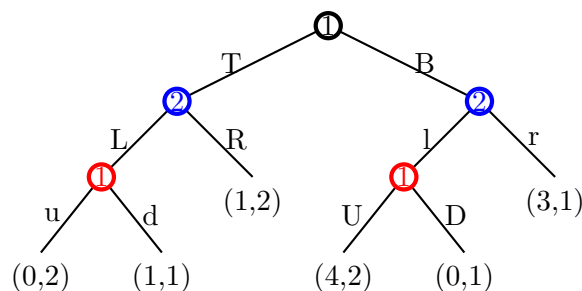# Game Theory: In-class Quiz 3, 2023 Fall, for TA

1. Consider the following extensive-form game.

(a) What is the number of pure strategies that player 1 has?

$\Rightarrow$ 8 (TuU, TuD, TdU, TdD, BuU, BuD, BdU, BdD)

(b) Represent this game in a normal form. Find all of the pure strategy Nash equilibria.

	P1 \ P2	L,l	L,r	R,l	R,r
	T,u,U	0,2	0,2	1,2	1,2
	T,u,D	0,2	0,2	1,2	1,2
	T,d,U	1,1	1,1	1,2	1,2
$\Rightarrow$	T,d,D	1,1	1,1	1,2	1,2
	B,u,U	4,2	3,1	4,2	3,1
	B,u,D	0,1	3,1	0,1	3,1
	B,d,U	4,2	3,1	4,2	3,1
	B,d,D	0,1	3,1	0,1	3,1

$\{$(B,u,U),(L,l)$\}$, $\{$(B,u,U),(R,l)$\}$, $\{$(B,u,D),(L,r)$\}$, $\{$(B,u,D),(R,r)$\}$, $\{$(B,d,U),(L,l)$\}$, $\{$(B,d,U),(R,l)$\}$, $\{$(B,d,D),(L,r)$\}$, $\{$(B,d,D),(R,r)$\}$ are Nash equilibria.

(c) Describe the subgame perfect equilibrium of this game.

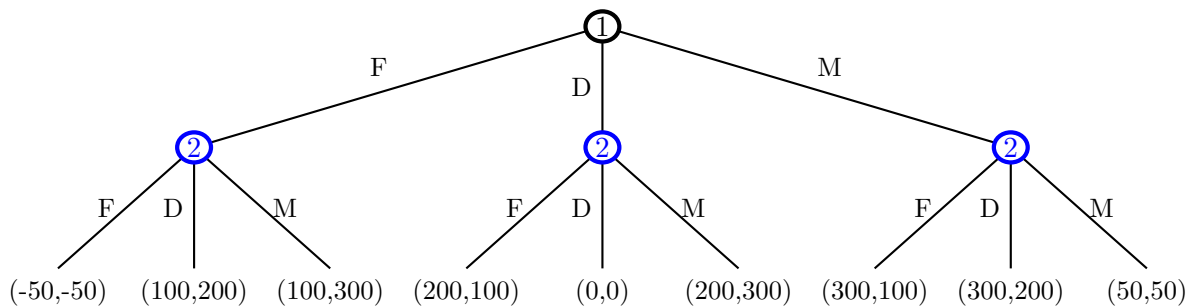$\Rightarrow$ $\{$(B,d,U), (R,l)$\}$ is the subgame perfect equilibrium.

2. Warner Brothers (P1) and Disney (P2) try to determine release dates for Wonder Woman 2 and Black Panther 2. There are three possible release months: February(F), December(D), and May(M).
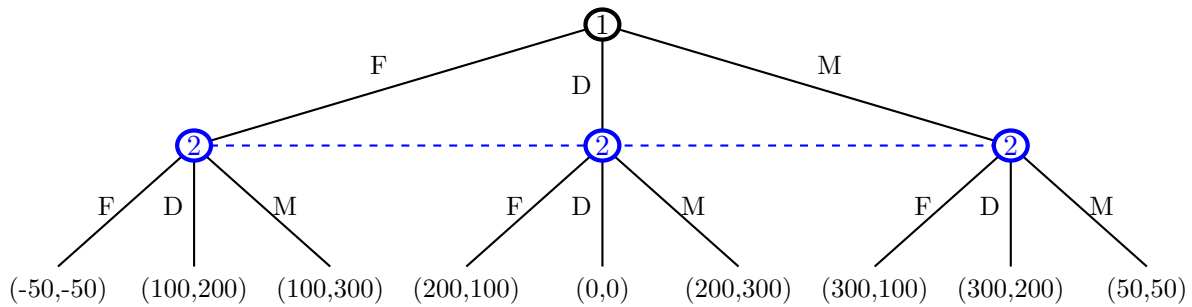
(a) Suppose Disney does not observe Warner Brothers' release decision, as described in the game tree above. Draw a payoff matrix of a normal form game, and find all of the pure strategy Nash equilibria. (There might be one, more than one, or none.)

$\Rightarrow$

P1 \ P2	F	D	M
F	-50,-50	100,200	100,300
D	200,100	0,0	200,300
M	300,100	300,200	50,50

{D, M} and {M, D} are Nash equilibria.

(b) Suppose P2 observes P1's release decision, as described in the game tree above. Describe the subgame perfect equilibrium of this game.

$\Rightarrow$ {M, (M,M,D)} is a SPE. [{M, D} is an INCORRECT answer.]

3. Consider a partnership between two players. When player 1 (P1) exerts effort of x and player 2 (P2) exerts effort of y, each player receives $2x + 2y + \frac{1}{2}xy$. They have an identical cost of effort: When exerting e, $C(e) = \frac{1}{2}e^2$. P1 exerts effort first. After observing P1's effort, P2 exerts her effort. Describe two players' effort levels in a subgame perfect equilibrium.
Sol: Given x, P2 chooses y to maximize her payoff, $2x + 2y + \frac{xy}{2} - \frac{y^2}{2}$. FOC: $2 + \frac{x}{2} - y = 0 \Rightarrow y = 2 + \frac{x}{2}$.

Knowing $y(x) = 2 + \frac{x}{2}$, P1 maximizes $2x + 2y(x) + \frac{xy(x)}{2} - \frac{x^2}{2} = 2x + 4 + x + x + \frac{x^2}{4} - \frac{x^2}{2} = 4x + 4 - \frac{x^2}{4}$. FOC: $4 = \frac{x}{2}$. $x = 8$. $y = 4$.