Unveiling the Failure of Positive Selection

*Positive Selection in Bargaining: Experiment

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Which is better for the seller?

Consider two-person bargaining. A buyer has a private value $v \sim F$. A seller makes an offer, then a buyer accepts it, takes an outside option if available, or rejects it to repeat the negotiation.

- Question: Which is a good situation to the seller? (A) The buyer has an outside option, and it is commonly known to both players. (B) The buyer does not have an outside option.

Which is better for the seller?

Consider two-person bargaining. A buyer has a private value $v \sim F$. A seller makes an offer, then a buyer accepts it, takes an outside option if available, or rejects it to repeat the negotiation.

- Question: Which is a good situation to the seller? (A) The buyer has an outside option, and it is commonly known to both players. (B) The buyer does not have an outside option.
- Board and Pycia (2014, BP henceforth): The seller enjoys the largest profit when $\exists$ a commonly-known outside option.
- This result is theoretically robust, in the sense that (i) it holds however small the value of the outside option, (ii) a key logical process works both on and off the equilibrium path, and (iii) the equilibrium strategy is the strongly rationalizable strategy (Cantonini, 2022).

Research Questions

- BP's result has a significant implication for the market design and regulatory policy in various markets: For the consumer surplus, the designer should prevent buyers from accessing outside options.
- This implication seems contrary to the conventional wisdom that restricting monopoly power usually makes the market more competitive and increases consumer surplus.

  Questions: Would the experiment participants exhibit the key logical process for the equilibrium in their belief updates? If not, when and in what sense do they fail?

Experimental Design: Two-Round Bargaining

Round 1

(*Only one treatment, called $M240$, is illustrated for simplicity.)
Each pair of participants consists of a seller and a buyer.

- Seller's value is normalized to zero.
- Buyer's value is $v \in V = \{70, 240, 500\}$ with equal prob.
- Buyer has an outside option whose value is 50.
- In Round 1, Seller offers $p_1 \in \{10, 180, 440\}$. Buyer can
  - accept p_1; Buyer earns $v - p_1$, Seller earns p_1, and the game ends(*);
  - take the outside option; Buyer earns 50, Seller earns 0, and the game ends(*);
  - reject p_1 and move on to round 2 with probability 0.8. With prob 0.2, the game ends, and both earn 0.

(*) With prob $\epsilon = 0.001$, the game moves to Round 2, when Buyer accepts p_1 or takes the outside option. Participants were instructed that ϵ is to theoretically guarantee the possibility of moving to Round 2, so it should be negligible.

Experimental Design: Two-Round Bargaining

Round 2

- At the beginning of Round 2, Seller reports her posterior belief of Buyer's value, by filling out the following sentence.

```
I believe that the value of the buyer paired in this match is
                70 with   a (____)% of chance,
               240 with   a (____)% of chance,
               500 with   a (____)% of chance.
```

- In Round 2, Seller offers $p_2 \in \{10, 180, 440\}$. Buyer can
  - accept p_2; Buyer earns $v - p_2$, Seller earns p_2, and the game ends;
  - take the outside option; Buyer earns 50, Seller earns 0, and the game ends;
  - ~~reject p_2; The game ends, and both earn 0.~~

Each participant has ten newly paired matches (periods).

Hypotheses (1/5)

Each step of the equilibrium characterization is associated with a testable hypothesis.

- The "minimal" rationality: The low type should never delay.
  (Outside option in R1 $= 50$. Largest possible payoff in delaying$= 48$.)
- If the game moves on to R2, then Seller must believe that the low type remains only because of ϵ. This leads to

Hypothesis (First-order positive selection)

No low-type buyers choose to delay. If ever moved to R2, Seller's posterior belief that Buyer is a high/middle type is weakly greater than the posterior belief that Buyer is a low type, for any p_1.

Hypotheses (2/5)

- Given the 1st-order positive selection, a rational Seller will never offer $p_L = 10$ in R2.

