A Theory of FAQs: Public Announcements with Rational Ignorance*

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Abstract

We study a model where one information sender communicates with many information recipients. The sender provides a public good in the form of an announcement. The announcement involves a set of answers to some potential queries (e.g., frequently asked questions (FAQs), product manuals, and user guides). The sender also provides a private good in the form of a private communication service. Information recipients learn about their heterogeneous query and the size of the FAQs. The recipients then decide whether or not to consult the FAQs, and, when necessary, purchase the private communication service, the price of which is ex-post determined by the number of people who purchase the service at the same time. It is difficult to achieve efficiency in this model, when the queries are not observable by the sender. The inefficiency can be summarized by an under-provided public good (i.e., FAQs) and an overpriced private good (i.e., private communication) in equilibrium. A marginal change in the private communication capacity does not affect the equilibrium size of the FAQs.

1 Introduction

L.A. Care Health Plan, a Medicaid Health Insurance provider for Los Angeles County residents, offers $10 to members who attend their orientation class.[1] Since the orientation

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[1] An example of such an offer can be found here.

class only covers rudimentary information that could be obtained online at no cost, it may not make sense for L.A. Care to hold such an orientation class, because that action is costly for both L.A. Care and its members, who spend time and energy attending it. However, we could infer that L.A. Care finds that informing its members is worth more than the $10 offered to them. Perhaps this is because L.A. Care would spend more than $10, on average, to individually handle members' queries that could have been covered in their online FAQs, information brochures, etc.

In general, efficiency is difficult to achieve via a public announcement or by any directed communication from one to many. Even though the government provides guidelines to help citizens cope with the Affordable Care Act, a customer service center in an online retail store provides online FAQs, product manuals articulate how to use new products, software developers provide user guides for those who seek help, and instructors of large classes provide a course syllabus, information senders still suffer from responding to queries made by uninformed individuals: Citizens directly contact the service agent to obtain information about their health plan, customers send emails without skimming the FAQs, the users of a product or service press 0 when calling the customer service center, the users of computer applications post a new online thread on a topic that has already been documented in the relevant manual (Novick and Ward, 2006), and students ask when the final exam is going to be held.

The downside of the public announcement is that the 'larger' announcement (in terms of its volume and/or details) requires the information recipients to pay more attention to the announcement. Although answers to the most typical queries may be found in the large announcement, it would take too much time and energy for the recipients to figure out the relevant answers if that announcement is too large. This is not only a problem from the perspective of the information sender, whose profits or performances significantly depend on the information recipients' overall experience,[2] but also a problem of the information recipients. The inefficiency of this communication causes those who have a specific legitimate inquiry to incur a cost of some kind: Customers who find that the announcement doesn't cover the answer to their question must contact the information sender and wait for some time (perhaps an inordinately long time) to obtain a response.

To describe the inefficiency of communications, such as those mentioned previously, we study a two-channel information-provision model with one information sender and many

[2] According to some business research, a 10% improvement in an enterprise company's customer experience score can translate into more than 1 billion dollars in increased revenue. See https://www.business2community.com/strategy/7-ways-customer-service-can-support-sales-0914164.

information recipients. The recipients can obtain relevant information from two different information channels offered by the sender: one public and the other private. To be more specific, the information sender provides a public good in the form of an announcement or FAQs. The announcement is a set of answers to some potential queries. The sender also offers a private communication service with a limited capacity. The information sender's goal is to determine the size of the public announcement that will minimize the overall communication cost. Information recipients draw a heterogeneous query from a known distribution. Their goal is to obtain an answer to their query. In the case of the public announcement channel, the cost of acquiring information (i.e., the time and energy spent in using the information to get an answer to their query) increases with the size of the announcement, that is, in the quality of the information contained therein.[3] Moreover, it is possible that the recipients fail to find the answers to their queries in the announcement. Thus, they may make use of the private communication channel, either in place of reading the announcement or in addition to doing so. The function that represents the probability that an information recipient will use the private channel, which is equal to the complement of the probability that the level of informativeness of an announcement of a given size will provide the answer to a certain query, is commonly known. When using the private communication channel, all recipients definitely find the answers to their queries. However, the cost per user to the private communication service is determined ex-post and increases with the number of private-channel users. This is due to the information sender's limited capacity for private communication. Hence, the private communication channel has cost externalities and the information recipients would use the private communication channel without knowing the ex-post price of acquiring the information. The information recipients' objective is to obtain information at a minimum cost.

We show that, in the Bayesian Nash equilibrium for this model, the public good (i.e., the public announcement or the FAQs) is under-provided and the private good (i.e., the private communication service) is overpriced, when compared to the situation where the information recipients' queries can be observed. A customer who has a specific nontrivial question, for example, may not find the answer to it in the FAQs, because the FAQs may not cover it well enough, and he may have to wait for an inordinately long time to be connected to a customer service agent and obtain an answer to his question.

The main driving force behind the inefficiency can be analogously compared to that of

[3] In the model, we implicitly assume that the information sender is always doing her best. That is, she has already streamlined the announcement to the point where it provides the most information possible for its size.

standard games in the provision of public goods. The larger the size of the announcement, with all other things being equal, the lower the expected price per subscriber to the private communication service. Thus, some information recipients may want to enjoy the luxury of the potentially cheaper private communication, even though their queries are likely to be covered by the announcement. This rational ignorance eventually leads the information sender to be reluctant to provide a public announcement of higher quality.[4] It is also worth noting that this under-provision of public goods turns out to be analogously connected to the underlying mechanism of free-riding behavior in the voluntary contributions of public goods.

We also find that the size of the FAQs should be *unchanged* and remain under-provided, even if the capacity of the private communication channel marginally changes. The increase in the capacity leads to the decrease in the cost of the private communication service, and the effect of the increased capacity is equally dissipated to all the information recipients who use the private communication service. Therefore, the marginal changes in the capacity scale up/down the objective function of the information sender, while the minimizing argument of the objective function, the size of the FAQs, would not be affected.

As regards the welfare of the information recipients, we find that it is not straightforward enough to identify who benefits and who suffers, though the overall communication cost is larger when the information recipients' queries are unobservable.

The rest of the paper is organized as follows. In the following subsection, we discuss the related literature. Section 2 describes the model. Section 3 describes an equilibrium and analyzes some of the properties. In Section 4, we conduct a welfare analysis. In Section 5, we discuss the marginal changes in the capacity for private communications. Section 6 concludes and discusses future work. Proofs of some of the lemmas and propositions that appear in the main text are provided in the Appendix.

1.1 Related Literature

Our paper is related to many private information acquisition studies (Angeletos and Pavan, 2007; Maćkowiak and Wiederholt, 2009; Myatt and Wallace, 2012). One of the most closely related studies is Colombo et al. (2014), which considered a model where agents take an action that has a payoff externality with the actions of others and a public signal is

[4] Novick and Ward (2006) interviewed 25 subjects to investigate why few users of computer applications seek help from the documentation, and some of the reasons are "difficulty of navigation (too basic to be useful)" and "inappropriate level of detail." We claim that these are indispensable features of the two-channel information-provision from one sender to many recipients.

given. In that paper, agents can choose the accuracy of their private signal before taking an action. They showed that public information and private information can substitute for each other. More specifically, the higher the accuracy of the public signal provided by a policy maker, the lower the precision of the private information acquired. Moreover, the strength of this relationship depends on that of the substitution effect. Similarly, in our paper, the information recipients decide whether or not to use the public announcement channel. In this way, private communication can be regarded as a substitute for the public announcement.

Our paper differs from Colombo et al. (2014) in many respects. First, in our paper, the information recipients can decide whether or not to ignore the public information and directly use the private communication channel. In Colombo et al. (2014), public information is always given to the recipients. In addition, in our paper, the quality of the public announcement is determined by the information sender, and the private communication channel guarantees that the correct answer will be given to the recipients. Moreover, the cost of the private communication channel is determined by the congestion externality, which makes for a strategic situation. Finally, in our paper, an increase in the size of the public announcement does not necessarily imply a decrease in the number of private channel users, because the cost of acquiring information from the public announcement increases with the size of it.

In the sense that some of the information recipients ignore the public announcement channel, this study is related to the literature on the rational inattention of consumers (Bordalo et al., 2016; de Clippel et al., 2014; Dessein et al., 2016; Sallee, 2014). However, a distinctive difference between this paper and the rational inattention literature is that those who choose not to pay attention to the public announcement do so, not because they have a limited capacity for digesting multi-dimensional information, but because they are rational enough to intentionally avoid the cost of acquiring information from the public announcement. In this regard, this paper is different from the literature on the rational inattention of decision-makers who determine how much to reduce uncertainty on the information flow. Since Sims (1998, 2003) started quantifying information as a reduction in uncertainty, where uncertainty is measured by entropy, the rational inattention has significantly impacted the studies of price setting problems (Reis, 2006; Maćkowiak and Wiederholt, 2009; Woodford, 2009; Matějka, 2016), consumption savings problems (Luo, 2008; Tutino, 2013; Maćowiak and Wiederholt, 2015), and portfolio choice problems (Van Nieuwerburgh and Veldkamp, 2009; Mondria, 2010). In this paper, however, decisions made by economic agents in this paper have nothing to do with reductions in the uncertainty of information flow. Rather, we call the problem that information recipients face in our model rational ignorance.

