Penalty Lottery*

Duk Gyoo Kim†

March 25, 2018

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Abstract

I propose a new institution to eliminate sequential public bad productions under imperfect monitoring for punishment: A violator (a citizen who produced a public bad and got monitored) hands over the fine to the next potential violator with some probability, and pays all the accumulated fines with the complementary probability. The overall fraction of violators approaches zero in finite time periods, and this prediction is robust to heterogeneities in citizen's attributes including risk preferences. The penalty lottery *self-selects* those who are more willing to produce a public bad, and hence endogenously imposes the larger expected fines to them. From lab experiments with abstract framing, I find strong support of the theoretical predictions.

Keywords: Institutional change, public bads, Misdemeanors, Imperfect monitoring

1 Introduction

Tickets issued for misdemeanors are a multi-billion industry. 10,803,028 citations for parking violations had been issued during the fiscal year 2017 in New York City.[1] The minimum fine is \$35, so the parking tickets alone are at least a 378-million-dollar deal. If that number represents the social costs incurred by negative externalities of wrongdoing, it is natural to consider what the good way to minimize misdemeanors is. Although a day fine system in which a unit of fine payment is based on the offender's daily personal income is employed in some countries and it has been considered as an alternative

*I thank Antoine Camous, Hans Peter Grüner, Harim Kim, Sang-Hyun Kim, Euncheol Shin, Thomas Tröger, Sangjun Ye, Minchul Yum, and Galina Zudenkova for their help comments.

†Department of Economics, University of Mannheim, *d.kim@uni-mannheim.de*

[1]Source: NYC Open Data

to the fixed fine (Hillsman, 1990), it is still debatable whether the wealth level—a factor not directly related to the offending action—justifies the different fines for the same wrongdoing, and whether it would prevent misdemeanors of the poor.[2] Spending more resources to enhance public monitoring, such as hiring more police officers, is not always implementable. Increasing the fine may not be justifiable. If the fine is precisely determined at the level to correct the negative externality of wrongdoing (Becker, 1968), there is no ground for increasing the fine.

Given that the fundamental reason of prohibiting misdemeanors is that it produces a public bad in the course of doing so, a broader goal is to seek efficient ways to reduce individual's sequential public bad production under imperfect monitoring. The main purpose of this paper is to propose a simple institutional change that helps to attain the goal, which I call a *penalty lottery*: A violator, a citizen who produced a public bad for his own sake and got monitored, hands over and accumulates the fine for the next violator with some probability q, and he pays all the accumulated fines with probability $1-q$. The standard rule of the game is nested as a special case with $q = 0$. This institutional change does involve neither a change of a public monitoring capacity nor an increase of the fines, but it would significantly reduce the number of public bad producers in the end. Even in a situation where public bad production is beneficial to every citizen in the beginning, this penalty lottery asymptotically prevents every citizen from producing a public bad after some finite time periods. This prediction is robust to heterogeneities in citizens' risk preferences and social preferences. There would be a minor loss from the perspective of the local government who intervenes to minimize the negative externalities induced by the public bad, but the loss would never exceed a certain finite level.

Another merit of the penalty lottery is that it endogenously imposes the larger expected fines to those who are more likely to produce a public bad. This is because those who are more likely to produce a public bad *self-select* to take larger accumulated fines. If people subjectively evaluate the fine relative to their income, then the penalty lottery could endogenously build up the day fine system. If individual's risk preferences are different, then risk seekers are willing to produce a public bad even when the size of accumulated fines is large, which makes them to share the burden of the large accumulated fines. This implies that when the penalty lottery is introduced, the expected benefit of risk seekers could be not larger than that of the risk averse.

I claim that the penalty lottery, or random handover of the penalty, would work in various situations in which the main challenge is to minimize the production of public

[2] Another practical issue is how the income level can be transparently measured: The fact that a day fine system is currently employed in countries where the degree public tax transparency is high suggests that establishing a high level of public tax transparency may be the first-order requirement for the implementation of the day fine system.

bads with imperfect monitoring capacity. In particular, this institution would work as a mechanism to prevent misdemeanors. A citizen decides whether to violate the law for his own sake, with knowing that his action is less likely monitored and even when it is disclosed, a fine is not charged but accumulated to the public account with some probability. With the complementary probability, however, he pays all the accumulated fines.

Testing the new institution in real-world jurisdictions is extremely challenging especially when we do not know whether there are unexpected negative outcomes from the institutional change. In particular, behavioral responses to the payment of the other's fines have not been reported, so we are unsure how people would actually behave under this new institution. This is the main reason why I propose to use laboratory experiments as a testbed.

I consider two treatments in each of them differs in the probability ($q \in \{0.1, 0.5\}$) of accumulating the fine rather than paying it. The treatment with $q = 0.1$ works as a baseline experiment, which mimics the current rule of the game. The experiment is designed to minimize the framing effects: I use neutral terms so that any of their actions are not labeled as bad behaviors. During the experiment, one subject per each round either choose a red ball or a blue ball, with knowing how many black stickers are in the common pool that is shared by the group members. When the subject chose a red ball, he keeps the ball and nothing further happens. When he choose a blue ball, a black sticker is attached to the blue ball with 30% of chance. In the case of drawing a blue ball with a black sticker, the black sticker is accumulated to the common pool with probability q, but with the other probability $1-q$, he will keep the black sticker as well as all the stickers that were in the common pool. A red ball is worth -1 token, where a token is a currency unit of the experiment, a blue ball is worth $+5$ tokens, and a black sticker is worth -8 tokens. Subjects get paid based on the number of red balls, blue balls, and black stickers that they have kept during the experiment. After the main experiment, subjects' risk preferences were measured by the Bomb Risk Elicitation Task (Crosetto and Filippin, 2013).

The experimental evidence is largely consistent with the theoretical predictions. First, as more black stickers are accumulated to the common pool, less subjects chose the blue ball. Although the treatment with $q = 0.5$ gives more incentives for them to choose the blue ball as the joint probability of getting the stickers is low, the accumulation of the black stickers leads them to choose the red ball about two times more than those in the treatment with $q = 0.1$. Second, risk-seeking behavior doesn't pay under the treatment with $q = 0.5$. When $q = 0.1$, taking risks of getting the black stickers is beneficial so there is a significant negative relationship between the risk averseness and the earnings from the main experiment. When $q = 0.5$, however, the negative relationship is completely nullified.

The rest of the paper is organized as follows. In the following subsection we discuss the related literature. Section 2 describes the model and provides the main theoretical implications. In Section 3 we describe the experiments and Section 4 reports experimental findings. Section 5 discusses the objective of the local government, and Section 6 concludes. Proofs of some of the lemmas and propositions that appear in the main text are provided in the Appendix.

1.1 Related Literature

This study is inspired by Gerardi et al. (2016), Duffy and Matros (2014) and Bhattacharya et al. (2014) who study how an institutional change affects voter turnout. Gerardi et al. (2016) compare the lottery (of giving a prize to one of turnout) with the fine (to those who do not turn out to vote), and Duffy and Matros (2014) consider a combined mechanism in which the fines imposed on non-participants are used to finance the prize of the lottery winner. In a similar vein, Bhattacharya et al. (2014) compare the compulsory voting with the voluntary voting. In a broader sense that we examine how an institutional change drives citizens to behave in a more desirable way, this paper contributes to the literature on the impact of institutional changes.

This study is also related to imperfect public monitoring on the behavior of producing public bads. Ambrus and Greiner (2012) report how 'imperfect' monitoring changes the contribution behavior in the public good contribution games with a costly punishment option. Their experimental evidence shows that access to a standard punishment technology significantly decreases net payoffs even in the long run, and access to a severe punishment technology leads to roughly the same payoffs as with no punishment option. This implies that how hard to make the individuals to act in a socially desirable way, even if a severe punishment is adopted. The main advantage of the penalty lottery over a simple severe punishment is that it works well under imperfect monitoring.

