

# Public Announcement with Rational Ignorance

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## Abstract

We study a two-channel information-search model where one information sender communicates with many information recipients. The information sender provides a public good in the form of an announcement—a set of answers to some potential queries—and also provides a private good in the form of a private communication service that directly responds to any particular query. Information recipients are heterogeneous in terms of the query they draw. They learn their query and the size of the public announcement, decide whether to use the public announcement channel, and, when necessary, purchase the private communication service, the price of which is ex-post determined by the number of people who purchase the service. Efficiency is hard to achieve with this type of information-search process. The inefficiency can be summarized by an under-provided public good and an overpriced private good in equilibrium. The main driving force behind the inefficiency is that agents who draw a query of intermediate specificity have an incentive to change their decisions of paying attention to the announcement, because of the possibility of incurring costs for both the public channel and the private channel, while other agents are more likely to incur a cost for only one of the two channels.

## 1 Introduction

L.A. Care Health Plan, one of the Medicaid Health Insurance providers, offers \$10 to a member who attends their orientation class.<sup>1</sup> Since the orientation class covers only rudimentary information that could be obtained online at no cost, at first blush it may not make

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<sup>1</sup>An example of such an offer can be found [here](#).

sense for L.A. Care to hold such an orientation class, because that action is costly for both L.A. Care and its members who avoidably spend time and energy attending it. However, we could infer that L.A. Care finds that informing members is worth more than the \$10 it offers them, perhaps because L.A. Care would spend more than \$10 on average to individually handle their members' queries that could have been covered in their online FAQs, information brochures, etc.

In general, efficiency is hard to achieve via a public announcement or by any directed communication from one to many. Even though the government provides guidelines to help citizens cope with the Affordable Care Act, a customer service center in an online retail store provides online FAQs, product manuals articulate how to use new products, online forums enable informative discussion on searchable threads, and instructors of large classes provide a course syllabus, information senders still suffer from responding to queries made by uninformed individuals: Citizens directly contact the service agent to get information about their health plan, customers send emails without skimming FAQs, users of a product or service press 0 when calling the customer service center, beginners post a new thread on a topic that has already been discussed, and students ask when the final exam is going to be held. This is not only a problem from the perspective of the information sender. The inefficiency of this communication causes those who have a specific legitimate inquiry to incur a cost of some kind: Customers who find that the announcement doesn't cover the answer to their question must contact the information sender and wait for some time (perhaps an inordinately long time) to get a response.

To describe the inefficiency of communications such as those mentioned above, we study a two-channel information-search model with one information sender and many information recipients. The recipients can obtain relevant information from two different information channels offered by the sender, one public and the other private. To be more specific, the information sender provides a public good in the form of an announcement. The announcement is a set of answers to some potential queries. The sender also offers a private communication channel with a limited capacity. The information sender's goal is to determine the size of the public announcement that will minimize the overall communication cost. Information recipients draw a heterogeneous query from a known distribution, and their goal is to obtain an answer to their query. In the case of the public announcement channel, the cost of acquiring information (the time and energy spent in using the information to get an answer to their query) increases in the size of the announcement, that is, in the quality of

the information contained therein.<sup>2</sup> Moreover, it is possible that recipients will fail to find the answers to their queries in the announcement. Thus they may make use of the private information channel, either in place of reading the announcement or in addition to doing so. The function that represents the probability that an information recipient will use the private channel, which is equal to the complement of the probability that the level of informativeness of an announcement of a given size will provide the answer to a certain query, is commonly known. When using the private communication channel, on the other hand, all the recipients definitely find the answers to their queries, but the cost per subscriber to the private communication service is determined ex-post and is increasing in the number of private-channel users. This is due to the information sender’s limited capacity for personal communication. Hence, the private communication channel has cost externalities, and the information recipients would use the private communication channel without knowing the ex-post price of acquiring information. The information recipients’ objective is to obtain information at minimum cost.

We show that in the Bayesian Nash equilibrium for this process, the public good (the announcement) is under-provided and the private good (the private communication channel) is overpriced—when compared to the situation where the information recipients’ queries can be observed. A customer who has a specific nontrivial question, for example, may not find the answer to it in the FAQs, because the FAQs may cover too little, and he may have to wait for an inordinately long time to be connected to a customer service agent and get an answer to his question. The main driving force behind this inefficiency is the free-riding incentive, which can be analogous to that of standard games in the provision of public goods. The larger the size of the announcement (*ceteris paribus*), the lower the expected price per subscriber to the private communication service. Thus some information recipients may want to enjoy the luxury of the potentially cheaper personal communication, even though their queries are like to be covered by the announcement. This rational ignorance leads the information sender to be reluctant to provide a public announcement of higher quality.

As regards the welfare of the information recipients, we found that it is not straightforward to figure out who benefits and who suffers, though the overall communication cost is larger when the information recipients’ queries are unobservable. We also found that the size of the public announcement should be unchanged if the capacity of the private communication channel increases.

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<sup>2</sup>In the model, we implicitly assume that the information sender is always doing her best. That is, she has already streamlined the announcement to the point where it provides the most information possible for its size.

The rest of the paper is organized as follows. In the following subsection we discuss the related literature. Section 2 describes the model. Section 3 describes an equilibrium and analyzes some properties. In Section 4 we do some welfare analysis. In Section 5 we discuss the cost parameter for the private information channel. Section 6 concludes. Proofs of some of the lemmas and propositions that appear in the main text are provided in the Appendix.

## 1.1 Related Literature

Many studies on private information acquisition, such as [Angeletos and Pavan \(2007\)](#), [Mackowiak and Wiederholt \(2009\)](#), and [Myatt and Wallace \(2012\)](#), are related to our paper. The one that is most closely related is [Colombo et al. \(2014\)](#), which considered a model where agents take an action that has a payoff externality with the actions of others and a public signal is given. In that paper, agents can choose the accuracy of their private signal before taking an action. [Colombo et al. \(2014\)](#) showed that public information and private information can substitute for each other: The higher the accuracy of the public signal provided by a policy maker, the lower the precision of the private information acquired; moreover, the strength of this relationship depends on that of the substitution effect. Similarly, in our paper the information recipients decide whether to use the public announcement channel or not, so private communication can be regarded as a substitute for the public announcement.

Our paper differs from theirs in many respects. First, in our paper the information recipients can decide to ignore the public information and directly use the private communication channel, while in their paper public information is always given to the recipients. In addition, in our paper the quality of the public announcement is determined by the information sender, and the private communication channel guarantees that the correct answer will be given to the recipients. Moreover, the cost of the private communication channel is determined by the congestion externality, which makes for a strategic situation. Finally, in our paper an increase in the size of the public announcement does not necessarily imply a decrease in the number of private channel users, because the cost of acquiring information from the public announcement is increasing in the size of it.

In the sense that some of the information recipients ignore the public announcement channel, this study can be related to the literature on the rational inattention of consumers ([Bordalo et al., 2016](#); [de Clippel et al., 2014](#); [Dessein et al., 2016](#); [Sallee, 2014](#)). However, a distinctive difference between this paper and the rational inattention literature is that those who choose not to pay attention to the public announcement do so not because they have a limited capacity for digesting multi-dimensional information, but because they are

rational enough to intentionally avoid the cost of acquiring information from the public announcement.

Though this paper deals with a model where one information sender sends a message to many recipients, it is the recipients who have private information, unlike in cheap-talk models (Farrell and Gibbons, 1989; Caillaud and Tirole, 2007; Goltsman and Pavlov, 2011), where the message sender knows more about the true state than the recipients do. Another difference is that in our model it is costly to digest information, while in the cheap-talk models the transfer of information is free.

Communication costs have been dealt with in some studies (Loder et al., 2006; Evans, 2012; Potters, 1992), but to the best of our knowledge, our study is the first one that incorporates the idea of communication costs into consumption of an announcement. In the sense that the information sender chooses the size of the announcement, this paper could also be remotely related to papers such as Bental and Spiegel (1995) that address quality-control problems under unknown customer types.

