omega}} = 1 - e^{-\lambda^{\underline{\omega},\overline{\omega}}\Delta}$
    - With remaining prob. $q^{\underline{\omega},\underline{\omega}}$ remains in $\underline{\omega}$
    - Arrival rates independent of currently available issues

Model: more general in paper

- **Bargaining Stage:** at start of period (of length Δ), each player selected as proposer w. prob $1/2$ iid
  - *Simple offer:* subset C of currently available issues and feasible utilities u
  - Other player accepts or rejects
    - $u \in \sum_{c \in C} U^{\theta_c}$ where θ_c is type of issue c
    - No transferable utility or contingent contracts
    - *Randomized offer:* randomize over simple offers if accepted
  - If issue set S agreed then transition to new state $\underline{\omega} = T(\omega, S)$
- Then **Arrivals Stage:** at most one new issue arrives stochastically:
  - State transitions from $\overline{\omega}$ to $\underline{\omega}$ with prob. $q^{\underline{\omega},\overline{\omega}} = 1 - e^{-\lambda^{\underline{\omega},\overline{\omega}}\Delta}$
    - With remaining prob. $q^{\underline{\omega},\underline{\omega}}$ remains in $\underline{\omega}$
    - Arrival rates independent of currently available issues
- In *finite issue* game: new issues eventually stop arriving
  - $\lambda^{\underline{\omega},\overline{\omega}} = 0$ if $L < \infty$ previous issues
- Payoffs: $U_i = \sum_{n=1}^{\infty} \delta^{n-1} u_i^n$
  - u_i^n =period n utility, $\delta = e^{-r\Delta}$, can normalize $r = 1$
- Focus on frequent offers $\Delta \to 0$

Stationary equilibrium and disagreement payoffs

- Equilibrium=SPNE
- Equilibrium is *Stationary* if behavior depends only on current state ω
  - Cont payoffs in ω in bargaining stage V^ω
- *Useful idea:* discounting as prob $(1-\delta)$ game ends w. payoffs $(0,0)$
- Disagreement payoffs d^ω conditional on an event (either game ends or new issue arrives), or equivalently from waiting for event

$$d^\omega = \frac{\sum_{\overline{\omega}\neq\omega} \delta q^{\omega,\overline{\omega}} V^{\overline{\omega}}}{1-\delta q^{\omega,\omega}}$$

Independent committees and stopping search

Independent committees

- Game G_θ identical to G *except negative utility if agree $\theta' \neq \theta$ issues*

  $\Rightarrow$ Payoffs: $V^{\omega,I} = \sum_\theta V^{\omega}_{G^\theta}$

Stopping search

- Game G_S identical to G *except no new issues arrive if some available*, $\lambda^{\omega,\overline{\omega}}_{G_S} = 0$

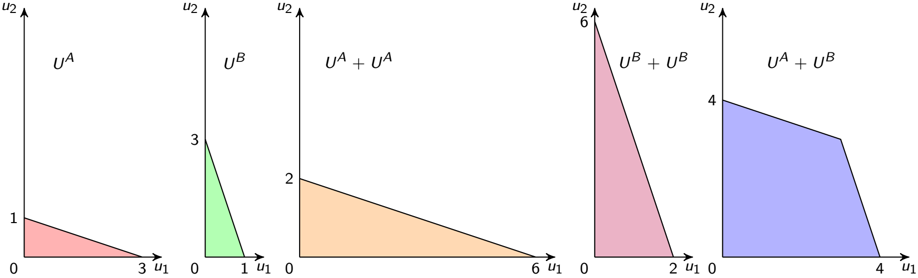
  $\Rightarrow$ Payoffs: $V^{\omega,S}$

  - Maybe due to institutional restrictions (e.g. papal conclave)
  - Consider endogenous search later

Baseline example: 2 independent linear issues, 2 types

- Exactly 2 issues can arrive w. 2 types $\Theta = \{A, B\}$, Apple/Banana
- Issue=unit of fruit: P1 gets z utils/unit of A, 1 util/unit of B
  - Focus: $z = 3$