Although this paper deals with a model where one information sender sends a message to many recipients, it is the recipients who have the private information. This is different to the cheap-talk models (Farrell and Gibbons, 1989; Caillaud and Tirole, 2007; Goltsman and Pavlov, 2011), where the message sender knows more about the true state than the recipients do. Another difference is that it is costly to transfer information from the information sender to the information recipients in our model. In the cheap-talk models, the transfer of information is free. Moreover, in the cheap-talk models, the sender is biased and wants the receiver to take an action in the desired direction of the sender, while in our paper, the sender wants the recipients to obtain the exact information they need.

Communication costs have been dealt with in some studies (Loder et al., 2006; Evans, 2012; Potters, 1992), but, to the best of our knowledge, our study is the first that incorporates the idea of communication costs into the consumption of an announcement. In the sense that the information sender chooses the size of the announcement, this paper could also be remotely related to papers such as Bental and Spiegel (1995) that addresses quality-control problems under unknown customer types.

2 The Model

In this section, we consider a model with $n+1$ players, indexed by $i \in \{0, 1, \dots, n\}$. When necessary, we call player 0 the information sender and the other players the information recipients. For the sake of pronominal clarity, the sender is female and the recipients are male. The number of players is commonly known.

2.1 The Information Provider

Player 0 produces a public good in the form of a public announcement at no cost. A public announcement, or the FAQs, is a set of information containing answers to potential queries which could be made by heterogeneous recipients. A public announcement covers only one dimension (one category of information) but varies in its size $a \in \mathbb{R}_+$.[5] By choosing the size of

[5]A unidimensional announcement may not adequately describe a typical announcement that covers answers to multidimensional questions (e.g., online FAQs, course syllabi, user guides, and handbooks). A general extension to this setup could be represented by a multidimensional unit ball, where each dimension

the announcement, the information provider can either only deal with some general issues or cover more detailed ones. The announcement is non-excludable and non-rivalrous, so by definition, it is a public good.

At the same time that the sender makes the announcement, she makes an offer of a private good in the form of a private communication. The sender answers all the private queries—and does so with a limited and fixed capacity K. Regardless of the specifics of the recipients' queries, she treats all the recipients equally. From the perspective of the sender, the cost of operating the communication service is fixed.

The sender's objective is to minimize the expected total communication cost incurred by each information recipient (hereinafter referred to as the total communication cost), by choosing an optimal size of the announcement. There are two reasons why the information sender is benevolent, or cares about, the total communication cost incurred by the recipients. Firstly, in many situations, the sender's potential benefits could depend on the information recipients' overall experience. In the retail sector, for example, the quality of customer care affects future profits.[6] Another example is that a government agency may want to signal citizens that a certain policy change is being managed efficiently—and to inform them of it at a low communication cost.

Secondly, this objective serves as a natural restriction that prevents the sender from intentionally increasing the cost of the private communication service. If, as an alternative, the sender's objective was to minimize her own communication cost, making everyone pay attention to the announcement by simply shutting down the communication service or making it harder to reach the service could solve the problem. Still another example is that the IRS may want to help taxpayers improve their level of tax compliance, by lowering the overall cost of finding answers regarding the filing of tax returns.

2.2 The Information Recipients

The recipients are heterogeneous in terms of the specifics of their queries. The query of recipient i is measured by the degree of the specifics, q_i, which is randomly drawn from a commonly known distribution F on $\mathbb{R}_+$ after the information sender sets the size of the public announcement. The probability density function for F, $f(q)$, is continuous and weakly decreasing in q, that is, the query would more likely be simple and general than complicated

represents a different category and the distance from the origin represents the degree of the specifics. This extension doesn't change the main conclusions of this study.

[6]See, for example, Sulek et al. (1995) that report customer satisfaction significantly affected sales performance.

and specific. Each recipient's goal is to find a complete answer to his query at a minimum cost.[7] Paying attention to the announcement is costly. The cost of attention, $C : \mathbb{R}_+ \to \mathbb{R}_+$, is continuous, increasing, and convex in the size of the announcement, with $C(0) = C'(0) = 0$.

Even if a recipient pays attention to the announcement, and the announcement may contain the answer to his query, there is still the chance that he may consult a private communication service. This could either be because the recipient's query is too specific to be answered by the FAQs or because the recipient unluckily misses the relevant answer in the FAQs. That is, there are some chances that the recipient with query q pays attention to the FAQs of size a, but does not have the query answered. The probability function $P : \mathbb{R}_+^2 \to [0,1]$ represents this chance. $P(a,q)$ is the probability that the recipient with query q reads the FAQs of size a but the query goes unanswered. It is decreasing and convex in a and increasing in q. That is, given the query q, the recipient is more likely to obtain an answer with a larger announcement, and given the announcement size a, the recipient is less likely to obtain an answer to a more specific query. Hence, it is natural to set $\lim_{a\to 0} P(a,q) = 1$ for all $q > 0$ and $\lim_{q\to 0} P(a,q) = 0$ for all $a > 0$. No matter what q is, if there are no announcements, the probability of getting answered by the announcement approaches zero, and no matter what a is, if the query is extremely general, the probability of getting answered by the announcement approaches 1. We also assume that $\lim_{a\to\infty} P(a,q) = 0$ and $\lim_{q\to\infty} P(a,q) = 1$. Hence, if the announcement is extremely detailed, the probability that any query is answered by the announcement approaches 1,[8] and if the query is extremely specific, the probability that any announcement answers the query approaches zero. Furthermore, $\lim_{q\to\infty} \frac{\partial P(a,q)}{\partial a} = 0$ for any finite a, which means that if a query is very unlikely to be covered by the announcement, any marginal change in the size of the announcement does not help. Two of the simple functional forms that satisfy those assumptions are $e^{-a/q}$ and $1 - \frac{2}{\pi}\arctan(a/q)$. We designed this $P(a,q)$ to capture many of the important heterogeneities in a parsimonious way.[9]

[7]We implicitly assume that the utility gain from obtaining the answer is large enough and that dropping out of this game is not in his interest. We also exclude the possibility that he will hire a "private consultant" to deal with his query (e.g., hiring a CPA instead of following the instructions given by the IRS for a complicated tax filing), or will organize a "forum" to circumvent the use of the private communication service provided by a firm.

[8]It could be more natural to assume that there is no complete announcement that answers everyone's specific question, that is, that $\lim_{a\to\infty} P(a,q) > 0$. This modification, however, does not change the main findings for the model.

[9]Alternatively, we could consider $C(a,q)$ increasing in q, which makes sense, because the more specific the query, the more they are required to read the fine print. Since both $P(a,q)$ and $C(a,q)$ increase the expected cost to read the announcement, technically, we can draw the same interpretation. We admit that $C(a,q)$ increasing in q may be easier to explain if a recipient with a low q does not need to read the full

The use of the private communication service is *ex-post* costly. Unlike the announcement, where the cost of acquisition is ex-ante known, the price of this private good is unknown at the time it is used. It features a congested good, so the price (waiting time, or service coarseness per unit of time in some contexts) increases linearly with the number of recipients who use the private communication channel. More specifically, the cost charged to each recipient is $t(d) = \kappa d$, where $\kappa = 1/K > 0$ is a known parameter and d is the number of information recipients who purchase the private communication service.[10] Note that the cost of the private communication service is decreasing in the information sender's capacity, K. In the sense that the information sender uses the whole capacity K for providing private communication services, K could be interpreted as a fixed amount of resources spent for maintaining the private communication channel.

The trade-off from the perspective of recipient i is summarized as follows: Paying attention to the public announcement will decrease his chances of purchasing the private communication service, but he will incur another cost if the announcement is not detailed enough to cover his specific query. On the contrary, not paying attention to the announcement is optimal if no one else uses the private communication service, while the price of using the private communication service could be substantial if a number of other players believed and behaved in the same way. The sequence of play is illustrated in Figure 1.

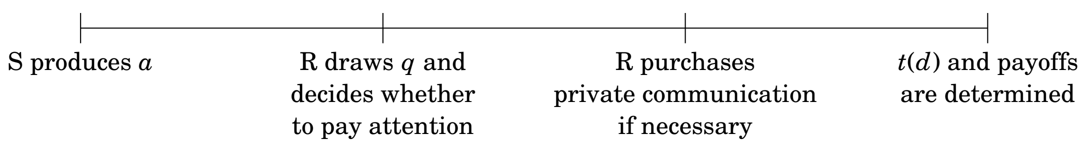

Figure 1: The Sequence of Play

Although the mechanism of the model is quite different to the voluntary contribution mechanism (VCM) of the public goods provision, it is worth mentioning the relationship between our model and that of the VCM. Many features in our model correspond to those in the VCM. In the VCM, the agents allocate their resources for the consumption of the

announcement. For example, when FAQs are sorted by the frequency of questions, a recipient with a low q can find his answer by spending a smaller amount of time and energy. Both functions $P(a,q)$ and $C(a,q)$ would be complicated, as all the results would depend on the shape of those two functions.

