Differential Geometry I – Lecture Notes

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1 Preparation

This section consists of some work done before the lectures started.
It is mostly to clean up my LaTeXtemplates and ensure they work nicely.

1.1 Motivation

Being able to measure the rate at which things change – that is differentiating them – is such a crucial part in many applications of mathematics.
The way we treated differentiability in Analysis I & II however restricted us to Euclidean spaces. But sadly, many models of the real world are not defined on nice subspaces of a Euclidean space.
Our first aim is to generalize the notion of a derivative to let us differentiate functions from "nice" topological spaces so called *smooth manifolds*.
These topological spaces should be general enough to apply to many situations but still strong enough to carry over some of our tools from ANalysis.

Definition

A topological space is called **metrisable**, if there exists a metric on the space that induces its topology.

Alternatively, one can call a topological space metrisable, if it is homeomorphic to a metric space.

Exercise

Find interesting examples of non-metrisable spaces that may come up in other fields.

- A simple example (and a source of many other counter-examples) is the Sierpinsky-space

  $$X = \{a, b, c\}, \quad \tau = \{\emptyset, \{a\}, X\}$$

  As $\{a\}$ is an open set, we know that $d(a, b), d(a, c) > 0$. But the open ball $B(b, \frac{d(a,b)}{2})$ does not contain a and should be open. But the only open set that does not contain a is $\emptyset$, which doesn't contain b.

Definition

A metrisable topological space is called **separable**, if there exists a countable dense subset.