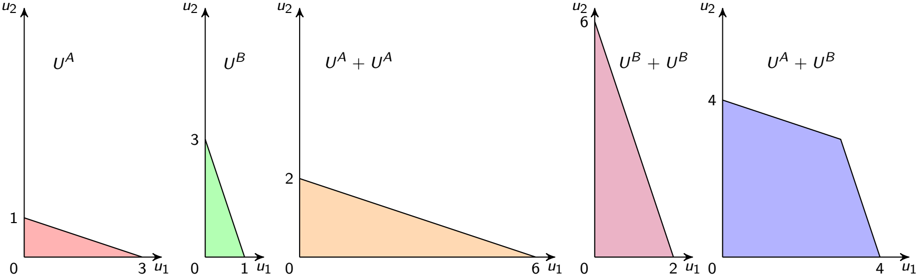
$$U^A = \{u \in \mathbb{R}^2_+ : u_1 \leq z(1 - u_2)\}, \quad U^B = \{u \in \mathbb{R}^2_+ : u_2 \leq z(1 - u_1)\}$$



Baseline example: 2 independent linear issues, 2 types

- Exactly 2 issues can arrive w. 2 types $\Theta = \{A, B\}$, Apple/Banana
- Issue=unit of fruit: P1 gets z utils/unit of A, 1 util/unit of B
  - Focus: $z = 3$

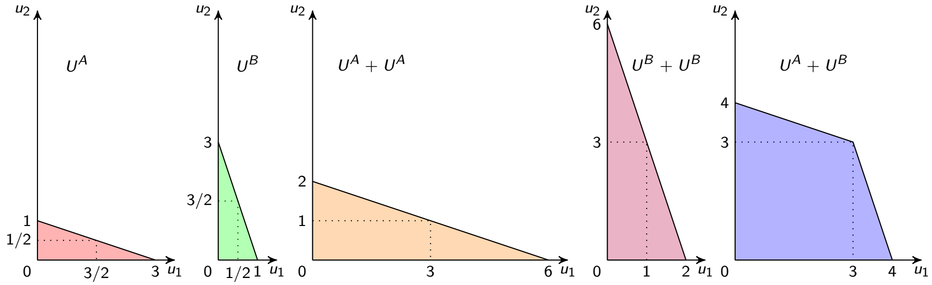
$$U^A = \{u \in \mathbb{R}^2_+ : u_1 \leq z(1 - u_2)\}, \quad U^B = \{u \in \mathbb{R}^2_+ : u_2 \leq z(1 - u_1)\}$$



- Equal arrival rate of A and B as 1^{st} issue $\check{\lambda} \in (0, \infty)$
  - As $\Delta \to 0$ discounted prob of arrival$\to$ $\check{p} = \check{\lambda}/(2\check{\lambda} + r) \in (0, 1/2)$
- Equal/independent arrival rate of A and B as 2^{nd} issue $\tilde{\lambda} \in (0, \infty)$
  - As $\Delta \to 0$ discounted prob of arrival$\to$ $\tilde{p} = \tilde{\lambda}/(2\tilde{\lambda} + r) \in (0, 1/2)$

What happens when 2^{nd} issue arrives?

As $\Delta \to 0$: agree on Nash solution for available utilites $\hat{V}^\omega = N(\hat{U}^\omega, \hat{d}^\omega)$, $\hat{d}^\omega = 0$

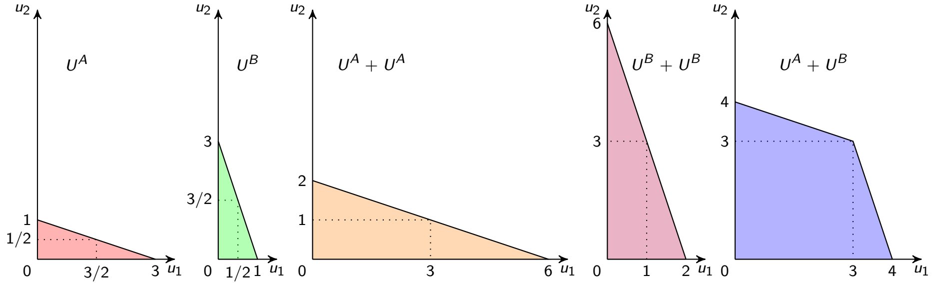


- $\omega_{\theta,\theta'}$ has 2 issues arrived w. types θ, θ', still available
- $\omega_{-\theta,\theta'}$ has 2 issues arrived w. types θ, θ', only θ' available

$$\hat{V}^{\omega_{A,A}} = (3, 1), \qquad \hat{V}^{\omega_{A,B}} = (3, 3)$$
$$\hat{V}^{\omega_{-A,A}} = (3/2, 1/2), \qquad \hat{V}^{\omega_{-A,B}} = (1/2, 3/2)$$