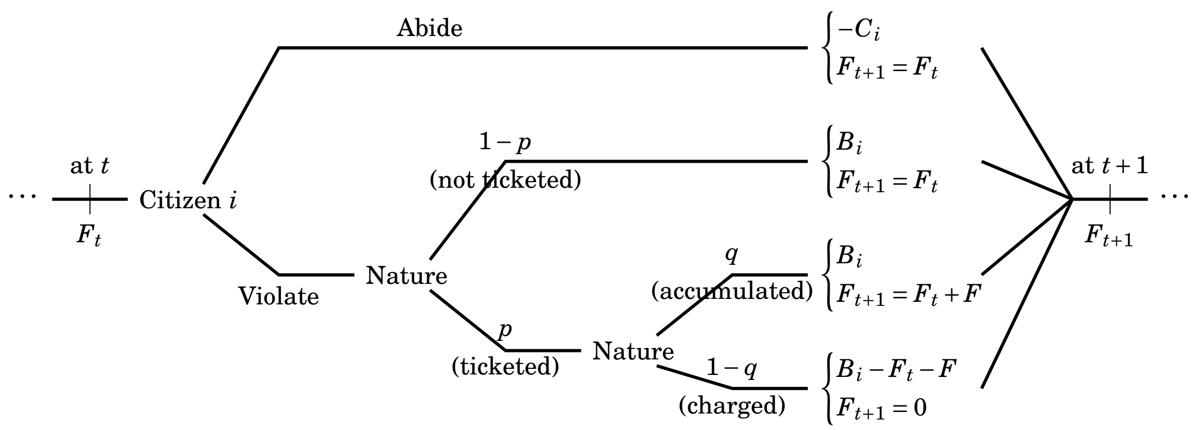
Using lotteries in a non-standard setting is not a new idea. Kearney et al. (2010) overview prize-linked savings (PLS) accounts which essentially provide 'no-lose' lottery tickets to savors, and Filiz-Ozbay et al. (2015) examine its validity using lab experiments. Morgan (2000) theoretically shows that funding public goods by means of lotteries outperform voluntary contributions in that lotteries increase the provision of the public good close to first-best level, and the theoretical predictions are consistent with experimental findings in Morgan and Sefton (2000).

2 Theoretical Framework and Implications

Although the model can be applied to general situations of sequential public bad production under imperfect monitoring for punishment, we consider a problem of conducting a misdemeanor for clearer illustration. Violating the law for one's own sake can be read as producing a public bad.

Consider a society with a continuum of citizens indexed by i. A random citizen sequentially[3] faces a problem of wrongdoing for his/her own benefit at a discrete time $t \in \mathbb{N}_+$.[4] For terminological clarity, I call those who violate the law as "wrongdoers", and some wrongdoers who got caught by a police officer as "violators". Let B_i denote the units of benefit from wrongdoing, C_i denote the units of cost to abide by the law, and F denote the amount of the fine. To make the problem nontrivial, assume that $F > 0$, $B_i > C_i > 0$, and $(1-p)u_i(B_i)+pu_i(B_i-F) > u_i(-C_i)$, where $p \in (0,1)$ is the common probability of getting monitored (hence ticketed), and $u_i(\cdot)$ is an increasing concave utility function of citizen i. That is, the expected utility of wrongdoing is greater than that of paying the cost of abiding by the law for all i. In this scenario, every citizen is better off by wrongdoing.

Figure 1: Timing of events



Now I consider one additional state variable. A violator is asked to pay the fine with probability $1-q$. With probability q, the violator hands over the fine to a (potential) next violator and the fine is accumulated to a public account. Let F_t denote the accumulated

[3]Their decisions are independent to what others are doing. This can also be interpreted that each citizen does not encounter others in the situation where wrongdoing is considered.

[4]Alternatively we can consider n citizens and each citizen sequentially arrives at $t = (\tau-1)n + i, \tau \in \mathbb{N}_+$, where the finite number of citizens and the circular time index are used for lab experiments. It doesn't mean that the model should assume that every citizen makes a decision in a circular queue.

fines at time t, with $F_0 = 0$. That is, at time t, a citizen faces a problem of choosing either B_i or C_i with knowing that the probability of paying the fine, $F_t + F$, is $p(1-q)$. The timing of event is described in Figure 1. Note that the typical rule of the game is nested in this institution as a special case with $q = 0$. I assume here that the local government's goal is to minimize infractions and misdemeanors.[5]

For any $q \in [0,1)$, a finite number of citizens would violate the law as the probability to pay the fine, $p(1-q)$, is lower than p. However, once the fines are sufficiently accumulated, the expected benefit of wrongdoing becomes no longer greater than the cost of abiding by the law. Specifically, for any τ such that $(1-p+pq)u_i(B_i)+p(1-q)u_i(B-(F_\tau+F)) \leq u_i(-C)$, citizen i will not violate the law. For risk neutral citizen i, there exists $k_i \in \mathbb{N}_+$ such that $k_i = \left\lceil \frac{B_i+C_i}{p(1-q)F} \right\rceil - 1$, where $\lceil x \rceil$ is the smallest integer larger than x, if there are k_i consecutive violators who have not paid the fines, citizen i will not violate the law afterwards. Since k_i is a cutoff level of fine accumulation for a risk-neutral citizen, k_i can be understood as the upper bound[6] of cutoffs for any degree of risk aversion. Although this model deals with three types of heterogeneities, all the other various dimensions of individual heterogeneities can also be summarized by k_i, and k_i is interpreted as a degree of willingness to violate the law. Let $\bar{k}$ denote $\max_i k_i$, and $G(k)$ denote the cumulative distribution of k_i. The actual number of wrongdoers varies by how many violators happened to pay the fines before reaching $F_t = \bar{k}F$. However, the probability of $\bar{k}$ consecutive accumulations approaches 1 as time goes by. Let $P(V|t;p)$ be the ex-ante probability that a violation is observed after t periods when the probability being monitored is $p \in (0,1)$. I use $P(V|t)$ whenever p is not of interest.

Proposition 1. *For $q \in (0,1)$, $P(V|t)$ is monotone decreasing in t. The expected time to reach $P(V|t) = 0$ is finite. If $q = 0$, $P(V|t) = 1$ for any t.*

Proof: See Appendix.

Proposition 1 becomes more intuitive if we consider a homogeneous economy where $u_i = u$, $B_i = B$, and $C_i = C$ for all i. Then $P(V|t;p)$ approximates to the probability that a success run of at least length k for an event with a probability of q occurs at least once within pt[7] number of trials. The nondecreasing probability of success run in the number of trials implies that in a longer period it is more likely to reach the

[5] Considering that the state court system's budget typically relies on funding from traffic citations as a source of revenue, this may not be a trivial assumption. I discuss this further in Section 5.

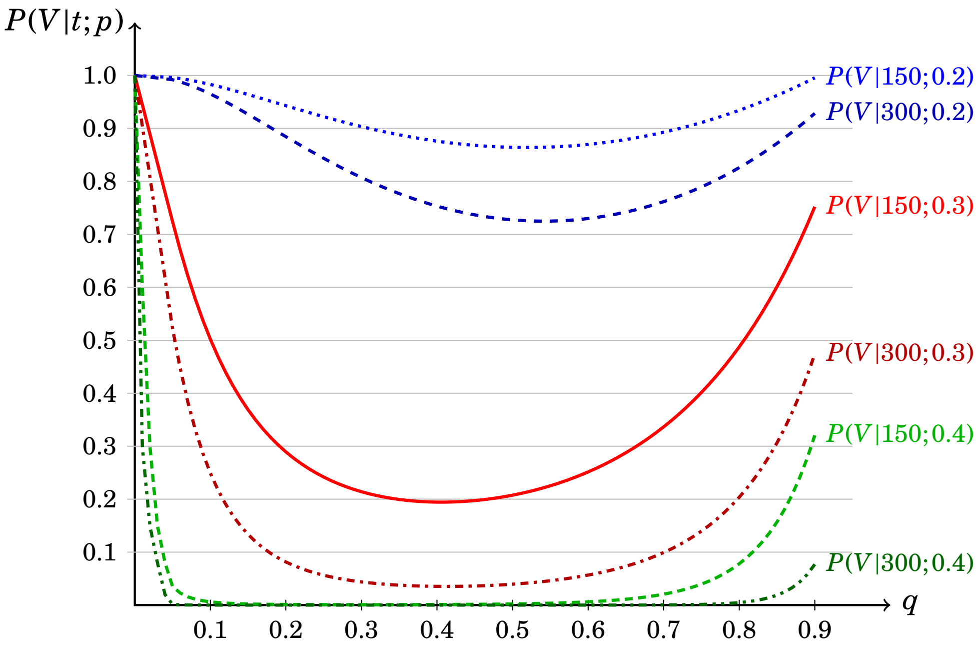
[6] The assumption of weak concavity of the utility function applies here. If some citizens have a convex utility function, which implies that they enjoy taking risks, then they may have a higher cutoff threshold than k_i.

