Collective Proofreading and the Optimal Voting Rule[*]

Duk Gyoo Kim[†] Jinhyuk Lee[‡] Euncheol Shin[§]

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Abstract

Policy decisions often involve a repeated proofreading process before implementation. We present a dynamic model of proofreading decisions by a heterogeneous committee, in which the committee decides when to stop proofreading and implement a risky policy. A proofreading process is costly but necessary because the risky project contains an unknown number of errors, and the value of the policy decreases by the number of undetected errors. The proofreading process continues as long as a qualified majority votes for continuation. Once the proofreading process ends, members receive heterogeneous penalties based on the remaining errors. We find that any qualified voting rule for proofreading results in an inefficient outcome. Unlike the result in Strulovici (2010), majority rule could have a bias not only toward under-experimentation but also toward over-experimentation.

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[†]Department of Economics, University of Mannheim, Germany. d.kim@uni-mannheim.de.

[‡]Department of Economics, Korea University, Republic of Korea. jinhyuklee@korea.ac.kr.

[§]College of Business, Korea Advanced Institute of Science and Technology, Republic of Korea. eshin.econ@kaist.ac.kr.

1 Introduction

On January 28, 1986, the Space Shuttle *Challenger* was destroyed 73 seconds after lift-off. The engineers warned that cold weather conditions might cause an O-ring seal to malfunction, but the launch, which had been delayed several times, could not be delayed further. All seven crew members were lost, and more than 4 billion dollars were wasted. We do not know the actual decision process of the launch, but it is clear that the decision is collectively made, and each committee member's interests are heterogeneous. This paper concerns a proofreading process before implementing a new policy or a project.

For implementing a new policy, agencies frequently run a proofreading process to verify its effectiveness. A primary goal of the proofreading process is to find and fix errors as many as possible to make sure that the policy is successful. For example, suppose that an environmental protection agency concerns about a new manufacturing process which may be harmful to the environment. The agency may conduct a series of investigations to detect violations of environmental laws such as overuse of toxic chemicals and pollutants in the process. The agency approves the manufacturing process only if the agency expects that the costs of further investigations outweigh the benefits of detecting potentially unobserved violations.

More complicated yet, the proofreading decisions are often made by a committee. When a decision committee consists of heterogeneous members, some members may want to conduct a stringent proofreading process, although it would incur a high cost of an investigation. On the contrary, others may be less involved with the investigation and may want fast approval of a new policy to enjoy potential benefits from policy implementation. To resolve this conflict of interest, the committee members might impose a collective decision rule, which is supposed to be designed to maximize social welfare. To understand the incentive structure of such a collective proofreading process and find an optimal voting rule, we build a model of collective proofreading decisions by heterogeneous members.

In our model, there is a committee consisting of n members, and it sequentially decides whether to continue a costly proofreading process or to stop it and implement the policy. The policy is risky in that it contains errors, and its value decreases by the number of undetected errors. The committee continues the proofreading process as long as a qualified majority of members agrees to do so.[1]

[1]It would be more realistic to consider two-stage voting procedures: one vote for the proofreading stringency, and the other one for approval of a (potentially) risky policy over a status quo policy. We mainly focus on the inefficiency involved in the proofreading stage, but we admit that additional voting stage with a status quo

We first consider a situation in which the errors are realized from a Poisson distribution so that the proofreading level preferred by each member can be constant regardless of the number of previously detected errors. To derive a clear policy implication, we assume further that there is at least one member whose optimal proofreading level as a single decision maker coincides with the socially optimal level. Hence, it is optimal for the committee to delegate the decision to this *representative* member. However, the representative member's preference over the optimal number of proofreading steps can be different from the committee's qualified number of proofreading steps. Due to this discrepancy, any qualified majority rule may result in a socially sub-optimal outcome.

The optimal voting rule that maximizes social welfare varies by the nature of the heterogeneity that each committee member has. By letting e be the number of members who prefer to stop the proofreading process no earlier than the representative member, we find that the e-qualified majority rule is welfare-maximizing. If such e is less than half of the full committee, then it means the proofreading procedure should continue even when a minority of members want to continue under the optimal voting rule. In order words, a majority rule may have a bias toward under-experimentation, which is along with the main finding of Strulovici (2010). However, we also find that a bias toward *over-experimentation* could exist. In this case, a supermajority rule is optimal, and the proofreading experimentation should be carried more conservatively. A simple majority rule could be socially optimal, but the conditions for it are on weak ground.

We extend our model to address a situation where there is a fixed number of issues that need to be evaluated. In particular, we assume that the errors are drawn from a Binomial distribution. In this case, the probability of detecting additional errors depends on the number of previously detected errors, so the optimal stopping decisions are no longer stationary. We can still characterize each committee member's optimal stopping rule as a function of the detected errors with respect to the proofreading levels. The same results as the Poisson model are retained.

1.1 Related Literature

Our model builds on seminal work on the single decision maker's optimal proofreading problem by Yang et al. (1982). Previous studies on optimal proofreading decisions (Chow and Schechner, 1985; Ferguson and Hardwick, 1989; Dalal and Mallows, 1988) have focused on a single decision maker's problem in which there are no concerns about the conflict of

policy can be another cause of potential inefficiency.

interest between heterogeneous committee members. In this paper, benefiting from a simple dynamic nature under the assumption of Poisson prior distribution of errors, we extend to a collective proofreading model without other behavioral and strategic concerns in collective decisions such as present-bias (Jackson and Yariv, 2015) or free-riding problems (Keller and Rady, 2010). As a result, we obtain a precise characterization of the welfare-maximizing voting rule.