  (Seller's expected payoff from $p_2 = 10$ is 10. Seller's expected payoff from $p_2 = 180$ is $180 * [\hat{q}(v_M) + \hat{q}(v_H)]$, where $\hat{q}(v_M) + \hat{q}(v_H) \geq 2/3$.)
- Then, by following similar reasoning for the no-delay of the low type, the middle type also finds it strictly suboptimal to delay the negotiation to R2.

Hypothesis (Second-order positive selection)

No middle-type buyers choose to delay. If ever moved to R2, Seller' posterior belief that Buyer is a high type is weakly greater than the posterior belief that Buyer is a low/middle type, for any p_1.

Hypothesis (3/5)

With full commitment power, it is optimal for Seller to commit to

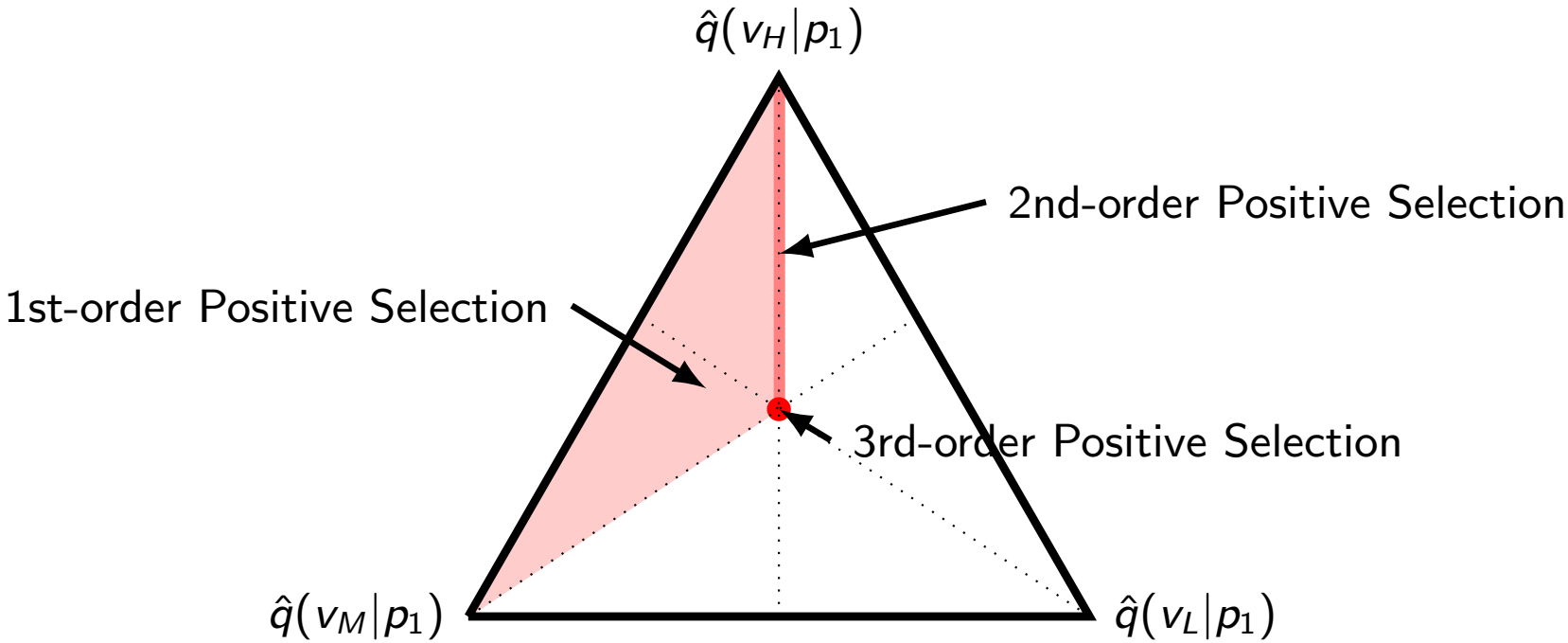
$$p_1 = p_2 = p_H = 440 := \arg \max_{p \in \{10,180,440\}} \sum_{v:u(v) \geq p} p \cdot q(v)$$

- Given the first- and second-order positive selections, the posterior belief $\hat{q}$ weakly FOSD the prior belief q.
- The seller's R2 (unrestricted) optimal price offer must be greater than p^*, but we have only three price alternatives. $p_2 = p_H$, implying that the high type has no reason to delay.