## 2 The Model

In this section, we consider a model with  $n+1$  players, indexed by  $i \in \{0, 1, \dots, n\}$ . When necessary, we call player 0 the information sender and the other players the information recipients. For the sake of clarity, we take the sender to be female and the recipients to be male. The number of players is commonly known.

### 2.1 The Information Provider

Player 0 produces a public good in the form of an announcement at no cost. An announcement is a set of information containing answers to potential queries which could be made by heterogeneous recipients. An announcement covers only one dimension (one category of information) but varies in its size  $a \in \mathbb{R}_+$ .<sup>3</sup> By choosing the size of the announcement, the information provider can either deal only with some general issues or cover more detailed ones. The announcement is non-excludable and non-rivalrous, so by definition it is a public good.

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<sup>3</sup>A unidimensional announcement may not adequately describe a typical announcement that covers answers to multidimensional questions, such as online FAQs, course syllabi, user guides, and handbooks. A general extension to this setup could be represented by a multidimensional unit ball where each dimension represents a different category and the distance from the origin represents the degree of the specifics. For the sake of simplicity, we do not explore this extension.

At the same time that the sender makes the announcement, she makes an offer of a private good in the form of personal communication. The sender answers all the private queries—and does so with a limited and fixed capacity  $K$ . Regardless of the specifics of the recipients’ queries, she treats all the recipients equally. From the perspective of the sender, the cost of operating the communication service is fixed.

Her objective is to minimize the expected total communication cost incurred by each information recipient (hereinafter referred to as the total communication cost), by choosing an announcement of optimal size. There are two reasons why the information sender is benevolent, or cares about the total communication cost. First, in many situations the sender’s potential benefits could depend on the information recipients’ overall experience. In the retail sector, for example, the quality of customer care affects future profits. Another example is that a government agency may want to signal citizens that a certain policy change is being managed efficiently—and to inform them of it at a low communication cost. Still another example is that the IRS may want to help taxpayers improve their level of tax compliance, by lowering the overall cost of finding answers regarding the filing of tax returns. Second, this objective serves as a natural restriction that prevents the sender from intentionally increasing the cost of the private communication service. If, as an alternative, the sender’s objective were to make everyone pay attention to the announcement, simply shutting down the communication service or making it harder to reach the service could solve the problem.

## 2.2 The Information Recipients

The recipients are heterogeneous in terms of the specifics of their queries. The query of recipient  $i$  is measured by the degree of its specifics,  $q_i$ , which is randomly drawn from a commonly known distribution  $F$  on  $\mathbb{R}_+$  after the information sender sets the size of the public announcement. The probability density function for  $F$ ,  $f(q)$ , is continuous and weakly decreasing in  $q$ . Each recipient’s goal is to find a complete answer to his query at minimum cost.<sup>4</sup> Paying attention to the announcement is costly. The cost of attention,  $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , is continuous, increasing, and convex in the size of the announcement, with  $C(0) = C'(0) = 0$ . Even if a recipient pays attention to the announcement, and the announcement does indeed contain the answer to his query, there is still a chance for him to purchase a ticket for

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<sup>4</sup>We implicitly assume that the utility gain from getting the answer is large enough that dropping out of this game is not in his interest. We also exclude the possibility that he will hire a “private consultant” to deal with his query, for example, by hiring a CPA instead of following the instructions given by the IRS for a complicated tax filing, or will organize a “forum” to circumvent use of the private communication service.

personal communication. The probability function  $P : \mathbb{R}_+^2 \rightarrow [0, 1]$  represents this chance.  $P(a, q)$  is decreasing convex in  $a$  and increasing in  $q$ . It is natural to set  $\lim_{a \rightarrow 0} P(a, q) = 1$  for all  $q > 0$  and  $\lim_{q \rightarrow 0} P(a, q) = 0$  for all  $a > 0$ . We also assume that  $\lim_{a \rightarrow \infty} P(a, q) = 0$ <sup>5</sup> and  $\lim_{q \rightarrow \infty} P(a, q) = 1$ . Further,  $\lim_{q \rightarrow \infty} \frac{\partial P(a, q)}{\partial a} = 0$  for any finite  $a$ , which means that if a query is totally unlikely to be covered by the announcement, any marginal change in the size of the announcement doesn't help. Two of the simple functional forms that satisfy those assumptions are  $e^{-a/q}$  and  $1 - \frac{2}{\pi} \arctan(a/q)$ .

Use of the personal communication service is ex-post costly. Unlike the announcement, where the cost of acquisition is ex-ante known, the price of this private good is unknown at the time it is used. It features a congested good, so the price (waiting time, or service coarseness per unit of time in some contexts) increases linearly with the number of recipients who use the private communication channel. Specifically, the cost charged to each recipient is  $t(d) = \kappa d$ , where  $\kappa = 1/K > 0$  is a known parameter and  $d$  is the number of information recipients who buy a ticket for the private communication service.<sup>6</sup> Note that the cost of personal contact is decreasing in the information sender's capacity  $K$ .

The trade-off from the perspective of recipient  $i$  can be summarized as follows: Paying attention to the announcement will decrease his chances of purchasing a ticket for the private communication service, but he will incur another cost if the announcement is not detailed enough to cover his specific query. On the contrary, not paying attention to the announcement is optimal if no one else uses the private communication service. However, the price of using the private communication service could be substantial if a number of other players believed and behaved in the same way. The sequence of play is shown in Figure 1.

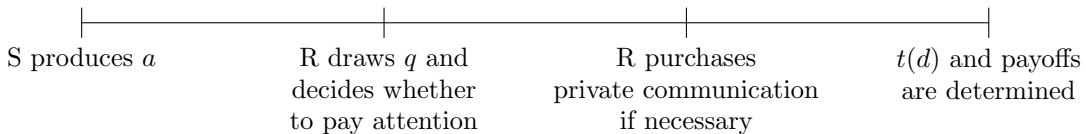


Figure 1: The Sequence of Play

<sup>5</sup>It could be more natural to assume that there is no complete announcement that answers everyone's specific question, that is, that  $\lim_{a \rightarrow \infty} P(a, q) > 0$ . This modification, however, does not change the main findings for the model.

<sup>6</sup>This specification is given for analytical simplicity, and can be interpreted as an (approximate) average time for a recipient’s query to be handled by private communication. Suppose, for example, that a concierge receives  $d$  queries simultaneously and handles them in a random order. If it takes one unit of time to handle each query, then the average waiting time for a recipient who approaches the concierge together with  $d - 1$  other recipients would be  $\sum_{i=1}^d i/d = (d + 1)/2$ . The difference between  $(d + 1)/2$  and  $d/2$  is  $1/2$ , which is small compared to  $d/2$  as  $d$  gets larger. Thus in this case  $t(d) \approx \kappa d$ , with  $\kappa = 1/2$ .

### 3 Analysis

#### 3.1 Benchmark: Complete information over $q$

Consider the first-best situation where the information sender can observe the values of  $q$  for all the recipients, and can therefore give them guidance on whether to pay attention to the announcement. Specifically, a cutoff  $q^o$  will work as a threshold: If  $q_i > q^o$ , the sender instructs recipient  $i$  to ignore the announcement; otherwise she urges him to pay attention to it. Then the optimal announcement quality  $a^o$  and the optimal cutoff  $q^o$  solve the optimization problem

$$(a^o, q^o) \in \arg \min_{(a, q) \in \mathbb{R}_+^2} F(q)C(a) + \kappa n \left( \int_0^q P(a, x) f(x) dx + \int_q^\infty f(x) dx \right), \quad (1)$$

where  $\int_0^q P(a, x) f(x) dx + \int_q^\infty f(x) dx := X(a, q)$  is the proportion of agents who will eventually buy a ticket for the private communication service; thus  $\kappa n X(a, q)$  is an individual's expected cost of purchasing the private communication service. The objective function represents the expected communication cost for one recipient.