This paper is also related to the recent literature on collective decisions in dynamic settings (Strulovici, 2010; Chan et al., 2018; Jackson and Yariv, 2015; Keller and Rady, 2010; Lizzeri and Yariv, 2017). In a modeling perspective, we consider agents who do not discount the future. This assumption is necessary for the existence of a representative agent in a dynamic model, as shown by Jackson and Yariv (2015). In our model, having a representative agent is not crucial, but it renders a more precise illustration of the welfare loss of a qualified majority rule compared with the socially optimal voting rule. In a setting of collective deliberation, Lizzeri and Yariv (2017) compare the performance of different voting rules in terms of the welfare of the committee under the presence of a self-control problem, while we find an optimal voting rule without a self-control problem. Strulovici (2010) considers situations where individuals learn their type more accurately by experimentation, but our model deals with experimentation decisions under complete information about the committee's heterogeneity. While the main finding of Strulovici (2010) is that majority rule has a downward bias in terms of experimentation, we found that majority rule could also yield a bias toward over-experimentation in our context. This observation is distinct from the recent studies which attempt to link collective decisions with present-biased outcomes.

In the sense that the committees decide to stop searching for potential errors by proofreading, this paper is also related to studies on collective search by committees. In the context of accepting a proposal or searching for alternative ones, Compte and Jehiel (2010) examine how each committee member affects the set of possible agreements. We could conduct the same exercise in our context, but our focus is more on the description of the socially optimal voting rule. Albrecht et al. (2010) compare how the collective search problem differs from a single-agent search problem with a focus on the case of symmetric agents, whereas our paper explicitly considers heterogeneous agents. In Moldovanu and Shi (2013), a committee decides whether to accept the current alternative with multiple attributes or to continue the costly search, and each committee member can privately assess the quality of only one attribute. Our model can be considered as a costly search of alternatives (with one attribute) whose quality is nondecreasing over time. Although information aggregation and adverse selection problems under private information of committee members as in Lauermann and Wolinsky (2016) are worth being investigated, we focus on the case of complete information.

We claim our paper contributes to advance theoretical models on environmental policy choice in different economic settings. Viscusi and Zeckhause (1976) provide a tractable framework to deal with a situation in which there is no clear ranking among environmental policies due to the presence of uncertainty. In particular, their model conveniently examines how a chance of irreversibility affects policy choice, as discussed in Viscusi (1985). Wirl (2006) examines how irreversibilities of the environmental policy affect the optimal intertemporal accumulation of greenhouse gases in the atmosphere under uncertainty. Gsottbauer and van den Bergh (2011) investigate the relationship between environmental policy decisions and other-regarding preferences. In our paper, we consider a different nature of the policy implementation: the uncertainty of the benefit (damage) of the policy and conflicts of interest between heterogeneous committee members resolved by voting. Although applications in environmental policy decisions motivate this paper, the model is applicable in many other cases such as optimal R&D investments (Moscarini and Smith, 2001; Weeds, 2002) decided by a committee.

2 Benchmark Model

In this section, we describe a single decision maker's decision problem. We introduce key assumptions simplifying the problem. Especially we assume that the number of errors is drawn from a Poisson distribution. Then, we investigate an optimal proofreading strategy, which is a building block for analyzing the equilibrium behavior in the committee decision.

2.1 Setup

There is a single risky project containing $\mathbf{M}$ errors, where $\mathbf{M}$ is distributed over $\mathbb{N}_0$ with $\mathbb{E}[\mathbf{M}] < \infty$.[2] There is a single decision-maker who tries to find and correct errors through a *proofreading process*. In each period t, the decision maker decides whether to stop or continue the proofreading process. If he decides to stop the process, he pays a penalty of $D > 0$ for each remaining error. Each error-finding step incurs a cost $c > 0$. To ignore the time preference of the decision maker, we assume that he does not discount the future, or

[2]Throughout the paper, $\mathbb{N}_0$ represents the set of natural numbers including zero.

assume that the time gap between each proofreading step is minuscule.

We denote by X_t the number of detected errors in period t, where $X_t \geq 0$ and $\sum_{j=1}^{t} X_j \leq \mathbf{M}$ for all $t \geq 1$. We assume the following for the number of errors $\mathbf{M}$ and the sequence of detected errors $(X_t)_{t \geq 1}$:

Assumption 1. *For $\mathbf{M}$ and $(X_t)_{t \geq 1}$,*

(1) the number of errors follows a Poisson distribution with $\lambda > 0$:

$$\mathbf{M} \sim Poisson(\lambda).$$

(2) the number of detected errors in period $t+1$ follows a Binomial distribution:

$$\mathbf{X}_{t+1} | \mathbf{M}, X_1, \ldots, X_t \sim Binomial\left(\mathbf{M} - \sum_{j=1}^{t} X_j, p\right),$$

where $p \in (0,1)$ is the probability of detecting an error.

As shown in Lemma 3.1 in Ferguson and Hardwick (1989), Assumption 1 dramatically simplifies the dynamic nature of the problem as the conditional probability of having errors in period $t+1$ is independent of the history of previous error findings, which is recapitulated by the following lemma.

Lemma 1. *The number of remaining errors after t steps of proofreading follows a Poisson distribution with $\lambda(1-p)^t$:*

$$\mathbf{M} - \sum_{j=1}^{t} X_j \sim Poisson(\lambda(1-p)^t).$$

Lemma 1 implies (1) that the mathematical expectation of the number of remaining errors is $\lambda(1-p)^t$, which is decreasing in t, and (2) that the expectation is independent of a history of error detection.