Hypothesis (Third-order positive selection)

Suppose $p^* = p_H$. No high-type buyers choose to delay. If ever moved to R2, Seller's posterior belief is the same as prior, $\hat{q}(v_L|p_1) = \hat{q}(v_M|p_1) = \hat{q}(v_H|p_1)$.

Summarizing the first three hypotheses



Positive Selections and Posterior Beliefs

- The earlier hypothesis nests the later one.
- If our experimental data do not support the theoretical predictions, these three hypotheses can provide us clear identification from where subjects fail.

Hypotheses (4/5)

This regards Seller's rationality.

- Given any posterior beliefs $\hat{q}(\cdot)$ the seller reported, the seller faces a static profit maximization problem.

$$\max_{p_2 \in \mathcal{P}} \sum_{v : u(v) \geq p_1} p_2 \cdot \hat{q}(v|p_1) \quad \forall p_1 \in \mathcal{P}. \tag{1}$$

- We expect Seller is, at least, best responding to her own (perhaps incorrect) belief.

Hypothesis

Given Seller's reported posterior belief $\hat{q}(\cdot)$, p_2 maximizes Seller's expected payoff.

Hypotheses (5/5)

This regards Buyer's rationality.

- When $v_M \in \{90, 240\}$, H1–H3 state that no buyers would choose to delay. This is the case when Buyer expects:

$$E[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\} \quad \forall v, p_1,$$

- which means that some buyers may reject p_1, based on her subjective (perhaps incorrect) belief.
- We have only three types and three price alternatives, so we check in which case the above inequality can be violated.

Hypotheses (5/5)

$E[\delta \max\{v - p_2, w\}] \leq \max\{w, v - p_1\}$?

$v \setminus p_1$	$p_L = 10$	$p_M = 180$	$p_H = 440$
$v_L = 70$	always hold	always hold	always hold
$v_M = 240$	always hold	can be violated if $p_2 = p_L$ is expected too much	can be violated if $p_2 \leq p_M$ is expected too much
$v_H = 500$	always hold		

Validity of not expecting "too low price" in Round 2

In words, if our experimental data shows some "rejections," then it is most likely in the v–p_1 pair shaded in red, somewhat likely in the pair shaded in blue, and not likely in other pairs. This leads

Hypothesis

Buyers with v_M or v_H are more likely to reject p_H, somewhat likely to reject p_M, and not likely to reject p_L. Buyers with v_L are not likely to reject any p_1.

Experiment: Basic Procedure

- oTree (Chen et al, 2016) + Zoom RTO experiment
- Turning on their video was a strict requirement
- HKUST, English
- 5 sessions each for M90, M240, and M420.

  ($M90$: $v_M = 90$, $M420$: $v_M = 420$)
- $106 + 100 + 88 = 294$ participants
- Ten matches
- Random matching, between-subject design
- On average, HKD 115 ($\approx$ USD 16) including HKD 40 show-up payment
- Online bank transfer via the autopay system of HKUST