[10] This specification is provided for its analytical simplicity. It can be interpreted as an (approximate) average time for a recipient's query to be handled by private communication. Suppose, for example, that a concierge receives d queries simultaneously and handles them in a random order. If it takes one unit of time to handle each query, then the average waiting time for a recipient who approaches the concierge, together with $d-1$ other recipients, would be $\sum_{i=1}^{d} i/d = (d+1)/2$. The difference between $(d+1)/2$ and $d/2$ is 1/2, which is small compared to $d/2$, as d gets larger. Thus, in this case, $t(d) \approx \kappa d$ with $\kappa = 1/2$.

private goods and the production of the public goods, where the latter involves a positive externality. In our model, agents spend their resources on the consumption of the public goods and the determination of the cost of the private goods, where the latter involves a negative externality (Table 1). This juxtaposition would be helpful to intuitively evaluate the results regarding the under-provision of the public good (the FAQs).

	Voluntary Contribution Mechanism	Model
Players	allocate resources to	spend money on
(Object 1)	the consumption of the private good	the consumption of the public good
	of which value is ex-ante known, and	of which cost is ex-ante known, and
(Object 2)	the production of the public good	the consumption of the private good
	of which value is ex-post determined,	of which cost is ex-post determined,
without knowing	who contributes how much in Object 2	how many people purchase Object 2
with knowing	a positive externality on Object 2.	a negative externality on Object 2.

Table 1: Juxtaposition of the model with the VCM of the public goods

3 Analysis

3.1 Optimum: Complete Information Over the Queries

Consider the first-best situation, where the information sender can observe the values of q for all the recipients and can provide them with guidance on whether to pay attention to the announcement. More specifically, a cutoff q^o will work as a threshold. In this way, if $q_i > q^o$, the sender instructs recipient i to ignore the announcement; otherwise, she urges him to pay attention to it.

The optimal announcement quality a^o and the optimal cutoff q^o solve the following optimization problem:

$$(a^o, q^o) \in \arg\min_{(a,q)\in\mathbb{R}_+^2} F(q)C(a) + \kappa n \left(\int_0^q P(a,x)f(x)dx + \int_q^\infty f(x)dx \right), \tag{1}$$

where $\int_0^q P(a,x)f(x)dx + \int_q^\infty f(x)dx := X(a,q)$ is the probability that a recipient will eventually use the private communication service, and $nX(a,q)$ is the expected number of recipients using the private communication service. Since the cost of the private communication service is ex-post determined by $nX(a,q)$, $\kappa nX(a,q)$ is an individual's expected cost of using the private communication service. The objective function represents the expected communication cost for one recipient.

In summary, a recipient with query $q_i \le q$ pays attention to the announcement, that is, he pays $C(a)$ with probability $F(q)$. If the query is not answered by the announcement with probability $P(a,q_i)$, he uses the private communication service with the other recipients who use the same service.

Since $C(a)$ is increasing and convex, and $\kappa n X(a,q)$ is decreasing and convex in a, the minimization problem will have a unique solution. The derivatives of the objective function with respect to a and q are:

$$\begin{aligned} a: &\quad F(q)\frac{dC(a)}{da} + \kappa n \int_0^q \frac{\partial P(a,x)}{\partial a} f(x)dx \\ q: &\quad C(a)f(q) + \kappa n f(q)(P(a,q)-1). \end{aligned}$$

If the solution is an interior point:

$$F(q^o)\left.\frac{dC(a)}{da}\right|_{a=a^o} + \kappa n \int_0^{q^o} \left.\frac{\partial P(a,x)}{\partial a}\right|_{a=a^o} f(x)dx = 0 \tag{2}$$

$$C(a^o) + \kappa n \left(P(a^o,q^o)-1\right) = 0 \tag{3}$$

Note that if the support of F is bounded above by $\overline{q}$, it is possible to have a corner solution, where $q^o = \overline{q}$, if n is sufficiently large. The intuition for this corner solution is straightforward. Since the price of using the private communication service increases with the number of information recipients who use it, the total communication cost can be reduced if the information sender urges every recipient to pay attention to the announcement.[11] To avoid this outcome, we'll consider the support of F to be unbounded on $\mathbb{R}_+$. Also, $(a,q) = (0,0)$ satisfies the first-order conditions, so it could actually minimize the total communication cost—for example, in a case where either n or κ is sufficiently small, that is, if there are few information recipients and an abundant private communication capacity. However, in the opposite case, where both n and κ are large, making no announcement at all (i.e., the case where $a = 0$) does not minimize the total communication cost, but maximizes it. Throughout this paper, we restrict our focus to cases where both n and κ are large, that is, where the number of information recipients is large and the capacity of the private communication

[11] For example, if $F(q)$ is a uniform distribution on [0,1] and the functional forms are specified as $P(a,q) = \frac{q}{1+a}$ and $C(a) = a^2$, then $a^o = \frac{\sqrt{4+5\kappa n}-2}{5}$ and $q^o = \frac{4a^o(1+a^o)^2}{\kappa n}$ without consideration of boundary conditions. One can check that $q^o > 1$ for any $n \ge 1$ and $\kappa > 0$, and therefore q^o is bounded above by 1.

service is limited. Simply put, we consider a typical business situation, where the number of customers who need assistance is substantially larger than the number of customer service agents.

The optimal announcement size, a^o, and the expected cost of using the private communication under the first-best setup, $\kappa n X(a^o, q^o)$, will be used as comparison point to show how information asymmetry leads to inefficiency. In the next section, we will examine a situation where the information sender does not observe the information recipients' queries.

3.2 The Recipients' Bayesian Game

We will now describe a symmetric Bayesian Nash equilibrium among the recipients in which the announcement size is fixed at some $a > 0$. Player i who learns the degree of the specifics of his query, q_i, and the announcement size a would have a threshold $q^*(a)$. That is, player i will pay attention to the announcement if $q_i \leq q^*(a)$, and will not pay attention otherwise. A Bayesian Nash equilibrium is defined as a strategy profile (either paying attention to the announcement or ignoring it based on q_i and $q^*(a)$), and the beliefs specified for each player about the types of the other players (that everyone else will follow the threshold strategy) that maximizes the expected payoff for each player, given their beliefs about the other players' types and given the strategies played by the other players.

Consider one recipient's decision threshold $\tilde{q}(a)$, when everyone else is acting according to $q^*(a)$. In order for this threshold rule to be an equilibrium, a recipient with $q^*(a)$ should be indifferent between paying attention to the announcement at $\tilde{q}(a)$ and not doing so:

$$C(a) + P(a, \tilde{q}(a))\kappa\left(1 + (n-1)X(a, q^*(a))\right) = \kappa(1 + (n-1)X(a, q^*(a))), \tag{4}$$

where $X(a, q^*(a)) := \int_0^{q^*(a)} P(a,x)f(x)dx + \int_{q^*(a)}^{\infty} f(x)dx$. The left-hand side of Equation (4) is the expected cost if a recipient pays attention to the announcement, while the right-hand side is the expected cost if he ignores it. $X(a, q^*(a))$ is the expected proportion of recipients who will purchase the private communication service if everyone follows the threshold rule. We restrict our attention to situations where the number of recipients is large enough that consuming the announcement is cheaper than using the private communication service if everyone else uses it. This case is summarized in Assumption 1.

Assumption 1. *n is sufficiently large. There exists $n(a)$ such that $C(a) = \lim_{q\to 0} \kappa(1 + (n(a) - 1)X(a, q)) = \kappa n(a)$, and $n > n(a)$.*

Rearranging Equation (4) yields:

$$1 - P(a, \tilde{q}(a)) = \frac{C(a)}{\kappa(1 + (n-1)X(a, q^*(a)))}, \tag{5}$$

which illustrates the existence of such an equilibrium.[12] Given a and n, the left-hand side of Equation (5), which represents the informativeness of the announcement (the probability that the announcement contains the answer to q) is monotone decreasing in q. The right-hand side, which represents the ratio of the cost of paying attention (to the announcement) to the expected cost of the private communication service, is monotone increasing in q, since $X(a,q)$ is decreasing in q.[13] In addition, by Assumption 1, $\lim_{q\to 0} 1 - P(a,q) > \lim_{q\to 0} \frac{C(a)}{\kappa(1+(n-1)X(a,q))}$. Therefore, there exists $\tilde{q}(a) > 0$, such that Equation (5) holds with $\tilde{q}(a) = q^*(a)$. Moreover, except for $a = 0$, there exists a unique $q^*(a)$ that satisfies Equation (5). For $a = 0$, when there is no announcement, $q^*(a)$ is indeterminate, since any q will satisfy Equation (5); in that case, we set $q^*(a) = \infty$. That is, every customer will check that there are no FAQs (and hence, there will be no cost). In a symmetric equilibrium, $\tilde{q}(a) = q^*(a)$.