To make the decision maker's problem non-trivial, we assume that (at least one) proofreading is ex-ante desirable. Since the expected penalty from approval of the risky project without proofreading is calculated as $D\lambda$, we assume that the total cost after one proofreading step is smaller than $D\lambda$:

Assumption 2. *Proofreading is desirable:*

$$D\lambda > D\lambda(1-p) + c \Leftrightarrow p\lambda > \frac{c}{D}.$$

2.2 Optimal Strategy

We analyze the single decision maker's optimal proofreading strategy. At time t, she observed the number of detected errors by $t-1$, and decides whether to continue or stop the proofreading process.

After t steps of the proofreading process, the expected penalty from implementing the policy is:

$$-D\,\mathbb{E}[\text{remaining errors}|\text{history}] = -D\,\mathbb{E}\left[\mathbf{M} - \sum_{j=1}^{t} X_j\right] = -D\lambda(1-p)^t,$$

where the second equality comes from Lemma 1. The total expected cost, the sum of the expected penalty and proofreading costs of t steps, is calculated as

$$C(t) = D\lambda(1-p)^t + tc.$$

The decision maker's key incentives are illustrated in Figure 1. The blue triangles denote the benefit of approving the risky project in period t after t error-finding trials. The expected penalty is strictly decreasing in t because the decision maker becomes more confident with the risky policy as a longer proofreading process is conducted.

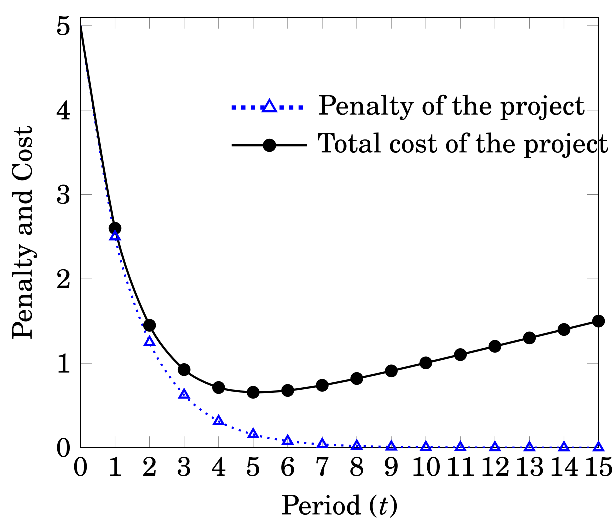

Figure 1: A numerical example of the costs: $D = 1$, $\lambda = 5$, $p = \frac{1}{2}$, and $c = 0.1$.

The black dots represent the total expected cost of approving the risky project after t steps of proofreading. It initially decreases in t by Assumption 2 because proofreading decreases the number of remaining errors and the probability of having errors, and this decrease dominates the constant marginal cost of additional proofreading. However, after a certain period ($t = 5$ in the figure), the total expected cost increases because the marginal benefit of having less errors is dominated by the marginal cost of additional proofreading. We denote t^* as the period minimizing the total expected cost.

Although it is obvious in the single decision-making problem, it will be useful in the context of the collective decisions to note that the decision maker has an incentive to continue the proofreading process during the periods in interval $[0, t^*]$ rather than stop and approve the risky project immediately because

$$D\lambda(1-p)^t \geq D\lambda(1-p)^{t^*} + (t^* - t)c.$$

This property will guarantee that sincere voting for the proofreading steps is a weakly dominant strategy.

The optimal stopping time is determined by the cost-penalty ratio, $\frac{c}{D}$. To see this point, note that the marginal decrease of the expected number of errors must be the same as the cost-penalty ratio at the optimal step t^* if it were to be on $\mathbb{R}_+$:

$$\lambda(1-p)^{t^*} \ln\left(\frac{1}{1-p}\right) = \frac{c}{D}.$$

Note that the interior solution of the above equation exists by Assumption 2. As the cost-penalty ratio decreases, the optimal stopping time t^* increases. Therefore, it follows that the cost-penalty ratio is a key characteristic that represents the decision maker's preference over the stopping times.

3 Collective Proofreading

We extend the previous single decision maker's problem to a collective deliberation problem. There is a decision committee consisting of n members, where n is assumed to be odd. Each member is indexed by $i \in N = \{1, \ldots, n\}$. We consider the same probabilistic proofreading environment. Specifically, members have the same prior distribution over the number of errors, and the committee members jointly find errors. That is, common values of λ and p apply to all members.

Each member's penalty per remaining error and the proofreading cost are heterogeneous. This heterogeneity may be due to the members' different positions in the agency or political interests. For example, politicians and policymakers may have small proofreading costs because they are less involved with the actual proofreading process compared to engineers and researchers who actually spend their time and effort to investigate the policy. The penalty per remaining error would also be heterogeneous: The investigator of the environmental policy would be less affected by the remaining misconduct of the new manufacturing process, while the engineers who made the Space Shuttle may be critically suffered from the remaining errors. We denote by $\{(D_i, c_i)\}_{i=1}^n$ the heterogeneous committee members. Without loss of generality, we assume that $D_{j+1} \geq D_j > 0$ for $j = 1, \ldots, n-1$. We denote by $\vec{t}^* = (t_i^*)_{i=1}^n$ the vector of each member's optimal stopping time if they were to behave as a single decision maker in the same proofreading environment, and we denote by $m(\vec{t}^*)$ the median of the optimal stopping times. We assume a nontrivial amount of heterogeneity so that $t_i^* \neq t_j^*$ for some $i \neq j$.

