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Assignment #2

STA 6856
TIME Series
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Problem A

1. $X_t = a + bZ_t + cZ_{t-2}$

Mean

$$E(X_t) = E(a + bZ_t + cZ_{t-2})$$
$$= E(a) + b\,\cancel{E(Z_t)} + c\,\cancel{E(Z_{t-2})}$$
$$= a$$

Autocovariance

$$\gamma_X(h) = Cov(X_t, X_{t+h})$$
$$= Cov(\cancel{a} + bZ_t + cZ_{t-2},\ a + bZ_{t+h} + cZ_{t-2+h})$$
$$= Cov(bZ_t, bZ_{t+h}) + Cov(bZ_t, cZ_{t-2+h}) + Cov(cZ_{t-2}, bZ_{t+h}) + Cov(cZ_{t-2}, cZ_{t-2+h})$$
$$= b^2 Cov(Z_t, Z_{t+h}) + bc\,Cov(Z_t, Z_{t-2+h}) + bc\,Cov(Z_{t-2}, Z_{t+h}) + c^2 Cov(Z_{t-2}, Z_{t-2+h})$$
$$= b^2 \gamma_Z(h) + bc\,\gamma_Z(h-2) + bc\,\gamma_Z(h-2) + c^2 \gamma_Z(h)$$

$$= (b^2+c^2)\,\gamma_Z(h) + 2(bc)\,\gamma_Z(h-2)$$

$h=0$ $\quad \gamma_X(0) = (b^2+c^2)\,\gamma_Z(0) + \cancel{2(bc)\,\gamma_Z(-2)}$
$$= (b^2+c^2)\,Var(Z_t)$$
$$= (b^2+c^2)\,\sigma^2$$
$$= b^2\sigma^2 + c^2\sigma^2$$

$h=1$ $\quad \gamma_X(1) = \cancel{(b^2+c^2)\,\gamma_Z(1)} + \cancel{2(bc)\,\gamma_Z(-1)} = 0$

$h=2$ $\quad \gamma_X(2) = \cancel{(b^2+c^2)\,\gamma_Z(2)} + 2(bc)\,\gamma_Z(0)$
$$2bc\,Var(Z_t) = 2bc\,\sigma^2$$

Any other lags $= 0$

Autocorrelation

$$\rho_x(h) = \frac{\gamma_x(h)}{\gamma_x(0)} = \frac{\gamma_x(0)}{\gamma_x(0)} = 1 \qquad ; h=0$$

$$= \frac{\gamma_x(1)}{\gamma_x(0)} = \frac{0}{b^2\sigma^2 + c^2\sigma^2} = 0 \qquad ; h=1$$

$$= \frac{\gamma_x(2)}{\gamma_x(0)} = \frac{2bc\cancel{\sigma^2}}{(b^2+c^2)\cancel{\sigma^2}} = \frac{2bc}{b^2+c^2} \qquad ; h=2$$

Any other $\rho_x(h) = \dfrac{0}{(b^2+c^2)\sigma^2} = 0$

the process is stationary because the mean, autocorrelation, autocovariance are constant. It is independent of time.

Problem A

2. $X_t = a + bZ_0$

Mean

$$E(X_t) = E(a + bZ_0)$$
$$= E(a) + b\cancel{E(Z_0)}$$
$$= a$$

Autocovariance

$$\gamma_X(h) = Cov(X_t, X_{t+h})$$
$$= Cov(a + bZ_0, a + bZ_0)$$
$$= \cancel{Cov(a,a)} + \cancel{Cov(a, bZ_0)} + \cancel{Cov(bZ_0, a)} + Cov(bZ_0, bZ_0)$$
$$= b^2 Cov(Z_0, Z_0)$$
$$= b^2 Var(Z_0)$$
$$= b^2 \sigma^2$$

Autocorrelation

$$\rho_X(h) = \frac{\gamma_X(h)}{\gamma_X(0)} = \frac{b^2\sigma^2}{b^2\sigma^2} = 1 \quad ; \forall h$$

Since it has a constant mean, autocovariance, autocorrelation, the process is stationary. It is independent of time.