Figure 2 illustrates how q^* varies with a and n, if the functional forms and parameters are specified as $P(a,q) = e^{-a/q}$, $C(a) = a^2$, $a \in \{0.3, 0.5\}$, $\kappa = 1/30$, $f(q) = e^{-q}$, and $n \in \{30, 60\}$. The shape of the figure is consistent with our intuition: As n increases, a larger proportion of recipients will pay attention to the announcement, because the cost of the private good will be substantial. For a given population size n, the larger the value of a, the less likely it is that the recipients will pay attention to the announcement, because the marginal cost of attention is increasing in a. The dependence of $q^*(a)$ on a, when $n \in \{30, 60\}$, is depicted in Figure 3.

Lemma 1. $\frac{\partial q^*(a,n)}{\partial n} > 0$ *and* $\frac{\partial q^*(a,n)}{\partial a} < 0$.

Proof: See Appendix.

These observations in Equation (4) and Lemma 1 make the information sender's problem nontrivial. In other words, decreasing the size of the announcement will prompt more recipients to pay attention to it. However, that reduces the chance that the announcement

[12] Since $P(a,q)$ is monotone in q, we have $\tilde{q}(a)$ explicitly. $\tilde{q}(a) = P^{-1}\left(a, 1 - \frac{C(a)}{\kappa(1+(n-1)X(a,q^*(a)))}\right)$, where $P^{-1}(a,x)$ is the inverse function of $P(a,q)$.

[13] $\frac{\partial X(a,q)}{\partial q} = \frac{\partial}{\partial q}\left[\int_0^q P(a,x)f(x)dx + \int_q^\infty f(x)dx\right] = P(a,q)f(q) - f(q) = [P(a,q)-1]f(q) \le 0$.

will provide answers to their queries, and therefore, increases the cost of the private communication service.

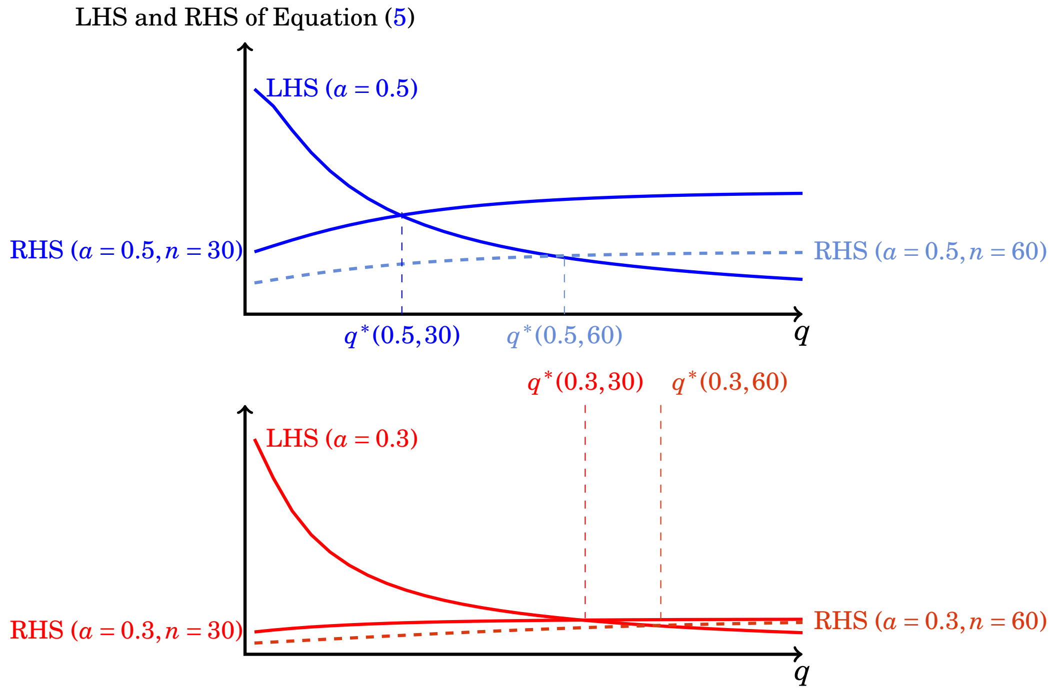

Figure 2: Illustration of $q^*(a, n)$, when $(a, n) \in \{0.3, 0.5\} \times \{30, 60\}$

q^* is the threshold that determines who will pay attention to the announcement. Player i whose query q_i is above the threshold q^* will not pay attention. Functional forms and parameters are specified as $P(a, q) = e^{-a/q}$, $f(q) = e^{-q}$, $C(a) = a^2$, $a \in \{0.3, 0.5\}$, $\kappa = 1/30$, and $n \in \{30, 60\}$. LHS means the left-hand side of equation (5), which represents the informativeness of the announcement with respect to q. RHS, the right-hand side of that equation, represents the ratio of the cost of paying attention (to the announcement) to the expected cost of private communication. When a increases from 0.3 to 0.5, q^* decreases. This means that a smaller proportion of information recipients will pay attention to the public announcement. When n increases from 30 to 60, q^* increases, and thus a larger proportion of recipients will pay attention to the announcement.

3.3 Sender's Decision about the Announcement Size

Knowing the information recipients' threshold $q^*(a)$ as a function of a, the information sender minimizes the expected total communication cost by solving the optimization problem:

$$
\begin{aligned}
a^* \in \arg\min_{a \in \mathbb{R}_+} & F(q^*(a))C(a) + \kappa n X(a, q^*(a)) \\
\text{s.t. } & C(a) + \kappa(1 + (n-1)X(a, q^*(a)))(P(a, q^*(a)) - 1) = 0,
\end{aligned} \tag{6}
$$

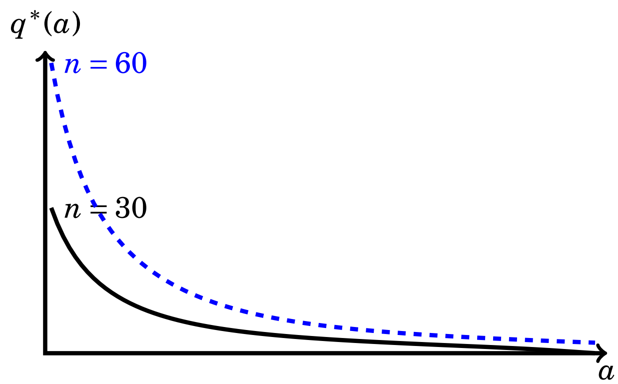

Figure 3: Illustration of $q^*(a)$, when $n \in \{30, 60\}$
$q^*(a)$ is the threshold that determines who will pay attention to the announcement. Functional forms and parameters are specified as in Figure 2. $q^*(a,n)$ decreases with a and increases with n.

where the constraint is a rearrangement of Equation (5) in such a way that renders it compatible with Equation (3). From this rearrangement, we can directly draw some predictions regarding the announcement size and the total communication cost.

Proposition 1. $F(q^*(a^*))C(a^*)+\kappa n X(a^*, q^*(a^*)) > F(q^o)C(a^o)+\kappa n X(a^o, q^o)$. *For sufficiently large n,* $a^* < a^o$.

Proof: See Appendix.

Now we know that a public announcement is under-provided when the queries are unobservable. This means that, in equilibrium, the cost of consuming the announcement is lower than when the queries are observable. Hence, the expected cost of using the private communication service must be higher. Otherwise, it contradicts the fact that (a^o, q^o) minimizes the total communication cost in the case of the unconstrained objective function.

Corollary 1. *The expected proportion of recipients who use the private communication service is higher when the queries are not observable by the information sender:*

$$X(a^*, q^*) > X(a^o, q^o).$$

Proof: See Appendix.

The ex-post price of the private communication channel is primarily determined by $X(a^*, q^*)$. In summary, when the information sender doesn't observe the queries, the public good (the announcement or the FAQs) is under-provided and the private good (the private communication service) is overpriced. From the perspective of a customer who has a non-trivial query, the FAQs contain too little information, and he also has to wait too long for a customer service agent to assist him.

It turns out that the inefficiency summarized in Proposition 1 and Corollary 1 is similar to that of the well-known free-riding behavior in regard to the voluntary contributions of public goods, even though the setups for the two scenarios are completely different. Roughly speaking, some people ignore the announcement (the public good), because they want to exploit the private communication service (the private good), while others pay attention to the announcement, to avoid incurring the extra cost of private communication. The information sender, who understands this free-riding incentive of the latter group of information recipients, has to reduce the size of the announcement, compared with that under the complete information.

4 Welfare Analysis

Corollary 1 implies that the expected cost of using the private communication channel is higher when the queries are unobservable: $\kappa[1+(n-1)X(a^o,q^o)] < \kappa[1+(n-1)X(a^*,q^*(a^*))]$. However, the direct cost of using the public announcement channel is lower in that case, $C(a^*) < C(a^o)$. As a result, there must be some information recipients who benefit (when the queries are unobservable, compared to when the queries are observable), and some who suffer. For the most part, recipients i, who have a low q_i, are better off. Those who have a high q_i are worse off.[14] This is because it is highly likely that information recipients i with a low q_i will find the answer to their query in the public announcement and can save the cost of using the public announcement channel, while it is highly likely that recipients i, with a high q_i, will use the private communication channel. Hence, they will pay more, because more recipients will eventually use the private channel, thereby causing it to become more costly.

