Homework # 3

Modern Coding Theory
Due: May 11, 2022

1. (Chapter 3 of Lecture Note, p.24)
   For Gallager Decoding B, the probability of error after processing of a digit in the ith tier is updated by

$$\begin{aligned} p_{i+1} = p_0 - p_0 \sum_{l=b}^{j-1} \binom{j-1}{l} \left[\tfrac{1+(1-2p_i)^{k-1}}{2}\right]^{l} \left[\tfrac{1-(1-2p_i)^{k-1}}{2}\right]^{j-1-l} \\ +(1-p_0) \sum_{l=b}^{j-1} \binom{j-1}{l} \left[\tfrac{1-(1-2p_i)^{k-1}}{2}\right]^{l} \left[\tfrac{1+(1-2p_i)^{k-1}}{2}\right]^{j-1-l}. \end{aligned}$$

   Show that the problem of finding the integer value of b minimizing p_i is equivalent to the problem of finding the smallest integer b for which

$$\frac{1-p_0}{p_0} \leq \left[\frac{1+(1-2p_i)^{k-1}}{1-(1-2p_i)^{k-1}}\right]^{2b-j+1}.$$

2. (Chapter 4 of Lecture Note, p.27)
   (a) Prove that $K = [0, \infty)$ is a semiring under the 'max' and 'product' operations.
   (b) Prove that $K = (-\infty, \infty)$ is a semiring under the 'min' and 'sum' operations.

3. (Chapter 4 of Lecture Note, p.31)
   Derive the weight distribution of a repeat-accumulate (RA) code of input length n.

4. (Chapter 4 of Lecture Note, p.33)
   Derive the updating rules for binary variable and parity-check nodes, when the messages are given as likelihood (LR), log-likelihood (LLR), likelihood difference (LD) and signed log-likelihood difference (SLLD), respectively.

5. (Chapter 4 of Lecture Note, p.33)
   Show that the updating rules at nodes of higher degree can be computed as

$$VAR(x_1, x_2, \dots, x_n) = VAR(x_1, VAR(x_2, \dots, x_n))$$
$$CHK(x_1, x_2, \dots, x_n) = CHK(x_1, CHK(x_2, \dots, x_n))$$