Implied continuation payoffs if 1^{st} issue is A

As $\Delta \to 0$*: agree on Nash solution for available utilites* $\hat{V}^{\omega} = N(\hat{U}^{\omega}, \hat{d}^{\omega})$*,*
$\hat{d}^{\omega} = 0$



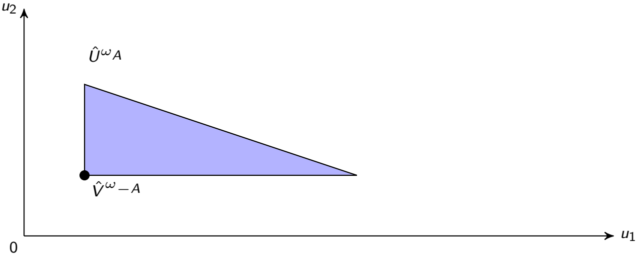
- $\omega_{\theta,\theta'}$ has 2 issues arrived w. types θ, θ', still available
- $\omega_{-\theta,\theta'}$ has 2 issues arrived w. types θ, θ', only θ' available
- ω_{θ} has 1 issue arrived w. type θ, still available
- $\omega_{-\theta}$ has 1 issue arrived w. type θ, no longer available

$$\hat{V}^{\omega_{A,A}} = (3,1), \quad \hat{V}^{\omega_{A,B}} = (3,3), \quad \Rightarrow \quad \hat{d}^{\omega_A} = \tilde{p}(6,4)$$
$$\hat{V}^{\omega_{-A,A}} = (3/2,1/2), \quad \hat{V}^{\omega_{-A,B}} = (1/2,3/2), \quad \Rightarrow \quad \hat{V}^{\omega_{-A}} = \tilde{p}(2,2)$$

What happens when 1^{st} issue arrives and is A?

- Feasible agreement utilities $\hat{U}^{\omega_A} = U^A + \hat{V}^{\omega_{-A}}$ as $\Delta \to 0$

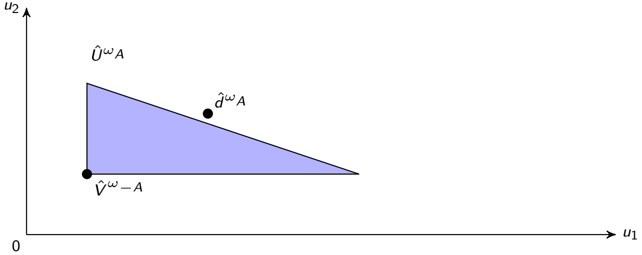
$$\hat{V}^{\omega_{-A,A}} = (3/2, 1/2), \quad \hat{V}^{\omega_{-A,B}} = (1/2, 3/2), \quad \Rightarrow \quad \hat{V}^{\omega_{-A}} = \tilde{p}(2,2)$$



What happens when 1^{st} issue arrives and is A?

- Feasible agreement utilities $\hat{U}^{\omega_A} = U^A + \hat{V}^{\omega_{-A}}$ as $\Delta \to 0$
- Compare to disagreement payoffs...

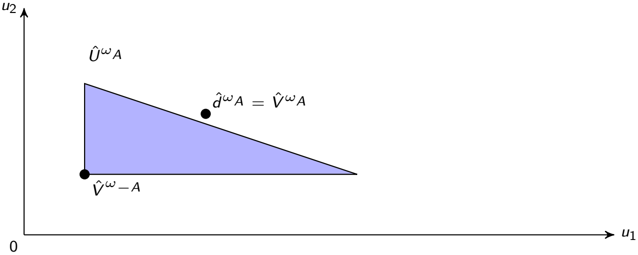
$$\hat{V}^{\omega_{A,A}} = (3,1), \quad \hat{V}^{\omega_{A,B}} = (3,3), \quad \Rightarrow \quad \hat{d}^{\omega_A} = \tilde{p}(6,4)$$
$$\hat{V}^{\omega_{-A,A}} = (3/2,1/2), \quad \hat{V}^{\omega_{-A,B}} = (1/2,3/2), \quad \Rightarrow \quad \hat{V}^{\omega_{-A}} = \tilde{p}(2,2)$$



What happens when 1^{st} issue arrives and is A?