[7] Since we consider an imperfect monitoring and only p proportion of the whole wrongdoings are monitored, only tp periods out of t can be considered as Bernoulli trials, on average.

threshold kF and no one will violate the law with a higher probability. For example, if $\{B,C,F,p,q\} = \{5,1,8,0.4,0.5\}$, $k = 3(= \lceil \frac{5+1}{0.4 \cdot 0.5 \cdot 8} \rceil - 1)$, and the probability of at least 3 consecutive accumulations in 100 periods is 95.66%. That is, the probability that there is still a wrongdoer after 100 periods is less than 5%. Such a probability approaches 0 as time passes by.

It is worth noting that q must be intermediate—neither too close to 0 nor to 1—for this institution to work in shorter time periods. If it were to be near 0, the threshold of the accumulated fines is not likely to be achieved. If it were to be near 1, no one would actually pay the accumulated fines as $\frac{B+C}{p(1-q)F}$ is convex increasing in q. If $\{B,C,F,p\} = \{5,1,8,0.3\}$, for example, the probability that there is a wrongdoer after 50 periods is near 0% with $q = 0.5$, while the probability is 41.18% with $q = 0.1$ and 25.21% with $q = 0.9$. Figure 2 illustrates theoretical predictions under a homogeneous economy with risk neutral citizens. This exercise would work as a benchmark for the experiments.

Figure 2: Summary of Some Theoretical Benchmarks, $F = 8, B = 5, C = 1$



$P(V|t;p)$ is the probability that a violation is observed after t periods when a wrongdoing is monitored with probability p. For smooth graphs, $k = \frac{B+C}{p(1-q)F} - 1$, instead of $\left\lceil \frac{B+C}{p(1-q)F} \right\rceil - 1$ is considered. To calculate this consecutive accumulation run, I used Feller's Success Run Approximation. See the proof of Proposition 1. Since we consider a situation where wrongdoing is beneficial to every citizen, such a probability is 100% for any time period if $q = 0$. As t increases, $P(V|t;p)$ monotone decreases with any given $q \in (0,1)$. In shorter time periods, having too high q does not function effectively because the probability that a wrongdoer pays a fine, $p(1-q)$, is lower with a higher q and it attracts citizens to violate the law. Having too low q does not work either because the probability of k consecutive fine accumulation is less likely.

In sum, after the society consecutively accumulates k fines, it reaches a 'steady state' where no one violates the law. Although we figured out that the penalty lottery leads to complete elimination of sequential public bad production in the long run, it is also important to understand what happens before reaching the steady state, because it may take longer time when individuals are more heterogeneous: Even a minuscule fraction of the population with a larger k_i being added to the society leads to a substantial delay to reach the steady state.

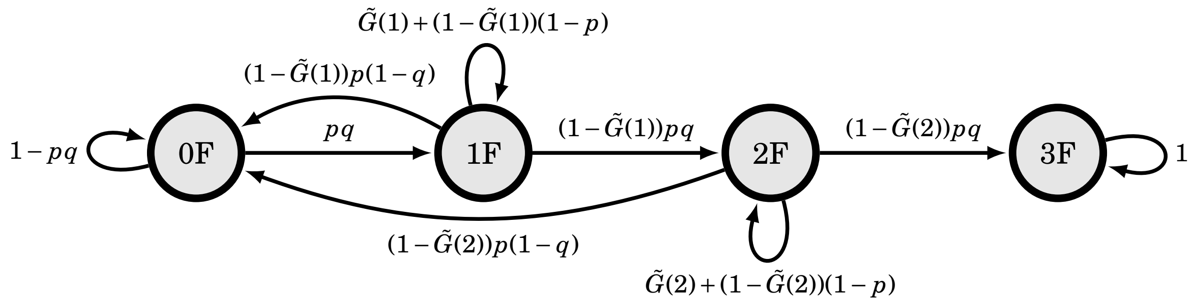
Proposition 2. *The expected number of time periods reaching the steady state starting from zero fine accumulations is*

$$\frac{1}{pq}\sum_{i=0}^{\bar{k}-1}\frac{1}{(1-G(i))q^i},$$

which leads to two observations. Given the class of the distributions with the same mean, $P(V|t)$ is decreasing in $g(\bar{k}) = 1-G(\bar{k}-1)$. If $\tilde{G}(k)$ is the mean-preserving spread of $G(k)$ with a larger $\bar{k}$, and $\tilde{P}(V|t)$ be the ex-ante probability under $\tilde{G}(k)$, $\tilde{P}(V|t) \geq P(V|t)$.

Proof: See Appendix.

Figure 3: Finite-State Absorbing Markov Chain and Transition Probability Matrices



$$T_G = \begin{bmatrix} 1-pq & pq & 0 \\ p(1-q) & 1-p & pq \\ 0 & 0 & 1 \end{bmatrix} \qquad T_{\tilde{G}} = \begin{bmatrix} 1-pq & pq & 0 & 0 \\ (p-pq)/2 & (2-p)/2 & pq/2 & 0 \\ (p-pq)/2 & 0 & (2-p)/2 & pq/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Consider $\tilde{G}(k)$, a mean-preserving spread of a degenerate distribution at 2, where $g(1) = g(3) = 0.5$. The probability of reaching the final node from the initial node after t periods, $(T^t_{\tilde{G}})_{1,k_h+1}$, is smaller than $(T^t_G)_{1,k+1}$.

For illustration of Proposition 2, consider a society where a half of the citizens have $k_l = 1$ and the other half have $k_h = 3$. Then, until k_l fines are consecutively accumulated everyone will violate the law, and for any given time t, pt violations on average will be monitored. However, when k_l fines are consecutively accumulated, only the half of the population would still violate the law, and for any given additional time τ, $\frac{1}{2}p\tau$ violations on average will be monitored. Two additional fines need to be consecutively accumulated with a smaller number ($\frac{1}{2}p\tau$ instead of $p\tau$) of probabilistic trials, so it must take longer time to reach the steady state, compared with a homogeneous society where $k_i = 2$ for all i. Worse yet, if someone with $k_h = 3$ pays all the accumulated fines before reaching $F_t = k_h F$, then the entire population starts violating the law again. Figure 3 illustrates the Markov chain and the corresponding transition probability matrices for the aforementioned example. The probability of observing a violation after t periods is the last element of the first column in T^t, where T is the transition probability matrix. Although the above example considered drastically different distributions, a minuscule change can leads to a significant delay. When the society consist of homogeneous citizens with $k_i = 2$, the probability that a violation is observed after 100 periods is 0.32% (with $p = 0.3$, $q = 0.5$), while if 1% of the population has $k_i = 3$ instead, such a probability skyrockets to 88.95%.

A naturally followed question is whether the penalty lottery is desirable for a substantively longer before-the-steady-state periods. The answer is yes, as it endogenously imposes the larger expected fines to those who are more likely to violate the law.

Corollary 1. *Let $\phi(k_i)$ be the ex-ante expected fines for an individual with k_i. $\phi(k_i)$ is increasing.*

Proof: $\phi(k_i) = \sum_{\kappa=0}^{k_i} p(\kappa)(\kappa+1)$ is the ex-ante expected fines, where $p(\kappa)$ is the ex-ante probability that κ fines have been accumulated. □

Among various dimensions of individual heterogeneities lead to different k_i, I focus on a few important dimensions: risk aversions, heterogeneous benefits, B_i, and some social preferences. Corollary 1 implies that those who have more benefits from wrongdoing and those who are more risk-seeking are *self-selecting* to pay the higher ex-ante expected fines. The risk averse citizens have a smaller k_i, as the fine gets accumulated the possible payoffs become more volatile, so their expect benefit of wrongdoing decreases with risk averseness. A similar logic applies to those who have a larger B_i. As the fine gets accumulated the smaller fraction of citizens with higher B_i would remain to be wrongdoers. Interpreting $\frac{B_i}{F}$ as a subjective value of the fine,[8] the higher ex-ante expected fines for those with a higher B_i would naturally implement a day fine system. Lying-averse citizens or sincere citizens can be understood as those with $B_i = 0$. Therefore,

[8]More often than not, the benefits of wrongdoing is not be measurable, while the fine, especially if it is a monetary penalty, is. From the perspective of the rich, F could be relatively minor compared with their wealth. In this case, a higher $\frac{B_i}{F}$ could mean that citizen i subjectively undervalues the fine.