$C_i(t)$ represents member i's expected total cost from t steps of proofreading. The aggregate cost is defined as the sum of members' costs. We assume that the socially optimal proofreading steps coincides with at least one member's cost minimizing proofreading steps:

Assumption 3. *There exists a representative member: For some $r \in N$,*

$$C_r(t) = \frac{1}{n} \sum_{i=1}^{n} C_i(t).$$

Let $t_r^* = \arg\min_{t \in \mathbb{N}_0} C_r(t)$ be the optimal stopping time of member r.[3]

Assumption 3 means that the aggregated cost is minimized if the committee delegates the decision to member r.[4] We call member r the representative member. All the intuitions and results do not rely on the index (identity) of the representative member.

We consider a qualified majority rule for committee decisions. Specifically, under a q-rule, the committee continues the proofreading process if at least q members of the committee want to do so. We call a $\frac{n+1}{2}$-rule as a simple majority rule.

For any voting strategies of the other committee members, sincere voting is a weakly dominant strategy. To understand this, observe that voting for continuation until member i's optimal stopping time t_i^* is at least as profitable as voting against continuation before t_i^*. Voting for continuation after t_i^* does not help her to attain the smallest expected cost either. The reasoning behind this observation is the same as why a truthful bidding strategy is

[3] Due to Lemma 1, without loss of generality, the optimal stopping time in the assumption is chosen over the set of non-negative integers instead of the set of stopping times.

[4] To guarantee existence of such a representative member, the assumption of no future discounting is crucial. Specifically, as shown in Jackson and Yariv (2017), it is impossible to have such a representative member if members are heterogeneous in their discount factors.

weakly dominant in a second-price auction. Therefore, behaving truthfully as if a committee member were a single-decision maker is weakly dominant.

Although we do not restrict our attention to a smaller set of strategies, such a weakly dominant strategy is symmetric, monotone, and stationary. A member's strategy is independent of her identity. The strategy is monotone in the sense that there is a threshold period t such that a member votes for the continuation of the proofreading process in or before period t and votes against after period t. A member's strategy does not depend on the history of previous voting outcomes. Therefore it is straightforward to characterize a subgame-perfect equilibrium of weakly dominant strategies.

Proposition 1. *In the subgame-perfect equilibrium of weakly dominant strategies, member i votes for continuation if and only if $t \leq t_i^*$.*

Unless otherwise stated, we call this subgame-perfect equilibrium the *equilibrium* throughout the paper. In the next section, assuming that voters act on the equilibrium, we find an optimal voting rule.

4 Optimal Voting Rule

In this section, we analyze the equilibrium outcomes under different q-rules. We identify the condition in which the q-rule is socially optimal.

Proposition 2. *Suppose $q = \#\{i | t_i^* \geq t_r^*\}$. The q-rule produces the socially-optimal outcome in the equilibrium.*

Proposition 2 states that the socially-optimal voting rule should make the representative member pivotal. In other words, in any situations where the representative member is not pivotal, collective decisions yield socially inefficient outcomes.

The key intuition for the inefficiency is captured by the fact that member i's optimal stopping time crucially depends on $\frac{c_i}{D_i}$, neither D_i nor c_i per se. Even if we impose a monotone rank of c_i, along with $D_{i+1} \geq D_i$ for $i = 1, \ldots, n-1$, the rank of $\frac{c_i}{D_i}$ is not sufficient to know the optimal voting rule.

The following two examples illustrate situations in which the committee continues the proofreading steps too much and too little under a simple majority rule, respectively.

The example in Table 1 considers a situation where the costs are negatively aligned with the penalties (that is, $c_i \geq c_{i+1}$). This cost-penalty relationship could be observed in

Member	D_i	c_i	t_i^*	$C_i(2)$	$C_i(3)$	$C_i(4)$	$C_i(5)$	$C_i(6)$	$C_i(7)$
1	1.0	0.60	3	2.45	**2.43**	2.71	3.16	3.68	4.24
2	1.1	0.30	4	1.98	1.59	**1.54**	1.67	1.89	2.14
3	1.2	0.15	**5**	1.80	1.20	0.98	**0.94**	0.99	1.10
4	1.7	0.10	6	2.33	1.36	0.93	0.77	**0.73**	0.77
5	2.0	0.05	7	2.60	1.40	0.83	0.56	0.46	**0.43**
			$\sum$	11.15	7.98	**6.99**	7.09	7.75	8.67

Table 1: Inefficiency: Over-proofreading. $\lambda = 5, p = 0.5$

the committee in which senior members or authoritative members take responsibilities of remaining errors while junior members exert effort to proofread. In this example, the order of $\frac{c_i}{D_i}$ is monotone, so is the optimal stopping time for each member. Although the preferences of each member's optimal stopping time are well ordered, and member 3 is seemingly a median member in every aspect, the simple majority rule involves over-proofreading.

Member	D_i	c_i	t_i^*	$C_i(2)$	$C_i(3)$	$C_i(4)$	$C_i(5)$	$C_i(6)$	$C_i(7)$
1	1.0	0.05	6	1.35	0.78	0.51	0.41	**0.38**	0.39
2	1.1	0.10	5	1.58	0.99	0.74	**0.67**	0.69	0.74
3	1.2	0.20	4	1.90	1.35	**1.18**	1.19	1.29	1.45
4	1.7	0.30	4	2.73	1.96	**1.73**	1.77	1.93	2.17
5	2.0	0.40	4	3.30	2.45	**2.23**	2.31	2.56	2.88
			$\sum$	10.85	7.53	6.39	**6.34**	6.85	7.62

Table 2: Inefficiency: Under-proofreading. $\lambda = 5, p = 0.5$

The example in Table 2 considers a situation where the costs are positively aligned with the penalties (that is, $c_i \leq c_{i+1}$). This cost-penalty relationship could be observed in the committee in which some members are more involved in the policy than other members. In this example, the simple majority rule involves under-proofreading.