Delay if 2^{nd} issue arrives quickly, $\tilde{p} > \overline{p}^L \in (0, 1/2)$

- Not worth giving up A as leverage if B might then arrive, expanding pie
  - $\overline{p}^L = 3/10$ given $z = 3$



- Give up $\hat{d}^{\omega_A} - \hat{V}^{\omega_{-A}}$ in continuation utility if reach agreement
- If frequent arrivals ($\tilde{\lambda} \to \infty, \tilde{p} \to 1/2$) compensating for this loss requires giving each player more than half Apple

$$\hat{d}^{\omega_A} - \hat{V}^{\omega_{-A}} = \tilde{p}(6,4) - \tilde{p}(2,2) = \tilde{p}(4,2) \to \Big(2, 1\Big) > \left(\frac{3}{2}, \frac{1}{2}\right)$$

Implication: Need for leverage in future negotiations can explain delay

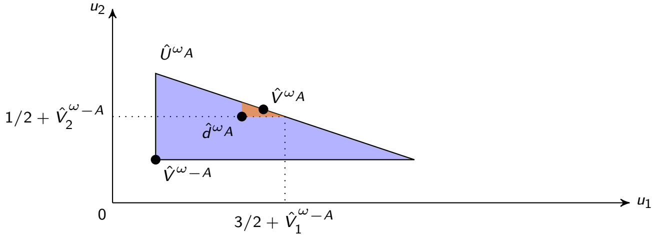
$\Rightarrow$ Delayed Dream Act

What happens when 1^{st} issue arrives and is A?

If 2^{nd} issue arrives slowly, low value P2 gets $> 1/2$ of Apple

- No delay if $\tilde{p} < \overline{p}^L$, Nash solution given feasible payoffs $N(\hat{U}^{\omega_A}, \hat{d}^{\omega_A})$
  - $\Rightarrow$ Divide residual Apple equally after compensating players for forgone utility from delay $\hat{d}_i^{\omega_A} - \hat{V}_i^{\omega_{-A}} = \tilde{p}(4,2)$

$$\hat{V}_2^{\omega_A} - \hat{V}_2^{\omega_{-A}} = \frac{1}{2}\left(1 + (\hat{d}_2^{\omega_{-A}} - \hat{V}_2^{\omega_{-A}}) - \frac{\hat{d}_1^{\omega_A} - \hat{V}_1^{\omega_{-A}}}{z}\right) = \frac{1}{2} + \frac{\tilde{p}}{3} > \frac{1}{2}$$



More surprising: Expanding utility frontier can *decrease* everyone's payoff!

- Initial payoffs can decrease in $\tilde{p}$ and in z for large z (=P1 value of A)
- Such expected payoffs weight for likelihood 1^{st} issue is A or B

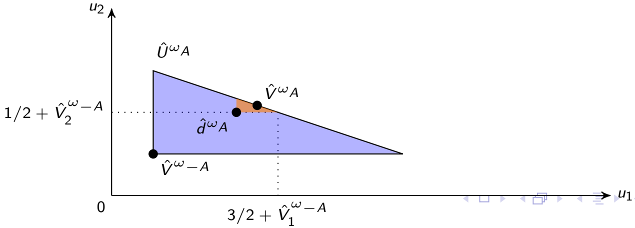
Why does P2 get $> 1/2$ of Apple?