Since  $C(a)$  is increasing convex, and  $\kappa n X(a, q)$  is decreasing convex in  $a$ , the minimization problem will have a unique solution. The derivatives of the objective function with respect to  $a$  and  $q$  are

$$\begin{aligned} a : \quad & F(q) \frac{dC(a)}{da} + \kappa n \int_0^q \frac{\partial P(a, x)}{\partial a} f(x) dx \\ q : \quad & C(a) f(q) + \kappa n f(q) (P(a, q) - 1). \end{aligned}$$

If the solution is an interior point,

$$F(q^o) \left. \frac{dC(a)}{da} \right|_{a=a^o} + \kappa n \int_0^{q^o} \left. \frac{\partial P(a, x)}{\partial a} \right|_{a=a^o} f(x) dx = 0 \quad (2)$$

$$C(a^o) + \kappa n (P(a^o, q^o) - 1) = 0 \quad (3)$$

Note that if the support of  $F$  is bounded above by  $\bar{q}$ , a corner solution where  $q^o = \bar{q}$  is possible if  $n$  is sufficiently large. The intuition for this corner solution is straightforward: Since the price of using the private communication service increases with the number of information recipients who use it, the total communication cost can be reduced if the in-



formation sender urges every recipient to pay attention to the announcement.<sup>7</sup> To avoid this outcome, we'll consider the support of  $F$  to be unbounded on  $\mathbb{R}_+$ . Also,  $(a, q) = (0, 0)$  satisfies the first-order conditions, so it could actually minimize the total communication cost—for example, in a case where either  $n$  or  $\kappa$  is sufficiently small, that is, if there are few information recipients and abundant private communication capacity. However, in the opposite case, where both  $n$  and  $\kappa$  are large, making no announcement at all (i.e., the case where  $a = 0$ ) does not minimize the total communication cost, but maximize it. Throughout this paper, we restrict our focus to cases where both  $n$  and  $\kappa$  are large, that is, where the number of information recipients is large and the capacity of the private communication service is limited.

The optimal announcement size,  $a^o$ , and the expected cost of using private communication under the benchmark setup,  $\kappa n X(a^o, q^o)$ , will be used as comparison points to show how information asymmetry leads to inefficiency. Now we examine a situation where the information sender does not observe the information recipients' queries.

### 3.2 The Recipients' Bayesian Game

First, we describe a symmetric Bayesian Nash equilibrium among the recipients in which the announcement size is fixed at some  $a > 0$ . Player  $i$  who learns the degree of specifics of his query,  $q_i$ , and the announcement size  $a$  would have a threshold  $q^*(a)$ , that is, player  $i$  will pay attention to the announcement if  $q_i \leq q^*(a)$ , and won't pay attention otherwise. Consider one recipient's decision threshold  $\tilde{q}(a)$  when everyone else is acting according to  $q^*(a)$ . In order for this threshold rule to be an equilibrium, a recipient with  $q^*(a)$  should be indifferent between paying attention to the announcement at  $\tilde{q}(a)$  and not doing so:

$$C(a) + P(a, \tilde{q}(a))\kappa(1 + (n-1)X(a, q^*(a))) = \kappa(1 + (n-1)X(a, q^*(a))), \quad (4)$$

where  $X(a, q^*(a)) := \int_0^{q^*(a)} P(a, x)f(x)dx + \int_{q^*(a)}^\infty f(x)dx$ . The left-hand side of equation (4) is the expected cost if a recipient pays attention to the announcement, while the right-hand side is the expected cost if he ignores it.  $X(a, q^*(a))$  is the expected proportion of agents who will buy a ticket for the private communication service if everyone follows the threshold rule. We restrict our attention to situations where the number of recipients is large enough

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<sup>7</sup>For example, if  $F(q)$  is a uniform distribution on  $[0, 1]$  and the functional forms are specified as  $P(a, q) = \frac{q}{1+a}$  and  $C(a) = a^2$ , then  $a^o = \frac{\sqrt{4+5\kappa n}-2}{5}$  and  $q^o = \frac{4a^o(1+a^o)^2}{\kappa n}$  without consideration of boundary conditions. One can check that  $q^o > 1$  for any  $n \geq 1$  and  $\kappa > 0$ , and therefore  $q^o$  is bounded above by 1.

that consuming the announcement is cheaper than using the private communication service if everyone else uses it.

**Assumption 1.**  $C(a) < \lim_{q \rightarrow 0} \kappa(1 + (n-1)X(a, q)) = \kappa n$

Solving for  $\tilde{q}(a)$ , we have

$$\tilde{q}(a) = P^{-1} \left( a, 1 - \frac{C(a)}{\kappa(1 + (n-1)X(a, q^*(a)))} \right), \quad (5)$$

where  $P^{-1}(a, x)$  is the inverse function of  $P(a, q)$ , that is,  $x = P(a, P^{-1}(a, x))$ . In a symmetric equilibrium,  $\tilde{q}(a) = q^*(a)$ . Rearranging equation (5) yields

$$1 - P(a, \tilde{q}(a)) = \frac{C(a)}{\kappa(1 + (n-1)X(a, q^*(a)))}, \quad (6)$$

which shows the existence of such an equilibrium. Given  $a$  and  $n$ , the left-hand side of equation (6), which represents the informativeness of the announcement (the probability that the announcement contains the answer to  $q$ ) is monotone decreasing in  $q$ , and the right-hand side, which represents the ratio of the cost of paying attention (to the announcement) to the expected cost of the private communication service, is monotone increasing in  $q$  since  $X(a, q)$  is decreasing in  $q$ .<sup>8</sup> Also, by Assumption 1,  $\lim_{q \rightarrow 0} 1 - P(a, q) > \lim_{q \rightarrow 0} \frac{c(a)}{\kappa(1 + (n-1)X(a, q))}$ . Therefore, there exists  $\tilde{q}(a) > 0$  such that equation (6) holds with  $\tilde{q}(a) = q^*(a)$ . Moreover, except for  $a = 0$ , there exists a unique  $q^*(a)$  that satisfies equation (6). For  $a = 0$ , when there is no announcement,  $q^*(a)$  is indeterminate since any  $q$  will satisfy equation (6); in that case, we set  $q^*(a) = \infty$ .

Figure 2 illustrates how  $q^*$  varies with  $a$  and  $n$  if the functional forms and parameters are specified as  $P(a, q) = e^{-a/q}$ ,  $C(a) = a^2$ ,  $a \in \{0.3, 0.5\}$ ,  $\kappa = 1/30$ ,  $f(q) = e^{-q}$ , and  $n \in \{30, 60\}$ . That is consistent with our intuition: As  $n$  increases, a larger proportion of agents will pay attention to the announcement, because the cost of the private good will be substantial. For a given population size  $n$ , the larger the value of  $a$ , the less likely it is that agents will pay attention to the announcement, because the marginal cost of attention is increasing in  $a$ . The dependence of  $q^*(a)$  on  $a$  when  $n \in \{30, 60\}$  is depicted in Figure 3.

**Lemma 1.**  $\frac{\partial q^*(a, n)}{\partial n} > 0$  and  $\frac{\partial q^*(a, n)}{\partial a} < 0$ .

**Proof:** See Appendix.

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<sup>8</sup>  $\frac{\partial X(a, q)}{\partial q} = \frac{\partial}{\partial q} \left[ \int_0^q P(a, x) f(x) dx + \int_q^\infty f(x) dx \right] = P(a, q) f(q) - f(q) = [P(a, q) - 1] f(q) \leq 0$ .