However, the changes in welfare might not be monotonically increasing/decreasing for the case where a recipient draws a query close to one of the cutoff points q^*, q^o. More specifically, if $P(a^*,q^*(a^*))$, the probability that a recipient with $q = q^*(a^*)$ fails to find the answer in the public announcement is sufficiently small and $P(a^o,q^o)$, the probability that a recipient with $q = q^o$ is sufficiently large, we determine three critical points, $\bar{q}_1 < q^*(a^*)$,

[14]The complete analysis for the welfare changes is presented in the Appendix.

$\bar{q}_2 \in (q^*(a^*), q^o)$, and q^o, where recipients i with $q_i \in (0, \bar{q}_1)$ are better off (when the queries are unobservable, rather than when they are observable), those with $q_i \in (\bar{q}_1, \bar{q}_2)$ are worse off, those with $q_i \in (\bar{q}_2, q^o)$ are better off, and those with $q_i \in (q^o, \infty)$ are worse off.

Proposition 2. *If $q^*(a^*) < q^o$ and $P(a^o, q^*(a^*)) < \pi < P(a^o, q^o)$, where*

$$\pi = \frac{P(a^*, q^*(a^*))\kappa[1+(n-1)X(a^*, q^*(a^*))] + C(a^*) - C(a^o)}{\kappa[1+(n-1)X(a^o, q^o)]},$$

then information recipients i with

1. *$q_i \in (0, \bar{q}_1) \cup (\bar{q}_2, q^o)$ are better off (when the queries are unobservable than when they are observable),*

2. *$q_i \in (\bar{q}_1, \bar{q}_2) \cup (q^o, \infty)$ are worse off,*

where $\bar{q}_1 < q^*(a^*) < \bar{q}_2 < q^o$.

Proof: See Appendix.

Since the probability distribution function of q is assumed to be weakly decreasing, the result of Proposition 2 may suggest that the proportion of information recipients who are better off could be higher when the queries are unobservable than when they are observable. However, the sum of the marginal benefits of those who enjoy the lowered expected cost must be strictly smaller than the sum of the marginal costs of those who suffer from the increased expected cost. Thus, the overall communication cost is always higher when the queries become unobservable.

5 Changes in the Capacity for Private Communication

In the model, the parameter for the private communication channel, κ, is exogenously determined. The parameter κ can be interpreted as a cost parameter for the use of the private communication channel. As the private communication service offered by the information sender becomes more accessible, maybe due to the increase in the capacity K, or due to some technological advancement, κ could get smaller. In this section, we consider the dependence of a and q on κ in equilibrium; we do this for the case where the queries are observable and for the case where they are not.

We show that when the queries are observable, both the equilibrium size of the public announcement and the threshold query are positively affected by the cost parameter. In the case where the private channel becomes more costly, the sender would like to make the public announcement more informative, so she will increase the size of the public announcement and force the recipients to be more patient. That is, $\frac{\partial a^o}{\partial \kappa} > 0$ and $\frac{\partial q^o}{\partial \kappa} > 0$.

When the queries are not observable, the equilibrium size of the public announcement is *not affected* by the parameter κ, that is, $\frac{\partial a^*}{\partial \kappa} = 0$. This is because the sender has to consider the constraint. For an information recipient with the threshold query q^*, the expected cost of using the public announcement is the same as that of ignoring it. Thus, the parameter κ does not affect the sender's minimization problem or the equilibrium size of the public announcement; however, in equilibrium the threshold query has a higher value of κ: $\frac{\partial q^*(a^*)}{\partial \kappa} > 0$. As the private channel becomes more costly, the recipients strategically become more patient.

Proposition 3. *As the private channel becomes more costly, the following hold:*

1. *When the queries are observable, the public announcement becomes larger and the recipients become more patient in equilibrium:* $\frac{\partial a^o}{\partial \kappa} > 0$ *and* $\frac{\partial q^o}{\partial \kappa} > 0$.

2. *When the queries are not observable, the public announcement remains the same but the recipients become more patient:* $\frac{\partial a^*}{\partial \kappa} = 0$ *and* $\frac{\partial q^*(a^*)}{\partial \kappa} > 0$.

Proof: See Appendix.

The second result of Proposition 3 may guide a business strategy for information providers. Even when marginally increasing the capacity for private communication services, increasing (or decreasing) the size of a public announcement is not a good idea, as long as the increased capacity is equally dissipated to the information recipients using the private communication channel.[15] Rather, the marginal increase in the capacity makes recipients pay attention to the announcement with less information. Since our results hold under Assumption 1, which could be violated if κ is small enough, the results in Proposition 3 do not necessarily imply that even a huge change in K (hence κ) cannot affect the size of the announcement.

[15]The linearity of the ex-post cost function, $t(\kappa, d) = \kappa d$, does not lead to this result. As long as we maintain the assumption that the information sender treats all the information recipients equally, the generalization of the functional form to $t(\kappa, d) = h(\kappa)r(d)$, where both $h(\cdot)$ and $r(\cdot)$ are increasing, does not change the result.

6 Concluding Remarks

We studied a two-channel information-provision model for a situation where one message sender communicates with many message recipients. One information channel, a public announcement, features a predetermined cost of acquiring the information and incomplete informativeness. The other information channel, a private communication service, is equipped with full informativeness, but the cost of acquiring information from it is determined ex-post by the number of people who use this channel. Each information recipient draws his own query and decides whether to acquire the information from the public announcement and whether to purchase the private communication service. The information sender takes the recipients' potential strategies into account when determining the size (or the quality) of the announcement.

We showed that if the recipients' queries are unobservable by the sender, the public information is under-provided and the private communication service is overpriced, when compared to a situation where the information sender knows who has which query. From the perspective of the information recipients, it is not straightforward enough to determine who benefits and who suffers, though the overall communication cost is definitely larger when the queries are unobservable. We also find that the information provider should not change the size of the announcement, even if the capacity of the private communication channel increases.

For those who are particularly interested in designing FAQs to maximize the expected customer satisfaction, our results are summarized in the following way. The size of the FAQs must be smaller than what one thinks that it is pertinent. In addition, and the marginal change in the capacity of the customer care unit should not lead to the changes in the size of the FAQs.

6.1 Discussion of Future Work

There are many directions for extending this study, as we imposed many simplifying assumptions. First, we assumed in this paper that the announcement is made only once and the announcement is the same for all recipients, as we believe this is practically true in many real-life situations: Recall, for example, product manuals and printed FAQs. However, if a new technology allows the information sender to tailor an announcement for each information recipient separately, it could be worth investigating the benefits of multiple announcements. Second, it is possible that the information recipients have different abilities to comprehend the same announcement. Thus, an extension of this study would be to consider two types of information recipients, which leads to an observational equivalence (from the perspective of the information sender) between those who don't pay attention to the announcement, and those who do pay attention but don't understand it even if the answers to their specific queries are included in the announcement. Another possible extension regarding the comprehension heterogeneity is to consider the heterogeneous costs of paying attention to the announcement. Although it is clear that recipients with high attention costs will ignore the announcement more than ones with low costs, it is unclear whether the size of the announcement should be larger or smaller than the case with the homogeneous attention cost, as it should depend on the distribution of the cost heterogeneity. Third, we considered a static game in this study. It could be extended to a two-period game, where in the second period the information sender can utilize the decisions made by the information recipients in the first period. In this case, high-ability information recipients may endogenously choose their attention level.

References

Angeletos, George-Marios and Alessandro Pavan, "Efficient Use of Information and Social Value of Information," *Econometrica*, 2007, *75* (4), 1103–1142.

Bental, Benjamin and Menahem Spiegel, "Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities," *Journal of Industrial Economics*, 1995, *43* (2), 197–208.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Competition for Attention," *The Review of Economic Studies*, 2016, *83* (2), 481–513.

Caillaud, Bernard and Jean Tirole, "Consensus Building: How to Persuade a Group," *American Economic Review*, December 2007, *97* (5), 1877–1900.

Colombo, Luca, Gianluca Femminis, and Alessandro Pavan, "Information Acquisition and Welfare," *The Review of Economic Studies*, 2014, *81* (4), 1438–1483.

de Clippel, Geoffroy, Kfir Eliaz, and Kareen Rozen, "Competing for Consumer Inattention," *Journal of Political Economy*, 2014, *122* (6), 1203–1234.

Dessein, Wouter, Andrea Galeotti, and Tano Santos, "Rational Inattention and Organizational Focus," *American Economic Review*, June 2016, *106* (6), 1522–1536.

Evans, Robert, "Mechanism Design With Renegotiation and Costly Messages," *Econometrica*, 2012, *80* (5), 2089–2104.

Farrell, Joseph and Robert Gibbons, "Cheap Talk with Two Audiences," *The American Economic Review*, 1989, *79* (5), 1214–1223.