Intuition: Share of Apple must compensate P2 for giving up logrolling leverage. Must get lots because Apple is not valuable for him

- Divide residual Apple equally after compensating players for forgone utility from delay $\hat{d}^{\omega_A} - \hat{V}^{\omega_{-A}} = \tilde{p}(4,2)$
- P1 forgone utility $\times 2$ as much as P2
  - Players benefit equally from delay if issue B arises (logrolling)
  - No benefit to delay if 2^{nd} issue is also A (split both equally)
- But Apple is worth $\times 3$ as much to P1 as P2 $\Rightarrow$ P2 must get more of it
  - P2 gives up more utility when normalized by value of Apple

$$\hat{d}_2^{\omega_A} - \hat{V}_2^{\omega_{-A}} > \frac{\hat{d}_1^{\omega_A} - \hat{V}_1^{\omega_{-A}}}{z}$$

$$\Rightarrow \hat{V}_2^{\omega_A} - \hat{V}_2^{\omega_{-A}} = \frac{1}{2}\left(1 + (d_2^{\omega_{-A}} - \hat{V}_2^{\omega_{-A}}) - \frac{\hat{d}_1^{\omega_A} - \hat{V}_1^{\omega_{-A}}}{z}\right) > \frac{1}{2}$$



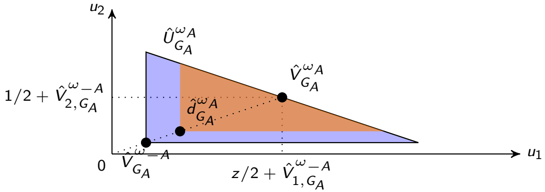
Independent committees G_A if 1^{st} issue is A

Players get $1/2$ of each fruit

- Intuitive! Will equally split any additional Apples, so split Apple now
  - No benefit from delay/logrolling

$$\hat{V}^{\omega_A}_{2,G_A} - \hat{V}^{\omega_{-A}}_{2,G_A} = \frac{1}{2}\left(1 + (\hat{d}^{\omega_{-A}}_{2,G_A} - \hat{V}^{\omega_{-A}}_{2,G_A}) - \frac{\hat{d}^{\omega_A}_{1,G_A} - \hat{V}^{\omega_{-A}}_{1,G_A}}{z}\right) = \frac{1}{2}$$

- $\hat{d}^{\omega_A}_{G_A} = \tilde{p}(z,1)$, $\hat{V}^{\omega_{-A}}_{G_A} = \tilde{p}(z,1)/2$



New justification for independent committees

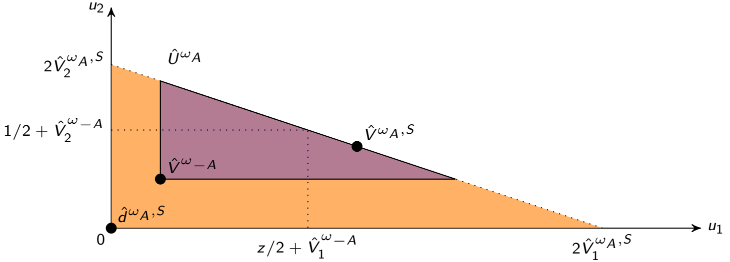
- Higher payoffs if (and only if) issues arrive infrequently: $1/2$ of favorite fruit better than less than $1/2$
- Prevents opponent holding it up for leverage in future negotiations
- Payoff difference can be large: $V_i^{\omega_0}/V_i^{\omega_0,I} \approx 0$

Stopping search if 1^{st} issue is A

Low value P2 gets $< 1/2$ of Apple

- $\hat{d}_i^{\omega_A,S} = 0$ and $\hat{V}^{\omega_{-A},S} = \tilde{p}(2,2) \Rightarrow$ No delay. Nash solution:

$$\hat{V}_2^{\omega_A,S} = \left\{ \frac{1}{2}\left(1 + \hat{V}_2^{\omega_{-A}} + \frac{\hat{V}_1^{\omega_{-A}}}{z}\right) = \frac{1}{2} + \hat{V}_2^{\omega_{-A}} - \tilde{p}\frac{2}{3} \right.$$



- P2 gets **no Apple** if $z > 2 + \sqrt{5}$ and 2^{nd} issue arrives quickly (efficient!)
  - Half orange surplus would be outside purple set (can't get ¡0 Apple)
- Always more efficient than independent committees
- More efficient than logrolling if issues arrive slowly or $z > 2 + \sqrt{5}$