Table 1: Experimental Design and Summary of Hypotheses

Treatment	p	q	$p(1-q)$	k	$P(\text{V}\|t=60)$
Mq	0.3	0.5	0.15	4	56.19%
Lq	0.3	0.1	0.27	2	85.44%

Each session consists of 60 consecutive rounds. In each round, one subject among a group of four chooses either a red ball or a blue ball. A red ball has a value of −1. A blue ball has a value of 5, but with probability p a black sticker of which value is −8 is attached to the blue ball. The black sticker is collected to the common pool with probability q.

lying-averse citizens would not be wrongdoers no matter what q is, and the institutional change does not affect their decisions. If citizens have heterogeneous other-regarding preferences, especially inequity aversions, then those with a higher inequity aversion would have a lower k_i. When $F_t > 0$, a wrongdoer is voluntarily taking the possibility of paying previous violators' fines, which exacerbate the inequity between him and the previous violators (who did the same action). All in all, the effect of the institutional change would be significant, and is robust to heterogeneities in individual characteristics.

3 Experimental Design and Procedure

To the best of my knowledge, behavioral responses to the (potential) payment of the other's penalties have not been reported, so it is unclear whether the institutional change would work in the desired direction. This is why the controlled lab experiments are used as a testbed.

Having abstract framing[9] in mind, I consider two experimental treatments which differ in the probability of penalty accumulation (q). For notational simplicity, the treatment with $q = 0.1$ is called Lq (Low q), and the other treatment with $q = 0.5$ is called Mq (Middle q). Table 1 summarizes the design of the experiments.

The basic procedure of an experimental session is as follows: Each subject is endowed with 100 tokens (the experimental currency units) in his/her account. Each subject is randomly grouped with three other subjects and sequentially makes a decision within the group. The experiment consists of 60 decision-making rounds, so each subject made 15 decisions.[10] When a subject is on her turn, she is asked to choose either a red ball or

[9] Although the design of experiments to test the effect of the institutional change might be straightforward, one of key challenges in this particular context is to maintain abstract framing. I avoid using terms such as "wrongdoing", "violations", "fines", and "misdemeanors": Otherwise interpretations of the experimental results could be confounded as there could exist an experimenter-demand effect (Zizzo, 2010). It is also known that subjects are heterogeneous in terms of internalizing the social norms (Kimbrough and Vostroknutov, 2016), so the statistical analysis would have less power to draw conclusions when the particular frame of violating a law is considered.

[10] There are two reasons why a subject makes 15 separate decisions rather than 15 consecutive decisions

a blue ball, with knowing that how many black stickers are accumulated in the common pool shared by the group members. In the beginning, there are no black stickers in the common pool. Keeping a red ball ends her turn. When choosing a blue ball, with probability 0.3 a black sticker is attached to the blue ball, and with the complementary probability 0.7 no black sticker is followed. If a black sticker is attached, the computer determines where the black sticker goes. With probability q, the black sticker is removed from the blue ball and added to the common pool. With probability $1-q$, the subject keeps the black sticker as well as all the black stickers from the common pool. In each turn, a subject doesn't know which balls other subjects have chosen, but know how many black stickers are accumulated in the common pool. At the end of the session, the total payoff of a subject who has x red balls, y blue balls, and z black stickers is $100-x+5y-8z$. That is, each red ball (the cost of abiding by the law) is worth -1 token, each blue ball (the benefits of wrongdoing) is worth $+5$ tokens, and each black sticker (the fine) is worth -8 tokens.

Two treatments which I do not conduct may be worth mentioning. First, when a subject makes a decision, how other subjects had acted is not informed. Although I believe that knowing other subjects' decisions would affect the subject to some degree and it would be interesting to know to what extent an indirect social pressure impacts, the main purpose of the experiments is not on examining such an impact. Second, instead of having $q = 0$, I consider treatments with $q = 0.1$ as a baseline experiment. Several studies including Tversky and Kahneman (1981) and Martinez-Marquina et al. (2018) have reported that individual's behavior under zero probability (or certainty) is distinctively different to that under near-zero probability (or near-certainty), and it is also known that people tend to treat complex lotteries different to simple lotteries even though those two lotteries are equivalent (Huck and Weizsäcker, 1999). Thus I believe that individual decisions on probabilistic events should be compared with those on other probabilistic events, not deterministic events.[11]

Experiments were conducted at the Mannheim Laboratory for Experimental Economics (mLab) in February 2018. I ran two sessions for each treatment, with 20 to 24 subjects (or 5 to 6 independent groups of four) each. A total of 84 subjects partici-

as a single player. First, if 15 consecutive decisions were to be made, then the state variable of the next round (the number of black stickers in the common pool) is endogenously affected by the previous decision, while having 3 decisions between a single subject's actions dilutes such endogeneity. Second, although it would be ideal if the size of the group is larger, with considering the typical capacity of a laboratory, the group size of four was practically optimal as it is large enough for diluting endogeneity as well as small enough for collecting many observations within a limited time period.

[11] Given that q must not be zero in the baseline, another practical reason of considering $q = 0.1$, not lower than 0.1, is that subjects may find the joint probability complicated than the Mq treatment. During the pilot session, $q = 0.05$ was used and many subjects struggled to instantly figure out the joint probability $p(1-q) = 0.285$.

pated in one of the sessions. The participants were drawn from the mLab subject pool. Python and its application Pygame were used to computerize the games and to establish a server–client platform. All experimental sessions were organized along the same procedure: After the subjects were randomly assigned to separate desks equipped with a computer interface, they were asked to carefully read the instructions (see Appendix B) for the experiment. After reading the instructions, subjects took a quiz to prove their understanding of the experiment. Those who failed the quiz were asked to read the instructions and to take the quiz again until they passed. An instructor answered all questions until every participant thoroughly understood the experiment. Although each of subjects belonged to a group of four, there was no physical reallocation of the subjects, and they only knew that they were randomly grouped. They were not allowed to communicate with other participants during the experiment, nor allowed to head up to look around the room. It was also emphasized to participants that their decisions would be anonymous and no deception is employed. At the end of the experiment, they were asked to fill out a survey asking their gender and age. The subjects' risk preferences were also measured by an additionally paid Bomb Risk Elicitation Task (BRET) (Crosetto and Filippin, 2013). BRET is known to be a parsimonious but accurate way to elicit one's risk preference: One among 36 boxes hides a bomb, and a subject collects as many as boxes she wants without knowing where the bomb is hidden. If the bomb was in one of the collected boxes, she earns zero. If the bomb was not in the collected boxes, she earns €0.11 for each chosen box. The total amount of tokens that each subject earned during the main session was converted into Euro at the rate of €0.08/token.[12] Payments (€11.80 on average, including the earnings from BRET) were made in private at the end of the session, and subjects were asked not to share their payment information. Each session lasted approximately 1 hour.

4 Experimental Results

To summarize first, I found that the experimental evidence is largely consistent with the theoretical predictions. First, the proportion of wrongdoers approaches 0 as the size of the accumulated fines increases. Second, risk-seekers are not better off than the risk averse agents in the Mq treatment, while risk-seekers are better off in the Lq treatment.

Table 2 summarizes the data. In total of 84 subjects (44 in Mq, and 40 in Lq) have participated in one of the sessions. Red Balls refer to the percentage of the actual choices

[12] I instructed subjects that the currency exchange should not be a concern as it would be handled by the server computer. The exchange rate €0.08/token is chosen to make the average earning to be close to the desired size of payments.

Table 2: Data Summary

Treatment	Number of Subjects	Red Balls (%)	Blue Balls (%)		
			w/o Sticker	Common	Subject
Mq	44	16.52	61.52	9.24	12.72
Lq	40	8.67	65.33	2.83	23.17

of red balls. Apparently the number of black stickers in the common pool affect subjects' choices, which will be examined shortly. There are three cases when subject chose a blue ball. Blue Balls w/o Sticker refers to the case where the subject chose a blue ball and it came without a black sticker. Blue Balls Common refers to the case where the subject chose a blue ball, it came with a black sticker, and the sticker is added to the common pool. Blue Balls Subject refers to the case where the subject chose a blue ball, it came with a black sticker, and the sticker as well as all the stickers in the common pool are added to the subject's account. The results are well randomized. In the Lq treatment, for example, given that a blue ball was chosen for 91.33% of the whole decision-making rounds, the fraction of those three cases would be close to 63.93% (=0.7*91.33), 2.74% (=0.3*91.33*0.1), and 24.66% (=0.3*91.33*0.9), respectively. Also, it seems that abstract framing worked properly: For 96.67% of the whole decision-making rounds in the Mq treatment, choosing a blue ball maximizes the subject's (risk neutral) expected payoff, and about 84.80% of the cases the blue ball was chosen. Similarly, among 95.5% of the decision-making rounds in the Lq treatment, 94.07% of the cases the blue ball was chosen. If subjects viewed the problems they faced during the experiment as a situation of violating the law for the sake of their own benefits, the fraction of the blue ball choices would have been much lower.