Goltsman, Maria and Gregory Pavlov, "How to talk to multiple audiences," *Games and Economic Behavior*, 2011, *72* (1), 100–122.

Loder, Theodore, Marshall Van Alstyne, and Rick Wash, "An Economic Response to Unsolicited Communication," *The B.E. Journal of Economic Analysis & Policy*, 2006, *6* (1), 1–38.

Luo, Yulei, "Consumption Dynamics under Information Processing Constraints," *Review of Economic Dynamics*, 2008, *11* (2), 366–385.

Maćkowiak, Bartosz and Mirko Wiederholt, "Optimal Sticky Prices under Rational Inattention," *The American Economic Review*, 2009, *99* (3), 769–803.

Maćowiak, Bartosz and Mirko Wiederholt, "Business Cycle Dynamics under Rational Inattention," *The Review of Economic Studies*, 2015, *82* (4), 1502–1532.

Matějka, Filip, "Rationally Inattentive Seller: Sales and Discrete Pricing," *The Review of Economic Studies*, 2016, *83* (3), 1125–1155.

Mondria, Jordi, "Portfolio Choice, Attention Allocation, and Price Comovement," *Journal of Economic Theory*, 2010, *145* (5), 1837–1864.

Myatt, David P. and Chris Wallace, "Endogenous Information Acquisition in Coordination Games," *The Review of Economic Studies*, 2012, *79* (1), 340–374.

Nieuwerburgh, Stijn Van and Laura Veldkamp, "Information Immobility and the Home Bias Puzzle," *The Journal of Finance*, 2009, *64* (3), 1187–1215.

Novick, David G. and Karen Ward, "Why Don't People Read the Manual?," in "Proceedings of the 24th Annual ACM International Conference on Design of Communication" SIGDOC '06 ACM New York, NY, USA 2006, pp. 11–18.

Potters, Jan, "Fixed Cost Messages," *Economics Letters*, 1992, *38* (1), 43–47.

Reis, Ricardo, "Inattentive Producers," *The Review of Economic Studies*, 2006, *73* (3), 793–821.

Sallee, James M., "Rational Inattention and Energy Efficiency," *The Journal of Law and Economics*, 2014, *57* (3), 781–820.

Sims, Christopher A., "Stickiness," in "Carnegie-Rochester Conference Series on Public Policy," Vol. 49 Elsevier 1998, pp. 317–356.

_ , "Implications of Rational Inattention," *Journal of Monetary Economics*, 2003, *50* (3), 665–690.

Sulek, Joanne M., Mary R. Lind, and Ann S. Marucheck, "The Impact of a Customer Service Intervention and Facility Design on Firm Performance," *Management Science*, 1995, *41* (11), 1763–1773.

Tutino, Antonella, "Rationally Iattentive Consumption Choices," *Review of Economic Dynamics*, 2013, *16* (3), 421–439.

Woodford, Michael, "Information-Contrained State-Dependent Pricing," *Journal of Monetary Economics*, 2009, *56* (S), 100–124.

Appendix: Omitted Proofs

Proof of Lemma 1

Define $W(a,q^*(a)) = \kappa(1-P(a,q^*(a)))(1+(n-1)X(a,q^*(a)))-C(a)$. In equilibrium, $W(a,q^*(a)) = 0$. First, using the Implicit Function Theorem we want to show that $\frac{\partial q}{\partial n} = -\frac{\partial W/\partial n}{\partial W/\partial q} > 0$. For notational simplicity, we will omit $(a,q^*(a))$ from $P(a,q^*(a))$ and $X(a,q^*(a))$ whenever unnecessary. It can be easily shown that $\frac{\partial W}{\partial n} = \kappa(1-P)X > 0$, and that $\frac{\partial W}{\partial q} = -\kappa P_2(1+(n-1)X)+\kappa(1-P)(n-1)X_2 < 0$, where P_i, $i = 1,2$, is the partial derivative of $P(a,q^*(a))$ with respect to the ith argument. X_i, $i = 1,2$, is defined similarly. Note that $P_2 > 0$ and $X_2 < 0$. Thus, $\frac{\partial q^*(a,n)}{\partial n} > 0$.

Next, we want to show that $\frac{\partial q}{\partial a} = -\frac{\partial W/\partial a}{\partial W/\partial q} < 0$. Since $\frac{\partial W}{\partial q} < 0$, what remains to be shown is that $\frac{\partial W}{\partial a} < 0$, that is, that $\frac{\partial}{\partial a}[\kappa(1-P+(n-1)X(1-P))-C(a)] = \kappa(n-1)(1-P)X_1 - \kappa(1+(n-1)X)P_1 - C'(a) < 0$. Note that $C'(a) > 0$, $X_1 < 0$, and $P_1 < 0$ for any positive a and q, so the sign of $\frac{\partial W}{\partial a}$ cannot immediately be determined by simply checking the sign of each of the three additively separable terms. To this end, we make three claims, and these complete the proof. First, when a is large, $\frac{\partial W}{\partial a} < 0$. When a approaches ∞, both $P_1(a,q)$ and $X_1(a,q)$ approach 0 regardless of what happens to $q^*(a)$. Since $C'(a)$ is increasing in a, for large a we have $\frac{\partial W}{\partial a} < 0$. Second, when $a = 0$, $\frac{\partial W}{\partial a} = 0$ because $1 - P(0,\infty) = 0$, $X_1(0,\infty) < 0$, $X(0,\infty) = 1$, $P_1(0,\infty) = 0$, and $C'(0) = 0$. Third, when a approaches 0, $X(a,q^*(a))$ approaches 1 faster than $P(a,q^*(a))$ does, so $\frac{\partial W}{\partial a} < 0$. Both $P(a,q^*(a))$ and $X(a,q^*(a))$ approach 1 when a gets sufficiently close to 0; however, the slope of $P(a,q^*(a))$ near $a = 0$ is close to 0 since $\lim_{a\to 0} P_1(a,q^*(a)) = 0$, while that of $X(a,q^*(a))$ is negative since $X_1(a,q^*(a)) = \int_0^{q^*(a)} P(a,x)f(x)dx$ and $\lim_{a\to 0} X_1(a,q^*(a)) = \int_0^{\infty} P_1(0,x)f(x)dx < 0$. This guarantees that when a is near 0, $(1-P)X_1 - XP_1 < 0$; therefore, $(n-1)\{(1-P)X_1 - XP_1\} < 0$. Also, for sufficiently large n we find that $(n-1)\{(1-P)X_1 - XP_1\} < P_1$. Furthermore, $C'(a)$ is positive, so $\kappa\left[(n-1)\{(1-P)X_1 - XP_1\} - P_1\right] - C'(a) < 0$.

Proof of Proposition 1

A direct comparison of equations (1) and (6) tells us that a^o is a solution of the unconstrained objective function, while a^* is a solution of the constrained one. Thus the indirect utility at $(a^*, q^*(a^*))$ is greater than or equal to that at (a^o, q^o), with equality holding if and only if $(a^*, q^*(a^*)) = (a^o, q^o)$. Since $X(a^o, q^o) < 1$, the constraint evaluated at (a^o, q^o) is strictly greater than 0, and therefore the strict inequality holds in the indirect utility. What remains is to show that a^* is strictly smaller than a^o for sufficiently large n. Suppose, for the sake of contradiction, that $a^o \le a^*$. Since $q^*(a)$ is decreasing in a, $q^*(a^*) \le q^*(a^o)$, where $q^*(a^o)$ satisfies the constraint in equation (6). Though the sign of $F(q^*(a^*))C(a^*) - F(q^*(a^o))C(a^o)$ is indeterminate, $\int_0^{q^*(a^*)} P(a,x)f(x)dx + \int_{q^*(a^*)}^{\infty} f(x)dx$ is larger than $\int_0^{q^*(a^o)} P(a,x)f(x)dx + \int_{q^*(a^o)}^{\infty} f(x)dx$ because $f(x) > P(a,x)f(x)$ for any x. Therefore, there exists $\hat{n}$ such that for any $n \ge \hat{n}$,

$$\frac{1}{\kappa n}\left[F(q^*(a^*))C(a^*) - F(q^*(a^o))C(a^o)\right]$$
$$> \int_0^{q^*(a^o)} P(a,x)f(x)dx + \int_{q^*(a^o)}^{\infty} f(x)dx - \int_0^{q^*(a^*)} P(a,x)f(x)dx - \int_{q^*(a^*)}^{\infty} f(x)dx.$$

If this is the case, $(a^o, q^*(a^o))$ yields a smaller total communication cost than $(a^*, q^*(a^*))$, which is contradictory. $\square$

Proof of Corollary 1

Though we could easily show that $q^*(a^o) \leq q^o$ and $q^*(a^o) < q^*(a^*)$, it is possible that $q^*(a^*) > q^o$, depending on the shape of $f(q)$ and $P(a,q)$. Thus we consider both cases for $q^o \geq q^*(a^*)$ and $q^o < q^*(a^*)$.