Two remarks are worth mentioning. First, the positive (resp. negative) relationship between D_i and c_i does not necessarily imply under-proofreading (resp. over-proofreading). The opposite cases are also possible. Second, the optimal voting rule may require a smaller number of votes than the simple majority to continue the proofreading process.

One remaining question would be under what conditions the simple majority rule is socially optimal.

Proposition 3. *The super-(sub-)majority rule is socially optimal if and only if*

$$\frac{c_m}{D_m} < (>) \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} D_i},$$

where the subscript m denotes the committee member whose $\frac{c}{D}$ is the median.

Proposition 3 also implies that the simple majority rule is socially optimal if and only if $\frac{c_m}{D_m} = \frac{\sum c_i}{\sum D_i}$. For example, if the cost parameters and the damage parameters are symmetrically distributed, then c_m and D_m coincide with the average of c_i and D_i, respectively, and the simple majority rule is socially optimal. Proposition 3 indirectly illustrates how fragile the foundation of the simple majority rule is in the context of collective proofreading decisions. It is well known that the asymmetric intensities among voters may make the simple majority rule socially undesirable.[5] It is even harder to achieve efficiency in the context of collective proofreading because we need symmetry in both dimensions.

Similar to the fact that a simple majority rule cannot be optimal for most of the cases, any q-rule cannot be a panacea for all cases. A naturally followed question is whether the committee could endogenously choose the optimal voting rule. In the ex-ante stage, where every member knows the joint distribution of the damages and the costs but does not know their realized values, the committee unequivocally prefers the optimal voting rule the most, as it renders them the highest expected payoff. Therefore, for any voting rules and protocols for mapping individuals' preference orders to the committee's preference order, the optimal voting rule will be selected. However, in the interim stage, where committee members privately learn their own values (D_i and c_i), their preferred stopping times, hence their preferred voting rules, vary. In this case, the primitive voting rule to determine the voting rule (Barbera and Jackson, 2004) may make a deviation from the optimal voting rule. Thus, a normative suggestion we can draw from this section is that the voting rule should either be made at the ex-ante stage or be exogenously determined by an unbiased executor.

5 Binomial Model

In the previous sections, thanks to the memoryless property of Poisson distributions, each member's preferred proofreading level was constant regardless of the number of the previously detected errors. Since the support of the Poisson distribution is unbounded, the model we considered in the previous sections, although it is clearer to show the inefficiency of

[5]For the reviews of this line of research, see Posner and Weyl (2017).

the collective proofreading decisions, may be different from more realistic situations where a committee collectively proofread finite issues. In this case, the number of previously detected (and fixed) errors do affect further proofreading decisions. In this section, we assume that the number of errors is drawn from a Binomial distribution.[6]

There is a single risky project containing $\mathbf{M} \sim Binomial(N, \pi)$ errors, where N is the number of issues that affect the overall return of the project, and $\pi > 0$ is the probability that each issue contains either one error or none.

Let $\mathbf{Y}_t = \mathbb{E}[\mathbf{M}_t | X_1, \ldots, X_t] D + ct$ denote the total expected cost after t steps of proofreading, where X_j is the number of errors detected in period $1 \le j \le t$, and $\mathbf{M}_t = \mathbf{M} - \sum_{j=1}^{t} X_j$ is the number of remaining errors.

We assume the independence of the error-detecting probability:

Assumption 4. *In each proofreading step, each error is detected with probability $p > 0$, and the detection is independent of the detection history and detection of other errors:*

$$\mathbf{X}_{t+1} | \mathbf{M}, X_1, \ldots, X_t \sim Binomial(\mathbf{M}_t, p)$$

Then, by Lemma 4.1 in Ferguson and Hardwick (1989), we obtain the following:

Lemma 2.

$$\mathbf{M}_t | X_1, \ldots, X_t \sim Binomial(N - \sum_{j=1}^{t} X_j, \pi_t),$$

where the updated error probability π_t is calculated as

$$\pi_t = \frac{\pi(1-p)^t}{(1-\pi) + \pi(1-p)^t}.$$

There are several notable properties driven from Lemma 2. First, the updated error probability is independent of the previously detected errors. Second, the error probability π_t is monotone *decreasing* in t.[7] Third, the expected number of remaining errors after t

[6] A model with this assumption nests a situation where a committee collectively decides to draw additional signal to learn the binary state of the world, as it could be understood as an error (bad state) drawn from a Bernoulli distribution.