Results: Overview

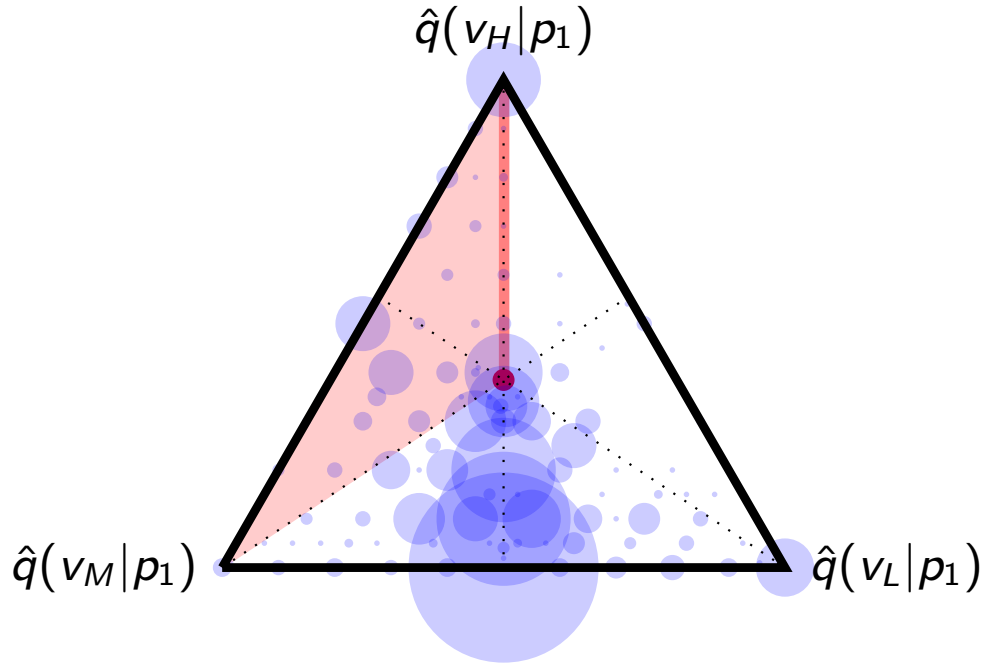
	$M90$	$M240$	$M420$
Avg.Offer (Theory)	346.25 (440)	295.86 (440)	378.50 (360)
%Reject_v_L (Theory)	39 (0)	35 (0)	32 (0)
%Reject_v_M (Theory)	51 (0)	60 (0)	74 (0)
%Reject_v_H (Theory)	63 (0)	50 (0)	59 (0)
Avg.SellerPayoffs (Theory)	54.43 (146.67)	87.94 (146.67)	165.16 (240)
Avg.BuyerPayoffs (Theory)	111.60 (53.33)	119.20 (53.33)	86.25 (83.33)

Table 1: Summary of Experimental Findings

Substantial differences:

- Avg.Offer is largest in $M420$. Theory predicts the opposite.
- Avg.Offer in $M90$ is significantly larger than that in $M240$. The equilibrium price offers are the same.
- Avg.SellerPayoffs is much smaller than the equilibrium payoff.
- Avg.BuyerPayoffs in $M420$ was the smallest, opposing theory.

Result 1



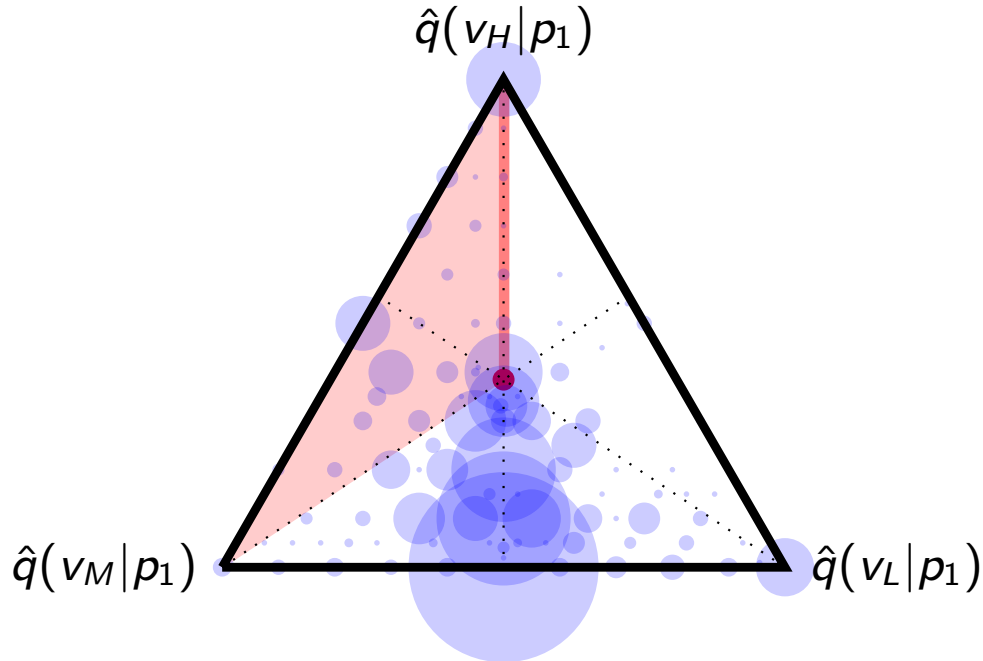
Observed posterior beliefs

Result

51.22% (753 out of 1,470) of the first-round price offers were rejected. Among 614 reported posterior beliefs, 36.64% of them are rationalized in the first-order positive selection.