4.1 Responses to Penalty Accumulations

One desirable observation is that subjects respond to the number of the black stickers in the common pool. Figure 4 shows how many red balls were chosen at different numbers of black stickers in the common pool. In both treatments it is clearly upward sloping, which implies that the more fines are accumulated, the less subjects choose to produce a public bad. In the Mq treatment, if the number of accumulated black stickers in the common pool exceeds 4, it is certainly not beneficial for risk neutral subjects to choose a blue ball. Except for one subject who consistently chose a blue ball when 5 stickers are in the common pool, everyone else chose a red ball. Unlike realistic situations the experiment has the last round, and subjects may choose a riskier option in their last round. Such a "last round effect" is observed (see the blue lines in Figure 4), but it is statistically insignificant. From now on, I examine all the decisions from the whole rounds unless otherwise stated.

Figure 4: Proportions of Red Ball Choices

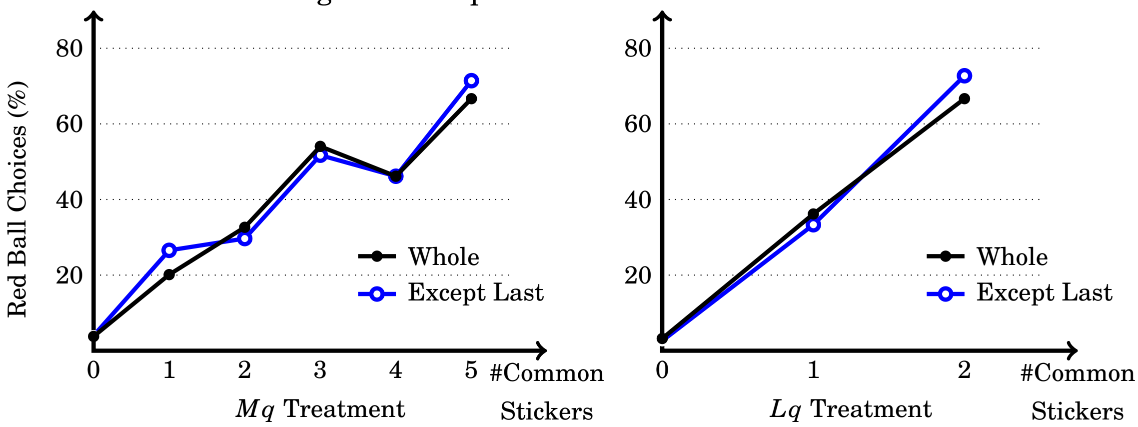


Table 3 summarizes some regression results. Throughout all the regression models being considered, two explanatory variables are statistically significant at the 1% level: the number of black stickers in the common pool (ComBlack) and the subject's previous action of choosing a red ball (prevRed). Consistent with the theoretical prediction and the trend shown in Figure 4, subjects strongly respond to the number of the black stickers in the common pool after controlling for other explanations. It is also reasonable that subjects respond more vigilantly under the Lq treatment, which is captured by negative estimates of the coefficient of ComBlack×Mq, where Mq is a binary variable indicating whether the treatment of the session was Mq. This is because k, the theoretical threshold for a risk neutral agent to start choosing red balls, is 4 in the Mq treatment, while it is 2 in the Lq treatment. The positive relationship between a previous red ball choice and a current red ball choice implies that there were some subjects who consistently chose red balls: Perhaps they are more risk averse, more reluctant to take an action that has a potential to negatively impact the other members' payoffs, and/or more inequity averse.

The previous realization of a probabilistic event also affects subjects' decisions in the following round especially when the realization is a loss, as many previous experimental studies regarding decision making under uncertainty report so (Imas, 2016). When a subject chose a blue ball in the previous round, and it led him to keep the black stickers including ones in the common pool (labeled as prevBSubj in the regression table), he tends to choose a blue ball again in the following round. However, this risk-taking behavior after a loss is not as significant as the affect of the number of black stickers in the common pool. Also, some individual characteristics collected by the after-experiment

Table 3: Red Ball Choices

	LPM			Logit	
	(1)	(2)	(3)	(4)	(5)
ComBlack	0.3245***	0.3245***	0.2119***	2.3669***	1.7400***
	(0.0526)	(0.0541)	(0.0502)	(0.4143)	(0.3980)
SubjBlack	−0.0102	−0.0073	−0.0024	−0.0736	−0.0273
	(0.0086)	(0.0089)	(0.0066)	(0.1001)	(0.0747)
Mq	0.0082	0.0155	0.0025	0.5942	0.4058
	(0.0334)	(0.0343)	(0.0213)	(0.5749)	(0.4004)
ComBlack×Mq	−0.1826***	−0.1797***	−0.0984*	−1.3706***	−0.9320**
	(0.0578)	(0.0602)	(0.0504)	(0.4884)	(0.4292)
prevRed			0.3650***		2.1169***
			(0.0830)		(0.4497)
prevBCom			0.0393		0.4052
			(0.0444)		(0.3425)
prevBSubj			−0.0266*		−1.2159**
			(0.0142)		(0.5663)
cons.	0.0525**	−0.1067	−0.0449	−4.8794**	−4.1715**
	(0.0225)	(0.1958)	(0.1228)	(2.4121)	(1.6359)
Indiv.Chars.	Excluded	Included	Included	Included	Included
R^2	0.2343	0.2518	0.3474	0.2680	0.3538
N	1,260	1,260	1,092	1,260	1,092

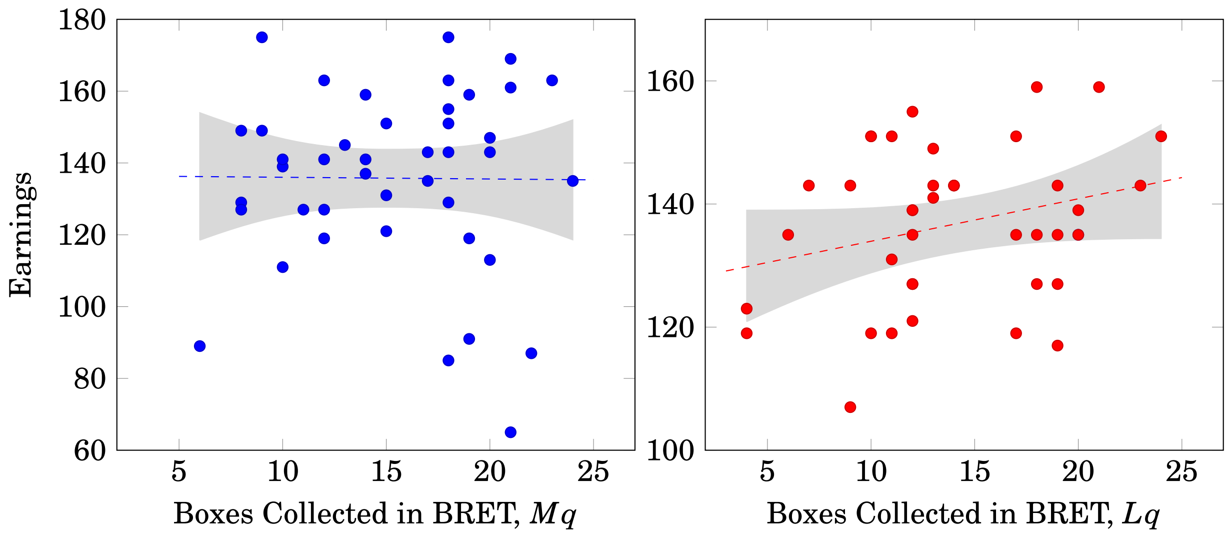
The dependent variable is the choice of a red ball. ComBlack is the number of black stickers in the common pool, and SubjBlack is the number of black stickers that the subject has kept. Mq is a binary variable indicating whether the treatment of the session was Mq. prevRed, prevBCom, and prevBSubj are binary variables respectively indicating whether the subject chose a red ball in the previous turn, whether she chose a blue ball followed by a black sticker added to the common pool, and whether she chose a blue ball followed by a sticker and kept all the stickers including ones in the common pool, respectively. Heteroskedasticity-robust standard errors clustered by subject id are in parenthesis. *, **, and *** indicate statistical significances at the 10% level, 5% level, and 1% level, respectively.

survey—gender, age, self-confidence in their performance of the experiment, and familiarity of this type of experiment—were insignificant.