Implication: Contradicts typical negotiation advice to always search for ways to grow pie

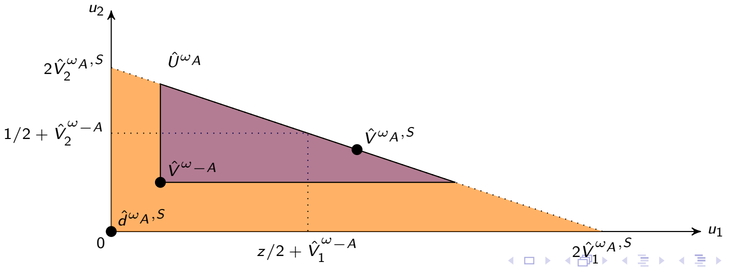
Why does low value P2 get $< 1/2$ of Apple?

Intuition: P2 more desperate for agreement o.w. no Banana can arrive

- P2 can't use A as leverage in future logrolling, $\hat{d}^{\omega_A,S} = (0,0)$
- P1 get $\times 3$ more payoff than P2 in Nash solution.
  - Impossible if each got $1/2$ Apple $\Rightarrow 3/2 + \hat{V}_2^{\omega-A} < 3(1/2 + \hat{V}_2^{\omega-A})$ since players get same post agreement cont. payoff $\hat{V}^{\omega-A} = \tilde{p}(2,2)$:
- P2's post agreement cont. payoff$>$P1's when normalized by Apple's value

$$\hat{V}_2^{\omega-A} > \frac{\hat{V}_1^{\omega-A}}{z}$$

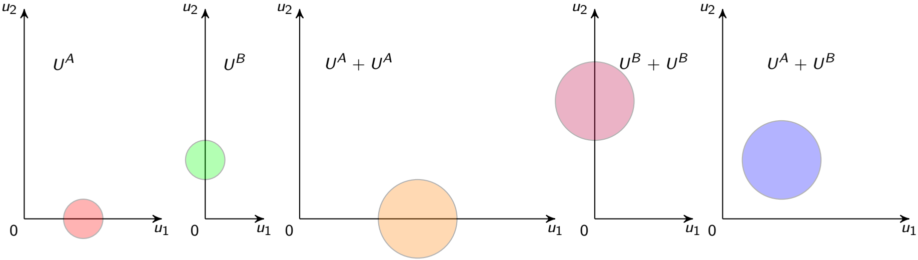
$$\hat{V}_2^{\omega_A,S} - \hat{V}_2^{\omega-A} = \frac{1}{2}\left(1 - \hat{V}_2^{\omega-A} + \frac{\hat{V}_1^{\omega-A}}{z}\right) < \frac{1}{2}$$



Findings extend

Stationary eq. payoffs: Stopping search>Independent Committees>Logrolling if

- Symmetric, 2 types but many issues, arbitrarily positive utility sets $U^\theta \cap \mathbb{R}^2_{++} \neq \emptyset$, arbitrary correlation and *infrequent arrivals*

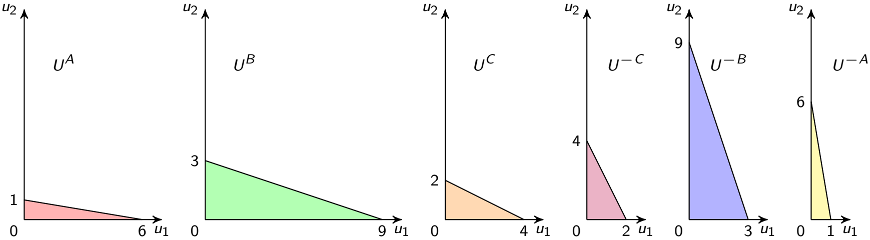


- $\lambda^{\omega_{\theta'},\theta,\omega_\theta} \approx 0$: small arrival rate of type $\theta' \in \{A, B\}$ as 2^{nd} issue when 1st issue was type θ
- $\lambda^{\omega_{\theta''},\theta',\theta,\omega_{\theta'},\theta} \approx 0$: small arrival rate of type $\theta'' \in \{A, B\}$ as 3^{rd} issue when first two issues were types θ and θ'