Note: we interpret our observations in the most "favorable" way.

Result 2

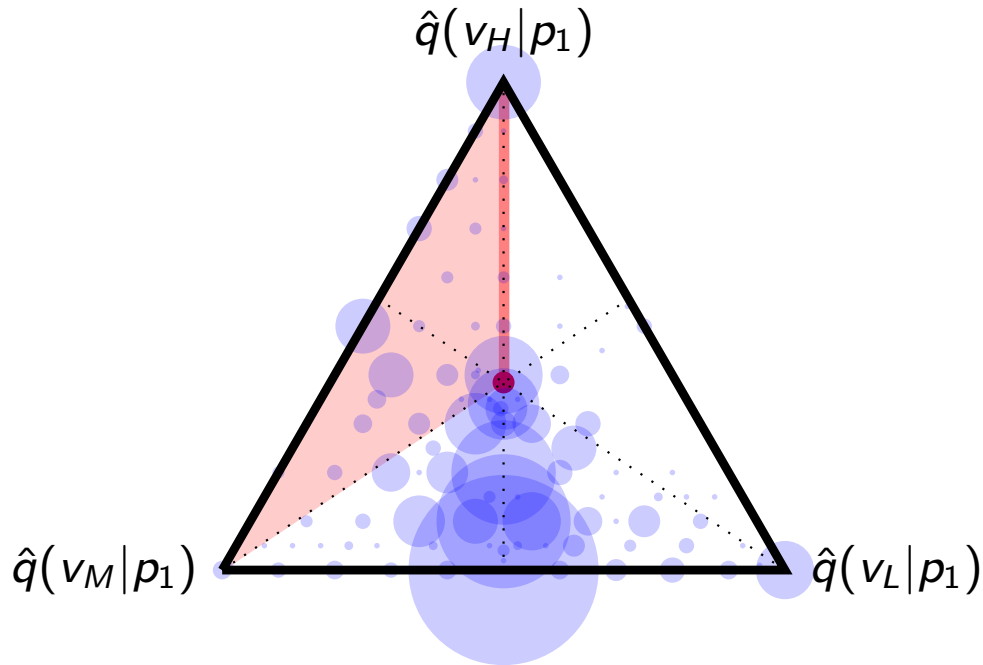


Observed posterior beliefs

Result

24.59% of the posterior beliefs (151 out of 614) are rationalized in the second-order positive selection.

Result 3



Observed posterior beliefs

Result

Only 6.51% of the posterior beliefs (40 out of 614) are rationalized in the third-order positive selection.

Caveat: The third-order positive selection leads the posterior belief to be identical to the prior belief. They might be completely naïve.

Result 4

- 66.94% (411 out of 614) of p_2s were optimal from their subjective beliefs.
- Among sellers on the 1st-order positive selection, 70.22% of p_2s were optimal. Among sellers on the 2nd-order positive selection, 76.82% of p_2s were optimal.
- Among sellers on the 3rd-order positive selection, only 55% of p_2s were optimal. More than half of them seem to be the most naïve ones who kept the prior.

Result

Majority (66.94%) of p_2s were optimal in the sense that the offer maximizes the expected profit calculated with their subjective beliefs. Higher-order reasoning on positive selection is positively associated with pricing optimality.