4.2 Risk Seeking Doesn't Pay

Another important theoretical prediction is regarding self-selection: Those who are more risk-seeking are self-selecting to pay the higher ex-ante expected fines. This implies that risk-taking actions will be paid less under the penalty lottery. The experimental evidence supports this claim. Figure 5 shows scatter plots and fitted lines of earnings (from the main experiment) on the number of boxes collected in BRET. A subject's risk preference is elicited by the number of collected boxes in BRET: As she collects more boxes, both the probability that the bomb is included in one of the collected boxes and the payoff when the bomb is not included in one of them increase. Thus a natural interpretation is that those who collect more boxes in BRET are more risk seeking. In Lq treatment, those who are more risk-seeking are paid more: Those who collect one more box in BRET earns on average 0.69 tokens ($p = 0.072$). In Mq treatment, however, there is no linear relationship ($p = 0.961$) between earnings and the number of boxes collected in BRET.

Figure 5: Risk Preferences and Earnings



This figure shows scatter plots and fitted lines of earnings on the number of boxes collected in BRET. In Mq treatment, there is no linear relationship between earnings and the number of boxes collected in BRET. In Lq treatment, however, those who are more risk-seeking are paid more.

This result, the null relationship between risk preferences and earnings under the penalty lottery, must require careful interpretation. Choosing a risky option (a blue ball) can increase the expected payoff when the expected loss (the number of black stickers in the common pool) is not large. That is, even with the penalty lottery, still some risk takers would be better off than the risk averse. In this regard, a more accurate but verbose title for this subsection should be "risk seeking pays less than what it used to pay without the penalty lottery". What we should look further is the variance of the earnings. Table 4 shows the F-test results of equality of variances. In the Mq treatment, the variance of the subjects whose risk preferences are in the top half is significantly larger than the variance of those in the bottom half. In the Lq treatment, such a difference in variances is not found. This demonstrates how the penalty lottery works in a way to prevent risk takers to earn more: Although it is in general true that a high risk yields a high return, with the penalty lottery, a high risk brings a higher volatility in payoffs.

Table 4: F-test of Equality of Variances

Mq		Lq	
St.dev.(Earnings $\mid B_i \leq \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i > \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i \leq \text{Med}(B_j)$)	St.dev.(Earnings $\mid B_i > \text{Med}(B_j)$)
18.92	30.81	13.20	12.22
$F_{20,20}(2.6504) = 0.0173^{**}$		$F_{21,16}(1.1678) = 0.3808$	

5 Discussions

5.1 The purpose of the local government

Although I claim that the penalty lottery would work in general situations of sequential public bad production under imperfect monitoring for punishment, I should admit that reducing misdemeanors is one of the most applicable cases. If we narrow down our focus to the elimination of misdemeanors, the main objective of the local government would matter because it affects the size of the fine and therefore the validity of the penalty lottery.

The main purpose of this study is not on investigating the optimal size of the fine. This is because I assume that a policymaker deliberately measures the negative externality of wrongdoing so the fine is given at a legitimate level (Becker, 1968). That is, I assume that the fine is equal to a Pigouvian tax to correct the negative externality incurred by the wrongdoing[13] plus the cost of executions. If the local government has a

[13] In this regard of interpreting the fines as indirect taxes, it makes more sense that speeding tickets are more issued to drivers who are not local constituents, as Makowsky and Stratmann (2009) find: This is a way to effectively increase tax revenues from the outside.

balanced budget on misdemeanors, having no revenues with few violations in the end is as good as having full (budget balanced) revenues with everyone's violation. It is a plus that in the course of attaining the steady state of no violations, the revenues are almost the same while imposing larger fines to those who are more willing to violate the law.

However, from many anecdotal evidences and some rigorous reports[14] we know that the fundamental goal of the municipalities may not be on eliminating infractions and misdemeanors: Speeding and parking tickets to drivers could be considered as an important revenue source of the local government. If the government's purpose is not to maximize the social welfare, but to maximize the revenue accrued from the fines, minimizing misdemeanors is not of their interests at all.

5.2 Sincere vs. rational non-violators

So far I have assumed that $B_i > C_i > 0$ and $(1-p)u_i(B_i)+pu_i(B_i-F) \geq u_i(-C_i)$ for all i, to start from a benchmark case where everyone is better off by producing a public bad. If we relax these assumptions, there could be some citizens who will not produce a public bad under the standard rule of the game. In the context of conducting misdemeanors, I call citizen i with $u_i(B_i) \leq u_i(-C_i)$ as a *sincere* non-violator because he doesn't violate the law regardless of p, and citizen j whose utility function satisfies $u_j(B_j) > u_j(-C_j)$ and $(1-p)u_j(B_j)+pu_j(B_j-F) \leq u_j(-C_j)$ as a *rational* non-violator. That is, while the penalty lottery doesn't give an incentive to sincere non-violators to violate the law, it does give an incentive to some rational non-violators if q is too large. For a rational non-violator j, there exists $q_j \in (0,1)$ such that for all $q \geq q_j$, $(1-p+pq)u_j(B_j)+p(1-q)u_j(B_j-F) \geq u_j(-C_j)$.

Even when a large fraction of the population are the rational non-violators, the penalty lottery will work unless q is not too high. Consider a 'marginal' citizen among rational non-violators such that $(1-p)u_j(B_j)+pu_j(B_j-F) = u_j(-C_j)$. This citizen is called marginal because he is indifferent between violating and not violating the law when $q = 0$, but for all $q > 0$ he violates the law if $F_t = 0$. Even if the marginal citizen is risk neutral, he will again abide by the law immediately after $\lceil q/(1-q) \rceil$ fines are accumulated. Thus, if $q \leq 0.5$, then all the rational non-violators will not violate the law when one fine is accumulated in the public account.

Also, I claim it is not a bad idea to distinguish sincere non-violators from those who strategically choose not to violate the law. Indeed, their violations can be increased without the introduction of the penalty lottery: For example, if the monitoring capacity

[14] Garrett and Wagner (2009) find that significantly more tickets are issued in the year following a decline in revenue, but an increase in revenue does not lead to smaller issuance of tickets. Makowsky and Stratmann (2011) also find that budgetary shortfalls lead to more frequent issuance of tickets to drivers.

were to be $p(1-q)$ instead of p, they would have violated the law.

6 Concluding Remarks

I propose using lotteries to control public bad production under imperfect monitoring for punishment. The key idea is to randomly accumulate the penalties to the public account, rather than to ask each violator to pay the penalty. The penalty lottery has many advantages over both the fixed fine system and the day fine system. With the penalty lottery, the society approaches zero public bad production in the long run, without increasing the monitoring capacity or the size of the penalty. With an appropriate level of probability of accumulating the penalties, the steady state (of zero public bad production) can be reached faster. Even if it takes long time to reach the steady state, the penalty lottery is desirable because it endogenously imposes larger expected fines to those who are more willing to produce a public bad. This is particularly important if people believe the current system, often cynically described as "evil pays better", needs to be changed. Experimental evidence largely supports the theoretical predictions.

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Appendix A. Proofs

Approximation of k Consecutive Accumulations

First I introduce the method for calculating the probability that a success run of at least length k for an event with a probability of q occurs at least once within n trials. Regarding a success run of length $\kappa \in \{0, 1, \ldots, k\}$ as a state, we can construct the following transition probability matrix, T of order $k+1$.

$$T = \begin{bmatrix} 1-q & q & 0 & \cdots & 0 \\ 1-q & 0 & q & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1-q & 0 & 0 & \cdots & q \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix},$$

where the probability of moving from state i to state j in one time period is $Pr(j|i) = T_{i+1,j+1}$. For example, $T_{23} = q$ is the probability of moving from a success run of length 1 to a success run of length 2. Then the entities of T^n are the probabilities of transitioning from one given state to another state in n Bernoulli trials. We call a Markov chain absorbing if there is at least one state such that the chain can never leave that state once entered. That is, k is an absorbing state. A stationary probability vector π, which does not change under application of the transition matrix, is $(0, 0, \ldots, 1)$. That is, $\pi T = \pi$ and $\lim_{n\to\infty}(T^n)_{1,k+1} = 1$.