These observations make the information sender's problem nontrivial: Decreasing the size of the announcement will prompt more recipients to pay attention to it, but that reduces the chance that the announcement will provide answers to their queries and therefore increases the cost of the private communication service.

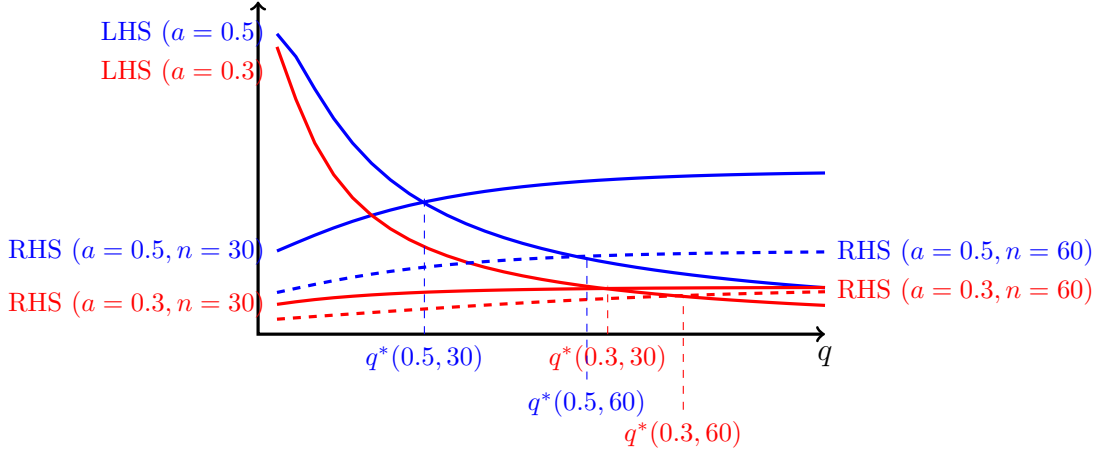


Figure 2: Illustration of  $q^*(a, n)$  when  $(a, n) \in \{0.3, 0.5\} \times \{30, 60\}$ .  $q^*$  is the threshold that determines who will pay attention to the announcement. Player  $i$  whose query  $q_i$  is above the threshold  $q^*$  will not pay attention. Functional forms and parameters are specified as  $P(a, q) = e^{-a/q}$ ,  $f(q) = e^{-q}$ ,  $C(a) = a^2$ ,  $a \in \{0.3, 0.5\}$ ,  $\kappa = 1/30$ , and  $n \in \{30, 60\}$ . LHS means the left-hand side of equation (6), which represents the informativeness of the announcement with respect to  $q$ . RHS, the right-hand side of that equation, represents the ratio of the cost of paying attention (to the announcement) to the expected cost of private communication. When  $a$  increases from 0.3 to 0.5,  $q^*$  decreases. This means that a smaller proportion of information recipients will pay attention to the public announcement. When  $n$  increases from 30 to 60,  $q^*$  increases, and thus a larger proportion of recipients will pay attention to the announcement.

### 3.3 Sender's Decision about the Announcement Size

Knowing the information recipients' threshold  $q^*(a)$  as a function of  $a$ , the information sender minimizes the expected total communication cost by solving the optimization problem

$$\begin{aligned} a^* \in \arg \min_{a \in \mathbb{R}_+} & F(q^*(a))C(a) + \kappa n X(a, q^*(a)) \\ \text{s.t. } & C(a) + \kappa(1 + (n-1)X(a, q^*(a)))(P(a, q^*(a)) - 1) = 0, \end{aligned} \quad (7)$$

where the constraint is a rearrangement of equation (6) in a way that renders it compatible with equation (3). From this rearrangement, we can directly draw some predictions regarding the announcement size and the total communication cost.

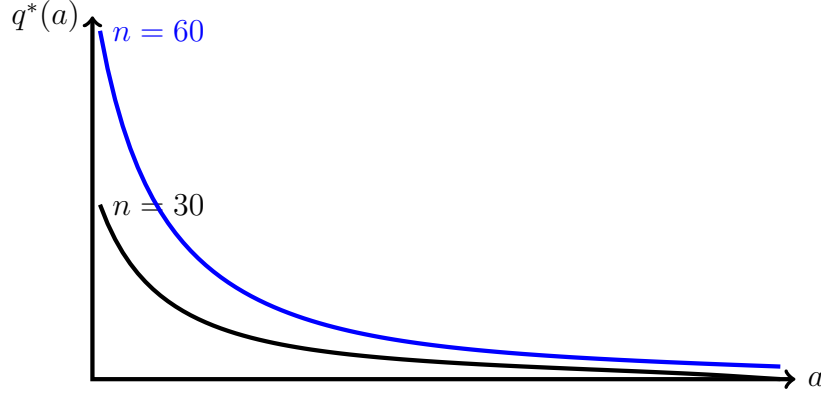


Figure 3: Illustration of  $q^*(a)$  when  $n \in \{30, 60\}$

$q^*(a)$  is the threshold that determines who will pay attention to the announcement. Functional forms and parameters are specified as in Figure 2.  $q^*(a, n)$  decreases with  $a$  and increases with  $n$ .

**Proposition 1.**  $F(q^*(a^*))C(a^*) + \kappa n X(a^*, q^*(a^*)) > F(q^o)C(a^o) + \kappa n X(a^o, q^o)$ . For sufficiently large  $n$ ,  $a^* < a^o$ .

**Proof:** A direct comparison of equations (1) and (7) tells us that  $a^o$  is a solution of the unconstrained objective function, while  $a^*$  is a solution of the constrained one. Thus the indirect utility at  $(a^*, q^*(a^*))$  is greater than or equal to that at  $(a^o, q^o)$ , with equality holding if and only if  $(a^*, q^*(a^*)) = (a^o, q^o)$ . Since  $X(a^o, q^o) < 1$ , the constraint evaluated at  $(a^o, q^o)$  is strictly greater than 0, and therefore the strict inequality holds in the indirect utility. What remains is to show that  $a^*$  is strictly smaller than  $a^o$ . Suppose, for the sake of contradiction, that  $a^o \leq a^*$ . Since  $q^*(a)$  is decreasing in  $a$ ,  $q^*(a^*) \leq q^*(a^o)$ , where  $q^*(a^o)$  satisfies the constraint in equation (7). Though the sign of  $F(q^*(a^*))C(a^*) - F(q^*(a^o))C(a^o)$  is indeterminate,  $\int_0^{q^*(a^*)} P(a, x)f(x)dx + \int_{q^*(a^*)}^\infty f(x)dx$  is larger than  $\int_0^{q^*(a^o)} P(a, x)f(x)dx + \int_{q^*(a^o)}^\infty f(x)dx$  because  $f(x) > P(a, x)f(x)$  for any  $x$ . Therefore, there exists  $\hat{n}$  such that for any  $n \geq \hat{n}$ ,  $\frac{1}{\kappa n} [F(q^*(a^*))C(a^*) - F(q^*(a^o))C(a^o)] > \int_0^{q^*(a^o)} P(a, x)f(x)dx + \int_{q^*(a^o)}^\infty f(x)dx - \int_0^{q^*(a^*)} P(a, x)f(x)dx - \int_{q^*(a^*)}^\infty f(x)dx$ . If this is the case,  $(a^o, q^*(a^o))$  yields a smaller total communication cost than  $(a^*, q^*(a^*))$ , which is contradictory.  $\square$

Now we know that a public announcement is under-provided when the queries are unobservable. This means that in equilibrium the cost of consuming the announcement is lower than when the queries are observable. Hence, the expected cost of using the private information channel must be higher. Otherwise, it contradicts the fact that  $(a^o, q^o)$  minimizes the total communication cost in the case of the unconstrained objective function.