Result 5

$v \setminus p_1$	p_L	p_M	p_H
v_L	9% (1/11)	36% (76/211)	36% (102/280)
v_M	0% (0/3)	57% (137/240)	67% (151/227)
v_H	0% (0/7)	31% (72/231)	82% (214/260)

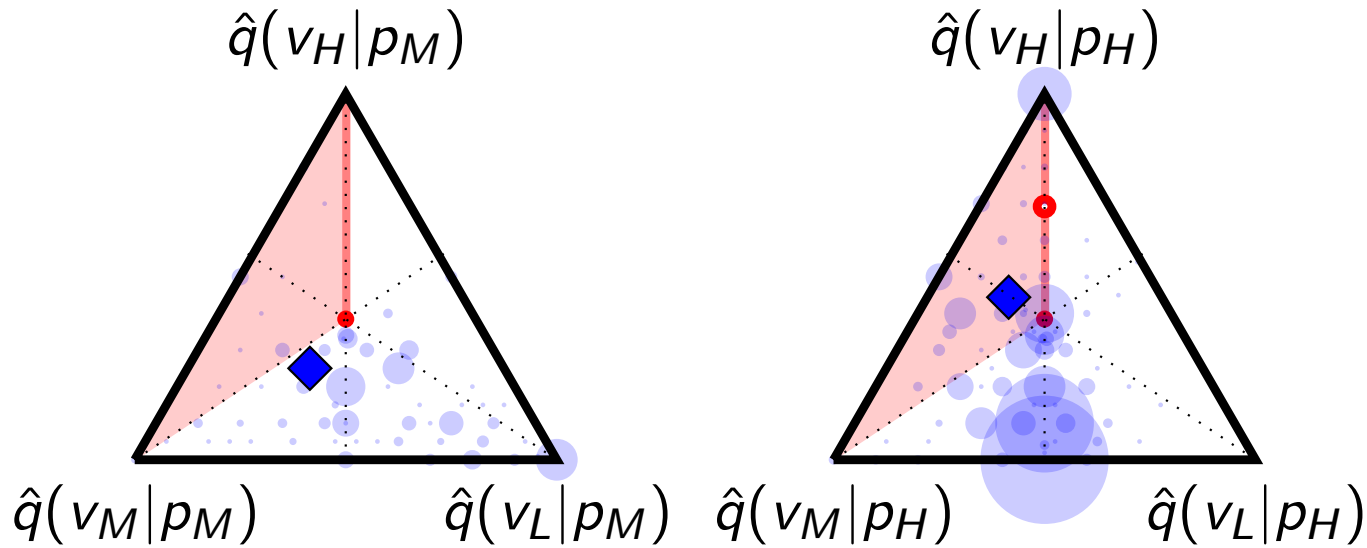
Proportions of Rejecting the First-Round Offer

Result

Some high- and middle-type buyers reject p_1 with expecting that p_2 would be more favorable to them. Some low-type buyers also reject p_1 where they could have been better off by exercising the outside option.

Further Result

Maybe Seller's posterior beliefs reflect Buyers' (perhaps irrational) rejection behaviors, and that may be the reason why they did not update the posterior belief in an equilibrium way? No.



Observed and Empirical posterior beliefs, by p_1

The blue circles on the left (right) depict the reported posterior beliefs after the first-round price offers of p_M (p_H) are rejected. A larger circle means more observations in the center of it. The diamond shape in each simplex is the posterior belief consistent with the empirically observed proportions of the rejections.

Take-away Messages

- BP's prediction is robust in theory, in many ways.
- It builds upon many layers of rational belief updating, positive selection of the remaining demand pool.
- We found that a substantial fraction (51.22%) of p_1s are rejected, which shouldn't be observed in theory.
- Only about 6.5% of Sellers report the posterior beliefs based on the 3rd-(or higher-)order positive selection.
- About half of them were naïve, meaning that few thought in an "equilibrium" way.
- Our contribution is not only checking the validity of BP but also presenting and utilizing a way to decipher which level of positive selection reasoning fails.

Related Literature

Theory

- Board and Pycia (2014), Tirole (2016)

Experiment

- Kneeland (2015), our companion paper Advertise

Research Questions

The sharp contrast in theoretical predictions inspires our research:

1. In the absence of outside option: Negative selection results in the minimum seller profit
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Would this stark difference be empirically valid, even when some players might not be entirely rational?

- We are interested in examining the treatment effect of the outside option, not in confirming or rejecting the Coase conjecture per se.