[7] It directly follows from the fact that

$$\pi_t = \frac{\pi(1-p)^t}{(1-\pi) + \pi(1-p)^t} = \frac{\pi}{\frac{1-\pi}{(1-p)^t} + \pi}.$$

proofreading steps is history dependent:

$$\begin{aligned}
\mathbb{E}[\mathbf{M}_{t+1}|X_1,\ldots,X_t] &= \mathbb{E}[\mathbf{M}_t - \mathbf{X}_{t+1}|X_1,\ldots,X_t] \\
&= \mathbb{E}[\mathbf{M}_t|X_1,\ldots,X_t] - \mathbb{E}[\mathbf{X}_{t+1}|X_1,\ldots,X_t] \\
&= \left(N - \sum_{j=1}^{t} X_j\right)\pi_t - \mathbb{E}[\mathbb{E}[\mathbf{X}_{t+1}|\mathbf{M}, X_1,\ldots,X_t]|X_1,\ldots,X_t] \\
&= \left(N - \sum_{j=1}^{t} X_j\right)\pi_t - \mathbb{E}[\mathbf{M}_t p|X_1,\ldots,X_t] \\
&= (1-p)\left(N - \sum_{j=1}^{t} X_j\right)\pi_t,
\end{aligned}$$

which depends on both (1) the updated error probability π_t and (2) the sum of detected errors up to period t. If other things are equal, the expected number of remaining errors is decreasing in period t and the number of detected errors in previous periods. Consequently, the expected cost in period $t+1$ is

$$\begin{aligned}
\mathbb{E}[\mathbf{Y}_{t+1}|X_1,\ldots,X_t] &= \mathbb{E}[\mathbf{M}_{t+1}|X_1,\ldots,X_t]D + (t+1)c \\
&= (1-p)\left(N - \sum_{j=1}^{t} X_j\right)\pi_t D + (t+1)c.
\end{aligned}$$

The expected cost is now history dependent, so is the optimal strategy.

One assured aspect is that the optimal strategy is still monotone. In fact, it is well-known in the optimal stopping literature that an optimal stopping strategy has a myopic form if a stopping problem is *monotone*.[8] The monotonicity requires that the sets

$$A_t = \{\mathbf{Y}_t < \mathbb{E}[\mathbf{Y}_{t+1}|X_1,\ldots,X_t]\}$$

are monotone increasing as $A_t \subseteq A_{t+1}$ almost surely for any t. In words, the condition $A_t \subseteq A_{t+1}$ means that if immediate stop at time t is optimal in period t, then it is also optimal to stop at all the following future periods, no matter how the future errors are detected.

In the current setting, the condition for monotonicity is satisfied. To see why, observe

[8] It is also called the *one-stage look-ahead rule*. See, for example, Bruss (2000).

that

$$\begin{aligned}
\mathbf{Y}_t < \mathbb{E}[\mathbf{Y}_{t+1}|X_1,\ldots,X_t] &\iff \mathbb{E}[\mathbf{M}_t|X_1,\ldots,X_t]pD < c \\
&\iff \left(N - \sum_{j=1}^{t} X_j\right)\pi_t pD < c \\
&\iff \left(N - \sum_{j=1}^{t} X_j\right)\frac{\pi}{\frac{1-\pi}{(1-p)^t}+\pi}pD < c.
\end{aligned}$$

The first term on the left-hand side of the last inequality, $N-\sum_{j=1}^{t} X_j$, is the difference between the total number of issues and the number of detected errors. This term is non-increasing for sure. The second term, $\frac{\pi}{\frac{1-\pi}{(1-p)^t}+\pi}$, is the updated belief of having an error in each remaining issue. This term is strictly decreasing and independent of the history of detected errors. Therefore, the whole expression on the left-hand side is decreasing.

The optimal stopping time t^* is the smallest integer t such that

$$\left(N - \sum_{j=1}^{t} X_j\right)\frac{\pi p}{\frac{1-\pi}{(1-p)^t}+\pi} \leq \frac{c}{D},$$

and the cost-penalty ratio, c/D is still a key characteristic. From this, we can characterize the optimal stopping threshold as a pair of the number of detected errors and the decision period. After rearranging the inequality, we have

$$\sum_{j=1}^{t} X_j \geq \kappa_0 - \frac{\kappa_1}{(1-p)^t},$$

where $\kappa_0 = N - \frac{c}{Dp}$ and $\kappa_1 = \frac{c(1-\pi)}{D\pi p}$. This stopping rule implies that proofreading should be stopped either when the number of detected errors is sufficiently large given the proofreading steps, or when they proofread for a sufficiently long time given the number of detected errors.

For a clearer illustration, we provide a numerical example that shows how a decision maker's optimal strategy depends on the detection history and steps when $(N,c,D,\pi,p) = (12,1,8,0.5,0.5)$. In Figure 2, the y-axis represents the number of detected errors up to period t, $\sum_{j=1}^{t} X_j$, and the x-axis represents the decision period t. With the set of parameters specified above, we have the following optimal stopping rule:

$$\sum_{j=1}^{t} X_j \geq 9.75 - 2^{t-2}.$$

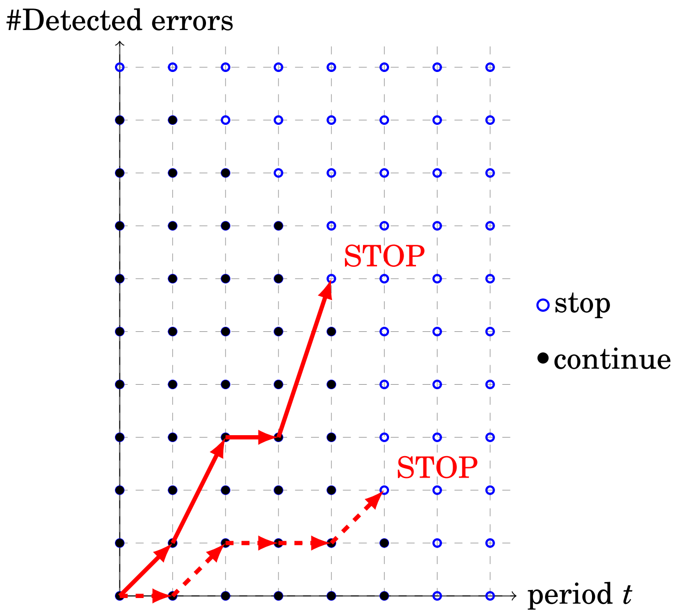

Figure 2: Illustration of the optimal stopping rule

The blue dots in Figure 2 represents the region in which the decision maker stops proofreading. Observe that the monotonicity is satisfied, that is, if (x, y) is contained in the blue region, so is (x, z) whenever $z \geq y$. This monotonicity implies that $\mathbf{Y}_t < \mathbb{E}[\mathbf{Y}_{t+1}|X_1, \ldots, X_t]$ is also satisfied. For this feature, the boundary between the blue dots and the black dots is decreasing in period t.

Each red line represents a sample path. Each path moves to the north-east direction in the lattice plane. The solid red line describes the scenario when $X_1 = 1, X_2 = 2, X_3 = 0, X_4 = 3$, and the decision maker stops at period 4. The dashed red line describes the scenario when $X_1 = 0, X_2 = 1, X_3 = 0, X_4 = 0, X_5 = 1$, and the decision maker stops at period 5.

If t and $\sum_{j=1}^{t} X_j$ were to be on $\mathbb{R}_+$, then we can delineate the boundary to continue proofreading on the plane of $(t, \sum_{j=1}^{t} X_j)$. Unless otherwise stated, we call this boundary as the optimal stopping rule. Although the optimal stopping decision for proofreading becomes history dependent, the same results as the original model are retained. Two key features of the optimal stopping rules are (1) that at any sample path (or history), the decision maker's optimal stopping rule is aligned in his/her cost-disadvantage ratio, and (2) that each heterogeneous committee's rule does not cross each other. Committee member i with a high $\frac{c_i}{D_i}$ will always have a weakly lower boundary in terms of the sum of the detected errors. Since this stopping problem is monotone, it is natural to restrict our attention to the class of monotone strategies. As long as everyone uses monotone strategies, it is a weakly undominated strategy to follow his/her own optimal stopping rule sincerely.

6 Concluding Remarks

Many serious policies and projects are implemented after rigorous proofreading steps. When a committee with heterogeneous members collectively makes the decisions about the proofreading process, the simple majority rule is likely to result in an inferior outcome. The socially optimal voting rule, which requires the committee to continue the proofreading process up to the socially optimal proofreading level can be constructed only after accounting for the members' heterogeneity. This construction is a nontrivial task: Although we are completely informed about each committee member's heterogeneity, there is no clear rule about which voting rule should be adopted. A simple majority rule could have a bias not only toward under-proofreading but also toward over-proofreading. The optimal voting rule should either be made at the ex-ante stage or be exogenously determined by an unbiased executor.

Two possible extensions are worth mentioning. First, one may consider a model in which some members prefer taking the safe (status quo) policy rather than choosing the risky policy after they agree to stop proofreading. That is, a committee follows two-stage voting procedures: one vote for the proofreading stringency, and the other one for approval of a risky policy over a status quo. In this case, the representative member or the decisive voter in the proofreading-decision stage may have to form a coalition with other members to implement the socially-optimal policy. For this analysis, one needs to model a dynamic bargaining process in line with the current proofreading process. Second, a simplified version of the Binomial model can be a theoretical benchmark for laboratory experiments. Although the theory has many interesting predictions, still many empirical questions remain unanswered: Do subjects optimally vote to stop proofreading when the expected benefit exceeds the cost they can afford? Do they rationally respond to the changes in the distribution of the errors of the risky project and the degree of the heterogeneity of the committee? How much is the actual welfare loss when we stick to the simple majority voting rules that are not maximizing welfare?[9] We may answer all the questions by conducting controlled laboratory experiments.

References

Albrecht, James, Axel Anderson, and Susan Vroman, "Search by committee," *Journal of Economic Theory*, 2010, *145* (4), 1386–1407. Search Theory and Applications.

Barbera, Salvador and Matthew O. Jackson, "Choosing How to Choose: Self-Stable Majority Rules and Constitutions," *The Quarterly Journal of Economics*, 2004, *119* (3), 1011–1048.

Bruss, F. Thomas, "Sum the odds to one and stop," *The Annals of Probability*, 06 2000, *28* (3), 1384–1391.

Chan, Jimmy, Alessandro Lizzeri, Wing Suen, and Leeat Yariv, "Deliberating Collective Decisions," *Review of Economic Studies*, 2018, *85* (2), 929–963.

Chow, Chung-Wen and Zvi Schechner, "On Stopping Rules in Proofreading," *Journal of Applied Probability*, 1985, *22*, 971–977.

Compte, Olivier and Philippe Jehiel, "Bargaining and Majority Rules: A Collective Search Perspective," *Journal of Political Economy*, 2010, *118* (2), 189–221.

Dalal, Siddhartha R. and Colin L. Mallows, "When Should One Stop Testing Software?," *Journal of the American Statistical Association*, 1988, *83* (403), 872–879.

Ferguson, T. S. and J. P. Hardwick, "Stopping Rules for Proofreading," *Journal of Applied Probability*, 1989, *26* (2), 304–313.

Gsottbauer, Elisabeth and Jeroen C. J. M. van den Bergh, "Environmental Policy Theory Given Bounded Rationality and Other-Regarding Preferences," *Environmental Resource Economics*, 2011, *49*, 263–304.