**Corollary 1.** *The expected proportion of recipients who use the private communication channel is higher when the queries are not observable by the information sender:*

$$X(a^*, q^*) > X(a^o, q^o).$$

**Proof:** See Appendix.

The ex-post price of the private information channel is determined mainly by  $X(a^*, q^*)$ . In sum, when the information sender doesn't observe the queries, the public good (the announcement) is under-provided and the private good (the private communication service) is overpriced.

It turns out that the inefficiency summarized in Proposition 1 and Corollary 1 is similar to that of the well-known free-riding behavior in regard to voluntary contributions of public goods, even though the setups for the two scenarios are completely different. Roughly speaking, some people ignore the announcement (the public good), because they want to exploit the private communication service (the private good), while others pay attention to the announcement, to avoid incurring the extra cost of private information. The information sender, who understands this free-riding incentive of the latter group of information recipients, reduces the size of the announcement.

## 4 Welfare Analysis

Corollary 1 implies that the expected cost of using the private communication channel is higher when the queries are unobservable:  $\kappa[1+(n-1)X(a^o, q^o)] < \kappa[1+(n-1)X(a^*, q^*(a^*))]$ . However, the direct cost of using the public announcement channel is lower in that case,  $C(a^*) < C(a^o)$ . As a result, there must be some information recipients who gain (when the queries are unobservable, compared to when the queries are observable) and some who lose. For the most part, recipients  $i$  who have a low  $q_i$  are better off, while those who have a high  $q_i$  are worse off.<sup>9</sup> This is because it is highly likely that information recipients  $i$  with a low  $q_i$  will find the answer to their query in the public announcement and can save the cost of using the public announcement channel, while it is highly likely that recipients  $i$  with a high  $q_i$  will use the private communication channel, and hence will pay more because more recipients will eventually use the private channel, thereby causing it to become more costly. However,

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<sup>9</sup>The general analysis for the welfare changes is given in the Appendix.

the changes in welfare might not be monotonic for the case where a recipient draws a query close to one of the cutoff points  $q^*$ ,  $q^o$ . Specifically, if  $P(a^*, q^*(a^*))$ , the probability that a recipient with  $q = q^*(a^*)$  fails to find the answer in the public announcement, is sufficiently small, and  $P(a^o, q^o)$ , the probability that a recipient with  $q = q^o$  is sufficiently large, we can find three critical points,  $\bar{q}_1 < q^*(a^*)$ ,  $\bar{q}_2 \in (q^*(a^*), q^o)$ , and  $q^o$ , where recipients  $i$  with  $q_i \in (0, \bar{q}_1)$  are better off (when the queries are unobservable than when they are observable), those with  $q_i \in (\bar{q}_1, \bar{q}_2)$  are worse off, those with  $q_i \in (\bar{q}_2, q^o)$  are better off, and those with  $q_i \in (q^o, \infty)$  are worse off.

**Proposition 2.** *If  $q^*(a^*) < q^o$  and  $P(a^o, q^*(a^*)) < \pi < P(a^o, q^o)$ , where*

$$\pi = \frac{P(a^*, q^*(a^*))\kappa[1 + (n-1)X(a^*, q^*(a^*))] + C(a^*) - C(a^o)}{\kappa[1 + (n-1)X(a^o, q^o)]},$$

*then information recipients  $i$  with*

1.  $q_i \in (0, \bar{q}_1) \cup (\bar{q}_2, q^o)$  *are better off (when the queries are unobservable than when they are observable),*
2.  $q_i \in (\bar{q}_1, \bar{q}_2) \cup (q^o, \infty)$  *are worse off,*

*where  $\bar{q}_1 < q^*(a^*) < \bar{q}_2 < q^o$ .*

**Proof:** See Appendix.

Since the probability distribution function of  $q$  is assumed to be weakly decreasing, the result of Proposition 2 may suggest that the proportion of information recipients who are better off could be higher when the queries are unobservable than when they are observable. However, the sum of the marginal benefits of those who enjoy the lowered expected cost must be strictly smaller than the sum of the marginal costs of those who suffer from the increased expected cost. Thus the overall communication cost is always higher when the queries are unobservable.

## 5 Cost Parameter for Private Information Channel, $\kappa$

In the model, the parameter for the private information channel,  $\kappa$ , is exogenously determined. The parameter  $\kappa$  can be interpreted as a cost parameter for use of the private information channel. As the private communication service offered by the information sender

becomes more convenient and accessible,  $\kappa$  could get smaller. In this section, we consider the dependence of  $a$  and  $q$  on  $\kappa$  in equilibrium; we do this for the case where the queries are observable and for the case where they are not.

First, we show that when the queries are observable, both the equilibrium size of the public announcement and the threshold query are positively affected by the cost parameter. In the case where the private channel becomes more costly, the sender would like to make the public announcement more informative, so she will increase the size of the public announcement and force the recipients to be more patient. That is,  $\frac{\partial a^o}{\partial \kappa} > 0$  and  $\frac{\partial q^o}{\partial \kappa} > 0$ .

When the queries are not observable, the equilibrium size of the public announcement is not affected by the parameter  $\kappa$ , that is,  $\frac{\partial a^*}{\partial \kappa} = 0$ . This is because the sender has to consider the constraint: For an information recipient with the threshold query  $q^*$ , the expected cost of using the public announcement is the same as that of ignoring it. Thus the parameter  $\kappa$  does not affect the sender's minimization problem or the equilibrium size of the public announcement; however, in equilibrium the threshold query has a higher value of  $\kappa$ :  $\frac{\partial q^*(a^*)}{\partial \kappa} > 0$ . As the private channel becomes more costly, the recipients strategically become more patient.

**Proposition 3.** *As the private channel becomes more costly, the following hold:*

1. *When the queries are observable, the public announcement becomes larger and the recipients become more patient in equilibrium:  $\frac{\partial a^o}{\partial \kappa} > 0$  and  $\frac{\partial q^o}{\partial \kappa} > 0$ .*
2. *When the queries are not observable, the public announcement remains the same but the recipients become more patient:  $\frac{\partial a^*}{\partial \kappa} = 0$  and  $\frac{\partial q^*(a^*)}{\partial \kappa} > 0$ .*

**Proof:** See Appendix.

The second result of Proposition 3 may provide a policy implication for information providers: Even when marginally increasing the capacity for private communication services, increasing (or decreasing) the size of a public announcement is not a good idea. Since our results hold under Assumption 1, which could be violated if  $\kappa$  is small enough, the results in Proposition 3 do not necessarily imply that even a huge change in  $K$  (hence  $\kappa$ ) cannot affect the size of the announcement.

## 6 Concluding Remarks

We studied a two-channel information-search model for a situation where one message sender communicates with many message recipients. One information channel, a public announcement, features a predetermined cost of acquiring the information and incomplete informativeness. The other information channel, a private communication service, is equipped with full informativeness, but the cost of acquiring information from it is determined ex post by the number of people who use this channel. Each information recipient draws his own query, and decides whether to acquire information from the public announcement and whether to purchase a ticket for the private communication service. The information sender takes the recipients' potential strategies into account when determining the size (or the quality) of the announcement. We showed that if the recipients' queries are unobservable by the sender, the public information is under-provided and the private communication service is overpriced (compared to a situation where the information sender knows who has which query). From the perspective of the information recipients, it is not straightforward to figure out who benefits and who suffers, though the overall communication cost is definitely larger when the queries are unobservable. We also found that the information provider should not change the size of the announcement even if the capacity of the private communication channel increases.