For the sake of contradiction, suppose $X(a^o,q^o) > X(a^*,q^*(a^*))$. From Proposition 1, we know that $a^o > a^*$. When $q^o \geq q^*(a^*)$, $X(a^o,q^o) > X(a^*,q^*(a^*))$ does not hold since $X(a,q)$ is decreasing in a and q.

Now let us consider the case for $q^o < q^*(a^*)$. There are three possible regions for recipients' queries; (i) $q_i \in (q^*(a^*),\infty)$, (ii) $q_i \in (q^o,q^*(a^*))$, and (iii) $q_i \in (0,q^o)$. Our goal is to show that the supposition leads to the contradiction to Proposition 1.

(i) $X(a^o,q^o) > X(a^*,q^*(a^*))$ implies that the private communication channel is less costly when the queries are unobservable. For $q_i \in (q^*(a^*),\infty)$, the recipients do not pay attention to the announcement no matter when the queries are observable or not. This means that the recipient i with $q_i \in (q^*(a^*),\infty)$ expects a lower cost when the queries are unobservable.

(ii) For $q_i \in (q^o,q^*(a^*))$, the recipients pay attention to the announcement when the queries are not observable because they expect a lower cost from it than that from directly using the private communication channel, $C(a^*)+P(a^*,q_i)\kappa(1+(n-1)X(a^*,q^*(a^*))) < \kappa(1+(n-1)X(a^*,q^*(a^*)))$. On the other hand, they do not pay attention to the announcement when the queries are observable. Since we suppose $X(a^o,q^o) > X(a^*,q^*(a^*))$, the expected cost from the private communication channel is greater when queries are observable, $\kappa(1+(n-1)X(a^o,q^o)) > \kappa(1+(n-1)X(a^*,q^*(a^*)))$, which implies that $C(a^*)+P(a^*,q_i)\kappa(1+(n-1)X(a^*,q^*(a^*))) < \kappa(1+(n-1)X(a^o,q^o))$. Thus, the recipient i with $q_i \in (q^o,q^*(a^*))$ expects a lower cost when the queries are not observable.

(iii) For $q_i \in (0,q^o)$, the recipients pay attention to the announcement no matter when the queries are observable or not. Their expected cost when the queries are observable is $C(a^o)+P(a^o,q_i)\kappa[1+(n-1)X(a^o,q^o)]$ and that when the queries are unobservable is $C(a^*)+P(a^*,q_i)\kappa[1+(n-1)X(a^*,q^*(a^*))]$. From equation (3), we know that $C(a^o) = \kappa n[1-P(a^o,q^o)]$ and since $X(a^o,q^o) < 1$, $C(a^o) > \kappa[1+(n-1)X(a^o,q^o)][1-P(a^o,q^o)]$. Hence, $C(a^o)+\kappa[1+(n-1)X(a^o,q^o)]P(a^o,q^o) > \kappa[1+(n-1)X(a^o,q^o)]$. In addition, $\kappa[1+(n-1)X(a^*,q^*(a^*))] > C(a^*)+\kappa[1+(n-1)X(a^*,q^*(a^*))]P(a^*,q^o)$, $C(a^o) > C(a^*)$, and $C(a)+\kappa[1+(n-1)X(a,q)]P(a,q_i)$ is increasing in q_i. Thus, the recipient i with $q_i \in (0,q^o)$ expects to pay a lower cost when the queries are unobservable.

Therefore, all recipients are better off when the queries are unobservable, which contradicts the results in Proposition 1.

Welfare Analysis

For notational convenience, we define $r(q_i|\tilde{a},\tilde{q})$ as the expected cost of use of the public announcement channel by information recipient i, given that the size of the public announcement is $\tilde{a}$ and the cutoff query lies at $\tilde{q}$, that is, $r(q_i|(\tilde{a},\tilde{q})) = C(\tilde{a}) + \kappa P(\tilde{a},q_i)[1+(n-1)X(\tilde{a},\tilde{q})]$. Similarly, we define $s(\tilde{a},\tilde{q})$ as the expected cost of use of the private channel, that is, $s(\tilde{a},\tilde{q}) = \kappa[1+(n-1)X(\tilde{a},\tilde{q})]$. Note that by construction, $r(q^*(a^*)|a^*,q^*(a^*)) = s(a^*,q^*(a^*))$ and $r(q^o|a^o,q^o) > s(a^o,q^o)$.

We consider six cases in total: three cases where $q^*(a^*) < q^o$, and another three cases where $q^*(a^*) > q^o$.

If $q^*(a^*) < q^o$, then $P(a^o,q^o) > P(a^o,q^*(a^*))$, from which it follows that $r(q^o|a^o,q^o) > r(q^*(a^*)|a^o,q^o)$. We have the following three cases, corresponding to the three possibilities for $s(a^*,q*(a^*))$ compared to $r(q^o|a^o,q^o)$ and $r(q^*(a^*)|a^o,q^o)$.

Case 1: $q^*(a^*) < q^o$ **and** $r(q^o|a^o,q^o) > r(q^*(a^*)|a^o,q^o) > s(a^*,q^*(a^*))$

In this case information recipients whose query is below q^o pay less (when the queries are unobservable than when they are observable), while those with a query above q^o pay more. Figure 4 illustrates this result. The red lines represent the expected cost when the queries are observable: As q_i gets larger, the expected cost increases—not necessarily linearly, but we're using a linear relationship in this appendix for the sake of illustration—until it reaches q^o, but beyond q^o the cost becomes constant because of the lack of additional informativeness of the public announcement. The blue line represents the expected cost when the queries are unobservable: From the construction of the Bayesian Nash equilibrium, at $q^*(a^*)$ the expected cost of using the public announcement channel is equal to the expected cost of ignoring it. We find it useful to represent each case with respect to $P(a,q)$. Case 1 is equivalent to $\pi < P(a^o,q^*(a^*)) < P(a^o,q^o)$, where $\pi = \frac{P(a^*,q^*(a^*))\kappa[1+(n-1)X(a^*,q^*(a^*))]+C(a^*)-C(a^o)}{\kappa[1+(n-1)X(a^o,q^o)]}$, that is, both $P(a^o,q^*(a^*))$ and $P(a^o,q^o)$ are large, and the difference between them is small.

Case 2: $q^*(a^*) < q^o$ **and** $s(a^*,q^*(a^*)) > r(q^o|a^o,q^o) > r(q^*(a^*)|a^o,q^o)$

In this case there exists $\bar{q} \in (0,q^*(a^*))$ such that $r(\bar{q}|(a^o,q^o)) = r(\bar{q}|(a^*,q^*(a^*))$. Information recipients whose query is below $\bar{q}$ pay less (when the queries are unobservable than

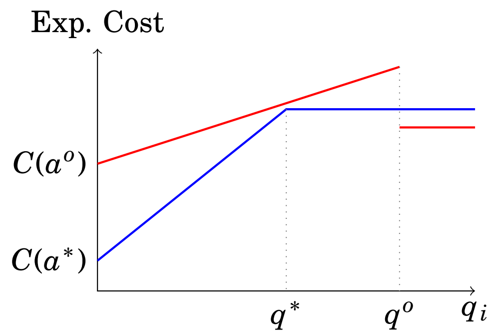

Figure 4: Case 1

when they are observable), while those with a query above $\bar{q}$ pay more. Figure 5 illustrates this result. Case 2 is equivalent to $\pi > P(a^o, q^o) > P(a^o, q^*(a^*))$, that is, both $P(a^o, q^*(a^*))$ and $P(a^o, q^o)$ are small, and the difference between them is also small.

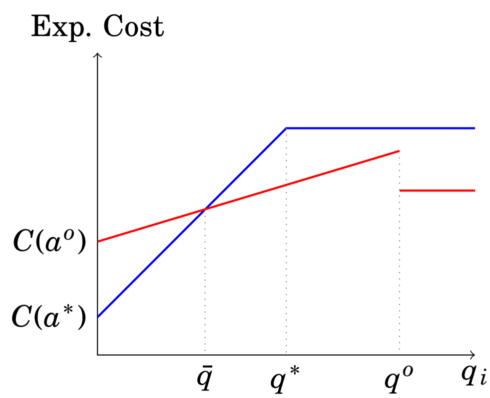

Figure 5: Case 2

Case 3 (Proposition 2): $q^*(a^*) < q^o$ **and** $r(q^o|a^o, q^o) > s(a^*, q^*(a^*)) > r(q^*(a^*)|a^o, q^o)$

In this case there exist $\bar{q}_1 \in (0, q^*(a^*))$ such that $r(\bar{q}_1|(a^o, q^o)) = r(\bar{q}_1|(a^*, q^*(a^*))$, and $\bar{q}_2 \in (q^*(a^*), q^o)$ such that $r(\bar{q}_2|(a^o, q^o)) = r(q^*(a^*)|a^*, q^*(a^*))$. Information recipients whose query is in $(0, \bar{q}_1) \cup (\bar{q}_2, q^o)$ pay less (when the queries are unobservable than when they are observable), while the others pay more. This is because information recipients with $q_i \in (0, \bar{q}_1)$ use the public announcement channel regardless of whether the information sender observes the queries, and it is highly likely that they will find the answer to their query in the public announcement. Hence, they enjoy the lower cost of using the public announcement channel. Second, recipients with $q_i \in (\bar{q}_1, q^*(a^*))$ also use the public announcement channel, even though the probability that they cannot find their answer in the public announcement is quite a bit higher (when the queries are unobservable than when they are observable), and thus they will pay more. Third, those with $q_i \in (q^*(a^*), \bar{q}_2)$ are asked to use the public announcement channel when the queries are observable but to ignore that channel when they are not. The expected cost of using the private communication channel is high enough that they will pay more. Fourth, the case for those with $q_i \in (\bar{q}_2, q^o)$ is the same as the case for those with $q_i \in (q^*(a^*), \bar{q}_2)$, but the expected cost of using the public announcement channel is sufficiently high that they need not read the announcement when the queries are not observable, hence they will pay less. Finally, those with $q_i \in (q^o, \infty)$ use the private channel regardless of whether the queries are observable, and the expected cost of using it becomes higher, thus they pay more. Figure 6 illustrates this result. Case 3 is equivalent to $P(a^o, q^o) > \pi > P(a^o, q^*(a^*))$, that is, $P(a^*, q^*(a^*))$ is sufficiently small and $P(a^o, q^o)$ is sufficiently large.