Jackson, Matthew O. and Leeat Yariv, "Collective Dynamic Choice: The Necessity of Time Inconsistency," *American Economic Journal: Microeconomics*, 2015, *7* (4), 150–178.

[9] In a similar line of this question, Martinelli et al. (forthcoming) provide experimental evidence that a simple majority rule, which is supposed to be sub-optimal, performs better than the (theoretically) optimal voting rule.

— **and** — , “The Non-Existence of Representative Agents,” *Working Paper*, 2017.

Keller, Godfrey and Sven Rady, “Strategic experimentation with Poisson bandits,” *Theoretical Economics*, 2010, *5* (2), 275–311.

Lauermann, Stephan and Asher Wolinsky, “Search With Adverse Selection,” *Econometrica*, 2016, *84* (1), 243–315.

Lizzeri, Alessandro and Leeat Yariv, “Collective Self-Control,” *American Economic Journal: Microeconomics*, 2017, *9* (3), 213–244.

Martinelli, Cesar, Mikhail Freer, and Siyu Wang, “Collective Experimentation: A Laboratory Study,” *Journal of Economic Behavior & Organization*, forthcoming.

Moldovanu, Benny and Xianwen Shi, “Specialization and partisanship in committee search,” *Theoretical Economics*, 2013, *8* (3), 751–774.

Moscarini, Giuseppe and Lones Smith, “The Optimal Level of Experimentation,” *Econometrica*, 2001, *69* (6), 1629–1644.

Posner, Eric A. and E. Glen Weyl, “Quadratic voting and the public good: introduction,” *Public Choice*, Jul 2017, *172* (1), 1–22.

Strulovici, Bruno, “Learning While Voting: Determinants of Collective Experimentation,” *Econometrica*, 2010, *78* (3), 933–971.

Viscusi, W. Kep, “Environmental Policy Choice with an Uncertain Chance of Irreversibility,” *Journal of Environmental Economics and Management*, 1985, *12*, 28–44.

Viscusi, W. Kip and Richard Zeckhause, “Environmental Policy Choice under Uncertainty,” *Journal of Environmental Economics and Management*, 1976, *3*, 97–112.

Weeds, Helen, “Strategic Delay in a Real Options Model of R&D Competition,” *The Review of Economic Studies*, 07 2002, *69* (3), 729–747.

Wirl, Franz, “Consequences of irreversibilities on optimal intertemporal CO2 emission policies under uncertainty,” *Resource and Energy Economics*, 2006, *28* (2), 105–123.

Yang, Mark C. K., Dennis D. Wackerly, and Andrew Rosalsky, “Optimal Stopping Rules in Proofreading,” *Journal of Applied Probability*, 1982, *19* (3), 723–729.

A Proofs

Proof of Proposition 1

Proof. We first show that the strategy of voting for continuation if and only if $t \le t_i^*$ returns a payoff at least as high as any other strategies. Let t_{-i} be a fixed strategy profile of the members except member i. There are two cases to consider: (1) the proofreading never stops regardless of member i's decision, and (2) the proofreading may stop at some period t depending on member i's decision in period t. Thus, it suffices to consider the second case. Note that the members are assumed to use monotone strategies. Let t_1 be the first time in which member i's decision matters. If $t_i^* \le t_1$, then the desired strategy returns the highest payoff as it terminates the proofreading process at time t_1. If $t_1 < t_i^*$, then it is better for member i to continue the process until t_i^*. Hence, the desired strategy never returns a strictly smaller payoff than any other strategies.

We now show that there exists at least one strategy profile of other players such that the desired strategy returns a strictly higher payoff with respect to the given strategy profile. Let t_{-i} be the strategy of the members except member i in which there are $q-1$ number of members who vote for stop whenever $t \ge t_i^*$, and there are $n-q$ number of members who always vote for continuation. Then, it follows that member i's decision is pivotal and playing the desired voting strategy returns a strictly higher payoff than any other strategies. □

Proof of Proposition 2

Proof. Let $q = \#\{i | t_i^* \ge t_r^*\}$. To show that the q-rule produces the socially-optimal outcome in the equilibrium, it suffices to show that the proofreading process stops at time t_r^*. By Proposition 1, vote i votes for continuation whenever $t \le t_i^*$. For this, for any $t \le t_r^*$, there exist at least q members who vote for continuation. Thus, the proofreading stops at exactly period t_r^*, and so the proposition is proven. □

Proof of Proposition 3

Proof. Since $\sum_{i=1}^n C_i(t) = (\sum_{i=1}^n D_i)\lambda(1-p)^t + t(\sum_{i=1}^n c_i)$, t_r^*, the minimizing argument of $\sum C_i(t)$, satisfies the following:

$$\lambda(1-p)^{t_r^*} \ln\left(\frac{1}{1-p}\right) = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n D_i}.$$

It is straightforward to check if $\frac{c_m}{D_m} = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n P_i}$, then t_m^* coincides with t^* because

$$\lambda(1-p)^{t^*}\ln\left(\frac{1}{1-p}\right) = \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n P_i} = \frac{c_m}{P_m} = \lambda(1-p)^{t_m^*}\ln\left(\frac{1}{1-p}\right).$$

If $\frac{c_m}{D_m} < \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n P_i}$, then $t_m^* > t^*$. That is, the median voter prefers to proofread more than the socially optimal proofreading level, and a simple majority has a bias toward over-experimentation. The case with $\frac{c_m}{D_m} > \frac{\sum_{i=1}^n c_i}{\sum_{i=1}^n P_i}$ is analogous. □