There are many directions for extending this study, as we imposed many simplifying assumptions. First, we assumed in this paper that the announcement is made only once, as we believe this is practically true in many real-life situations. However, if a new technology allows the information sender to tailor an announcement for each information recipient separately, it could be worth investigating the benefits of multiple announcements. Second, it is possible that the information recipients have different abilities to comprehend the same announcement, thus one extension of this study would be to consider two types of information recipients, which leads to an observational equivalence (from the perspective of the information sender) between those who don't pay attention to the announcement and those who do pay attention but don't understand it, even if the answers to their specific queries are included in the announcement. We considered a static game here, but it could be extended to a two-period game where in the second period the information sender can utilize the decisions made by the information recipients in the first period. In this case, high-ability information recipients may endogenously choose their attention level. Third, another interesting extension could be to consider many information senders instead of a single sender, as in online discussion forums. Admittedly, it is hard to identify the objective of a forum



when it has a dual role: An online forum for a specific theme is a venue for the sharing of information, and at the same time it is a place where people can communicate and socialize with those who share their interests.

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## Appendix: Omitted Proofs

### Proof of Lemma 1

In equilibrium,  $C(a) = [1 - P(a, q^*(a))] \kappa [1 + (n - 1)X(a, q^*(a))]$ . Define  $W(a, q^*(a)) = \kappa(1 - P(a, q^*(a)))(1 + (n - 1)X(a, q^*(a))) - C(a)$ . First, using the Implicit Function Theorem we want to show that  $\frac{\partial q}{\partial n} = -\frac{\partial W / \partial n}{\partial W / \partial q} > 0$ . For notational simplicity, we will omit  $(a, q^*(a))$  from  $P(a, q^*(a))$  and  $X(a, q^*(a))$  whenever unnecessary. It can be easily shown that  $\frac{\partial W}{\partial n} = \kappa(1 - P)X > 0$ , and that  $\frac{\partial W}{\partial q} = -\kappa P_2(1 + (n - 1)X) + \kappa(1 - P)(n - 1)X_2 < 0$ , where  $P_i$ ,  $i = 1, 2$ , is the partial derivative of  $P(a, q^*(a))$  with respect to the  $i$ th argument.  $X_i$ ,  $i = 1, 2$ , is defined similarly. Note that  $P_2 > 0$  and  $X_2 < 0$ . Thus,  $\frac{\partial q^*(a, n)}{\partial n} > 0$ .

Next, we want to show that  $\frac{\partial q}{\partial a} = -\frac{\partial W / \partial a}{\partial W / \partial q} < 0$ . Since  $\frac{\partial W}{\partial q} < 0$ , what remains to be shown is that  $\frac{\partial W}{\partial a} < 0$ , that is, that  $\frac{\partial}{\partial a}[\kappa(1 - P + (n - 1)X(1 - P)) - C(a)] = \kappa(n - 1)(1 - P)X_1 - \kappa(1 + (n - 1)X)P_1 - C'(a) < 0$ . Note that  $C'(a) > 0$ ,  $X_1 < 0$ , and  $P_1 < 0$  for any positive  $a$  and  $q$ , so the sign of  $\frac{\partial W}{\partial a}$  cannot immediately be determined by simply checking the sign of each of the three additively separable terms. To this end, we make three claims, and these complete the proof. First, when  $a$  is large,  $\frac{\partial W}{\partial a} < 0$ . When  $a$  approaches  $\infty$ , both  $P_1(a, q)$  and  $X_1(a, q)$  approach 0 regardless of what happens to  $q^*(a)$ . Since  $C'(a)$  is increasing in  $a$ , for large  $a$  we have  $\frac{\partial W}{\partial a} < 0$ . Second, when  $a = 0$ ,  $\frac{\partial W}{\partial a} = 0$  because  $1 - P(0, \infty) = 0$ ,  $X_1(0, \infty) < 0$ ,  $X(0, \infty) = 1$ ,  $P_1(0, \infty) = 0$ , and  $C'(0) = 0$ . Third, when  $a$  approaches 0,  $X(a, q^*(a))$  approaches 1 faster than  $P(a, q^*(a))$  does, so  $\frac{\partial W}{\partial a} < 0$ . Both  $P(a, q^*(a))$  and  $X(a, q^*(a))$  approach 1 when  $a$  gets sufficiently close to 0; however, the slope of  $P(a, q^*(a))$  near  $a = 0$  is close to 0 since  $\lim_{a \rightarrow 0} P_1(a, q^*(a)) = 0$ , while that of  $X(a, q^*(a))$  is negative since  $X_1(a, q^*(a)) = \int_0^{q^*(a)} P(a, x)f(x)dx$  and  $\lim_{a \rightarrow 0} X_1(a, q^*(a)) = \int_0^\infty P_1(0, x)f(x)dx < 0$ . This

guarantees that when  $a$  is near 0,  $(1-P)X_1 - XP_1 < 0$ ; therefore,  $(n-1)\{(1-P)X_1 - XP_1\} < 0$ . Also, for sufficiently large  $n$  we find that  $(n-1)\{(1-P)X_1 - XP_1\} < P_1$ . Furthermore,  $C'(a)$  is positive, so  $\kappa[(n-1)\{(1-P)X_1 - XP_1\} - P_1] - C'(a) < 0$ .

## Proof of Corollary 1

Though we could easily show that  $q^*(a^o) < q^o$  and  $q^*(a^o) < q^*(a^*)$ , it is possible that  $q^*(a^*) > q^o$ , depending on the shape of  $f(q)$  and  $P(a, q)$ . Thus we consider both cases for  $q^o > q^*(a^*)$  and  $q^o < q^*(a^*)$ .

For the sake of contradiction, suppose  $X(a^o, q^o) > X(a^*, q^*(a^*))$ . From Proposition 1, we know that  $a^o > a^*$ . When  $q^o > q^*(a^*)$ ,  $X(a^o, q^o) > X(a^*, q^*(a^*))$  does not hold since  $X(a, q)$  is decreasing in  $a$  and  $q$ .

Now let us consider the case for  $q^o < q^*(a^*)$ . There are three possible regions for recipients' queries; (i)  $q_i \in (q^*(a^*), \infty)$ , (ii)  $q_i \in (q^o, q^*(a^*))$ , and (iii)  $q_i \in (0, q^o)$ . Our goal is to show that the supposition leads to the contradiction to Proposition 1.

- (i)  $X(a^o, q^o) > X(a^*, q^*(a^*))$  implies that the private communication channel is less costly when the queries are unobservable. For  $q_i \in (q^*(a^*), \infty)$ , the recipients do not pay attention to the announcement no matter when the queries are observable or not. This means that the recipient  $i$  with  $q_i \in (q^*(a^*), \infty)$  expects a lower cost when the queries are unobservable.
- (ii) For  $q_i \in (q^o, q^*(a^*))$ , the recipients pay attention to the announcement when the queries are not observable because they expect a lower cost from it than that from directly using the private communication channel. On the other hand, they do not pay attention to the announcement when the queries are observable, which is more costly than the case when they do not pay attention when the queries are not observable. Thus, the recipient  $i$  with  $q_i \in (q^o, q^*(a^*))$  expects a lower cost when the queries are not observable.
- (iii) For  $q_i \in (0, q^o)$ , the recipients pay attention to the announcement no matter when the queries are observable or not. Their expected cost when the queries are observable is  $C(a^o) + P(a^o, q_i)\kappa[1 + (n-1)X(a^o, q^o)]$  and that when the queries are unobservable is  $C(a^*) + P(a^*, q_i)\kappa[1 + (n-1)X(a^*, q^*(a^*))]$ . From equation (3), we know that  $C(a^o) = \kappa n[1 - P(a^o, q^o)]$  and since  $X(a^o, q^o) < 1$ ,  $C(a^o) > \kappa[1 + (n-1)X(a^o, q^o)][1 - P(a^o, q^o)]$ . Hence,  $C(a^o) + \kappa[1 + (n-1)X(a^o, q^o)]P(a^o, q^o) > \kappa[1 + (n-1)X(a^o, q^o)]$ . In addition,

$\kappa[1+(n-1)X(a^*, q^*(a^*))] > C(a^*) + \kappa[1+(n-1)X(a^*, q^*(a^*))]P(a^*, q^o)$ ,  $C(a^o) > C(a^*)$ , and  $C(a) + \kappa[1+(n-1)X(a, q)]P(a, q_i)$  is increasing in  $q_i$ . Thus, the recipient  $i$  with  $q_i \in (0, q^o)$  expects to pay a lower cost when the queries are unobservable.