Findings extend

Stationary eq payoffs: Stopping search$>$ max$\{$Independent Committees,Logrolling$\}$ if

- Symmetric, many types (all pie divisions), many issues and *infrequent arrivals*

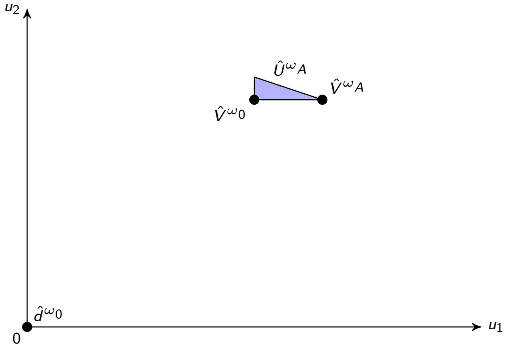


- 1^{st} issue arrival rate of type k and $-k$ equally likely
- 2^{nd} issue arrival rate small and equally likely type k and $-k$ likely if first issue type $k' \notin \{k, -k\}$
- 3^{rd} issue arrival rate small

Findings extend

Stationary eq payoffs: Stopping search is efficient $>$ max{Indep. Committees,Logrolling}

- Symmetric, many types (all pie divisions), *infinite issues* and *frequent arrivals* independent of state
  - Player with higher value of pie gets all of it (as lower normalized cost of delay)
  - Although also folk theorem with infinite issues and non-stationary eq



Endogenous search

- General finding that restricting (stopping) search during bargaining can be valuable:
  - Stops player extracting surplus on issues which her opponent values more, by threatening delay
  - Makes her more desperate for agrement (so can move onto finding issues he values)
- *So will players choose to restrict their search?*

Endogenous search

Adapted Arrivals stage

- Players simultaneously choose search efforts $e_i \in [0,1]$ at cost $c_i e_i \Delta \geq 0$
- Efforts $e = (e_1, e_2)$ rescale new issue arrival probability by $K(e) \in [0,1]$
  - State $\overline{\omega} \neq \omega$ arrives with probability $q^{\omega,\overline{\omega}} K(e)$
  - $K(e)$ =production function, where $K(1,1) = 1$, $\partial K(e)/\partial e_i > 0$, $\partial K(e)/\partial e_i \leq 0$
  - $q^{\omega,\overline{\omega}} = 1 - e^{-\lambda^{\omega,\overline{\omega}}\Delta}$ as previously
- *Stationary eq:* Player i exerts effort e_i^{ω}

Endogenous search

- Effort $e \Rightarrow$ Disagreement payoffs $d^{\omega,e}$ (U^{ω} independent of e) $\Rightarrow$ Bargaining payoffs $V^{\omega,e}$
- Cont. payoff at arrivals stage if deviate to $\tilde{e}_i$ for one period:

$$\underline{V}_i^{\omega,e}(\tilde{e}_i) = -\Delta c_i \tilde{e}_i + \delta\Big(V_i^{\omega,e} + K(\tilde{e}_i, e_j) \sum_{\overline{\omega} \neq \omega} q^{\omega,\overline{\omega}}(V_i^{\overline{\omega}} - V_i^{\omega,e})\Big)$$

- In equilibrium such deviations shouldn't be profitable

Strong effort incentives for small costs

- If $e_i < 1$ optimal given $c_i = 0$ then i's payoff decreases when new issue arrives!
  - Demanding condition as new issues expand feasible utilities

$$\frac{\partial \underline{V}_i^{\omega,e}(e)}{\partial \tilde{e}_i} > 0 \quad \text{iff} \quad \frac{\sum_{\overline{\omega}\neq\omega} q^{\omega,\overline{\omega}} V_i^{\overline{\omega}}}{\sum_{\overline{\omega}\neq\omega} q^{\omega,\overline{\omega}}} > V_i^{\omega,e}$$

- Cont. payoff at arrivals stage if deviate to $\tilde{e}_i$ for one period:

$$\underline{V}_i^{\omega,e}(\tilde{e}_i) = -\Delta c_i \tilde{e}_i + \delta\Big(V_i^{\omega,e} + K(\tilde{e}_i, e_j)\sum_{\overline{\omega}\neq\omega} q^{\omega,\overline{\omega}}(V_i^{\overline{\omega}} - V_i^{\omega,e})\Big)$$

