CSE 4360/5364 - *Autonomous Robots*

Homework 1- Fall 2023

Due Date: Sep. 19 2023, 5:30pm

Problems marked with * are mandatory only for students of CSE 5364 but will be graded for extra credit for students of CSE 4360.

Forward and Inverse Kinematics

For this Exercise you have to derive the forward kinematics and a partial inverse kinematic function for a manipulator and implement them in a robot simulator. To do this you are provided with a C library containing the simulator. All your code should be written in the file *kin_fncs.c* which contains 2 main functions, *fwd_kin(theta, x)* and *inv_kin(x, theta)*. These functions are directly called by the simulator.
To start with this part of the assignment you have to retrieve the compressed tar file for your computer architecture (either for Linux, or OSX with X-Windows) from Canvas and uncompress it to your account. This archive contains a directory named *kin* which contains the following files:

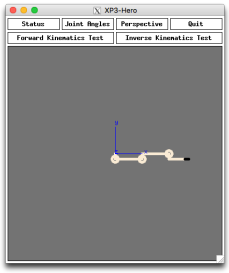
Imakefile This file is used to create a machine specific Makefile by typing *xmkmf*.

kin_fncs.c This is the file you have to edit in order to implement the kinematic functions.

lib/libKin.a This library contains the simulator and some routines to test your kinematic functions.

The Kinematic Simulator

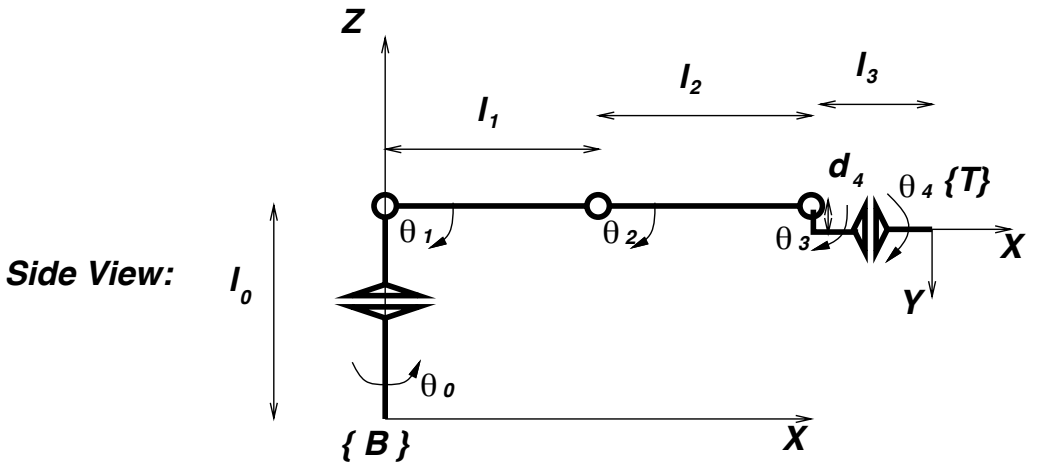
To generate the simulator with your kinematic functions you just have to type *make*. This creates the simulator executable *Kinematics*. The simulator should look as follows:

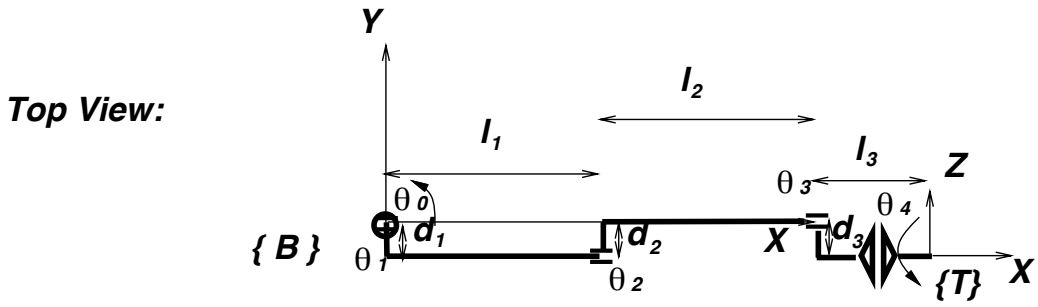


The *Joint Angles* and *Perspective* buttons at the top of the simulator window will open a set of sliders which allow you to move the individual joints of the robot and to change your viewing angle, respectively (the default perspective is a top view).
To test your forward and inverse kinematic functions you can press the *Status* button which will print the current joint angles, the actual location of the tool frame, and the results of your forward and inverse kinematic functions for the current robot configuration. In addition, the 2 buttons in the second row will test your functions in 100 random robot configurations and report if they differ from the expected result.

The Robot

In the first examples you are to determine the forward kinematics and a partial inverse kinematics for the following 5 degree-of-freedom robot manipulator:





The link length are defined as follows:

$$l_0 = 0.25m \quad l_1 = 0.2m$$
$$l_2 = 0.2m \quad l_3 = 0.15m$$

In addition there are multiple offsets that represent displacements at the joints. The offsets are $d_1 = -0.04m$ between joint 1 and joint 2, $d_2 = 0.04$ between joint 2 and joint 3, and offset $d_3 = d_4 = -0.04m$ between joint 3 and joint 4 of this robot. The picture above shows the robot in the configuration where all joint angles are 0. The simulator will actually show one additional joint (joint 5) corresponding to the opening of the gripper. This, however, is not relevant for the kinematic functions.

1. Determine and implement the forward kinematic function for this Robot.

   Here you are supposed to determine the transformation from configuration space (joint angles) to the Cartesian location of the tool frame ($\{T\}$) in base frame coordinates ($\{B\}$). You only have to provide the position. The orientation of the tool frame is not required. (HINT: What effect does the last joint, Θ_4, have on the location of the tool frame ?)

For this part of the assignment you are to hand in a written version of the forward kinematics and your derivation (either closed form or as a sequence of transformation matrices), and the code for the forward kinematic function you implemented.

For the implementation you have to write the function *fwd_kin()* inside the file *kin_fncs.c*. This function has the following structure:

```
fwd_kin(theta, x)
double theta[6];
double x[3];
{
...
}
```

It gets passed in an array containing the 6 joint angles *theta* (the last angle corresponds to the gripper and is of no interest here) and should calculate *x* , the 3 dimensional position of the tool frame (x[0] = x, x[1] = y, x[2] = z).

2. Determine a partial inverse kinematic function for the robot and implement it in the simulator.

   In this part of the assignment you are to derive and implement an inverse kinematic function for the robot manipulator for a specific orientation of the tool frame (if multiple solutions exist only one is required here). This orientation is given by $\Theta_4 = 0$ and the requirement that the X-axis of the tool frame is parallel to the Z-axis of the base frame but in the opposite direction. In other words, the last link of the manipulator is to point straight down. (HINT: To determine this inverse kinematics you should decompose the problem. You can determine Θ_0 and Θ_3 separately and calculate the other joint angles by analyzing the substructure formed by links 1 and 2 (l_1 and l_2). This requires moving the wrist frame to joint 3.)

   Again you are to hand in a written version of this inverse kinematics and its derivation and the code. The code here should be implemented in the function *inv_kin()*.

```
inv_kin(x, theta)
double x[3];
double theta[6];
{
...
}
```

   This function gets passed in the location of the tool frame (x) and has to compute a corresponding joint angle configuration (*theta*).

3.* Derive the Jacobian Matrix for the position components of the robot.

   In this part of the assignment you have to derive the mapping between joint velocities and the directional velocities of the tool frame of the robot. Since joint 4 ($Theta_4$) does not affect the position of the tool frame (it only changes its orientation), you only need to derive the terms for the first 4 joints of the robot (i.e. you can think of the robot as having only 4 joints and thus reduce the Jacobian to a 3×4 matrix).

   Note: Keep in mind the relation of the Kinematics and the Jacobian when deriving the 12 entires of the reduced Jacobian.