Therefore, all recipients are better off when the queries are unobservable, which contradicts the results in Proposition 1.

## Welfare Analysis

For notational convenience, we define  $r(q_i|\tilde{a}, \tilde{q})$  as the expected cost of use of the public announcement channel by information recipient  $i$ , given that the size of the public announcement is  $\tilde{a}$  and the cutoff query lies at  $\tilde{q}$ , that is,  $r(q_i|(\tilde{a}, \tilde{q})) = C(\tilde{a}) + \kappa P(\tilde{a}, q_i)[1 + (n-1)X(\tilde{a}, \tilde{q})]$ . Similarly, we define  $s(\tilde{a}, \tilde{q})$  as the expected cost of use of the private channel, that is,  $s(\tilde{a}, \tilde{q}) = \kappa[1 + (n-1)X(\tilde{a}, \tilde{q})]$ . Note that by construction,  $r(q^*(a^*)|a^*, q^*(a^*)) = s(a^*, q^*(a^*))$  and  $r(q^o|a^o, q^o) > s(a^o, q^o)$ .

We consider six cases in total: three cases where  $q^*(a^*) < q^o$ , and another three cases where  $q^*(a^*) > q^o$ .

If  $q^*(a^*) < q^o$ , then  $P(a^o, q^o) > P(a^o, q^*(a^*))$ , from which it follows that  $r(q^o|a^o, q^o) > r(q^*(a^*)|a^o, q^o)$ . We have the following three cases, corresponding to the three possibilities for  $s(a^*, q^*(a^*))$  compared to  $r(q^o|a^o, q^o)$  and  $r(q^*(a^*)|a^o, q^o)$ .

### Case 1: $q^*(a^*) < q^o$ and $r(q^o|a^o, q^o) > r(q^*(a^*)|a^o, q^o) > s(a^*, q^*(a^*))$

In this case information recipients whose query is below  $q^o$  pay less (when the queries are unobservable than when they are observable), while those with a query above  $q^o$  pay more. Figure 4 illustrates this result. The red lines represent the expected cost when the queries are observable: As  $q$  gets larger, the expected cost increases—not necessarily linearly, but we’re using a linear relationship in this appendix for the sake of illustration—until it reaches  $q^o$ , but beyond  $q^o$  the cost becomes constant because of the lack of additional informativeness of the public announcement. The blue line represents the expected cost when the queries are unobservable: From the construction of the Bayesian Nash equilibrium, at  $q^*(a^*)$  the expected cost of using the public announcement channel is equal to the expected cost of ignoring it. We find it useful to represent each case with respect to  $P(a, q)$ . Case 1 is equivalent to  $\pi < P(a^o, q^*(a^*)) < P(a^o, q^o)$ , where  $\pi = \frac{P(a^*, q^*(a^*))\kappa[1+(n-1)X(a^*, q^*(a^*))]+C(a^*)-C(a^o)}{\kappa[1+(n-1)X(a^o, q^o)]}$ , that is, both  $P(a^o, q^*(a^*))$  and  $P(a^o, q^o)$  are large, and the difference between them is small.

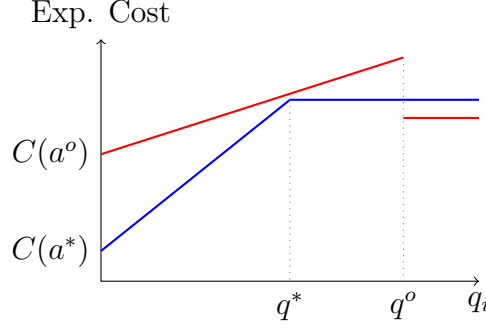


Figure 4: Case 1

**Case 2:**  $q^*(a^*) < q^o$  and  $s(a^*, q^*(a^*)) > r(q^o|a^o, q^o) > r(q^*(a^*)|a^o, q^o)$

In this case there exists  $\bar{q} \in (0, q^*(a^*))$  such that  $r(\bar{q}|(a^o, q^o)) = r(\bar{q}|(a^*, q^*(a^*)))$ . Information recipients whose query is below  $\bar{q}$  pay less (when the queries are unobservable than when they are observable), while those with a query above  $\bar{q}$  pay more. Figure 5 illustrates this result. Case 2 is equivalent to  $\pi > P(a^o, q^o) > P(a^o, q^*(a^*))$ , that is, both  $P(a^o, q^*(a^*))$  and  $P(a^o, q^o)$  are small, and the difference between them is also small.

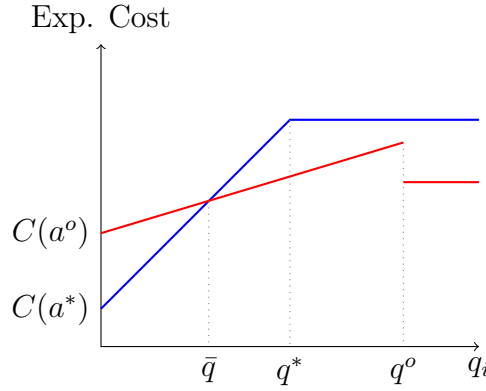


Figure 5: Case 2

**Case 3 (Proposition 2):**  $q^*(a^*) < q^o$  and  $r(q^o|a^o, q^o) > s(a^*, q^*(a^*)) > r(q^*(a^*)|a^o, q^o)$

In this case there exist  $\bar{q}_1 \in (0, q^*(a^*))$  such that  $r(\bar{q}_1|(a^o, q^o)) = r(\bar{q}_1|(a^*, q^*(a^*)))$ , and  $\bar{q}_2 \in (q^*(a^*), q^o)$  such that  $r(\bar{q}_2|(a^o, q^o)) = r(q^*(a^*)|a^*, q^*(a^*))$ . Information recipients whose query is in  $(0, \bar{q}_1) \cup (\bar{q}_2, q^o)$  pay less (when the queries are unobservable than when they are observable), while the others pay more. This is because information recipients with

$q_i \in (0, \bar{q}_1)$  use the public announcement channel regardless of whether the information sender observes the queries, and it is highly likely that they will find the answer to their query in the public announcement. Hence, they enjoy the lower cost of using the public announcement channel. Second, recipients with  $q_i \in (\bar{q}_1, q^*(a^*))$  also use the public announcement channel, even though the probability that they cannot find their answer in the public announcement is quite a bit higher (when the queries are unobservable than when they are observable), and thus they will pay more. Third, those with  $q_i \in (q^*(a^*), \bar{q}_2)$  are asked to use the public announcement channel when the queries are observable but to ignore that channel when they are not. The expected cost of using the private communication channel is high enough that they will pay more. Fourth, the case for those with  $q_i \in (\bar{q}_2, q^o)$  is the same as the case for those with  $q_i \in (q^*(a^*), \bar{q}_2)$ , but the expected cost of using the public announcement channel is sufficiently high that they need not read the announcement when the queries are not observable, hence they will pay less. Finally, those with  $q_i \in (q^o, \infty)$  use the private channel regardless of whether the queries are observable, and the expected cost of using it becomes higher, thus they pay more. Figure 6 illustrates this result. Case 3 is equivalent to  $P(a^o, q^o) > \pi > P(a^o, q^*(a^*))$ , that is,  $P(a^*, q^*(a^*))$  is sufficiently small and  $P(a^o, q^o)$  is sufficiently large.