- Similar logic if $c_i \approx 0$

Back to baseline example: 2 issues, 2 types

- Same setup as before, but now endogenous effort:
    - *Key finding:* for most parameters unique equilibrium. Maximal effort $e = (1, 1)$, payoffs match exogenous search with logrolling

Back to baseline example: 2 issues, 2 types

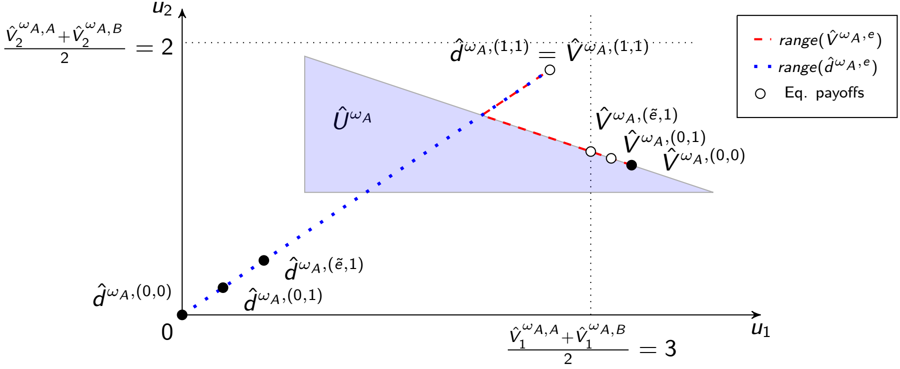
- Same setup as before, but now endogenous effort:
  - *Key finding:* for most parameters unique equilibrium. Maximal effort $e = (1, 1)$, payoffs match exogenous search with logrolling

Result: For all sufficiently small c_i, in all stationary eq as $\Delta \to 0$:

- Both players must exert maximal effort $e_i = 1$ if no available issues
- If 1^{st} issue is Apple
  - Low value P2 must exert maximal effort $e_2 = 1$
  - Eq exists where P1 exerts maximal effort $e_1 = 1$
  - Eq is unique if $K(0, 1) \geq 1/2$ *or* arrival rates are slow *or* fast
  - If $K(0, 1) \approx 0$ and intermediate arrival rates, Eq exists with $e_1 = 0$ and payoffs $\approx$ stopping search
    - Efficiency lower than $e_1 = 1$ if $z < 2.41$, higher if $z > 4.23$
    - Coordination on inefficiently high or low P1 effort
  - If $K(0, 1) < 1/2$ then for some $(z, \tilde{\lambda})$ Eq exists where $e_1 = 0$

Multiple eq. with endogenous effort if 1^{st} issue Apple

- $z = 3$, intermediate arrival rate, $\tilde{p} = 9/20$, small $K(0,1) = 1/81$
- If $e = (1,1) \Rightarrow$ large $\hat{d}^{\omega_A,e}$, new issue arrival benefits both players
- If $e = (0,1) \Rightarrow$ small $\hat{d}^{\omega_A,e}$, new issue arrival harms P1



Higher payoffs with larger search costs

- Linear search production $K(e) = \tilde{K}(e_1 + e_2) + (1 - 2\tilde{K})e_1 e_2$
- Cost of effort $c_i = \sqrt{\tilde{K}} > 0$
- **Observation:** If $\tilde{K} \approx 0$ then exists eq. with stopping search behavior (effort iff no issues currently available)
  - Positive search costs + strong complementarities in search
  - $\Rightarrow$ Larger search costs can increase payoffs
- But terrible Eq. also exist with no search in any state
- If $-i$ doesn't search then nor will i

$$\underline{V}_i^{\omega,e}(\tilde{e}_i) = -\Delta c_i \tilde{e}_i + \delta\left(V_i^{\omega,e} + K(\tilde{e}_i, e_j) \sum_{\overline{\omega} \neq \omega} q^{\omega,\overline{\omega}}(V_i^{\overline{\omega}} - V_i^{\omega,e})\right)$$

Conclusion

Please order lots of Apples on your Bananaphone, then eat them!