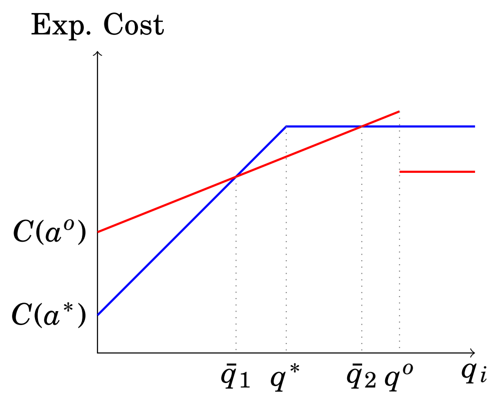

Figure 6: Case 3

When $q^*(a^*) > q^o$, we can find the cutoff query $\bar{q}$, and information recipients with $q \in (0, \bar{q})$ are better off (when the queries are unobservable than when they are observable), while those with $q \in (\bar{q}, \infty)$ are worse off. The cutoff $\bar{q}$ depends on how likely it is that a recipient with query q^o will find the answer to his query in the public announcement given a^*. In this regard, we have the following three cases, corresponding to the three possibilities for $r(q^o|a^*, q^*(a^*))$ compared to $r(q^*(a^*)|a^*, q^*(a^*))$ and $s(a^o, q^o)$.

Case 4: $q^*(a^*) > q^o$ **and** $r(q^o|a^*, q^*(a^*)) > r(q^*(a^*)|a^*, q^*(a^*)) > s(a^o, q^o)$

In this case there exists $\bar{q} \in (0, q^o)$ such that $r(\bar{q}|(a^o, q^o)) = r(\bar{q}|(a^*, q^*(a^*))$. Figure 7 illustrates this result. Case 4 is equivalent to $P(a^*,q^o) > \pi_2$, where $\pi_2 = \frac{\kappa[1+(n-1)X(a^o,q^o)]P(a^o,q^o)+C(a^o)-C(a^*)}{\kappa[1+(n-1)X(a^*,q^*(a^*))]}$, that is, $P(a^*,q^o)$ is sufficiently large.

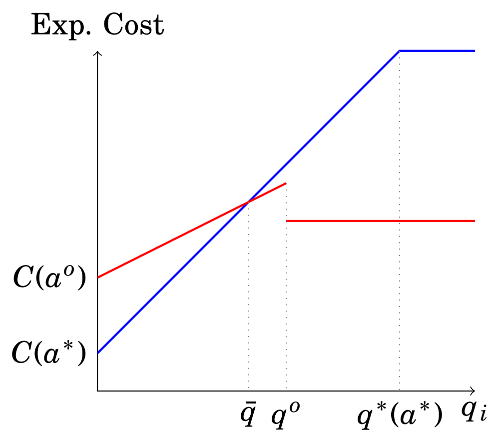

Figure 7: Case 4

Case 5: $q^*(a^*) > q^o$ **and** $r(q^*(a^*)|a^*,q^*(a^*)) > s(a^o,q^o) > r(q^o|a^*,q^*(a^*))$

In this case there exists $\bar{q} \in (q^o, q^*(a^*))$ such that $s(a^o,q^o) = r(\bar{q}|(a^*,q^*(a^*))$. Figure 8 illustrates this result. Case 5 is equivalent to $P(a^*,q^o) < \pi_3$, where $\pi_3 = \frac{\kappa[1+(n-1)X(a^o,q^o)]P(a^o,q^o)-C(a^*)}{\kappa[1+(n-1)X(a^*,q^*(a^*))]}$, that is, $P(a^*,q^o)$ is sufficiently small.

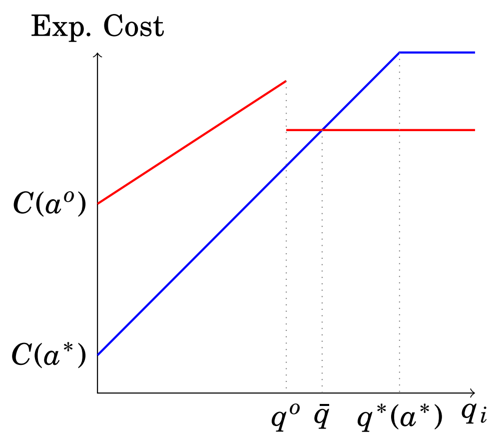

Figure 8: Case 5

Case 6: $q^*(a^*) > q^o$ **and** $r(q^*(a^*)|a^*,q^*(a^*)) > r(q^o|a^*,q^*(a^*)) > s(a^o,q^o)$

If $\pi_3 < P(a^*, q^o) < \pi_2$, q^o is the cutoff query. That is, for any $q < q^o$, $r(q|(a^o, q^o)) > r(q|(a^*, q^*(a^*))$, and for any $q > q^o$, $r(q|(a^o, q^o)) < r(q|(a^*, q^*(a^*))$. This result indirectly implies that analysis of changes in the welfare of the recipients is indeed nontrivial: Only under very specific conditions do we find a case where information recipients with $q < q^o$ are better off (when the queries are unobservable than when they are observable). Figure 9 illustrates this result.

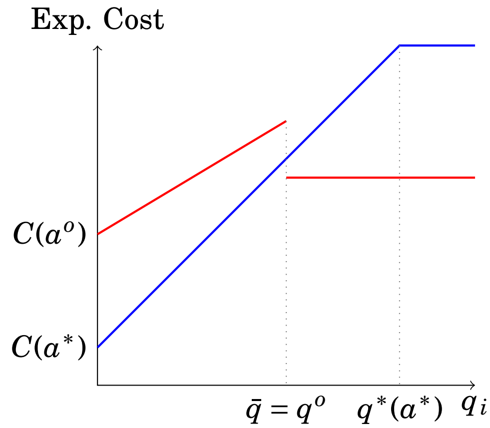

Figure 9: Case 6

Proof of Proposition 3

By equations (2) and (3), $F(q^o)C'(a^o) + \kappa n X_1(a^o, q^o) = 0$ and $f(q^o)C(a^o) + \kappa n X_2(a^o, q^o) = 0$. If we take the derivative of the expression on the left-hand side of equation (2) with respect to κ, then we get $[f(q^o)C'(a^o) + \kappa n X_{12}(a^o, q^o)]\frac{\partial q^o}{\partial n} + [F(q^o)C''(a^o) + \kappa n X_{11}(a^o, q^o)]\frac{\partial a^o}{\partial n} + nX_1(a^o, q^o) = 0$. Note that $f(q^o)C'(a^o) + \kappa n X_{12}(a^o, q^o) = 0$, since $C(a^o) = \kappa n[1 - P(a^o, q^o)]$ and $X_{12}(a, q) = f(q)P_1(a, q)$. In addition, $F(q^o)C''(a^o) + \kappa n X_{11}(a^o, q^o) > 0$ and $X_1(a^o, q^o) < 0$. Thus, $\frac{\partial a^o}{\partial \kappa} > 0$. We can use analogous reasoning to show that $\frac{\partial q^o}{\partial \kappa} > 0$.

When the queries are not observable, the sender has to consider the constraint, $C(a) = [1 - P(a, q^*(a^*))]\kappa[1 + (n-1)X(a, q^*(a))]$, when minimizing the total communication cost. Rewriting the sender's objective function as $F(q^*(a))[1 - P(a, q^*(a^*))]\kappa[1 + (n-1)X(a, q^*(a))] + \kappa n X(a, q^*(a)) = \kappa\{F(q^*(a))[1 - P(a, q^*(a^*))][1 + (n-1)X(a, q^*(a))] + nX(a, q^*(a))\}$, it is straightforward to see that in equilibrium κ does not affect the size of the public announcement. On the other hand, it is easy to show that by the constraint, $q^*(a^*)$ increases with κ.