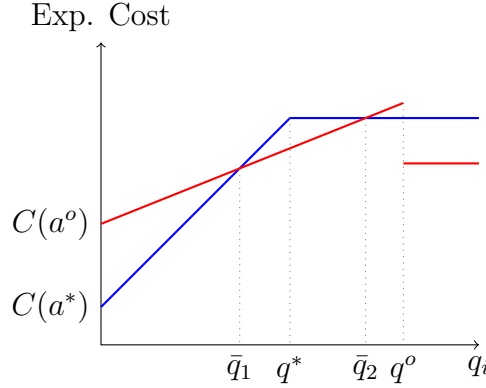


Figure 6: Case 3

When  $q^*(a^*) > q^o$ , we can find the cutoff query  $\bar{q}$ , and information recipients with  $q \in (0, \bar{q})$  are better off (when the queries are unobservable than when they are observable), while those with  $q \in (\bar{q}, \infty)$  are worse off. The cutoff  $\bar{q}$  depends on how likely it is that a recipient with query  $q^o$  will find the answer to his query in the public announcement given  $a^*$ . In this regard, we have the following three cases, corresponding to the three possibilities for  $r(q^o|a^*, q^*(a^*))$  compared to  $r(q^*(a^*)|a^*, q^*(a^*))$  and  $s(a^o, q^o)$ .

**Case 4:**  $q^*(a^*) > q^o$  and  $r(q^o|a^*, q^*(a^*)) > r(q^*(a^*)|a^*, q^*(a^*)) > s(a^o, q^o)$

In this case there exists  $\bar{q} \in (0, q^o)$  such that  $r(\bar{q}|(a^o, q^o)) = r(\bar{q}|(a^*, q^*(a^*)))$ . Figure 7 illustrates this result. Case 4 is equivalent to  $P(a^*, q^o) > \pi_2$ , where  $\pi_2 = \frac{\kappa[1+(n-1)X(a^o, q^o)]P(a^o, q^o) + C(a^o) - C(a^*)}{\kappa[1+(n-1)X(a^*, q^*(a^*))]}$ , that is,  $P(a^*, q^o)$  is sufficiently large.

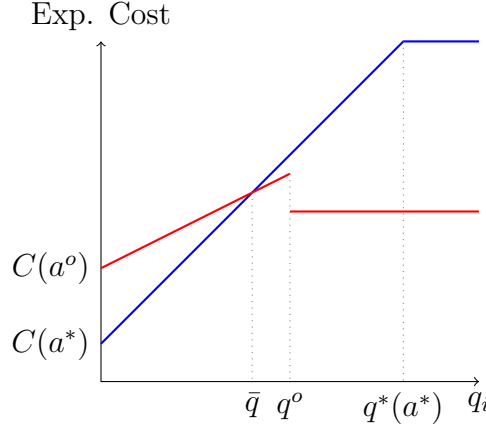


Figure 7: Case 4

**Case 5:**  $q^*(a^*) > q^o$  and  $r(q^*(a^*)|a^*, q^*(a^*)) > s(a^o, q^o) > r(q^o|a^*, q^*(a^*))$

In this case there exists  $\bar{q} \in (q^o, q^*(a^*))$  such that  $s(a^o, q^o) = r(\bar{q}|(a^*, q^*(a^*)))$ . Figure 8 illustrates this result. Case 5 is equivalent to  $P(a^*, q^o) < \pi_3$ , where  $\pi_3 = \frac{\kappa[1+(n-1)X(a^o, q^o)]P(a^o, q^o) - C(a^*)}{\kappa[1+(n-1)X(a^*, q^*(a^*))]}$ , that is,  $P(a^*, q^o)$  is sufficiently small.

**Case 6:**  $q^*(a^*) > q^o$  and  $r(q^*(a^*)|a^*, q^*(a^*)) > r(q^o|a^*, q^*(a^*)) > s(a^o, q^o)$

If  $\pi_3 < P(a^*, q^o) < \pi_2$ ,  $q^o$  is the cutoff query. That is, for any  $q < q^o$ ,  $r(q|(a^o, q^o)) > r(q|(a^*, q^*(a^*)))$ , and for any  $q > q^o$ ,  $r(q|(a^o, q^o)) < r(q|(a^*, q^*(a^*)))$ . This result indirectly implies that analysis of changes in the welfare of the recipients is indeed nontrivial: Only under very specific conditions do we find a case where information recipients with  $q < q^o$  are better off (when the queries are unobservable than when they are observable). Figure 9 illustrates this result.

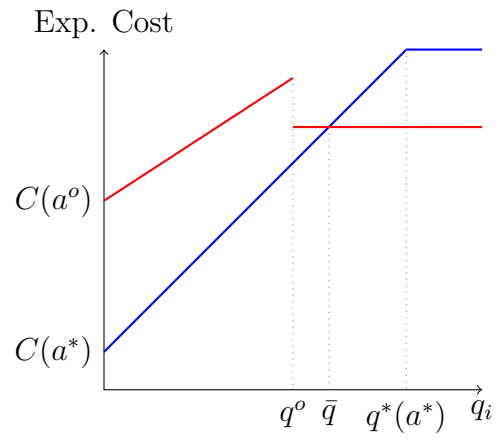


Figure 8: Case 5

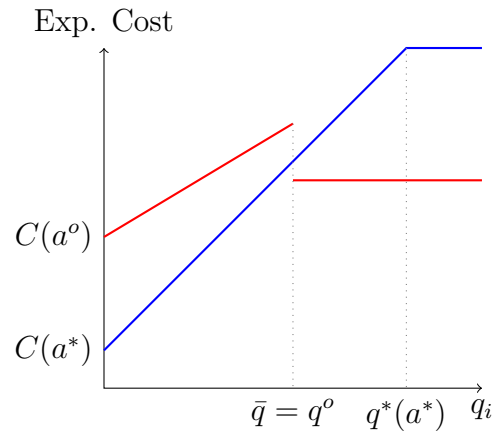


Figure 9: Case 6



### Proof of Proposition 3

By equations (2) and (3),  $F(q^o)C'(a^o) + \kappa n X_1(a^o, q^o) = 0$  and  $f(q^o)C'(a^o) + \kappa n X_2(a^o, q^o) = 0$ . If we take the derivative of the expression on the left-hand side of equation (2) with respect to  $\kappa$ , then we get  $[f(q^o)C'(a^o) + \kappa n X_{12}(a^o, q^o)] \frac{\partial q^o}{\partial \kappa} + [F(q^o)C''(a^o) + \kappa n X_{11}(a^o, q^o)] \frac{\partial a^o}{\partial \kappa} + n X_1(a^o, q^o) = 0$ . Note that  $f(q^o)C'(a^o) + \kappa n X_{12}(a^o, q^o) = 0$ , since  $C(a^o) = \kappa n [1 - P(a^o, q^o)]$  and  $X_{12}(a, q) = f(q)P_1(a, q)$ . In addition,  $F(q^o)C''(a^o) + \kappa n X_{11}(a^o, q^o) > 0$  and  $X_1(a^o, q^o) < 0$ . Thus,  $\frac{\partial a^o}{\partial \kappa} > 0$ . We can use analogous reasoning to show that  $\frac{\partial q^o}{\partial \kappa} > 0$ .

When the queries are not observable, the sender has to consider the constraint,  $C(a) = [1 - P(a, q^*(a^*))]\kappa[1 + (n - 1)X(a, q^*(a))]$ , when minimizing the total communication cost. Rewriting the sender's objective function as  $F(q^*(a))[1 - P(a, q^*(a^*))]\kappa[1 + (n - 1)X(a, q^*(a))] + \kappa n X(a, q^*(a)) = \kappa\{F(q^*(a))[1 - P(a, q^*(a^*))][1 + (n - 1)X(a, q^*(a))] + nX(a, q^*(a))\}$ , it is straightforward to see that in equilibrium  $\kappa$  does not affect the size of the public announcement. On the other hand, it is easy to show that by the constraint,  $q^*(a^*)$  increases with  $\kappa$ .