Homework # 2

Modern Coding Theory
Due: April 13, 2022

1. Consider the [15,11] binary Hamming code C with parity-check matrix given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Find the WEF of the code C.

   (b) Find the IOWEF of the code C.

   (c) Find the IRWEF of the code C.

   (d) Find the conditional IRWEF of the code C.

2. Consider the $[8, 4]$ binary systematic code $\mathcal{C}$ with parity-check matrix given by

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   (a) Find the generator matrix G of the code $\mathcal{C}$.

   (b) Find the weight enumerating function (WEF) $A(X)$ of the code $\mathcal{C}$.

   (c) Using $A(X)$ in (b), estimate an upper bound on the word error probability P_W over the AWGN channel.

   (d) Find the input-output weight enumerating function (IOWEF) $A^{IO}(W, X)$ of the code $\mathcal{C}$.

   (e) Compute the error coefficient B_i for $i = 1, \ldots, 8$.

   (f) Using the results in (e), estimate an upper bound on the bit error probability P_b over the AWGN channel.

   (g) For all $w \geq 0$, find the conditional IOWEF $A_w^{IO}(X)$ of the code $\mathcal{C}$

   (h) Find the input-redundancy weight enumerating function (IRWEF) $A^{IR}(W, Z)$ of the code $\mathcal{C}$.

(i) For all $w \geq 0$, find the conditional IRWEF $A_w^{IR}(Z)$ of the code $\mathcal{C}$.

(j) Find the minimum-distance asymptote of the code $\mathcal{C}$, when E_b/N_0 goes to infinity

3. Prove that the BER of the Turbo code is bounded as

$$P_b \leq \sum_{w=1}^{N} \frac{w}{2N} W^w A_w^{IR}(Z) \Bigg|_{W=Z=e^{-R\frac{E_b}{N_0}}}$$

where N is the interleaver size.