Although this is the exact method to find the probability, Feller (2008) provides the approximation of the probability that a success run of at least length k for an event with a probability of q occurs at least once within n trials:

$$q_n \sim 1 - \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}},$$

where q_n is the probability of a success run of at least length k in n trials, and x is the root of

$$Q(x) = 1 - x + (1-q)q^k x^{k+1} = 0,$$

other than $x = \frac{1}{q}$. The other root is smaller than $\frac{1}{q}$ if $(1-q)(k+1) > 1$. To check this, we need to show

$$x^* = \arg\min_x Q(x) < \frac{1}{q}$$

because for any $x > \frac{1}{q}$, $Q(x)$ is increasing. $x^* = \left(\frac{1}{(1-q)q^k(k+1)}\right)^{1/k}$, and this is strictly smaller than $\frac{1}{q}$ if and only if $\frac{1}{(1-q)(k+1)} < 1$. When $(1-q)(k+1) > 1$, the other root must be greater than 1 because $Q(1) = (1-q)q^k$, but it is typically close to 1. $Q(1) = (1-q)q^k$ tends to be small for a large k. This approximation renders an easier way to illustrate the theoretical predictions when citizens are homogeneous.

Proof of Proposition 1:

We apply the approximation of k consecutive accumulations to the relevant situations that we have in mind: a sequential public bad production in a society of risk-neutral homogeneous citizens. Suppose n violators (among n/p wrongdoers) sequentially arrive, and each of the violators hands over his/her fine to the next violator with probability q and pays all the accumulated fines with probability $1-q$. The probability that the fines are accumulated at least for k consecutive times is approximately $1 - \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}}$. Since every citizen violates the law whenever it is beneficial to do so, the probability that a wrongdoer exists after n/p periods is

$$P(V|t = n/p) = \frac{1-qx}{(k+1-kx)(1-q)} \frac{1}{x^{n+1}}.$$

For the cases where we consider, the condition $(1-q)(k+1) > 1$ always holds because

$$k = \left\lceil \frac{B+C}{Fp(1-q)} \right\rceil - 1 \geq \frac{B+C}{Fp(1-q)} - 1 \Leftrightarrow (1-q)(k+1) \geq \frac{B+C}{Fp} > 1.$$

Note $B + C > Fp$, or $(1-p)B + p(B-F) > -C$ must hold, otherwise the assumption $(1-p)u_i(B) + pu_i(B-F) > u_i(-C)$ is violated. Since x is greater than 1, $P(V|n/p)$ decreases as n gets larger. However, if $q = 0$, $Q(x) = 1 - x$, so $x = 1$. If $x = 1$, $P(V|t)$ is always 1 for any t.

Now we consider heterogeneous citizens. k_i is the smallest integer such that

$$(1-p+pq)u_i(B_i) - p(1-q)u_i(k_iF + F) \leq u_i(-C).$$

Since $u_i(\cdot)$ is weakly concave, the upper bound of such k_i is $\left\lceil \frac{B_i+C_i}{p(1-q)F} \right\rceil - 1$. Thus $\bar{k} = \max_i k_i$ is finite for any $q \in [0,1)$. Let $G(k)$ denote the cumulative density function of $k = 0,\dots,\bar{k}$, and $g(k) = G(k) - G(k-1), k = 1,\dots,\bar{k}$ is the probability mass of the citizens with $k_i = k$. By the assumption of the initial incentive of public bad production, $g(0) = G(0) = 0$.

The transition probability matrix with the density function G, T_G is a square matrix of order $\bar{k}+1$. For notational simplicity, denote $l = \bar{k}+1$. T_G is characterized as follows:

- $(T_G)_{1,1} = 1 - pq$.
- $(T_G)_{i,1} = (1 - G(i-1))p(1-q)$ for $i = 2,\dots,l$.
- $(T_G)_{i,i} = G(i-1) + (1 - G(i-1))(1-p)$ for $i = 2,\dots,l$.
- $(T_G)_{i,i+1} = (1 - G(i-1))pq$ for $i = 1,\dots,l-1$.
- All the other entities are zero.

Note that $G(l) = 1$ and l is an absorbing state, so the last row of T_G is a standard unit vector whose last entity is 1. Since $T^t_{1,l}$ is the probability of transitioning from state 0 to state $\bar{k}$ in t trials, $P(V|t) = 1 - T^t_{1,l}$. The stationary probability vector π is $(0,0,\dots,1)$. That is, $\lim_{n\to\infty}(T^t_G)_{1,l} = 1$, or $\lim_{t\to\infty} P(V|t) = 0$. Next we want to show the monotonicity: $(T^{t+1}_G)_{1,l} \geq (T^t_G)_{1,l}$ for any t. Since $(T^{t+1}_G)_{1,l} = (T^t_G)_{1,l-1}(T_G)_{l-1,l} + (T^t_G)_{1,l}(T_G)_{l,l} = (T^t_G)_{1,l-1}g(\bar{k})pq + (T^t_G)_{1,l}$, $(T^{t+1}_G)_{1,l} \geq (T^t_G)_{1,l}$ if and only if $(T^t_G)_{1,l-1}g(\bar{k})pq \geq 0$.

Last, we show that the expected time to reach the absorbing state is finite. Let

$$T_G = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & 1 \end{bmatrix},$$

where $\mathbf{Q}$ is a $\bar{k}$-by-$\bar{k}$ matrix, $\mathbf{R} = [0,\dots,0,g(\bar{k})pq]'$ is a $\bar{k}$-by-1 vector, and $\mathbf{0}$ is an 1-by-$\bar{k}$ zero vector. Thus, $\mathbf{Q}$ describes the probability of transitioning from some transient state to another, and $\mathbf{R}$ describes the probability of transitioning from some transient state to the absorbing state. Since state $\bar{k}-1$ is the only state that has a positive probability of transitioning to the absorbing state, all the other entities in $\mathbf{R}$ except for the last one are zeros.

The expected number of visits to a transient state j starting from a transient state i before being absorbed is described by the so-called fundamental matrix, denoted by $\mathbf{N}$.

$$\mathbf{N} = \sum_{k=0}^{\infty} \mathbf{Q}^k = (\mathbf{I}_{\bar{k}} - \mathbf{Q})^{-1},$$

where $(\mathbf{Q}^k)_{i,j}$ is the probability of transitioning from state i to j in exactly k periods, and $\mathbf{I}_{\bar{k}}$ is the $\bar{k}$-by-$\bar{k}$ identity matrix. Summing this for all k yields the expected number of visits to transient states. Since the determinant of $(\mathbf{I}_{\bar{k}} - \mathbf{Q})$ is nonzero, $\mathbf{N}$ is well-defined. The expected number of periods being absorbed when starting in state 0 is the first entry of the vector $\mathbf{t} = \mathbf{NJ}$, where $\mathbf{J}$ is a $\bar{k}$-by-1 vector of which entities are all 1, that is, $\sum_{i=1}^{\bar{k}} \mathbf{N}_{1,i}$, which is finite. □

Proof of Proposition 2:

Our goal is to characterize the first row of $\mathbf{N}$ in terms of $G(k)$, p and q. Since the sum of the first row of $\mathbf{N}$ yields the expected number of time periods to reach the absorbing state, and we showed the monotonicity in Proposition 1, characterizing $\mathbf{N}$ is the key to describe $P(V|t)$. One property of the absorbing Markov chain is the probability of being absorbed in the absorbing state is captured by

$$\mathbf{B} = \mathbf{NR},$$

where $\mathbf{B}_i$ is the probability of being absorbed when starting from transient state i. Since there is only one absorbing state, $\mathbf{B}$'s entries are all 1. Also, since $\mathbf{R} = [0,\ldots,0,g(\bar{k})pq]'$, the last column of $\mathbf{N}$ is $[\frac{1}{(1-G(\bar{k}-1))pq},\ldots,\frac{1}{(1-G(\bar{k}-1))pq}]'$. Another important feature of the transition probability matrix considered in this paper is that state $i-1$ is the only transient state to reach state i in the next period, $i = 1,\ldots,\bar{k}-1$. Therefore we can recursively calculate $\mathbf{N}_{1,i}$ from $\mathbf{N}_{1,\bar{k}}$. For example, the expected number of visits to state $\bar{k}-2$, $\mathbf{N}_{1,\bar{k}-1}$ must be equal to $\mathbf{N}_{1,\bar{k}}(1-G(\bar{k}-1))\frac{1}{(1-G(\bar{k}-2))q}$, which is $\frac{1}{1-G(\bar{k}-2)pq^2}$. Thus, we have

$$\mathbf{N}_{1,i} = \frac{1}{(1-G(i-1))pq^{\bar{k}-(i-1)}}, \quad \text{for } i = 1,\ldots,\bar{k}.$$

$N_{1,1} = \frac{1}{pq^{\bar{k}}}$ is intuitive because $\frac{1}{q}$ is the inverse of the probability of transitioning from the other $\bar{k}-1$ transient state to another state than state 0, and $\frac{1}{pq}$ is the inverse of the probability of leaving state 0. Therefore the expected number of time periods being absorbed when starting in state 0 is

$$\sum_{k=1}^{\bar{k}} \mathbf{N}_{1,k} = \frac{1}{pq}\sum_{i=0}^{\bar{k}-1}\frac{1}{(1-G(i))q^i}.$$

Now we preform two exercises regarding a mean-preserving spread (MPS) of $G(k)$: (1) a MPS with the same support, and (2) a MPS with a larger support. If $\tilde{G}(k)$ is a MPS of $G(k)$ with the same support, it must imply that $\tilde{G}(k)$ has fatter tails. That is, there exists $i \in [0,\bar{k}]$ such that $\tilde{G}(i) \le G(i)$ for $k \ge i$. Since the expected number of time periods depends on $\frac{1}{(1-G(i))q^i}$, having a larger $1-\tilde{G}(i)$ for some large i decreases the expected number of time periods. When it comes to a MPS with a larger support, Without loss of generality, we can construct $\tilde{G}(k)$ by

$$\{(\tilde{g}(1),\dots,\tilde{g}(\bar{k}),\tilde{g}(\bar{k}+1))\in\Delta|\tilde{g}(i)=g(i)-\varepsilon_i \text{ for } i\leq\bar{k},\tilde{g}(\bar{k}+1)=\frac{1}{\bar{k}+1}\sum_{i=1}^{\bar{k}}i\varepsilon_i\},$$

where $\varepsilon_i \geq 0$. Then $1-\tilde{G}(i)$ is slightly larger than $1-G(i)$ for $i=1,\dots,\bar{k}$, so $\sum_{i=0}^{\bar{k}-1}\frac{1}{(1-\tilde{G}(i))q^i} < \sum_{i=0}^{\bar{k}-1}\frac{1}{(1-G(i))q^i}$, but $\frac{1}{g(\bar{k}+1)q^{\bar{k}+1}}$ is substantially larger than $\sum_{i=0}^{\bar{k}-1}\left[\frac{1}{(1-G(i))q^i}-\frac{1}{(1-\tilde{G}(i))q^i}\right]$ due to Jensen's inequality. □

Appendix B.

B.1. Sample Instructions, Mq Treatment

This is an experiment in group decision making. Please pay attention to the instructions. You may earn a considerable amount of money which will be paid in cash at the end of the experiment. The currency in this experiment is called 'tokens'. In the beginning, you are endowed with 100 tokens.

There will be a quiz after the instructions, to ensure you understand the experiment.

Overview:

The experiment consists of 60 group decision-making 'rounds'. In each round, one of the group members decides (while others wait) to choose either a red ball or a blue ball, with knowing how many black stickers are in the common pool. The details follow.

How the groups are formed:

All subjects will be randomly assigned to groups of four. For example, if there are 20 subjects in this lab, there will be five groups of four subjects. You will belong to the same group throughout the whole experiment. There will be neither physical reallocation nor interactions. Only the server computer knows who are grouped with whom, and you input your choices to your computer interface. That is, you will not know who your group members are, and your group members will not know you either.

The balls:

In each round, one of the group members will choose either a red ball or a blue ball, while other members will see 'Please Wait' screen. What happens in your turn are as follows:

When you choose a red ball, nothing will happen further. You keep it and your group moves on to the next round.

When you choose a blue ball, nothing will happen with probability 0.7. With the other probability 0.3, a black sticker will be attached to the blue ball. If you have a black sticker, the computer will determine where black stickers go. With probability 0.5, the black sticker is removed from your blue ball, and added to the common pool shared by group members. With the other probability 0.5, you will keep your black sticker as well as all the black stickers from the common pool. That is, if you choose a blue ball, the probability that you will have black sticker(s) is 0.15 (= 0.3*0.5). Then your group moves on to the next round.

The balls and the sticker have different values. 1 token per each red ball will be deducted from your account, 5 tokens per each blue ball will be added to your account, and 8 tokens per each black sticker will be deducted from your account at the end of the session.

[Example] Suppose there are no black stickers in the common pool. If you choose a red ball in your turn, you lose 1 token. If you choose a blue ball and it does not have a black sticker, you earn 5 tokens. If the blue ball has a black sticker, then with probability 0.5 you earn 5 tokens and the sticker is added to the common pool. (Now 1 sticker is in the common pool.) With the other probability 0.5, you lose 3 tokens (earn 5 tokens from the blue ball, but lose 8 tokens from the black sticker).

Information in your turn:

When it is your turn, you will receive the following information:

- The number of black stickers in the common pool
- The numbers of red balls, blue balls, and black stickers you have kept

It will not be informed what other participants have done. If there are no black stickers in the common pool, it could mean either that no one has added black stickers to the common pool, or that someone ahead of you kept all the stickers from the common pool.

Final Earnings:

You are endowed with 100 tokens. If you have x red balls, y blue balls, and z black stickers at the end of the session, your total payoff is $100 - x + 5y - 8z$. Your earning will be converted into Euros at the rate of 8 eurocents per token. At the end of the main session, there might be an additional task of which result would be paid. The server computer will calculate the final payment. Please don't worry about this conversion.

Summary of the process:

1. The experiment will consist of 60 rounds. Everyone is endowed with 100 tokens.

2. In the beginning, all subjects will be randomly assigned to groups of four members. One of the group members will make a decision in each round.

3. When it is your turn, you will choose either a red ball or a blue ball, with knowing how many black stickers are in the common pool.

4. If you choose a red ball, your turn ends. If you choose a blue ball, with probability 0.3 a black sticker is followed. When a black sticker is followed by the blue ball, the black sticker will be detached from your blue ball and added to the common pool with probability 0.5. With the other probability 0.5, you will keep all the black stickers including those in the common pool.

5. A red ball is worth -1 token. A blue ball is worth +5 tokens. A black sticker is worth -8 tokens.

Quiz

Q1. Suppose 1 black sticker is in the common pool in your turn. You choose a red ball. Which of the followings is correct? (a) With some probability, it will come with a black sticker. (b) I keep the red ball. It will add 1 token to my account. (c) I keep the red ball. It will deduct 1 token from my account. (d) I keep the red ball and the black sticker in the common pool.

Q2. Suppose 2 black stickers are in the common pool in your turn. You choose a blue ball, and it doesn't come with a black sticker. Which of the followings is correct? (a) I keep the blue ball and one black sticker from the common pool. (b) I can decide how many black stickers I can add to the common pool. (c) I keep the blue ball. It will add 5 tokens to my account. (d) I keep the blue ball. It will deduct 8 tokens from my account.

Q3. Suppose 2 black stickers are in the common pool in your turn. You choose a blue ball, and it comes with a black sticker. Which of the followings is NOT correct? (a) With probability 0.5, I can change my choice. (b) With probability 0.5, I keep the blue ball, and the black sticker goes to the common pool. (c) With probability 0.5, I keep the blue ball and all the black stickers. (d) If the black sticker goes to the common pool, there are 3 stickers in the common pool.

Q4. If you choose a blue ball, what's the probability of keeping black sticker(s) in your account?

Q5. Suppose that you keep 10 blue balls and 2 black stickers at the end of the session. During the session, how many tokens have you gained/lost in total? (Don't count the endowment.)